

MATH 152 – PYTHON LAB 5

Directions: Use Python to solve each problem. ([Template link](#))

1. Given $f(x) = \frac{x^3 - 4x + 3}{(x - 5)^2(x^2 + 3)(x^2 + 5)}$

(a) The partial fraction decomposition of $f(x)$ is $\frac{A}{(x - 5)} + \frac{B}{(x - 5)^2} + \frac{Cx + D}{x^2 + 3} + \frac{Ex + F}{x^2 + 5}$. Using this, write and solve a system of equations to find A through F (as you would by hand). Integrate the resulting partial fraction decomposition.

(b) Use Python to find the partial fraction decomposition of f and integrate the result.

(c) Integrate f directly and indicate whether your integrals (a), (b), and (c) are the same or not.

2. (a) Calculate the value of a so that $\int_0^\infty \frac{x^2}{x^4 + a^2} dx = 0.1$. (Note: When defining x and a as symbolic variables, include `positive = True` to clear up some issues when calculating.)

(b) Find the value of a such that $\int_1^a x^6 e^{-x^7} dx = \int_a^\infty x^6 e^{-x^7} dx$.

(c) Evaluate $\int_1^a x^6 e^{-x^7} dx$ using the value found in part (b), then use your result to print the value of $\int_1^\infty x^6 e^{-x^7} dx$ without integrating again.

3. Let $f(x) = \frac{|x| \cos^2(x)}{x^3}$ and $g(x) = \frac{1}{x^2}$.

(a) Show $\int_1^\infty g(x) dx$ converges.

(b) Plot f and g on the same axes in the domain $x \in [1, 10]$ to show $f(x) \leq g(x)$ on the given interval.

(c) Evaluate $\int_1^\infty f(x) dx$. Give exact and approximate answers.