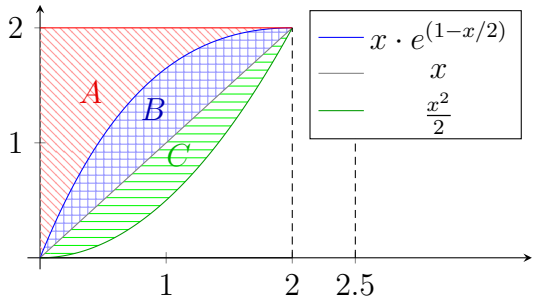
Math 152 – Python Lab 3

Directions: Use Python to solve each problem. (Template link)

1. Refer to the given figure and find the volume generated by rotating the given region about the indicated axis. May use whichever method you prefer, unless otherwise indicated.



   (a) Region A about x-axis

   (b) Region A about y-axis

   (c) Region B about x-axis

   (d) Region C about $x = 2.5$ using the washer method

2. A trough is 5 m long and has ends which are isosceles triangles with height 2 m and width (across the top) 3 m. The trough has a spout at the top of the tank with height 1 m. The tank is full of water.

   (a) How much work is required to pump all of the water out of the tank? Note that the density of water is $\rho = 1000$ kg/m^3 and the acceleration due to gravity is $g = 9.8$ m/s^2.

   (b) Suppose the pump breaks down after 3×10^4 J of work has been done. What is the depth of the remaining water in the tank?

3. Given $f(x) = \cos^2(x)$ and $g(x) = \cos^4(x)$ (give exact answers for all parts):

   (a) Plot the functions on the x-interval $\left[0, \frac{\pi}{2}\right]$. Find the volume when the region between the two curves is rotated about the line $x = \frac{\pi}{2}$.

   (b) Find the area of the region.

   (c) The **center of mass** of a region $[a, b]$ is the point $(\bar{x}, \bar{y})$, where $\bar{x} = \dfrac{1}{A}\displaystyle\int_a^b x(f(x) - g(x))\,dx$ and $\bar{y} = \dfrac{1}{A}\displaystyle\int_a^b \frac{1}{2}(f(x)^2 - g(x)^2)\,dx$, with A the area between the curves. Find the x-coordinate of the center of mass of the region. In a print statement, explain why this answer makes sense based on the graph in part a).

   (d) When the region rotates about the line $x = \frac{\pi}{2}$, how far does the center of mass travel? Multiply this value by the area. What do you notice when you compare your answer to part a)?