Math 152 – Python Lab 6

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0.1 MATH 152 Lab 6

MATH 152 Lab 6 Section Number: 571

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[1]:
```
from sympy import *
from sympy.plotting import plot, plot_parametric, plot3d_parametric_surface,␣
↪plot3d_parametric_line, plot3d
import numpy as np
import matplotlib.pyplot as plt
```

0.1.1 Question 1

1a

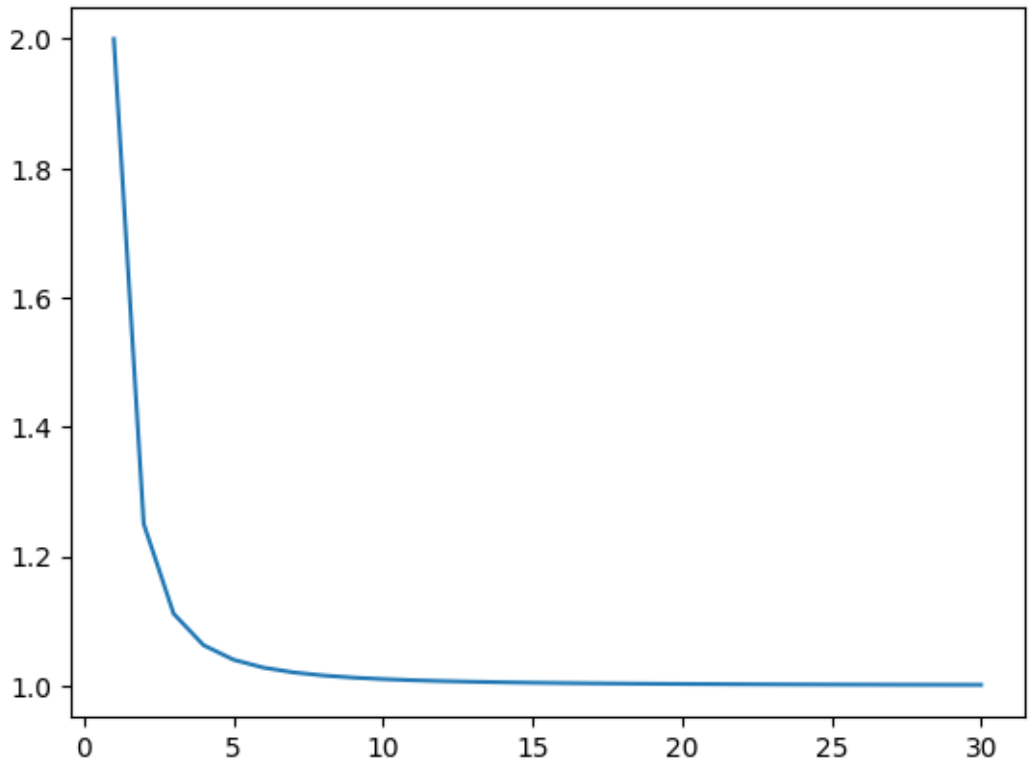
[2]:
```
n = symbols("n")
a_n = 1 + 1 / (n ** 2)
for i in range(1, 11):
    print(f"a_n = {a_n} for n={i} is {a_n.subs(n, i).evalf()}")

print("It looks like the limit goes to 1")
```

```
a_n = 1 + n**(-2) for n=1 is 2.00000000000000
a_n = 1 + n**(-2) for n=2 is 1.25000000000000
a_n = 1 + n**(-2) for n=3 is 1.11111111111111
a_n = 1 + n**(-2) for n=4 is 1.06250000000000
a_n = 1 + n**(-2) for n=5 is 1.04000000000000
a_n = 1 + n**(-2) for n=6 is 1.02777777777778
a_n = 1 + n**(-2) for n=7 is 1.02040816326531
a_n = 1 + n**(-2) for n=8 is 1.01562500000000
a_n = 1 + n**(-2) for n=9 is 1.01234567901235
a_n = 1 + n**(-2) for n=10 is 1.01000000000000
It looks like the limit goes to 1
```

1b

[3]:
```
# plot the first 30 points
x = np.arange(1, 31)
y = [a_n.subs(n, i) for i in x]
plt.plot(x, y)
plt.show()
print("The graph above is the first 30 points of the sequence")
print("It looks like the limit goes to 1")
```




```
The graph above is the first 30 points of the sequence
It looks like the limit goes to 1
```

1c

[4]:
```
limit_ = Limit(a_n, n, oo)
print(f"The limit of the sequence is {limit_.doit()}")
```

```
The limit of the sequence is 1
```

0.1.2 Question 2

2a

[5]:
```
n = symbols("n", integer=True)
b = (4 / 5) ** n / n
L = limit(b, n, oo)
print("The limit as n approaches infinity of b of n is:", L)
```

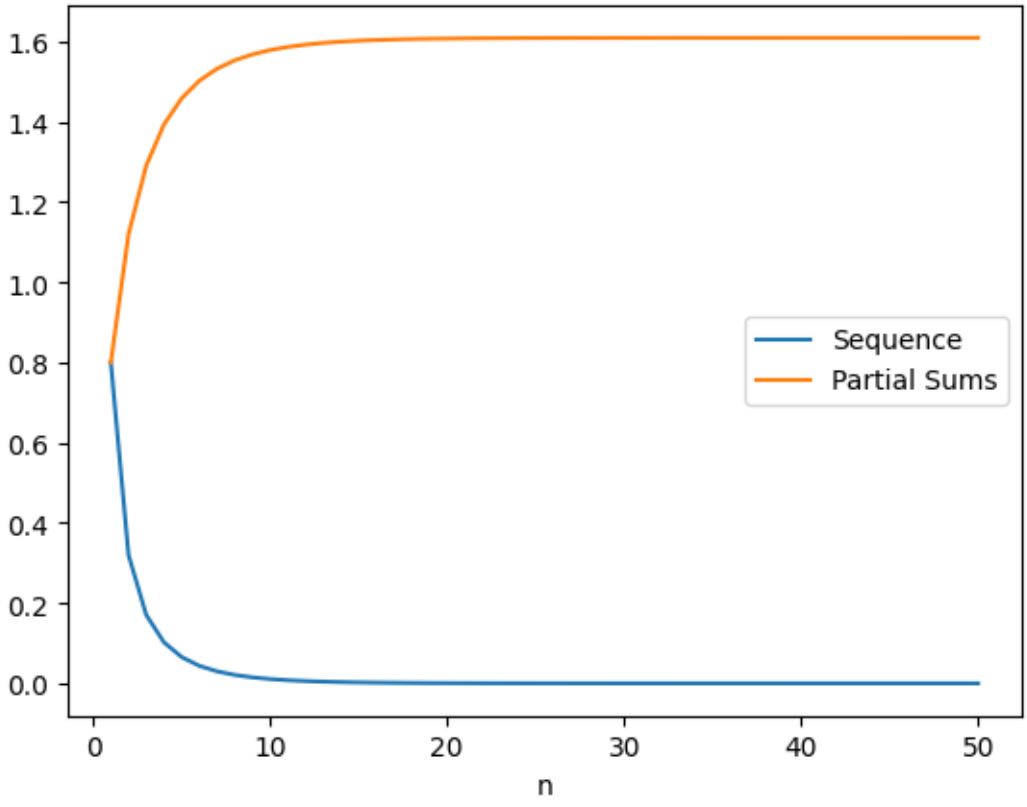
```
The limit as n approaches infinity of b of n is: 0
```

2b

[6]:
```
# Define the sequence
n = np.arange(1, 51)
bn = ((4 / 5) ** n) / n

# Calculate the partial sums
partial_sums = np.cumsum(bn)

# Plot the sequence and partial sums
plt.plot(n, bn, label="Sequence")
plt.plot(n, partial_sums, label="Partial Sums")
plt.xlabel("n")
plt.legend()
plt.show()
print(f"The graph above shows the sequence and partial sums for the first 50␣
 ↪terms")
```



The graph above shows the sequence and partial sums for the first 50 terms

2c

[7]:
```
S = np.cumsum(b)
print(f"Sum of series is {S[len(S) - 1]}, which is close to {L}")
```

```
Sum of series is 0.8**n/n, which is close to 0
```

0.1.3 Question 3

3a

[8]:
```
n = symbols("n")
an = 3 / (n ** 2 + 4 * n)
answer = limit(an, n, oo)
print(f"The limit of {an} as n approaches infinity is {answer}")
```

```
The limit of 3/(n**2 + 4*n) as n approaches infinity is 0
```

3b

[9]:
```
for i in range(1, 13):
    answer = limit(an, n, i)
    print("partial sums", answer)
s_n = Sum(an, (n, 1, Symbol("n")))
for i in range(1, 13):
    partial_sum = s_n.subs(Symbol("n"), i).doit()
    print(f"The {i}th partial sum is {partial_sum}")
```

```
partial sums 3/5
partial sums 1/4
partial sums 1/7
partial sums 3/32
partial sums 1/15
partial sums 1/20
partial sums 3/77
partial sums 1/32
partial sums 1/39
partial sums 3/140
partial sums 1/55
partial sums 1/64
The 1th partial sum is 3/5
The 2th partial sum is 17/20
The 3th partial sum is 139/140
The 4th partial sum is 1217/1120
The 5th partial sum is 775/672
The 6th partial sum is 4043/3360
```

```
The 7th partial sum is 6559/5280
The 8th partial sum is 1681/1320
The 9th partial sum is 7431/5720
The 10th partial sum is 10575/8008
The 11th partial sum is 4873/3640
The 12th partial sum is 39439/29120
```

3c

[10]:
```
partial_frac = apart(an)
print(f"The partial fraction decomposition of a_n is {partial_frac}")
```

```
The partial fraction decomposition of a_n is -3/(4*(n + 4)) + 3/(4*n)
```

3d

[11]:
```
partial_sum = s_n.doit()
print(f"The general formula for the partial sum is {partial_sum}")
```

```
The general formula for the partial sum is 25/16 - 3/(4*(n + 4)) - 3/(4*(n + 3))
- 3/(4*(n + 2)) - 3/(4*(n + 1))
```

3e

[12]:
```
# find the sum of the series
print(f"The sum of the series is {partial_sum.subs(Symbol('n'), oo)}")
```

```
The sum of the series is 25/16
```

0.1.4 Question 4

4a

[13]:
```
a = 1
An = 1 / (3 * a ** 2) + 2 * a / 3

for i in range(0, 10):
    print(An)
    a = An
    An = 1 / (3 * a ** 2) + 2 * a / 3

print("as the sequence doesn't change from the starting number, it limits to 1␣
 ↪at infinity")
```

```
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0
```

```
1.0
1.0
1.0
as the sequence doesn't change from the starting number, it limits to 1 at
infinity
```

4b

[14]:
```
a = 100
An = 1 / (3 * a ** 2) + 2 * a / 3

print(a)
for i in range(0, 25):
    print(An)
    a = An
    An = 1 / (3 * a ** 2) + 2 * a / 3
print("This is the process repeated from 100")
```

```
100
66.6667
44.44454166659167
29.629863193656167
19.753621810618192
13.169935457645675
8.781878786203064
5.858908047611485
3.915649283118309
2.6321734377751667
1.8028938852987562
1.3044799059949463
1.065539523202045
1.0039485733107845
1.0000155095502647
1.0000000002405411
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0
1.0
This is the process repeated from 100
```

4C

[15]:
```
print("Based off parts a and b, the sequence is converging to 1")
```

Based off parts a and b, the sequence is converging to 1