Math 152 – Python Lab 4

Directions: Use Python to solve each problem. (Template link)

1. Given the integral $\int_0^{\pi/2} \sqrt[4]{5+\sin(x)}dx$

   (a) Plot the function on the domain $x \in [0, \pi/2]$.

   (b) Recall the **Left Endpoint Riemann Sum** from MATH 151:

   $$\int_a^b f(x)dx \approx \sum_{i=1}^{n} f(x_{i-1})\Delta x$$

   where $\Delta x = \dfrac{b-a}{n}$ and $x_i = a + i \cdot \Delta x$ This is easy to compute in Python:

      i. Define a list x from a (inclusive) to b (exclusive) with stepsize Δx.

      ii. Define a list $y = f(x)$.

      iii. Sum the list y and multiply by Δx

   Compute the Left Endpoint approximation using $n = 200$ subintervals.

2. The **Right Endpoint Riemann Sum** is the same process, but starting at $a + \Delta x$ and ending at b (inclusive).

   Compute the Right Endpoint approximation using $n = 200$ subintervals.

3. The Midpoint Sum is again the same process, but starting at $a + \dfrac{\Delta x}{2}$ and ending at $b - \dfrac{\Delta x}{2}$ (inclusive).

   (a) Compute the Midpoint approximation using $n = 200$ subintervals.

   (b) Compute the average of the Left and Right Endpoint approximations. Is this equal to the Midpoint approximation?

(More questions on page 2!!!!)

4. Another approximation for integrals is the Trapezoid Rule:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{2}(f(x_0) + 2f(x_1) + 2f(x_2) + \ldots + 2f(x_{n-1}) + f(x_n))$$

There is a built-in function **trapz** in the package **scipy.integrate** (See the Overview for more information).

   (a) Compute the Trapezoid approximation using $n = 200$ subintervals.

   (b) Is the Trapezoid approximation equal to the average of the Left and Right Endpoint approximation?

   (c) Run the following code to illustrate the trapezoid method with 4 trapezoids:

```
from numpy import *
import sympy as sp

x=sp.symbols('x')
f=(5+sp.sin(x))**sp.Rational(1,4)
sp.plot(f,(x,0,pi/2))
xp=[0,pi/8,pi/4,3*pi/8,pi/2]
yp=[f.subs(x,i) for i in xp]
import matplotlib.pyplot as plt
plt.plot(xp,yp)
```

   Notice that the trapezoid approximation is obtained by using lines to estimate $f(x)$ on each subinterval.

5. Simpson's Method is another approximation to the integral which uses parabolas instead of lines to approximate $f(x)$:

$$\int_a^b f(x)dx \approx \frac{\Delta x}{3}(f(x_0) + 2f(x_1) + 4f(x_2) + ... + 4f(x_{n-2}) + 2f(x_{n-1}) + f(x_n))$$

   (a) Use the function **simps** in the **scipy.integrate** package to compute Simpson's Approximation using $n = 200$ subintervals.

6. The value of the integral to 10 decimal places is 2.4196410881. Use this value to estimate the error |actual − estimate| in each of the five approximations.