

MATH 152 – PYTHON LAB 7

Directions: Use Python to solve each problem. ([Template link](#))

1. Euler found the sum of the p -series with $p = 4$:

$$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$$

- (a) Find the partial sum s_{10} of the series $\sum_{n=1}^{\infty} \frac{1}{n^4}$. Estimate the error in using s_{10} as an approximation to the sum of the series.
- (b) A variation of the Remainder Estimate tells us that:

$$s_n + \int_{n+1}^{\infty} f(x) dx \leq s \leq s_n + \int_n^{\infty} f(x) dx$$

Use $n = 10$ to give an improved estimate of the sum.

- (c) Compare your estimates in part (b) with Euler's estimate.
- (d) Find a value of n so that s_n is within 10^{-6} of the sum.

2. Given the series $\sum_{n=2}^{\infty} n^2 e^{-n}$ and the function $f(x) = x^2 e^{-x}$:

- (a) Compute $\int f(x) dx$ and $\int_1^{\infty} f(x) dx$.
- (b) In a print statement, state your conclusion about the convergence or divergence of the series based on your answer to a).
- (c) Compute s_{10} , s_{50} , s_{100} , and s .
- (d) Use the Remainder Estimate for the Integral Test to estimate $s - s_{100}$. Compare the actual value of $s - s_{100}$. Does the actual error fall in the expected range?
- (e) According to the Remainder Estimate, how many terms are needed to sum the series to within 10^{-10} ? Compute the sum to confirm $|s - s_N| < 10^{-10}$. (NOTE: To expedite the computation, convert the terms to floating point before summing)

(NOTE: #3 on the next page)

3. Given the series $\sum_{n=1}^{\infty} \frac{n \sin^2(n)}{1 + n^3}$:

- (a) Let $a_n = \frac{n \sin^2(n)}{1 + n^3}$. Define a series b_n with which to compare it.
- (b) Plot the first 50 terms of a_n and b_n on the same graph to determine which is larger. If the graph is not clear, use the logical test $a_n < b_n$ to test the logical value comparing each term.
- (c) Determine whether $\sum_{n=1}^{\infty} b_n$ converges or not, and state whether any conclusion can be made about the convergence of $\sum_{n=1}^{\infty} a_n$ as a result.
- (d) If (c) is conclusive, skip to (e). If (c) is inconclusive, determine whether $\frac{a_n}{b_n}$ converges or not, and state your conclusion about the convergence of $\sum_{n=1}^{\infty} a_n$. (NOTE: If you still cannot conclude anything, start over with a different b_n !)
- (e) Repeat parts (a) – (d) for $a_n = \frac{e^n + 1}{ne^n + 1}$