

MATH 152 – PYTHON LAB 6

Directions: Use Python to solve each problem. ([Template link](#))

1. Given the sequence $a_n = \left(1 + \frac{1}{n^2}\right)$:

- (a) Evaluate a_1, a_2, \dots, a_{10} to numerically estimate the limit.
- (b) Plot the first 30 points of the sequence to graphically estimate.
- (c) Use Python to calculate $\lim_{n \rightarrow \infty} a_n$.

2. Given $b_n = \frac{(4/5)^n}{n}$:

- (a) Show that $\lim_{n \rightarrow \infty} b_n = 0$.
- (b) Plot the first 50 terms of the sequence and the first 50 partial sums on the same graph.
- (c) Evaluate the exact and approximate sum of the series.

3. Given the series $\sum_{n=1}^{\infty} a_n$, where $a_n = \frac{3}{n^2 + 4n}$:

- (a) Evaluate $\lim_{n \rightarrow \infty} a_n$
- (b) Write the first 12 terms of the sequence and the first 12 partial sums of the series.
- (c) Calculate the partial fraction decomposition of a_n .
- (d) Based on your answer in (c), state a general formula for s_n , the n^{th} partial sum, and find the sum of the series or show it diverges.

4. Given $a_0 = 1$ and $a_{n+1} = \frac{1}{3a_n^2} + \frac{2a_n}{3}$

- (a) Evaluate the first ten terms of the sequence. Evaluate the limit as $n \rightarrow \infty$.
- (b) Repeat the process given $a_0 = 100$.
- (c) Based on parts (a) and (b), can you say anything about the sequence behavior regardless of the starting term?