

A Certified Core Policy Language

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Abstract

We present a Certified Core Policy Language (ACCPL) that can be used to express access-control rules in a minimal way. Full-blown access-control policy languages such as eXtensible Access Control Markup Language (XACML) [ANP⁺03] already exist however because access rules in such languages are often expressed in a declarative manner using fragments of a natural language like English, it isn't always clear what the intended behaviour of the system encoded in the access rules should be. To remedy this ambiguity, formal specification of how an access-control mechanism should behave is typically given in some sort of logic, often a subset of first order logic. To show that an access-control system actually behaves correctly with respect to its specification, proofs are needed, however the proofs that are often presented in the literature are hard or impossible to formally verify. The verification difficulty is partly due to the fact that the language used to do the proofs while mathematical in nature, utilizes intuitive justifications to derive the proofs.

In this thesis, we describe the design and implementation of ACCPL in Coq. The Coq proof assistant is used to encode and model the behaviour of ACCPL. We will use certain properties described in [TK06] to study how amenable to analysis and reasoning ACCPL policies are with respect to other access-control policy languages and how the design choices that were made contributed to this ease of reasoning. In particular decidability criteria for ACCPL are expressed and proofs are developed in the Coq proof assistant.

Table of Contents

List of Listings	6
1 Introduction	1
1.1 Access-Control Models [NIS09]	1
1.1.1 Access Control Lists	1
1.1.2 Capabilities	1
1.1.3 RBAC	1
1.1.4 ABAC	2
1.1.5 PBAC	2
1.2 A Core Policy Language for PBAC Systems	2
1.3 Formal Semantics for PBAC Languages	4
1.4 Logic Based Semantics	4
1.5 Specific Problem	5
1.6 Contributions	5
2 ACCPL Syntax	7
2.1 Introduction	7
2.2 Environmental Facts	8
2.3 Productions	8
3 ACCPL Syntax In Coq	12
3.1 Introduction to Coq	12
3.2 ACCPL Syntax	12

4	ODRL0 Semantics	18
4.1	Introduction	18
4.2	Agreement Translation	18
4.3	Policy Set Translation Definition	19
4.3.1	PrimitivePolicySet Translation Definition	19
4.3.2	PrimitiveExclusivePolicySet Translation Definition	19
4.3.3	AndPolicySet Translation Definition	20
4.3.4	getId Translation	20
4.3.5	action Translation	20
4.4	prin Translation Cases	21
4.4.1	Single Subject Translation Definition	21
4.4.2	List of Subjects Translation Definition	21
4.5	Policy Translation Cases	21
4.5.1	PrimitivePolicy Translation Definition	22
4.5.2	AndPolicy Translation Definition	22
4.6	Prerequisite Translation Cases	22
4.6.1	True Prerequisite Translation Definition	23
4.6.2	Constraint Prerequisite Translation Definition	23
4.6.3	ForEachMember Prerequisite Translation Definition	24
4.6.4	NotCons Prerequisite Translation Definition	24
4.6.5	AndPrqs Prerequisite Translation Definition	24
4.6.6	OrPrqs Prerequisite Translation Definition	24
4.6.7	XorPrqs Prerequisite Translation Definition	25
4.7	Constraint Translation Cases	25
4.7.1	Principal Constraint Translation	25
4.7.2	Count Constraint Translation	25
4.7.3	CountByPrin Constraint Translation	26
5	ACCPL Semantics In Coq	27
5.1	Introduction	27
5.2	Translations	28
5.3	Decision Procedures	37

6	Queries	41
6.1	Introduction	41
6.2	Queries	41
6.3	Answering Queries	42
7	Examples	44
7.1	Introduction	44
7.2	Agreement 2.1	44
7.3	Agreement 2.5	45
8	Some Simple Theorems	47
8.1	Introduction	47
8.2	Theorem One	48
8.3	Theorem Two	49
8.4	Theorem Three	50
9	Proposed Future Work	53
9.1	Summary	53
9.2	Machine-Checked Proof of Decidability of Queries	53
9.3	SELinux	54
9.4	SELinux Policy Language	55
9.5	Agreements in Security Enhanced Linux (SELinux)	56
9.6	Environments	56
9.7	Queries in SELinux	57
9.8	Decidability of Queries in SELinux	57
	APPENDICES	58
	References	58

Listings

2.1	Agreement for Mary Smith in XML	7
2.2	Agreement for Mary Smith as BNF (as used in [PW06])	8
2.3	agreement	9
2.4	prin	9
2.5	subject	9
2.6	asset	9
2.7	policySet	9
2.8	primPolicySet	9
2.9	primInclusivePolicySet	10
2.10	primExclusivePolicySet	10
2.11	policy	10
2.12	primPolicy	10
2.13	act	10
2.14	policyId	10
2.15	preRequisite	11
2.16	constraint	11
3.1	ODRL0: Coq Version of Agreement	14
3.2	ACPL: Coq Version of Agreement	15
3.3	nonemptylist	16
3.4	preRequisite	17
4.1	Agreement Translation	19
4.2	Policy Set Translation Cases	19
4.3	Policy Set Translation Definition : PrimitivePolicySet	19
4.4	Policy Set Translation Definition : PrimitiveExclusivePolicySet	20
4.5	Policy Set Translation Definition : AndPolicySet	20

4.6	getId for a Single policy	20
4.7	getId for Policies Definition	20
4.8	act Translation Definition	20
4.9	Prin Translation Cases	21
4.10	Prin Translation Definition : Single subject	21
4.11	Prin Translation Definition : List of subjects	21
4.12	Policy Translation Cases	22
4.13	PrimitivePolicy Translation Definition	22
4.14	Policy Translation Definition : AndPolicy	22
4.15	Prerequisite Translation Cases	23
4.16	Prerequisite Translation Definition : Always True Prerequisite	23
4.17	Prerequisite Translation Definition : Constraint	23
4.18	Prerequisite Translation Definition : ForEachMember	24
4.19	Prerequisite Translation Definition : Not Constraint	24
4.20	Prerequisite Translation Definition : Conjunction	24
4.21	Prerequisite Translation Definition : Inclusive Disjunction	24
4.22	Prerequisite Translation Definition : Exclusive Disjunction	25
4.23	Constraint Translation : Count	25
4.24	Constraint Translation : Count by Principal	26
5.1	Environments and Counts	27
5.2	Defining Environments	28
5.3	getCount Function	28
5.4	Translation of Agreement	29
5.5	Query Answers and Results	29
5.6	Translation of unregulated policies	29
5.7	Translation of negative policies	30
5.8	Translation of list of policies: positive, negative and unregulated use the same pattern	30
5.9	Translation of positive policies	31
5.10	Translation of PIPS and PEPS	31
5.11	Translation of Policy Set	32
5.12	Translation of a Prin	33

5.13	Translation of a Positive Policy	33
5.14	Translation of a Negative Policy	34
5.15	Translation of a PreRequisite	35
5.16	Translation of a Constraint	35
5.17	Translation of forEachMember	36
5.18	Translation of NotCons	36
5.19	Translation of count	37
5.20	Translation of count TEMP	37
6.1	Queries	42
6.2	Answerable Queries: Error	42
6.3	Answerable Queries: Permit	42
6.4	Answerable Queries: Deny	42
6.5	Answerable Queries: Not Applicable	42
6.6	f_q^+	43
6.7	f_q^-	43
6.8	Answerable Queries: Query Inconsistent	43
6.9	Answerable Queries: Permission Granted	43
6.10	Answerable Queries: Permission Denied	43
6.11	Answerable Queries: Permission Unregulated	43
7.1	Agreement 2.1 (as used in [PW06])	44
7.2	Agreement 2.1 in Coq	45
7.3	Agreement 2.5 (as used in [PW06])	45
7.4	Example 2.5	46
8.1	Proof Example	48
8.2	Hypothesis for Theorem One	48
8.3	Theorem One	48
8.4	Hypothesis for Theorem Two	49
8.5	Theorem Two	49
8.6	Inconsistent Count Formulas	50
8.7	Inconsistent Count Formula And Environment	50
8.8	Inconsistent Environment	50
8.9	Inconsistent Environment	51

9.1	Environments and Counts	54
9.2	Declarations	55
9.3	AV Rule	55
9.4	Type Transition and Role-Allow Rules	56
9.5	Constrain Rule	56
9.6	'allow'/'deny' Rule as a Mapping	56
9.7	SELinux Agreement	56
9.8	f_q^+ for SELinux	57
9.9	f_q^- for SELinux	57
9.10	f_q^+ for SELinux	57
9.11	f_q^- for SELinux	57

Chapter 1

Introduction

1.1 Access-Control Models [NIS09]

Authorization refers to the process of rendering a decision to allow access to a resource or asset of interest. By the same token all unauthorized access requests to resources must be controlled and ultimately denied, hence the term “access-control”.

Various access-control models exist and below we give a short summary of some of the most important ones leading up to our own core language, ACCPL.

1.1.1 Access Control Lists

Access Control List (ACL) is perhaps the oldest and the most basic access-control model. A list of subjects along with their rights are kept per resource or object of interest. Every time a subject makes an access request on an object, the ACL of the object is consulted and access is either granted or denied to the requester based on whether the requester is listed in the ACL and has the correct set of rights.

1.1.2 Capabilities

Capabilities based access-control works based on a list of objects and associated rights. The list of objects and the associated rights comprise an “unforgeable” ticket that a reference monitor checks to allow access (or not). Capabilities based systems don’t need to authenticate users as ACL based systems do.

1.1.3 RBAC

In Role-based Access Control (RBAC) systems a requester’s role determines whether access is granted or denied. In this model users belong to roles and rights are associated with roles so no direct association between users and rights exist. Roles are meant to group users

and add flexibility when assigning rights. RBAC systems naturally solve the problem of assigning ACLs for a large group of users and manage the administration cost of changing users' rights in ACL based systems.

1.1.4 ABAC

Despite many advantages of RBAC systems, some disadvantages also exist. Often a role needs to be decomposed into sub-roles based on the type of resource to be administered, and perhaps also based on the location that the resource serves [NIS09]. Basically RBAC suffers from a lack of a sub-typing mechanism whereby individual members of a group/role may be differentiated and access is granted or denied based on a more granular set of attributes. Attribute-based Access Control (ABAC) model was proposed to fulfill this granularity requirement. In ABAC access control decisions are made based on a set of attributes, associated with the subject making the request, the environment, and/or the resource itself [NIS09].

1.1.5 PBAC

In order to harmonize access control in large environments with many subjects and objects and disparate attributes, Policy-based Access Control (PBAC) model has been proposed [NIS09]. PBAC allows for a more uniform access-control model across the system. PBAC systems help create and enforce policies that define who should have access to what resources, and under what circumstances [NIS09]. There is also a need for large organizations to put in place mechanisms such that access-control rules can be easily audited. This calls for a data-driven approach to access-control where the data, in this case the access-control rules, are available to read and analyze. A data-driven approach like PBAC additionally helps with modularity of the system as changes in access control rules will have almost no impact to the underlying system the rules are meant to protect.

1.2 A Core Policy Language for PBAC Systems

Does there already exist a suitable policy language that can be used for expressing general access-control expressions? How about XACML, Open Digital Rights Language (ODRL) [Ian02] or eXtensible rights Markup Language (XrML) [WLD⁺02]?

Currently the most popular Rights Expression Languages (REL)s are the XrML, and the ODRL. Both of these languages are XML based and are considered declarative languages. XrML has been selected to be the REL for *MPEG-21* which is an ISO standard for multimedia applications. ODRL is also a standards based REL which has been accepted as part of the W3C community with the mandate of standardizing how rights and policies, related to the usage of digital content on the Open Web Platform, *OWP* [Wik15b], are expressed. ODRL 2.0 supports expression of rights and also privacy rules for social media

while ODRL 1.0 was only dealing with the mobile ecosystem – ODRL 1.0 was adopted by the Open Mobile Alliance, *OMA* in 2000.

RELs, or more precisely Digital Rights Expression Languages (DREL)s when dealing with digital assets deal with the “rights definition” aspect of the Digital Rights Management (DRM) ecosystem. A DREL, allows the expression and definition of digital asset usage rights such that other areas of the DRM ecosystem, namely the enforcement mechanism and the usage tracking components can function correctly.

As popular as both XrML and ODRL are, their adoption and usage is still somewhat limited in practice. Both Apple and Microsoft for example have defined their own lightweight RELs [JHM06] in *Fair Play* [Wik15a] and in *PlayReady* [Wik15c]. The authors of [JHM06] argue that both these RELs (XrML and ODRL) and others are simply too complex to be used effectively (for expressing rights) since they also try to cover much of the the enforcement and tracking aspects of DRMs.

DRM refers to the digital management of rights associated with the access or usage of digital assets. There are various aspects of rights management however. According to the authors of the white paper “A digital rights management ecosystem model for the education community,”([CPR04]) digital rights management systems cover the following four areas: 1) defining rights 2) distributing/acquiring rights 3) enforcing rights and 4) tracking usage.

XACML is a high-level and platform independent access control system that is also XML based. XACML is an OASIS standard which defines a language for the definition of policies and access requests, and a workflow to achieve policy enforcement [MPT12]. According to Masi et al [MPT12] designing XACML access control policies is difficult and error prone. Furthermore XACML comes without a formal semantics as do both XrML and ODRL. The XACML standard is written in prose and contains quite a number of loose points that may give rise to different interpretations and lead to different implementation choices [MPT12].

XACML, ODRL and XrML are all PBAC based languages where ODRL and XrML differ from XACML by their focus on digital assets protection and in general DRM, hence the term REL. All three are full-blown and custom languages that have one thing in common; they suffer from a lack of formal semantics. Additionally all of these languages cover much more than policy expressions leading to access decisions; they also address enforcement of the policies (ODRL and XrML specifically and DRM in general distinguish themselves from general access-control languages by additionally addressing enforcement of policies beyond where the policies were generated). A third reason that made these custom languages unsuitable as a core policy language was the fact that they are limited in terms of what can be built on top of them; for example expressing hierarchical role-based access-control in XACML requires a fairly complex encoding [TK06].

Rights expressions in DRELs and specifically ODRL are used to arbitrate access to assets under conditions. The main construct in ODRL is the *agreement* which specifies users, asset(s) and policies whereby controls on users’ access to the assets are described. This is very similar to how access control conditions are expressed in access control policy languages such as XACML [ANP+03] and SELinux [SVLS02]. In fact several authors have

worked on interoperability between RELs and access control policy languages, specifically between ODRL and XACML, [PRD05, MRD09] and also on translation from high level policies of XACML to low-level and fine grained policies of SELinux [ASLZ08].

A policy language that was based on some sort of logic with formal semantics and also one that was minimal and extendible was clearly needed. We started by looking at Lithium [HW08] and subsequently Pucella and Weissman’s subset of ODRL [PW06] as potential core languages. Lithium uses DRM as the main application of its access-control system whereas Pucella and Weissman’s primary goal is to define formal semantics for ODRL whose main application in turn is DRM. We will use Pucella and Weissman’s subset of ODRL as the basis for ACCPL and in doing so treat digital rights as our main access-control application without loss of generality with respect to other applications, with the final goal of performing formal verification on policies written in ACCPL.

1.3 Formal Semantics for PBAC Languages

Formal methods help ensure that a system behaves correctly with respect to a specification of its desired behavior [Pie02]. This specification of the desired behavior is what’s referred to as *semantics* of the system. Using formal methods requires defining precise and formal semantics, without which analysis and reasoning about properties of the system in question would become impossible. For example, an issue with the current batch of RELs are due to their semantics being expressed in a natural language (e.g. English) which by necessity results in ambiguous and open to interpretation behavior.

To formalize the semantics of PBAC languages several approaches have been attempted by various authors. Most are logic based [HW08, PW06] while others are based on finite-automata [HKS04], operational semantics based interpreters [SS09] and web ontology (from the Knowledge Representation Field) [KG10].

1.4 Logic Based Semantics

Formal logic can represent the statements and facts we express in a natural language like English. Propositional logic is expressive enough to express simple facts as propositions and uses connectives to allow for the negation, conjunction and disjunction of the facts. In addition simple facts can be expressed conditionally using the implication connective. Propositional logic however is not expressive enough to express policies of the kind used in languages like ODRL and XrML. For example, a simple policy expressed in English like “All who pay 5 dollars can watch the movie Toy Story” cannot be expressed in propositional logic because the concept of variables doesn’t exist in propositional logic.

A richer logic such as “Predicate Logic”, also called “First Order Logic” (*FOL*), is more suitable and has the expressive power to represent policies written in English. Moreover, FOL can be used to capture the meaning of policies in an unambiguous way.

Halpern and Weissman [HW08] propose a fragment of FOL to represent and reason about policies. The fragment of FOL they arrive at is called *Lithium* which is decidable and allows for efficiently answering interesting queries. Lithium restricts policies to be written based on the concept of “bipolarity” which disallows by construction policies that both permit and deny an action on an object. Pucella and Weissman [PW06] specify a predicate logic based language that represents a subset of ODRL.

1.5 Specific Problem

Policy languages and the agreements written in those languages are meant to implement specific goals such as limiting access to specific assets. The tension in designing a policy language is usually between how to make the language expressive enough, such that the design goals for the policy language may be expressed, and how to make the policies verifiable with respect to the stated goals.

As stated earlier, an important part of fulfilling the verifiability goal is to have formal semantics defined for policy languages. For ODRL, authors of [PW06] have defined a formal semantics based on which they declare and prove a number of important theorems (their main focus is on stating and proving algorithm complexity results). However as with many paper-proofs, the language used to do the proofs while mathematical in nature, uses a lot of intuitive justifications to show the proofs. As such these proofs are difficult to verify or more importantly to “derive”. Furthermore the proofs can not be used directly to render a decision on a sample policy (e.g. whether to allow or deny access to an asset). Of course one may (carefully) construct a program based on these proofs for practical purposes but we will have no way of certifying those programs correct, even assuming the original proofs were in fact correct.

1.6 Contributions

In this thesis we have built a language called ACCPL based on ODRL and starting with definitions in [PW06]. The ACCPL framework has been formalized in *Coq* [BC04] which is both a programming language and a proof-assistant. We have declared and proved decidability results for ACCPL in *Coq* which will allow us to extract programs from the proofs; the executable programs can be used on specific policies to render a specific decision such as “a permission has been granted”.

We originally started with a specific subset of [PW06] so that we could concentrate on what we believed to be the essence or core of the language. For example, we started with only one of three different kinds of “facts” (that may affect the permit/deny type decisions). We also had to change some of the language productions to allow for *Coq*’s requirement for clearly terminating recursion. However we maintained the central semantic definitions including “Closed World Assumptions” [PW06] where the semantics only specify explicitly Permitted and notPermitted answers so the semantics as stated by Pucella and Weissman [PW06] are not complete and therefore not decidable. We have made

major modifications to the semantics of Pucella and Weissman’s language to make the new ACCPL language decidable.

Decidability results subsume an important sub-category, namely inconsistency or conflict-detection in policy expressions or rules. Authors of [SM12] [SMF13] describe and implement in Coq a conflict detection algorithm for detecting conflicts in XACML access control rules. XACML is an expressive and at the same time complex policy language which makes conflict detection a difficult task. Authors of [SM12] then prove the conflict detection algorithm correct (or certified) by developing a formal proof in Coq. The proof is rather complex and involves a large number of cases, including many corner cases that were difficult to get right [SM12]. For ACCPL we have formally proven that conflicts are not possible.

ACCPL being a core policy language with certified semantics could be used to implement various policy languages and reason about interoperability between those languages [PRD05,MRD09]. In this manner our Coq based language ACCPL can be viewed as *abstract syntax*, complete with defined semantics, that can be used for implementing various policy languages with more concrete syntax (e.g. W3C’s ODRL and SELinux).

Chapter 2

ACCPL Syntax

2.1 Introduction

We follow the style of [PW06] by using abstract syntax to express policy statements in ACCPL. Abstract syntax is a more compact representation than XML which is what all the XML-based policy languages such as ODRL use. Furthermore abstract syntax simplifies specifying the semantics as we shall see later. As an example the agreement “If Mary Smith pays five dollars, then she is allowed to print the eBook ‘Treasure Island’ twice and she is allowed to display it on her computer as many times as she likes” written in ODRL’s XML encoding is illustrated in Listing 2.1 [PW06].

Listing 2.1: Agreement for Mary Smith in XML

```
<agreement>
  <asset> <context> <uid> Treasure Island </uid> </context> </
    asset>
  <permission>
    <display>
      <constraint>
        <cpu> <context> <uid> Mary’s computer </uid> </context> <
          /cpu>
        </constraint>
      </display>
      <print>
        <constraint> <count> 2 </count> </constraint>
      </print>
      <requirement>
        <prepay>
          <payment> <amount currency="AUD"> 5.00</amount> </payment>
        </prepay>
      </requirement>
    </permission>
```

```
<party> <context> <name> Mary Smith </name> </context> </
  party>
</agreement>
```

The agreement in Listing 2.1 is shown in 2.2 using the syntax from [PW06].

Listing 2.2: Agreement for Mary Smith as BNF (as used in [PW06])

```
agreement
  for Mary Smith
  about Treasure Island
  with prePay[5.00] -> and[cpu[Mary's Computer] => display,
                                count[2] => print].
```

In the following we will cover the *abstract syntax* of ACCPL that we later express using Coq’s constructs such as *Inductive Types* and *Definitions*.

2.2 Environmental Facts

In ACCPL, agreements and facts (i.e. environments) will refer to a count of how many times each policy should be and has been used to justify an action. This is the only fact that ACCPL will cover although we conjecture adding other facts and the machinery to support those facts should not change verification goals and results so far of ACCPL.

In ACCPL a *prerequisite* is either *true*, a *constraint*, the negative of a *constraint* or a conjunction of *prerequisites*. *true* is the prerequisite that always holds. Constraints are facts that are outside of control of users. For example, there is nothing *Alice* can do to satisfy the constraint “user must be Bob”.

We will describe ACCPL in a *BNF* grammar that looks more like Pucella and Weissman’s subset grammar [PW06]. BNF style grammars are more abstract as they only give suggestions about the surface syntax of expressions without getting into lexical analysis and parsing related aspects such as precedence order of operators [PCG⁺11]. The Coq version in contrast is more formal and could be directly used for building compilers and interpreters. We will present both the BNF version and the Coq version for each construct of ACCPL.

2.3 Productions

The top level ACCPL production is the *agreement*. An agreement expresses what actions a set of subjects may perform on an object and under what conditions. Syntactically an agreement is composed of a set of subjects/users called a *principal* or *prin*, an *asset* and a *policySet*.

Listing 2.3: agreement

```
<agreement> ::=
    'agreement' 'for' <prin> 'about' <asset> 'with' <
        policySet>
```

Principals or prins are composed of *subjects* which are specified based on the application e.g. Alice, Bob, etc.

Listing 2.4: prin

```
<prin> ::= { <subject1>, ..., <subjectm> }
```

Listing 2.5: subject

```
<subject> ::= N
```

Assets are also application specific but similar to subjects we will continue using specific ones for the DRM application (taken from [PW06]). *ebook*, *The Report* and *latestJingle* are examples of specific subjects we will be using throughout. Syntactically an asset is represented as a natural number (N). Similarly for subjects.

Listing 2.6: asset

```
<asset> ::= N
```

Agreements express who may perform an action on an asset. They include a set of subjects (i.e. a *prin*), an asset and a policy set. A policy set is a primitive policy set implying non-nested policy sets. Note that we could define various combining operators for policy sets such as conjunctions and disjunctions but we keep ACCPL’s policy sets limited to the primitive kind but use a policy combining operator when it comes to dealing with policies later on. Each primitive policy set specifies a *prerequisite* and a *policy*. In general if the prerequisite “holds” the policy is taken into consideration. Otherwise the policy will not be looked at. Some primitive policy sets are specified as *inclusive* as opposed to others that are explicitly specified as *exclusive*. The *Primitive Exclusive Policy Sets* are exclusive to agreement’s users in that only those users may perform the actions specified in the policy set. The implication is that all other users who are not specified in the agreement’s principal (prin) are forbidden from performing the specified actions, no matter whether the *prerequisite* holds or not. Not surprisingly we also define *Primitive Inclusive Policy Sets* that don’t enforce any exclusivity to the agreement’s users.

Listing 2.7: policySet

```
<policySet> ::=
    <primPolicySet> ; primitive policy set
```

Listing 2.8: primPolicySet

```
<primPolicySet> ::=
    <primInclusivePolicySet> ; primitive inclusive policy set
    <primExclusivePolicySet> ; primitive exclusive policy set
```

Listing 2.9: primInclusivePolicySet

```
<primInclusivePolicySet> ::=
  <preRequisite> → <policy>          ; primitive inclusive policy set
```

Listing 2.10: primExclusivePolicySet

```
<primExclusivePolicySet> ::=
  <preRequisite> ↦ <policy>          ; primitive exclusive policy set
```

A primitive policy specifies an action to be performed on an asset, depending of whether the policy’s prerequisite holds or not. If the prerequisite holds the agreement’s user is permitted to perform the action on the agreement’s asset; otherwise permission is denied. Pucella and Weissman’s subset of ODRL [PW06] specify a unique identifier for each policy to help the translation (from agreements to formulas). ACCPL has maintained the identifier for future work and we include it here in our definition of the policy construct however as far as the proofs are concerned the policy identifier could be removed without a loss to the obtained results. Primitive policies could also be grouped together using the conjunction combining operator.

Listing 2.11: policy

```
<policy> ::=
  'and' [ <primPolicy1>, ...,
          <primPolicym> ]      ; conjunction
```

Listing 2.12: primPolicy

```
<primPolicy> ::=
  <preRequisite> ⇒<policyId> <act>      ; primitive policy
```

An *Action* (*act*) is represented as a natural number. Similar to assets and subjects, actions are application specific. Some example actions taken from [PW06] are *Display* and *Print*.

Listing 2.13: act

```
<act> ::= N
```

A *Policy Id* (*policyId*) is a unique identifier specified as (increasing) positive integers.

Listing 2.14: policyId

```
<policyId> ::= N
```

In ACCPL a *prerequisite* is either true or it is a *constraint*. The *true* prerequisite always holds. A constraint is an intrinsic part of a policy and cannot be influenced by agreement’s user. Minimum height requirements for popular attractions and rides are examples of what we would consider a constraint. *NotCons* is a negation of a constraint. Finally the set of prerequisites are closed under conjunction operator (*AndPrqs*).

Listing 2.15: preRequisite

```

<preRequisite> ::=
    'True'                                ; always true
    <constraint>                          ; constraint
    'not' [ <constraint> ]                 ; suspending constraint
    'and' [ <preRequisite1>, ..., <preRequisitem> ]
    ; conjunction

```

Constraints are either *Principal*, *Count* or *CountByPrin*. Principal constraints basically require matching to specified prins. For example, the user being Alice is a Principal constraint. A count constraint refers to a set of policies P and specifies the number of times the user of an agreement has invoked the policies in P to justify her actions. If the count constraint is part of a policy then the set P is composed of the single policy. In the case that the count constraint is part of a policy set, the set P is the set of policies specified in the policy set.

Listing 2.16: constraint

```

<constraint> ::=
    <prin>                                ; principal
    'Count' [N]                          ; number of executions
    <prin> ('Count' [N])                  ; number of executions by prin

```

Chapter 3

ACCPL Syntax In Coq

3.1 Introduction to Coq

Coq is known first and foremost as a proof-assistant. The underlying formal language that Coq uses is a much more expressive version of typed lambda calculus called Calculus of (Co)Inductive Constructions (CIC) where proofs and programs can both be represented. For example, CIC adds polymorphism (terms depending on types), type operators (types depending on types) and dependent types (types depending on terms).

Specifications of programs in Coq may be expressed using the specification language *Gallina* [Hue92]. Coq is then used to develop proofs to show that a program's run-time behaviour satisfies its specification. Such programs are called *certified* because they are formally verified and confirmed to conform to their specifications [BC04].

Assertions or propositions are statements about values in Coq such as $3 < 8$ or $8 < 3$ that may be true, false or even be only conjectures. To verify that a proposition is true a proof needs to be constructed. While paper-proofs use a combination of mathematics and natural language to describe their proofs, Coq provides a formal (and therefore unambiguous) language that is based on proof-theory to develop proofs in. Verification of complex proofs is possible because one can verify the intermediate proofs or sub-goals in steps, each step being derived from the previous by following precise derivation rules. The Coq proof engine solves successive goals by using predefined *tactics*. Coq tactics are commands to manipulate the local context and to decompose a goal into simpler goals or sub-goals [BC04].

3.2 ACCPL Syntax

YACPL productions were presented as high level abstract syntax in 2.3. Below we present the corresponding encodings in Coq.

An agreement is a new inductive type in Coq by the same name. The constructor *Agreement* takes a *prin*, an *asset* and a *policySet*. *prin* is defined to be a non empty list of *subjects* (see listing 3.1).

Types *asset*, *subject*, *act* and *policyId* are simply defined as *nat* which is the datatype of natural numbers defined in coq’s library module *Coq.Init.Datatypes* (*nat* is itself an inductive datatype). We use Coq constants to refer to specific objects of each type. For example, the subject ‘Alice’ is defined as *DefinitionAlice* : *subject* := 101. and the act ‘Play’ as *DefinitionPlay* : *act* := 301.. For each “nat” type in YACPL we have also used constants that play the role of “Null” objects (see “Null Object Pattern” [MRB98]), for example *NullSubject*. This is needed partly because of the way YACPL language elements are defined which corresponds to the need to use *nonemptylist* exclusively even though at intermediate stages during the various algorithms Coq’s *list* is a better fit because it allows empty lists.

Next we define the *policySet* datatype. Note the close/one-to-one mapping to its counterpart in listing 2.7. A *policySet* is constructed only one way: by calling the *PPS* constructor which takes a *primPolicySet* as input. There are two ways a *primPolicySet* can be constructed (see listing 2.8) corresponding to two constructors: *PIPS* and *PEPS*.

PIPS takes a *primInclusivePolicySet* as input while *PEPS* takes a *primExclusivePolicySet*. Both *primInclusivePolicySet* and *primExclusivePolicySet* types are constructed by taking a *preRequisite* and a *policy* as parameters (see listing 2.9 and 2.10).

A *policy* is defined as a datatype with the constructors *Policy* which takes a *nonemptylist* (see listing 3.3) of *primPolicys* (see listing 2.11). *primPolicy* is constructed by calling *PrimitivePolicy* which takes a *preRequisite*, a *policyId* and an action *act* (see listing 2.12). Ignoring the *policyId* for a moment, a primitive policy consists of a prerequisite and an action. If the prerequisite holds the action is allowed to be performed on the asset.

The data type *nonemptylist* reflects the definition of “policy conjunction” in listing listing 2.11 in chapter 2 (see definition of *nonemptylist* in listing 3.3). Essentially *nonemptylist* represents a list data structure that has at least one element and it is defined as a new *polymorphic* inductive type in its own section.

Listing 3.1: ACCPL: Coq Version of Agreement

```

Inductive agreement : Set :=
| Agreement : prin → asset → policySet → agreement.

Definition prin := nonemptylist subject.

Definition asset := nat.

Definition subject := nat.

Definition act := nat.

Definition policyId := nat.

Inductive policySet : Set :=
| PPS : primPolicySet → policySet.

Inductive primPolicySet : Set :=
| PIPS : primInclusivePolicySet → primPolicySet
| PEPS : primExclusivePolicySet → primPolicySet.

Inductive primInclusivePolicySet : Set :=
| PrimitiveInclusivePolicySet : preRequisite → policy →
  primInclusivePolicySet.

Inductive primExclusivePolicySet : Set :=
| PrimitiveExclusivePolicySet : preRequisite → policy →
  primExclusivePolicySet.

Inductive policy : Set :=
| Policy : nonemptylist primPolicy → policy.

Inductive primPolicy : Set :=
| PrimitivePolicy : preRequisite → policyId → act → primPolicy.

```


Listing 3.2: ACPL: Coq Version of Agreement

```

Inductive agreement : Set :=
| Agreement : prin → asset → policySet → agreement.

Definition prin := nonemptylist subject.

Definition asset := nat.

Definition subject := nat.

Definition act := nat.

Definition policyId := nat.

Inductive primPolicy : Set :=
| PrimitivePolicy : preRequisite → policyId → act → primPolicy.

Inductive policy : Set :=
| Policy : nonemptylist primPolicy → policy.

Inductive primInclusivePolicySet : Set :=
| PrimitiveInclusivePolicySet : preRequisite → policy →
  primInclusivePolicySet.

Inductive primExclusivePolicySet : Set :=
| PrimitiveExclusivePolicySet : preRequisite → policy →
  primExclusivePolicySet.

Inductive primPolicySet : Set :=
| PIPS : primInclusivePolicySet → primPolicySet
| PEPS : primExclusivePolicySet → primPolicySet.

Inductive policySet : Set :=
| PPS : primPolicySet → policySet.

```

Listing 3.3: nonemptylist

```

Section nonemptylist.

Variable X : Set.

Inductive nonemptylist : Set :=
| Single : X → nonemptylist
| NewList : X → nonemptylist → nonemptylist.

End nonemptylist.

```

In listing 3.4 *preRequisite* is defined as a new datatype with constructors *TruePrq*, *Constraint*, *NotCons* and *AndPrqs* (see listing 2.15 for the abstract syntax equivalent).

TruePrq represents the always true prerequisite. The *Constraint* prerequisite is defined as the type *constraint* so its description is deferred here. Intuitively a constraint is a prerequisite to be satisfied that is outside the control of the user(s). For example, the constraint of being 'Alice' if you are 'Bob' (or 'Alice' for that matter). The constructor *NotCons* is defined the same way the *Constraint* constructor is. This constructor is defined as the type *constraint* and it is meant to represent the negation of a *constraint* as we shall see in the translation (see listing 5.18). The remaining constructor *AndPrqs* takes as parameters non empty lists of prerequisites. This constructor represents the conjunction combining operator.

Finally a *constraint* (see listing 2.16 for the abstract syntax equivalent) is defined as a new datatype with constructors *Principal*, *Count* and *CountByPrin*.

Principal constraint takes a *prin* to match. For example, the constraint of the user being Bob would be represented as "Principal constraint". The *Count* constructor takes a *nat* which represents the number of times the user of an agreement has invoked the corresponding policies to justify her actions. If the count constraint is part of a policy then the corresponding policies is basically the single policy, whereas in the case that the count constraint is part of a policy set, the corresponding policies would be the set of those policies specified in the policy set. The *CountByPrin* is similar to *Count* but it takes an additional *prin* parameter. In this case the subjects specified in the *prin* parameter override the agreements' user(s).

Listing 3.4: preRequisite

```

Inductive preRequisite : Set :=
| TruePrq : preRequisite
| Constraint : constraint → preRequisite
| NotCons : constraint → preRequisite
| AndPrqs : nonemptylist preRequisite → preRequisite.

Inductive constraint : Set :=
| Principal : prin → constraint
| Count : nat → constraint
| CountByPrin : prin → nat → constraint.

```

We started with the encoding of the agreement for Mary Smith in chapter 2/listing ?? but we deferred the definition of the Coq constructs used to define that agreement. All the definitions needed to encode the agreement for Mary Smith except for the prerequisite *PrePay* (which is not part of YACPL) have now been defined. More agreement examples will be given in Chapter 7.

Chapter 4

ODRL0 Semantics

4.1 Introduction

In this section, we describe the semantics of ODRL0 language by a translation from agreements to a subset of many-sorted first-order logic formulas with equality. The semantics will help answer queries of the form “may subject s perform action act to asset a ?”. If the answer is yes, we say permission is granted. Otherwise permission is denied. Note that in the listings in this chapter we use $\llbracket \cdot \rrbracket$ (double square brackets) notation as a mapping of ODRL0 syntactic elements to their translations as many-sorted first-order logic formulas. \triangleq are used between a translation and its corresponding formula to mean the translation is “mapped to” the formula. This translation is derived from the one in [PW06] mainly by adapting it to the ODRL0 subset and in particular syntax, which covered in chapter 2.

Unless specifically mentioned, we use superscripts to denote parameters to translation mappings. However we make a distinction on whether the translation notation is used on the right hand side (rhs) of a \triangleq or the left hand side (lhs). The right side occurrence is similar to a function call where we pass actual parameters to the function call - here to the mapping. The left side occurrence is similar to function declarations/definitions where we define formal parameters - here when we define the translation for a construct for the first time.

4.2 Agreement Translation

At a high-level, an agreement is translated into a conjunction of formulas of the form $\forall x(prerequisites(x) \rightarrow P(x))$ where $P(x)$ itself is a conjunction of formulas of the form $prerequisites(x) \rightarrow (\neg)Permitted(x, act, a)$, where $Permitted(x, act, a)$ means the subject x is permitted to perform action act on asset a . More concretely, given an agreement and observing that the only way an agreement can be built is by passing a *prin* (typically $prin_u$ the agreement’s user(s)), an *asset*, and a *policySet* to the constructor of the type agreement (see 3.2), the given agreement is translated by invoking the agreement translation mapping with the passed in arguments: $\llbracket agreement \rrbracket^{policySet, prin_u, a}$.

The agreement translation mapping declares its three formal arguments as a *policySet*, a set of users, *prin*, and an asset, *a* (see 4.1).

Listing 4.1: Agreement Translation

$$\llbracket agreement \rrbracket^{policySet, prin, a} \triangleq \llbracket policySet \rrbracket^{prin, a}$$

4.3 Policy Set Translation Definition

The translation for a *policySet* declares two formal arguments: a set of users, *prin* and an asset, *a*. The translation is defined by cases, one for each clause of the grammar in listing 2.7. Recall that a *policySet* is either a *PrimitivePolicySet*, a *PrimitiveExclusivePolicySet* or a *AndPolicySet*, a conjunction of policy sets (see 4.2).

Listing 4.2: Policy Set Translation Cases

$$\begin{aligned} \llbracket policySet \rrbracket^{prin, a} \triangleq & \\ & \llbracket preRequisite \rightarrow policy \rrbracket^{prin, a} \quad ; \text{ primitive policy set} \\ & \llbracket preRequisite \mapsto policy \rrbracket^{prin, a} \quad ; \text{ primitive exclusive policy set} \\ & \llbracket and[policySet_1, \dots, policySet_m] \rrbracket^{prin, a} \quad ; \text{ conjunction} \end{aligned}$$

Each of the *policySet* kinds has its own translation function, which will be defined in the next 3 subsections. See 4.3.4 for the translation of *getId* and 4.3.5 for the translation of *act* mentioned below.

4.3.1 PrimitivePolicySet Translation Definition

Translation of a *PrimitivePolicySet* (*preRequisite* \rightarrow *policy*) declares two formal arguments: a set of users, *prin* and an asset, *a* and yields a formula that includes a test on whether the subject is in the set of agreements' users (translation of *prin*), the translation of the *policy* and the translation of the *preRequisite*. Basically if the subject in question is a user of the agreement and the policySet's prerequisites hold, then the policy holds. Translation of the policy for a *PrimitivePolicySet* is called a *positive translation*. A positive translation is one where the actions described by the policies are permitted (see 4.3).

Listing 4.3: Policy Set Translation Definition : PrimitivePolicySet

$$\begin{aligned} \llbracket preRequisite \rightarrow policy \rrbracket^{prin, a} \triangleq & \forall x ((\llbracket prin \rrbracket_x \wedge \\ & \llbracket preRequisite \rrbracket_x^{getId(policy), prin, a}) \rightarrow \llbracket policy \rrbracket_x^{positive, prin, a}) \end{aligned}$$

4.3.2 PrimitiveExclusivePolicySet Translation Definition

Translation of a *PrimitiveExclusivePolicySet* (*preRequisite* \mapsto *policy*) declares two formal arguments: a set of users, *prin* and an asset, *a* and yields the conjunction of

two implications. The first implication, is the same as one found in the translation of *PrimitivePolicySet*. The second implication however restricts access (to make the policy set exclusive) to only those subjects that are in the agreement's user(s). Translation of the policy in the second implication is called a *negative translation*. A negative translation is one where the actions described by the policies are not permitted (see 4.4).

Listing 4.4: Policy Set Translation Definition : PrimitiveExclusivePolicySet

$$\begin{aligned} \llbracket preRequisite \mapsto policy \rrbracket^{prin,a} &\triangleq \forall x ((\llbracket prin \rrbracket_x \wedge \\ &\llbracket preRequisite \rrbracket_x^{getId(policy),prin,a}) \rightarrow \llbracket policy \rrbracket_x^{positive,prin,a}) \wedge \forall x \\ &(\neg \llbracket prin \rrbracket_x \rightarrow \llbracket policy \rrbracket_x^{negative,prin,a}) \end{aligned}$$

4.3.3 AndPolicySet Translation Definition

Translation of *AndPolicySet* declares two formal arguments: a set of users, *prin* and an asset, *a* and yields the conjunctions of the corresponding policy set translations (see 4.5).

Listing 4.5: Policy Set Translation Definition : AndPolicySet

$$\llbracket and[policySet_1, \dots, policySet_m] \rrbracket^{prin,a} \triangleq \llbracket policySet_1 \rrbracket^{prin,a} \wedge \dots \wedge \llbracket policySet_m \rrbracket^{prin,a}$$

4.3.4 getId Translation

The *getId* function when applied to a single *policy* (see 2.11) is defined to return the *policyId* of the *policy*.

Listing 4.6: getId for a Single policy

$$\llbracket getId(policy) \rrbracket \triangleq \llbracket getId(preRequisite \Rightarrow_{policyId} act) \rrbracket \triangleq policyId$$

getId function when applied to a set of policies is the set union of the translations for each individual policy (see 4.6).

Listing 4.7: getId for Policies Definition

$$\llbracket getId(and[policy_1, \dots, policy_m]) \rrbracket \triangleq getId(policy_1) \cup \dots \cup getId(policy_m)$$

4.3.5 action Translation

Translation of actions, such as *Play* and *Display* is simply the constant corresponding to each action instance.

Listing 4.8: act Translation Definition

$$\llbracket act \rrbracket \triangleq N$$

4.4 prin Translation Cases

Translation for a *prin* ($\llbracket prin \rrbracket_x$) declares no formal arguments and is a formula that is true if and only if the subject x is in the prin set. A *prin* is either a single subject or a list of subjects so the translation covers both cases (see 4.9).

Listing 4.9: Prin Translation Cases

$$\llbracket prin \rrbracket_x \triangleq \begin{array}{l} \llbracket subject \rrbracket_x \\ \llbracket \{subject_1, \dots, subject_m\} \rrbracket_x \end{array}$$

Each of the *prin* kinds has its own translation function, which will be defined in the following 2 subsections.

4.4.1 Single Subject Translation Definition

If the *prin* is a single subject, the translation is a formula that is true if and only if the subject x is the same as the single subject *subject* (see 4.10).

Listing 4.10: Prin Translation Definition : Single subject

$$\llbracket subject \rrbracket_x \triangleq x = subject$$

4.4.2 List of Subjects Translation Definition

Translation of a list of subjects is the disjunction of the translations for each subject (see 4.11).

Listing 4.11: Prin Translation Definition : List of subjects

$$\llbracket \{subject_1, \dots, subject_m\} \rrbracket_x \triangleq \llbracket subject_1 \rrbracket_x \vee \dots \vee \llbracket subject_m \rrbracket_x$$

4.5 Policy Translation Cases

The translation for a *policy* declares three formal arguments: *polarity*, which indicates whether the policy translation is positive or negative, a set of users, *prin* and an asset, *a*. The translation is defined by cases, one for each clause of the grammar in listing 2.11. Recall that a *policy* is either a *PrimitivePolicy* or a *AndPolicy*, a conjunction of policies (see 4.12).

Listing 4.12: Policy Translation Cases

$\llbracket policy \rrbracket^{polarity, prin, a} \triangleq$	
$\llbracket preRequisite \Rightarrow_{policyId} act \rrbracket_x^{polarity, prin, a}$; primitive policy
$\llbracket and[policy_1, \dots, policy_m] \rrbracket_x^{polarity, prin, a}$; conjunction

Each of the *policy* kinds has its own translation function, which will be defined in the next 2 subsections.

4.5.1 PrimitivePolicy Translation Definition

Translation of a *PrimitivePolicy* ($preRequisite \Rightarrow_{policyId} act$) declares three formal arguments: *polarity*, which indicates whether the policy translation is positive or negative, a set of users, *prin* and an asset, *a*. In the case of positive polarity, the translation yields a formula that 'permits' *x* to *act* on *a* if translation for the *preRequisite* holds. In the case that the polarity is negative, the translation yields a formula indicating that *x* is not permitted to *act* on *a* regardless of whether the translation for *preRequisite* holds or not (see 4.13).

Listing 4.13: PrimitivePolicy Translation Definition

$\llbracket preRequisite \Rightarrow_{policyId} act \rrbracket_x^{polarity, prin, a} \triangleq$	
$(\llbracket preRequisite \rrbracket_x^{\{policyId\}, prin, a}) \Rightarrow Permitted(x, \llbracket act \rrbracket, a)$	
; polarity=positive	
$\neg(Permitted(x, \llbracket act \rrbracket, a))$; polarity=negative

4.5.2 AndPolicy Translation Definition

Translation of *AndPolicy* declares three formal arguments: *polarity*, which indicates whether the policy translation is positive or negative, a set of users, *prin* and an asset, *a* and yields the conjunctions of the corresponding policy translations (see 4.14).

Listing 4.14: Policy Translation Definition : AndPolicy

$\llbracket and[policy_1, \dots, policy_m] \rrbracket^{polarity, prin, a} \triangleq \llbracket policy_1 \rrbracket^{polarity, prin, a} \wedge \dots \wedge \llbracket policy_m \rrbracket^{polarity, prin, a}$

4.6 Prerequisite Translation Cases

Translation for a *preRequisite* declares three formal arguments: a set of *policyIds* (identifiers for policies that are implied by the prerequisites), a *prin* and an asset, *a*. The translation is defined by cases, one for each clause of the grammar in listing 2.15. Recall that a *preRequisite* is either always *TruePrq*, a *Constraint*, a *ForEachMember*, a *NotCons*, a *AndPrqs*, a *OrPrqs* or a *XorPrqs* (see 4.15).

Listing 4.15: Prerequisite Translation Cases

```

 $\llbracket preRequisite \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq$ 
   $\llbracket true \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$  ; always true
   $\llbracket constraint \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$  ; constraint
   $\llbracket forEachMember[prin'; \{constraint_1, \dots, constraint_n\}] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$ 
; constraint distribution
   $\llbracket not\ constraint \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$  ; suspending constraint
   $\llbracket and\ [preRequisite_1, \dots, preRequisite_k] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$ 
; conjunction
   $\llbracket or\ [preRequisite_1, \dots, preRequisite_k] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$ 
; disjunction
   $\llbracket Xor\ [preRequisite_1, \dots, preRequisite_k] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$ 
; exclusive disjunction

```

Each of the *preRequisite* kinds has its own translation function, which will be defined in the following subsections.

4.6.1 True Prerequisite Translation Definition

The translation for a *TruePrq* yields a formula that is always *true* (see 4.16).

Listing 4.16: Prerequisite Translation Definition : Always True Prerequisite

```

 $\llbracket true \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq \text{True}$ 

```

4.6.2 Constraint Prerequisite Translation Definition

The translation for a *constraint* declares three formal arguments: a set of *policyIds*, a set of users, *prin* and an asset, *a*. The translation is defined by cases, one for each clause of the grammar in listing 2.16. Recall that a *constraint* is either a *Principal*, a *Count* or a *CountByPrin* (see 4.17).

Listing 4.17: Prerequisite Translation Definition : Constraint

```

 $\llbracket constraint \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq$ 
   $\llbracket prin \rrbracket_x$  ; principal
   $\llbracket count[N] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$  ; number of executions
   $\llbracket (count[N])^{prin'} \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$  ; number of executions by prin'

```

Each of the *constraint* kinds has its own translation function, which will be defined in the subsections following 4.7.

4.6.3 ForEachMember Prerequisite Translation Definition

The translation for a *forEachMember* declares three formal arguments: a set of *policyIds*, a set of users, *prin* and an asset, *a*. Note that *forEachMember* takes additional formal arguments: a *prin'* that overrides the *prin* that is passed at the *preRequisite* level and a set of *constraints* (see 4.18).

Listing 4.18: Prerequisite Translation Definition : ForEachMember

$$\llbracket \text{forEachMember}[prin'; \{constraint_1, \dots, constraint_n\}] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq \bigwedge_{(subject, i) \in prin' \times \{1, \dots, n\}} \llbracket constraint_i \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, \{subject\}, a}$$

4.6.4 NotCons Prerequisite Translation Definition

The translation for a *NotCons* yields a formula that is simply the negation of the translation for a constraint (see 4.17).

Listing 4.19: Prerequisite Translation Definition : Not Constraint

$$\llbracket \text{not constraint} \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq \neg \llbracket constraint \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$$

4.6.5 AndPrqs Prerequisite Translation Definition

The translation for a *AndPrqs* yields a formula that is the conjunction of the translation for each *preRequisite* (see 4.20).

Listing 4.20: Prerequisite Translation Definition : Conjunction

$$\llbracket \text{and } [preRequisite_1, \dots, preRequisite_k] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq \llbracket preRequisite_1 \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \wedge \dots \wedge \llbracket preRequisite_k \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$$

4.6.6 OrPrqs Prerequisite Translation Definition

The translation for a *OrPrqs* yields a formula that is the inclusive disjunction of the translation for each *preRequisite* (see 4.21).

Listing 4.21: Prerequisite Translation Definition : Inclusive Disjunction

$$\llbracket or \ [preRequisite_1, \dots, preRequisite_k] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq \llbracket preRequisite_1 \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \vee \dots \vee \llbracket preRequisite_k \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$$

4.6.7 XorPrqs Prerequisite Translation Definition

The translation for a *XorPrqs* yields a formula that is the exclusive disjunction of the translation for each *preRequisite* (see 4.22).

Listing 4.22: Prerequisite Translation Definition : Exclusive Disjunction

$$\llbracket Xor \ [preRequisite_1, \dots, preRequisite_k] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq \llbracket preRequisite_1 \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \oplus \dots \oplus \llbracket preRequisite_k \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$$

4.7 Constraint Translation Cases

Translation for a constraint is a formula $\llbracket constraint \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a}$, where the set of *policyIds* are identifiers for policies that are implied by the constraint, *prin* is the set of users (typically the agreement's user(s) denoted by $prin_u$) to which the constraint applies, *x* is a variable of type *subject*.

4.7.1 Principal Constraint Translation

The translation for a *Principal* is defined to be the translation of the corresponding *prin* defined earlier in listing 4.9.

4.7.2 Count Constraint Translation

The translation for a *Count* declares three formal arguments: a set of *policyIds*, a set of users, *prin* and an asset, *a* and it is a formula that is true when the sum of the number of times that *subject_i* (taken from *prin*) has invoked a policy with policy identifier *id_j* is smaller than *N* (see listing 4.23).

Listing 4.23: Constraint Translation : Count

$$\llbracket count[N] \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq (\sum_{(id, subject) \in (\{policyId_1, \dots, policyId_m\} \times prin)} getCount(subject, id)) < N$$

4.7.3 CountByPrin Constraint Translation

The translation for a *CountByPrin* declares the same formal arguments as the translation for *Count* and it is a formula that is true when the sum of the number of times that *subject_i* has invoked a policy with policy identifier *id_j* is smaller than *N*. The difference from the *Count* case, is that *prin'* in *CountByPrin* overrides the passed in *prin* (typically agreement's user(s) or *prin_u*). See listing 4.24.

Listing 4.24: Constraint Translation : Count by Principal

$$\llbracket (count[N])^{prin'} \rrbracket_x^{\{policyId_1, \dots, policyId_m\}, prin, a} \triangleq (\sum_{(id, subject) \in (\{policyId_1, \dots, policyId_m\} \times prin')} getCount(subject, id)) < N$$

Chapter 5

ACCPL Semantics In Coq

5.1 Introduction

The translation functions plus the auxiliary types and infrastructure which implement the semantics have been encoded in Coq. Translation functions build Coq terms of type *Prop*. Well-formed propositions (or *Props*) are assertions one can express about values such as mathematical objects or even programs (e.g. $3 < 8$) in Coq.

Whether a permission is granted or denied depends on the agreements in question but also on the facts recorded in the environment. For ODRL0 those facts revolve around the number of times a policy has been used to justify an action (see section 2.2 for more details on odrl0). We encode this information in an *environment* which is a conjunction of equalities of the form $count(s, policyId) = n$.

The Coq version of the count equality is a new inductive type called *count_equality*. An environment is defined to be a non-empty list of *count_equality* objects (see listing 5.1). Function *make_count_equality* in listing 5.1 is simply a convenience function that builds *count_equality*s. For an example of how environments are created see listing 5.2.

Listing 5.1: Environments and Counts

```
Inductive count_equality : Set :=
| CountEquality : subject → policyId → nat → count_equality.

Definition make_count_equality
(s:subject)(id:policyId)(n:nat): count_equality :=
CountEquality s id n.

Inductive environment : Set :=
| SingleEnv : count_equality → environment
| ConsEnv : count_equality → environment → environment.
```

Listing 5.2: Defining Environments

```

Definition e1 : environment :=
  (SingleEnv (make_count_equality Alice id1 8)).

```

We also define a *getCount* function (see listing 5.3) that given a pair consisting of a subject and policy id, looks for a corresponding count in the environment. Note the use of the keyword *Fixpoint*. *Fixpoint* is the Coq keyword for recursive functions and we use it instead of 'Definition' when the function being defined is recursive such as is the case for function *getCount*. *getCount* assumes the given environment is consistent, so it returns the first matched *count* it sees for a *(subject, id)* pair. If a *count* for a *(subject, id)* pair is not found it returns 0.

Listing 5.3: getCount Function

```

Fixpoint getCount
  (e:environment)(s:subject)(id: policyId): nat :=
  match e with
  | SingleEnv f =>
    match f with
    | CountEquality s1 id1 n1 =>
      if (beq_nat s s1)
      then if (beq_nat id id1) then n1 else 0
      else 0
    end
  | ConsEnv f rest =>
    match f with
    | CountEquality s1 id1 n1 =>
      if (beq_nat s s1)
      then if (beq_nat id id1) then n1 else (getCount rest s id)
      else (getCount rest s id)
    end
  end.

```

5.2 Translations

Translation of the top level *agreement* element proceeds by case analysis on the structure of the agreement. However an agreement can only be built one way; by calling the constructor *Agreement*. The translation proceeds by calling the translation function for the corresponding *policySet* namely the parameter to *Agreement* called *ps*. See listing 5.4 which is the Coq implementation of 4.1. The formal argument *e* of type *environment* is passed along many translation function but will only eventually be used to get the count information from the *getCount* function.

Listing 5.4: Translation of Agreement

```

Definition trans_agreement
  (e:environment)(ag:agreement)(action_from_query:act)
  (subject_from_query:subject)(asset_from_query:asset) : nonemptylist result :=

  match ag with
  | Agreement prin_u a ps  $\Rightarrow$ 
    (trans_ps e action_from_query subject_from_query asset_from_query ps prin_u
     a)
  end.

```

Translation of a *policySet* (called *trans_ps* in listing 5.11), takes as input *e*, the environment, *ps*, the policy set, *prin_u*, the agreement's user, and *a*, the asset, and proceeds by case analysis of different policySet constructors and recursing into translation functions for the composing elements. A policySet is either a *PrimitivePolicySet*, *PrimitiveExclusivePolicySet* or a *AndPolicySet*.

Note that to implement the translation for an *AndPolicySet* a local function *trans_ps_list* has been defined where for a single *policySet*, *trans_ps* is called, and for a list of *policySets*, the conjunction of *trans_{ps}* are returned.

Listing 5.5: Query Answers and Results

```

Inductive answer : Set :=
  | Permitted : answer
  | Unregulated : answer
  | NotPermitted : answer.

Inductive result : Set :=
  | Result : answer  $\rightarrow$  subject  $\rightarrow$  act  $\rightarrow$  asset  $\rightarrow$  result.

Definition makeResult
  (ans:answer)(s:subject)(ac:act)(ass:asset) : result :=
  (Result ans s ac ass).

```

Listing 5.6: Translation of unregulated policies

```

Definition process_single_pp_trans_policy_unregulated
  (pp: primPolicy)(x:subject)
  (a:asset)(action_from_query: act) : nonemptylist result :=
  match pp with
  | PrimitivePolicy prq' policyId action  $\Rightarrow$ 
    (Single (makeResult Unregulated x action_from_query a))
  end.

Fixpoint trans_policy_unregulated
  (e:environment)(x:subject)(p:policy)(a:asset)

```

```

(action_from_query: act){struct p} : nonemptylist result :=

match p with
  | Policy pp_list ⇒ trans_pp_list_trans_policy_unregulated pp_list x a
  action_from_query
end.

```

Listing 5.7: Translation of negative policies

```

Definition process_single_pp_trans_policy_negative
  (pp: primPolicy)(x:subject)
  (a:asset)(action_from_query: act) : nonemptylist result :=
match pp with
  | PrimitivePolicy prq' policyId action ⇒

    if (eq_nat_dec action_from_query action)
    then
      (Single
        (makeResult NotPermitted x action_from_query a))
    else
      (Single
        (makeResult Unregulated x action_from_query a))

end.

Fixpoint trans_policy_negative
  (e:environment)(x:subject)(p:policy)(a:asset)
  (action_from_query: act){struct p} : nonemptylist result :=

match p with
  | Policy pp_list ⇒ trans_pp_list_trans_policy_negative pp_list x a
  action_from_query
end.

```

Listing 5.8: Translation of list of policies: positive, negative and unregulated use the same pattern

```

Fixpoint trans_pp_list_trans_policy_<positive | negative | unregulated>
  (pp_list:nonemptylist primPolicy)(e:environment)(x:subject)
  (prin_u:prin)(a:asset)(action_from_query: act){struct pp_list}:=
match pp_list with
  | Single pp1 ⇒ process_single_pp_trans_policy_<positive | negative | unregulated>
    > pp1 e x prin_u a action_from_query
  | NewList pp pp_list' ⇒ app_nonempty
  (process_single_pp_trans_policy_<positive | negative | unregulated> pp e x prin_u
    a action_from_query)
  (trans_pp_list_trans_policy_<positive | negative | unregulated> pp_list' e x
    prin_u a action_from_query)

```



```
end.
```

Listing 5.9: Translation of positive policies

```
Definition process_single_pp_trans_policy_positive
(pp: primPolicy)(e:environment)(x:subject)(prin_u:prin)
(a:asset)(action_from_query: act) : nonemptylist result :=
match pp with
| PrimitivePolicy prq' policyId action =>
  if (trans_preRequisite_dec e x prq' (Single policyId) prin_u)
  then (* prin /\ prq /\ prq' *)
    if (eq_nat_dec action_from_query action)
    then
      (Single
        (makeResult Permitted x action_from_query a))
    else
      (Single
        (makeResult Unregulated x action_from_query a))
    else (* prin /\ prq /\ ~prq' *)
      (Single
        (makeResult Unregulated x action_from_query a))
  end.

Definition trans_policy_positive
(e:environment)(x:subject)(p:policy)(prin_u:prin)(a:asset)
(action_from_query: act) : nonemptylist result :=
match p with
| Policy pp_list => trans_pp_list_trans_policy_positive pp_list e x prin_u a
  action_from_query
end.
```

Listing 5.10: Translation of PIPS and PEPS

```
Definition trans_policy_PIPS
(e:environment)(prq: preRequisite)(p:policy)(x:subject)
(prin_u:prin)(a:asset)(action_from_query:act) : nonemptylist result :=

  if (trans_prin_dec x prin_u)
  then (* prin *)
    if (trans_preRequisite_dec e x prq (getId p) prin_u)
    then (* prin /\ prq *)
      (trans_policy_positive e x p prin_u a action_from_query)
    else (* prin /\ ~prq *)
      (trans_policy_unregulated e x p a action_from_query)
  else (* ~prin *)
    (trans_policy_unregulated e x p a action_from_query).

Definition trans_policy_PEPS
(e:environment)(prq: preRequisite)(p:policy)(x:subject)
```

```

(prin_u:prin)(a:asset)(action_from_query:act) : nonemptylist result :=

if (trans_prin_dec x prin_u)
then (* prin *)
  if (trans_preRequisite_dec e x prq (getId p) prin_u)
  then (* prin /\ prq *)
    (trans_policy_positive e x p prin_u a action_from_query)
  else (* prin /\ ~prq *)
    (trans_policy_unregulated e x p a action_from_query)
else (* ~prin *)
  (trans_policy_negative e x p a action_from_query).

```

The listing 5.11 is the Coq implementation of 4.2.

Listing 5.11: Translation of Policy Set

```

Fixpoint trans_ps
  (e:environment)(action_from_query:act)(subject_from_query:subject)(asset_from_query
    :asset)
  (ps:policySet)
  (prin_u:prin)(a:asset){struct ps} : nonemptylist result :=

let process_single_ps := (fix process_single_ps (pps: primPolicySet):=

match pps with
| PIPS pips =>
  match pips with
  | PrimitiveInclusivePolicySet prq p =>
    (trans_policy_PIPS e prq p subject_from_query prin_u a action_from_query)
  end
| PEPS peps =>
  match peps with
  | PrimitiveExclusivePolicySet prq p =>
    (trans_policy_PEPS e prq p subject_from_query prin_u a action_from_query)
  end
end) in

if (eq_nat_dec asset_from_query a)
then (* asset_from_query = a *)
  match ps with
  | PPS pps => process_single_ps pps
  end
else (* asset_from_query <> a *)
  (Single
    (makeResult
      Unregulated subject_from_query action_from_query asset_from_query)).

```

Translation of a *prin* (called *trans_prin* in listing 5.12) takes as input x , the *subject* in question, p , the principal or the *prin*, and proceeds based on whether p is a single subject or a list of subjects. If p is a single subject, s , the *Prop* $x = s$ is returned. Otherwise the disjunction of the translation of the first subject in p (s) and the *rest* of the subjects is returned. The listing 5.12 is the Coq implementation of 4.9.

Listing 5.12: Translation of a Prin

```
Fixpoint trans_prin
  (x:subject)(p: prin): Prop :=

  match p with
  | Single s  $\Rightarrow$  (x=s)
  | NewList s rest  $\Rightarrow$  ((x=s)  $\vee$  trans_prin x rest)
  end.
```

A positive translation for a policy (called *trans_policy_positive* in listing 5.13) takes as input e , the *environment*, x , the *subject*, p , the *policy* to translate, $prin_u$, the agreement's user, and a , the asset and proceeds based on whether we have a *PrimitivePolicy* or a *AndPolicy*. If the policy is a *PrimitivePolicy* an implication is returned which indicates x is *permitted* to do *action* to a , if the *preRequisite* holds.

Permitted is a predicate specified as *ParameterPermitted* : *subject* \rightarrow *act* \rightarrow *asset* \rightarrow *Prop*. So *Permitted* predicate takes a *subject*, an *act* (an action) and an *asset* and builds a term of type *Prop*.

Note that to implement the translation for an *AndPolicy* a local function *trans_p_list* has been defined where for a single *policy*, *trans_policy_positive* is returned, and for a list of *policies*, the conjunction of *trans_policy_positives* are returned.

The listing 5.13 is the Coq implementation of 4.13 and 4.14 when *polarity* = *positive*.

Listing 5.13: Translation of a Positive Policy

```
Fixpoint trans_policy_positive
  (e:environment)(x:subject)(p:policy)(prin_u:prin)(a:asset){struct p} : Prop :=

let trans_p_list := (fix trans_p_list (p_list:nonemptylist policy)(prin_u:prin)(a:
  asset){struct p_list}:=
  match p_list with
  | Single p1  $\Rightarrow$  trans_policy_positive e x p1 prin_u a
  | NewList p p_list'  $\Rightarrow$ 
    ((trans_policy_positive e x p prin_u a) /\
     (trans_p_list p_list' prin_u a))
  end) in

match p with
```

```

| PrimitivePolicy prq policyId action  $\Rightarrow$  ((trans_preRequisite e x prq (Single
policyId) prin_u)  $\rightarrow$ 
                                (Permitted x action a))
| AndPolicy p_list  $\Rightarrow$  trans_p_list p_list prin_u a
end.

```

A negative translation for a policy (called *trans_policy_negative* in listing 5.14) takes as input e , the *environment*, x , the *subject*, p , the *policy* to translate, and a the asset and proceeds based on whether we have a *PrimitivePolicy* or a *AndPolicy*. If the policy is a *PrimitivePolicy* an implication is returned which indicates x is forbidden to do *action* to a regardless of whether *preRequisite* holds. As is the case for the positive translation, to implement the translation for an *AndPolicy* a local function *trans_p_list* has been defined where for a single *policy*, *trans_policy_negative* is returned, and for a list of *policies*, the conjunction of *trans_policy_negatives* are returned.

The listing 5.14 is the Coq implementation of 4.13 and 4.14 when *polarity* = *negative*.

Listing 5.14: Translation of a Negative Policy

```

Fixpoint trans_policy_negative
(e:environment)(x:subject)(p:policy)(a:asset){struct p} : Prop :=
let trans_p_list := (fix trans_p_list (p_list:nonemptylist policy)(a:asset){
    struct p_list}:=
    match p_list with
    | Single p1  $\Rightarrow$  trans_policy_negative e x p1 a
    | NewList p p_list'  $\Rightarrow$  ((trans_policy_negative e x p a) /\
                                (trans_p_list p_list' a))
    end) in

match p with
| PrimitivePolicy prq policyId action  $\Rightarrow$  not (Permitted x action a)
| AndPolicy p_list  $\Rightarrow$  trans_p_list p_list a
end.

```

The translation of a *prerequisite* (called *trans_preRequisite* in listing 5.15) takes as input e , the *environment*, x , the *subject*, prq , the *preRequisite* to translate, *IDs*, the set of identifiers (of policies implied by the *prq*), $prin_u$, the agreement's user, and proceeds by case analysis on the structure of the *prerequisite*. A *prerequisite* is either a *TruePrq*, a *Constraint*, a *ForEachMember*, a *NotCons*, a *AndPrqs*, a *OrPrqs* or a *XorPrqs*.

In listing 5.15 the translation for *TruePrq* is the Prop *True*, the translations for *Constraint*, *ForEachMember* and *NotCons* simply call respective translation functions for corresponding types *constraint* and *forEachMember* (namely *trans_constraint*, *trans_forEachMember* and *trans_notCons*). Note that the translation for *AndPrqs*, *OrPrqs* and *XorPrqs* have not yet been implemented but based on the their many-sorted-logic formulas' specifications (listing 2.15) they will be conjunctions, disjunctions and exclusive disjunctions of translations for each *prerequisite*.

The listing 5.15 is the Coq implementation of 4.15.

Listing 5.15: Translation of a PreRequisite

```

Definition trans_preRequisite
  (e:environment)(x:subject)(prq:preRequisite)(IDs:nonemptylist policyId)(prin_u:
    prin) : Prop :=

  match prq with
  | TruePrq ⇒ True
  | Constraint const ⇒ trans_constraint e x const IDs prin_u
  | ForEachMember prn const_list ⇒ trans_forEachMember e x prn const_list IDs
  | NotCons const ⇒ trans_notCons e x const IDs prin_u
  | AndPrqs prqs ⇒ True
  | OrPrqs prqs ⇒ True
  | XorPrqs prqs ⇒ True
  end.

```

The translation of a *constraint* (called *trans_constraint* in listing 5.16) takes as input *e* the *environment*, *x* the *subject*, *const*, the *constraint* to translate, *IDs*, the set of identifiers (of policies implied by the parent *preRequisite*) and *prin_u*, the agreement's user and proceeds by case analysis on the structure of the *constraint*. A *constraint* is either a *Principal*, a *Count* or a *CountByPrin*. The translation for *Principal* returns the translation function (namely *trans_prin*) for the *prn* (the *prin* that accompanies the *const* constraint). The translation for *Count* and *CountByPrin* return the translation function *trans_count*. For *Count* the *prin* used is the agreement's user, whereas the *prin* used is the one passed to *CountByPrin* namely *prn*.

The listing 5.16 is the Coq implementation of 4.17.

Listing 5.16: Translation of a Constraint

```

Fixpoint trans_constraint
  (e:environment)(x:subject)(const:constraint)(IDs:nonemptylist policyId)
  (prin_u:prin){struct const} : Prop :=
  match const with
  | Principal prn ⇒ trans_prin x prn

  | Count n ⇒ trans_count e n IDs prin_u

  | CountByPrin prn n ⇒ trans_count e n IDs prn
  end.

```

The translation of a *forEachMember* (called *trans_forEachMember* in listing 5.17) takes as input *e* the *environment*, *x* the *subject*, *principals*, the set of subjects that override the agreement's user(s), *const_list* the set of constraints and *IDs*, the set of identifiers (of policies implied by the parent *preRequisite*).

To implement the translation for a *forEachMember* we start by calling an auxiliary function *process_two_lists* that effectively returns a new list composed of pairs of members of the first list and the second list (the cross-product of the two input lists). In the case of a *forEachMember* translation, the call is “*process_two_lists principals const_list*” which returns a list of pairs of subject and constraint namely *prins_and_constraints*. *prins_and_constraints* is then passed to a locally defined function *trans_forEachMember_Aux* where for a single pair of subject and constraint *trans_constraint* is called and for a list of pairs of subject and constraints, the conjunction of *trans_constraints* (for the first pair) and *trans_forEachMember_Auxs* (for the rest of the pairs) are returned.

The listing 5.17 is the Coq implementation of 4.18.

Listing 5.17: Translation of *forEachMember*

```

Fixpoint trans_forEachMember
  (e:environment)(x:subject)(principals: nonemptylist subject)(const_list:
  nonemptylist constraint)
  (IDs:nonemptylist policyId){struct const_list} : Prop :=

let trans_forEachMember_Aux
:= (fix trans_forEachMember_Aux
  (prins_and_constraints : nonemptylist (Twos subject constraint))
  (IDs:nonemptylist policyId){struct prins_and_constraints} : Prop :=

  match prins_and_constraints with
  | Single pair1 ⇒ trans_constraint e x (right pair1) IDs (Single (left pair1))
  | NewList pair1 rest_pairs ⇒
    (trans_constraint e x (right pair1) IDs (Single (left pair1))) /\
    (trans_forEachMember_Aux rest_pairs IDs)
  end) in

let prins_and_constraints := process_two_lists principals const_list in
trans_forEachMember_Aux prins_and_constraints IDs.

```

The translation of a *NotCons* (called *trans_notCons* in listing 5.18) takes as input *e* the *environment*, *x* the *subject*, *const*, the *constraint* to translate, *IDs*, the set of identifiers (of policies implied by the parent *preRequisite*) and *prin_u*, the agreement’s user and proceeds to return the negation of *trans_constraint* (see listing 5.16).

The listing 5.18 is the Coq implementation of 4.19.

Listing 5.18: Translation of *NotCons*

```

Definition trans_notCons
  (e:environment)(x:subject)(const:constraint)(IDs:nonemptylist policyId)(prin_u:prin) :
  Prop :=
  ~ (trans_constraint e x const IDs prin_u).

```

The translation of a *Count* or a *CountByPrin* (called *trans_count* in listing 5.19) takes as input *e* the *environment*, *n* the total number of times the subjects mentioned in *prin_u* (last parameter) may invoke the policies identified by *IDs* (third parameter).

To implement the translation for a *Count* or a *CountByPrin* we start by calling an auxiliary function *process_two_lists* that effectively returns a new list composed of pairs of members of the first list and the second list (the cross-product of the two input lists). In the case of *trans_count*, the call is “*process_two_lists IDs prin_u*” which returns a list of pairs of *policyId* and *subject* namely *ids_and_subjects*. *ids_and_subjects* is then passed to a locally defined function *trans_count_aux*.

trans_count_aux returns the current count for a single pair of *policyId* and *subject* (the call to *getCount* which looks up the environment *e* and returns the current count per each *subject* and *policyId*) and for a list of pairs of *policyId* and *subjects*, the addition of *get_count* (for the first pair) and *trans_count_auxs* (for the rest of the pairs) is returned.

A local variable *running_{total}* has the value returned by *trans_count_aux*. Finally the proposition *running_{total} < n* is returned as the translation for a *Count* or a *CountByPrin*.

Note that the only difference between translations for a *Count* and a *CountByPrin* is the additional *prn* parameter for *CountByPrin* which allows for getting counts for subjects not necessarily the same as *prin_u*, the agreement’s user(s).

The listing 5.19 is the Coq implementation of 4.23 and of 4.24.

Listing 5.19: Translation of count

```

Fixpoint trans_count
  (e:environment)(n:nat)(IDs:nonemptylist policyId)
  (prin_u:prin) : Prop :=

  let trans_count_aux
    := (fix trans_count_aux
      (ids_and_subjects : nonemptylist (Twos policyId subject)) : nat :=
        match ids_and_subjects with
        | Single pair1 => getCount e (right pair1) (left pair1)
        | NewList pair1 rest_pairs =>
          (getCount e (right pair1)(left pair1)) +
          (trans_count_aux rest_pairs)
        end) in

  let ids_and_subjects := process_two_lists IDs prin_u in
  let running_total := trans_count_aux ids_and_subjects in
  running_total < n.

```

5.3 Decision Procedures

Listing 5.20: Translation of count TEMP

```

Theorem double_neg_constraint:
  ∀ (e:environment)(x:subject)
  (const:constraint)(IDs:nonemptylist policyId)(prin_u:prin),
  (trans_constraint e x const IDs prin_u) →
  ~~(trans_constraint e x const IDs prin_u).
Proof.
intros e x const IDs prin_u.
intros H. unfold not. intros H'. apply H'. exact H. Qed.

Theorem trans_negation_constraint_dec :
  ∀ (e:environment)(x:subject)
  (const:constraint)(IDs:nonemptylist policyId)(prin_u:prin),
  {~trans_constraint e x const IDs prin_u} +
  {~~trans_constraint e x const IDs prin_u}.

Proof.
intros e x const IDs prin_u.
pose (j:= trans_constraint_dec e x const IDs prin_u).
destruct j. apply double_neg_constraint in t. right. exact t. left. exact n. Defined.

Theorem trans_notCons_dec :
  ∀ (e:environment)(x:subject)
  (const:constraint)(IDs:nonemptylist policyId)(prin_u:prin),
  {trans_notCons e x const IDs prin_u} +
  {~ trans_notCons e x const IDs prin_u}.

Proof.
intros e x const IDs prin_u.
unfold trans_notCons. apply trans_negation_constraint_dec. Defined.

Theorem trans_preRequisite_dec :
  ∀ (e:environment)(x:subject)
  (prq:preRequisite)(IDs:nonemptylist policyId)(prin_u:prin),
  {trans_preRequisite e x prq IDs prin_u} +
  {~ trans_preRequisite e x prq IDs prin_u}.

Proof.
intros e x prq IDs prin_u.
induction prq.
simpl. auto.
simpl. apply trans_constraint_dec.
simpl. apply trans_notCons_dec.
simpl. auto.
simpl. auto.
simpl. auto.
Qed.

```



```
Theorem subject_in_prin_dec :
   $\forall (a:\text{subject}) (l:\text{prin}), \{\text{is\_subject\_in\_prin } a \ l\} + \{\sim \text{is\_subject\_in\_prin } a \ l\}.$ 
```

Proof.

```
induction l as [| a0 l IH1].
```

```
apply eq_nat_dec.
```

```
destruct (eq_nat_dec a0 a); simpl; auto.
```

```
destruct IH1; simpl; auto.
```

```
right; unfold not; intros [Hc1| Hc2]; auto.
```

```
Defined.
```

```
Theorem trans_prin_dec :
```

```
 $\forall (x:\text{subject})(p:\text{prin}),$ 
   $\{\text{trans\_prin } x \ p\} + \{\sim \text{trans\_prin } x \ p\}.$ 
```

Proof.

```
apply subject_in_prin_dec.
```

```
Defined.
```

f

Chapter 6

Queries

6.1 Introduction

We first mentioned queries in chapter 4 on page 18. Ultimately policy statements describing an agreement will be used to enforce those agreements. To enforce policy agreements, access queries are asked from the policy engine and access is granted or denied based on the answer.

By defining formal semantics for ODRL the authors of [PW06] were able to prove that answering a query on whether access should be granted or not, is decidable and NP-hard for the full ODRL.

In this chapter we will review our encoding of queries in Coq and Coq representations of other definitions used in [PW06] which we will use to prove decidability results of our own.

6.2 Queries

Queries are tuples of the form $(A, s, action, a, e)$ in [PW06]. The tuple corresponds to the question of determining whether a set A of agreements imply that a subject s may perform action $action$ on an asset a given the environment e . The Coq representation is listed in listing 6.1. We distinguish single agreement queries from multiple agreement queries by defining two separate types: *single_query* and *general_query*.

Listing 6.1: Queries

```

Inductive single_query : Set :=
| SingletonQuery : agreement → subject → act → asset → environment →
  single_query.

Inductive general_query : Set :=
| GeneralQuery : nonemptylist agreement → subject → act → asset →
  environment → general_query.

```

6.3 Answering Queries

Answering a query as defined earlier can lead to one of four outcomes: error(listing 6.2), permitted(listing 6.3), denied(listing 6.4) and “not applicable” (listing 6.5) defined in [TK06]. Note that e denotes ‘an environment being consistent’ (see section 5.1) in the following listings.

Listing 6.2: Answerable Queries: Error

$$\begin{aligned}
 (\bigwedge \llbracket \text{agreement} \rrbracket) \wedge e &\implies \text{Permitted}(s, \text{act}, a) \text{ and} \\
 (\bigwedge \llbracket \text{agreement} \rrbracket) \wedge e &\implies \neg \text{Permitted}(s, \text{act}, a)
 \end{aligned}$$

Listing 6.3: Answerable Queries: Permit

$$\begin{aligned}
 (\bigwedge \llbracket \text{agreement} \rrbracket) \wedge e &\implies \text{Permitted}(s, \text{act}, a) \text{ and} \\
 (\bigwedge \llbracket \text{agreement} \rrbracket) \wedge e &\not\Rightarrow \neg \text{Permitted}(s, \text{act}, a)
 \end{aligned}$$

Listing 6.4: Answerable Queries: Deny

$$\begin{aligned}
 (\bigwedge \llbracket \text{agreement} \rrbracket) \wedge e &\not\Rightarrow \text{Permitted}(s, \text{act}, a) \text{ and} \\
 (\bigwedge \llbracket \text{agreement} \rrbracket) \wedge e &\implies \neg \text{Permitted}(s, \text{act}, a)
 \end{aligned}$$

Listing 6.5: Answerable Queries: Not Applicable

$$\begin{aligned}
 (\bigwedge \llbracket \text{agreement} \rrbracket) \wedge e &\not\Rightarrow \text{Permitted}(s, \text{act}, a) \text{ and} \\
 (\bigwedge \llbracket \text{agreement} \rrbracket) \wedge e &\not\Rightarrow \neg \text{Permitted}(s, \text{act}, a)
 \end{aligned}$$

In [PW06] a slightly different formulation is used to denote the same four decision types. “Query Inconsistent”, “Permission Granted”, “Permission Denied” and “Permission Unregulated”. They define the formulas f_q^+ and f_q^- as below (see listings 6.6 and 6.7).

Listing 6.6: f_q^+

$$f_q^+ \triangleq (\wedge \llbracket agreement \rrbracket) \implies Permitted(s, act, a)$$

Listing 6.7: f_q^-

$$f_q^- \triangleq (\wedge \llbracket agreement \rrbracket) \implies \neg Permitted(s, act, a)$$

Now answering the queries will depend on the *E-validity* of f_q^+ and f_q^- . E-validity or the consistency of the environment is not captured explicitly in [PW06] (see listings 6.8, 6.9, 6.10 and 6.11) however the decision of which answer a query results in takes the consistency of the environment into account. In this thesis we will encode and use the decision algorithms in [PW06].

Listing 6.8: Answerable Queries: Query Inconsistent

f_q^+ and f_q^- both hold

Listing 6.9: Answerable Queries: Permission Granted

f_q^+ holds and f_q^- does not hold

Listing 6.10: Answerable Queries: Permission Denied

f_q^+ does not hold and f_q^- holds

Listing 6.11: Answerable Queries: Permission Unregulated

f_q^+ does not hold and f_q^- does not hold

Chapter 7

Examples

7.1 Introduction

In this chapter we will take a tour of the syntax and semantics we have so far developed by examining some example agreements. In the following we will start by reviewing some of the examples used in [PW06].

Ultimately the goal of specifying all the syntax and semantics is to declare some interesting theorems about policy expressions and proving them in Coq. However we will first start with some specific propositions/theorems about these examples to get a feel for how proofs are done in Coq.

7.2 Agreement 2.1

Consider example 2.1 (from [PW06]) where the *policySet* is a *AndPolicySet* with *p1* and *p2* as the individual *policySets*. Let *p1* be defined as *Count*[5] \rightarrow *print* and *p2* as *and*[*Alice*, *Count*[2]] \rightarrow *print*.

The agreement is that the asset *The Report* may be printed a total of five times by either *Alice* or *Bob*, and twice more by *Alice*. So if *Alice* and *Bob* have used policy *p1* to justify their printing of the asset m_{p1} and n_{p1} times, respectively, then either may do so again if $m_{p1} + n_{p1} < 5$. If they have used *p2* to justify their printing of the asset m_{p2} and n_{p2} times, respectively, then only *Alice* may do so again if $m_{p2} + n_{p2} < 2$. Note that since *Bob* doesn't meet the prerequisite of being *Alice*, n_{p2} is effectively 0, so we have $m_{p2} < 2$ as the condition for *Alice* being able to print again (*Alice* does meet the prerequisite of being *Alice*).

Listing 7.1: Agreement 2.1 (as used in [PW06])

```
agreement
for {Alice, Bob}
about TheReport
with and [p1, p2].
```

The Coq version of the agreement 2.1 (listing 7.1) and its sub-parts is listed below. It is best to start with the agreement itself called *A2.1* in the listing and compare to the agreement 2.1 listed in 7.1.

Listing 7.2: Agreement 2.1 in Coq

```

Definition p1A1:policySet :=
  PrimitivePolicySet
    TruePrq
    (PrimitivePolicy (Constraint (Count 5)) id1 Print).

Definition p2A1prq1:preRequisite := (Constraint (Principal (Single Alice))).
Definition p2A1prq2:preRequisite := (Constraint (Count 2)).

Definition p2A1:policySet :=
  PrimitivePolicySet
    TruePrq
    (PrimitivePolicy (AndPrqs (NewList p2A1prq1 (Single p2A1prq2))) id2 Print).

Definition A2.1 := Agreement (NewList Alice (Single Bob)) TheReport
  (AndPolicySet (NewList p1A1 (Single p2A1))).

```

7.3 Agreement 2.5

Consider example 2.5 (from [PW06]) where the *policySet* is a *PrimitivePolicySet* with a *Count* constraint as prerequisite and a *AndPolicy* as the policy. The *AndPolicy* is the conjunction of two *PrimitivePolicies*. Both policies have prerequisites of type *ForEachMember* with actions *display* and *print* respectively. The *prin* component for both *ForEachMembers* is *Alice, Bob*, whereas the constraint for the first *ForEachMember* is *Count[5]* and for the second is *Count[2]*.

Listing 7.3: Agreement 2.5 (as used in [PW06])

```

agreement
for {Alice, Bob}
about ebook
with Count [10] → and [forEachMember[{Alice, Bob}; Count[5]] ⇒id1 display,
                      forEachMember[{Alice, Bob}; Count[1]] ⇒id2 print].

```

The Coq version of the agreement 2.5 (listing 7.3) and its sub-parts is listed below. See agreement 2.5 listed in 7.3 for comparison.

The agreement is that the asset *ebook* may be displayed up to five times by Alice and Bob each, and printed once by each. However the total number of actions (either *display* or *print*) justified by the two policies by either Alice and Bob is at most 10.

Listing 7.4: Example 2.5

```
Definition tenCount:preRequisite := (Constraint (Count 10)).
Definition fiveCount:constraint := (Count 5).
Definition oneCount:constraint := (Count 1).

Definition prins2_5 := (NewList Alice (Single Bob)).
Definition forEach_display:preRequisite := ForEachMember prins2_5 (Single fiveCount)
.
Definition forEach_print:preRequisite := ForEachMember prins2_5 (Single oneCount).

Definition primPolicy1:policy := PrimitivePolicy forEach_display id1 Display.
Definition primPolicy2:policy := PrimitivePolicy forEach_print id2 Print.

Definition policySet2_5:policySet :=
  PrimitivePolicySet tenCount (AndPolicy (NewList primPolicy1 (Single primPolicy2))).

Definition A2.5 := Agreement prins2_5 ebook policySet2_5.
```


Chapter 8

Some Simple Theorems

8.1 Introduction

In this chapter we will declare and prove some very simple theorems about the examples from chapter 7. This simple introduction is only meant to give us a feel for how theorems are stated in Coq and how proofs are constructed using Coq *tactics*.

As mentioned earlier, propositions are types in Coq whose type is the sort *Prop*. Any term t whose type is a proposition is a proof term or, for short, a proof. A *Hypothesis* is a local declaration $h : P$ where h is an identifier and P is a proposition. An *Axiom* is similar to a hypothesis except it is declared at the global scope and so it is always available.

A *Theorem* or *Lemma* is stated by giving an identifier whose type is a proposition ([BC04]). The proposition is the statement of the theorem or lemma. It must be followed by a proof. Keywords “Hypothesis”, “Axiom” and “Theorem” or “Lemma” are used in each case respectively.

To build a proof in Coq the user states the proposition to prove; this is called a goal to be proved or discharge, along with some hypothesis that makes up the local context. The user then uses commands called tactics to manipulate the local context and to decompose the goal into simpler goals. The goal simplification into sub-goals will continue until all the sub-goals are solved.

In listing 8.1 we have declared a theorem called *example1* and the corresponding proposition *forallx : nat, x < x + 1*.

Note that the notation $P : T$ is also used to declare program P has type T . This duality of notation is due to Curry-Howard isomorphism which relates the two worlds of type theory and structural logic together [BC04]. Once the Theorem has been declared Coq displays the proposition to be proved under a horizontal line written ——— , and displays the context of local facts and hypothesis, if any, above the horizontal line. At this point one can enter proof mode by using *Proof*. upon which Coq is ready to accept tactics. Entering tactics that can break the stated goal (under the horizontal line) into one or more sub-goals is how one progresses until no goals left at which point Coq responds with “No more subgoals” [Ber10].

Listing 8.1: Proof Example

Theorem `example1`: $\forall x:\text{nat}, x < x + 1$.

In the following listings, for the sake of completeness of the presentation, we will include the Coq commands that complete the proof of the respective theorem. We will not however explain the individual commands further here. Showing the commands in the listings, is meant as an indication of the size of the proof in terms of lines of Coq script.

8.2 Theorem One

In listing 8.3 we define a *policySet* with a *constraint* such that if *Alice* has used the policy with *id1* to justify her printing a_1 times, she may do so again if $a_1 < 5$. The agreement *AgreeCan* simply links the asset *TheReport* with the subject *Alice* and the *policySet* previously defined.

We capture the fact that *Alice* has used the policy with *id1* to justify her printing 2 times in an environment called *eA1*. Recall that environments are defined to be non-empty lists of *count_equality* objects (see listing 5.1).

We also declare a hypothesis *H* with the proposition that results from the translation of the agreement (see definition of *trans_agreement* in listing 5.4) and the environment. The proposition can be shown in Coq after some clean-up (e.g. replaced 101 by *Alice*) and using the form *Eval compute* :

Listing 8.2: Hypothesis for Theorem One

$\forall x : \text{subject}, x = \text{Alice} \wedge \text{True} \rightarrow 2 < 5 \rightarrow \text{Permitted } x \text{ Print TheReport}.$

The theorem *One* that we are going to prove is trivial but nonetheless in English it states that *Alice* is Permitted to Print *TheReport*. The proof comes after the command 'Proof.' and ends with 'Qed'.

Listing 8.3: Theorem One

```

Definition psA1:policySet :=
  PrimitivePolicySet
    TruePrq
    (PrimitivePolicy (Constraint (Count 5)) id1 Print).

Definition AgreeCan := Agreement (Single Alice) TheReport psA1.

Definition eA1 : environment :=
  (SingleEnv (make_count_equality Alice id1 2)).

Hypothesis H: trans_agreement eA1 AgreeCan.

Theorem One: Permitted Alice Print TheReport.

```

```
Proof. simpl in H. apply H. split. reflexivity. auto. omega. Qed.
```

8.3 Theorem Two

In listing 8.5 we define an exclusive policy set *policySet* containing a policy *pol* that allows printing. The agreement *AgreeA5* includes the exclusive policy set to express that Bob may print *LoveAndPeace*. However any subject that is not the agreement's user (e.g. Bob) is forbidden from printing *LoveAndPeace*.

Notice that due to the fact that environments are defined as non-empty lists, we have added a Null count to it (see *eA5*). We continue to capture the relevant facts from the environment and the agreement through defining a hypothesis (e.g. *H*). The hypothesis is shown in listing 8.5.

Listing 8.4: Hypothesis for Theorem Two

```

∀ x : subject,
  (x = Bob /\ True → True → Permitted x Print LoveAndPeace) /\
  ((x = Bob → False) → Permitted x Print LoveAndPeace → False).

```

Theorem *T1_A5* states the exclusivity of the policy set, namely that any subject that is not Bob is not permitted to print the asset *LoveAndPeace*. Theorem *T2_A5* uses *T1_A5* to prove Alice is not permitted to print the asset.

Listing 8.5: Theorem Two

```

Definition prin_bob := (Single Bob).
Definition pol:policy := PrimitivePolicy TruePrq id3 Print.
Definition pol_set:policySet := PrimitiveExclusivePolicySet TruePrq pol.
Definition AgreeA5 := Agreement prin_bob LoveAndPeace pol_set.
Definition eA5 : environment := (SingleEnv (make_count_equality NullSubject NullId 0))
.

Hypothesis H: trans_agreement eA5 AgreeA5.

Theorem T1_A5: ∀ x, x <> Bob → ~Permitted x Print LoveAndPeace.
Proof. simpl in H. apply H. Qed.

Theorem T2_A5: ~Permitted Alice Print LoveAndPeace.
Proof. simpl in H. apply T1_A5. apply not_eq_S. omega. Qed.

End A5.

```

8.4 Theorem Three

In listing 5.1 we defined environments as non-empty lists of *count_equality* objects which are in turn defined as counts per each subject, policy-id pair. These count formulas represent how many times each policy has been used to justify an action by a subject wrt a policy (specified by the policy id) and semantically it makes sense that they are unique in time. When and if two *count_equality* objects with the same subject and policy id refer to different counts, we say we have *inconsistent* count formulas. The listing 8.6 defines the binary predicate *inconsistent*.

Listing 8.6: Inconsistent Count Formulas

```

Definition inconsistent (f1 f2 : count_equality) : Prop :=
  match f1 with (CountEquality s1 id1 n1) =>
    match f2 with (CountEquality s2 id2 n2) =>
      s1 = s2 -> id1 = id2 -> n1 <> n2
    end
  end.

```

Next we would like to expand the notion of inconsistency to more than two count formulas. We first define a predicate over a count formula and an environment as in listing 8.7. If the environment is a singleton then we just compare the two count formulas for inconsistency, else we build the disjunction of the inconsistency between the count formula on one hand and the head of the environment and the rest of the environment, respectively.

Listing 8.7: Inconsistent Count Formula And Environment

```

Fixpoint formula_inconsistent_with_env (f : count_equality)
  (e : environment) : Prop :=
  match e with
  | SingleEnv g => inconsistent f g
  | ConsEnv g rest => (inconsistent f g) ∨ (formula_inconsistent_with_env f rest)
  end.

```

Finally we define a new inductive data type that represents *consistent* environments (see listing 8.8). An environment is consistent if it is a singleton count formula, if it consists of only two consistent count formulas and finally if the environment consists of a consistent environment and the consistent composition of a count formula and the consistent environment (see constructor *consis_more*.

Listing 8.8: Inconsistent Environment

```

Inductive env_consistent : environment -> Prop :=
| consis_1 : ∀ f, env_consistent (SingleEnv f)
| consis_2 : ∀ f g, ~(inconsistent f g) -> env_consistent (ConsEnv f (SingleEnv g))
| consis_more : ∀ f e,

```

```
env_consistent e → ~(formula_inconsistent_with_env f e) → env_consistent (
  ConsEnv f e).
```

We will now pose several small theorems about consistency of count formulas and environments and provide proofs for them (see listing 8.9).

Listing 8.9: Inconsistent Environment

```
Theorem f1_and_f2_are_inconsistent: inconsistent f1 f2.
Proof.
unfold inconsistent. simpl. omega. Qed.

Theorem f1_and___env_of_f2_inconsistent: formula_inconsistent_with_env f1 (
  SingleEnv f2).
Proof.
unfold formula_inconsistent_with_env. apply f1_and_f2_are_inconsistent. Qed.

Theorem two_inconsistent_formulas_imply_env_inconsistent:
  ∀ f g, inconsistent f g → ~env_consistent (ConsEnv f (SingleEnv g)).
Proof.
intros. unfold not. intros H'.
inversion H'. intuition. intuition. Qed.

Theorem e2_is_inconsistent: ~env_consistent e2.
Proof.
apply two_inconsistent_formulas_imply_env_inconsistent.
apply f1_and_f2_are_inconsistent. Qed.

Theorem env_consistent_implies_two_consistent_formulas:
  ∀ (f g: count_equality),
    env_consistent (ConsEnv f (SingleEnv g)) → ~inconsistent f g.
Proof.
intros. inversion H. exact H1. intuition. Qed.

Theorem two_consistent_formulas_imply_env_consistent:
  ∀ (f g: count_equality),
    ~inconsistent f g → env_consistent (ConsEnv f (SingleEnv g)).
Proof.
intros. apply consis_2. exact H. Qed.

Theorem env_inconsistent_implies_two_inconsistent_formulas:
  ∀ (f g: count_equality),
    ~env_consistent (ConsEnv f (SingleEnv g)) → inconsistent f g.
```

```

Proof.
induction f.
induction g.
unfold inconsistent.
intros.
subst.
generalize (dec_eq_nat n n0).
intro h; elim h.
intro; subst.
elim H.
apply consis_2.
unfold inconsistent.
intro.
assert (s0=s0); auto.
assert (p0=p0); auto.
specialize H0 with (1:=H1) (2:=H2).
elim H0; auto.
auto.
Qed.

```

Theorem same_subjects_policyids_different_counts_means_inconsistent : $\forall (s1\ s2:$
 subject),
 $\forall (id1\ id2: policyId),$
 $\forall (n1\ n2: nat),$

$$(s1 = s2 \wedge id1 = id2 \wedge n1 <> n2) \rightarrow$$

$$inconsistent\ (CountEquality\ s1\ id1\ n1)\ (CountEquality\ s2\ id2\ n2).$$

```

Proof.
intros. unfold inconsistent. intros. intuition. Qed.

```

Chapter 9

Proposed Future Work

9.1 Summary

We started off by looking at DRELs and specifically at ODRL and considered its formal semantics as captured by [PW06]. We presented the encodings and semantics of the constructs for a significant subset of ODRL in Coq and then defined what queries looked like and what the decision problem was in this context. We have also encoded the decision algorithms as presented in [PW06] in Coq in order to perform formal verification of theorems of interest. We noted the common thread between RELs and policy languages for access control systems such as those between ODRL and SELinux. We discussed the goal of generalizing the concept of a policy language from strictly representing subsets of ODRL, to representing (subsets of) both ODRL and SELinux policy languages with the goal of applying the decision algorithms to both types of policies, in a unified manner.

9.2 Machine-Checked Proof of Decidability of Queries

By defining formal semantics for ODRL authors of [PW06] were able to show some important results. First result is that answering the question of whether a set of ODRL statements imply a permission, denial or other possibilities is decidable and also that its complexity is NP-hard.

The authors of [PW06] then prove that by removing the construct *not[policySet]* from ODRL's syntax answering the same query remains decidable and efficient (polynomial time complexity).

We will prove equivalent results as above starting with the decidability result of answering a query in ODRL0 (which does not include *not[policySet]*). The theorem in listing 9.1 states that for all environments, all single agreements, all subjects, all actions and all assets, either permission is granted, permission is denied, permission is unregulated or query is inconsistent.

Listing 9.1: Environments and Counts

```

Theorem queriesAreDecidable:  $\forall$  (e:environment),
     $\forall$  (agr: agreement),
     $\forall$  (s:subject),
     $\forall$  (action:act),
     $\forall$  (a:asset),

    (permissionGranted e [agr] s action a)  $\vee$ 
    (permissionDenied e [agr] s action a)  $\vee$ 
    (queryInconsistent e [agr] s action a)  $\vee$ 
    (permissionUnregulated e [agr] s action a).

```

We will then augment ODRL0 with the constructs we omitted from the full ODRL (resulting in what we have earlier called ODRL1 or ODRL2) including the troublesome construct *not[policySet]* and attempt to prove that the decidability results remain intact. There is a chance that a proof is not possible due to particulars of the Coq encoding we have used, in which case, we will adjust our encoding.

9.3 SELinux

We started out by looking at DRELS and specifically ODRL where rights expressions are used to arbitrate access to assets under conditions. Recall that the main construct in ODRL is the agreement which specifies users, asset(s) and policies (as part of policy sets) whereby controls on users' access to the assets are described. This is reminiscent of how access control conditions are expressed in access control policy languages such as XACML and SELinux.

While XACML is a high-level and platform independent access control system SELinux is platform dependent (e.g. Linux based) and low-level. SELinux enhances the Discretionary Access Control (DAC) that most unix based systems employ by Mandatory Access Control (MAC) where designed access control policies are applied throughout the system possibly overriding whatever DAC is in place by the system users.

SELinux uses Linux's extended file attributes to attach a *security context* to passive entities (e.g. files, directories, sockets) and also to each active entity typically a Linux user space process. Security context is a data structure that is composed of a user, a role and a domain (or type). While users can map directly to ordinary user names they can also be defined separately. Roles are meant to group users and add flexibility when assigning permissions and are the basis for RBAC. Finally domains or types are the basis for defining common access control requirements for both passive and active entities.

The enforcement of SELinux policies are performed by the *security server*. Whenever a security operation is requested from user space by a system call, the security server is invoked to arbitrate the operation and either allow the operation or to deny it. Each operation is identified by two pieces of information: an object class (e.g. file) and a

permission (e.g. read, write). When an operation is requested to be performed on an object, the class and the permissions associated with the object along with security contexts of the source (typically the source entity is a process) and the object are passed to the security server. The security server consults the loaded policy (loaded at boot time) and allows or denies the access request [SSS04].

9.4 SELinux Policy Language

The SELinux policy has four different kinds of statements: declarations, rules, constraints and assertions [ALP03]. Assertions are compile time checks that the *checkpolicy* tool performs at compile time. The other three kinds of statements however are evaluated at run-time.

Declaration statements are used to declare a type, a role and a type. First types are declared using a type declaration statement. Roles are declared and authorized for particular domains (types) through role declarations, and finally user declarations are used to define each user and to specify the set of authorized roles for each of these users (see 9.2). Note that in the listings below we will present a simplified and modified version of the official SELinux syntax in order to (re-)use the ODRL Coq framework.

Listing 9.2: Declarations

<code>'type' T ';</code>	<code>; type T</code>
<code>'role' R T ';</code>	<code>; role R is associated with type T</code>
<code>'user' U R ';</code>	<code>; user U is associated with role R</code>

Rule statements define access vector rules. Access vector (AV) rules (see listing 9.3) specify which operations are allowed and whether to audit (log). Any operation not covered by AV rules are denied and all denied operations are logged. The semantics of the AV rule with *avkind allow* is: processes with type *T1* are allowed to perform operations in *P* on objects with class *C* and type *T2* (with *avkind=deny* meaning not to allow).

Listing 9.3: AV Rule

<code><avRule> → <avkind> T1 T2:C P ';</code>
<code><avkind> → 'allow' 'deny'</code>

When a process changes security context, the role may change, assuming a *role transition* exists relating the old and the new roles. There is a related AV rule called the *type transition* rule where a process with type *T1* is allowed/denied to transition to type *T2* when *C*=process and *P*=transition (see 9.4).

Listing 9.4: Type Transition and Role-Allow Rules

```
<avkind> T1 T2:process transition ';'

<role_transition_rule> → <avkind> R1 R2 ';'
    ; when a process changes security contexts this rule must hold
```

Constraints are additional conditions on permissions in the form of boolean expressions that must hold in order for the specified permissions to be allowed (see listing 9.5. Whenever a permission is requested on an object class C , the security server checks that the constraints hold.

Listing 9.5: Constrain Rule

```
<constraint> → 'constrain' C, P, <expr> ';'

<expr> → 'not' <expr> | <expr> 'and' <expr> | <expr> or <expr> | U1 <
    op> U2 | T1 <op> T2 | R1 <op> R2

<op> → '==' | '!='
```

9.5 Agreements in SELinux

As with ODRL we will start by limiting the policy language to only allow AV rules. As mentioned earlier an operation not covered by a allow rule is denied by SELinux. We will make up explicit *deny* rules therefore an agreement is defined to be a combination of allow and deny rules. Allow and deny rules as mappings are defined in listing 9.6.

Listing 9.6: 'allow'/'deny' Rule as a Mapping

```
AV rule :  $T \times (T \times C) \rightarrow 2^P$ 
```

Listing 9.7: SELinux Agreement

```
<agreement> ::= <avRule> ';' <agreement>
```

9.6 Environments

Environments are collections of *role-type* and *user-role* relations. A role-type relation $role(R, T)$ simply associates a role with a type. A user-role relation $user(U, R)$ associates a user with a role. An environment is consistent with respect to a security context $\langle T, R, U \rangle$, if and only if $role(R, T)$ and $user(U, R)$ relations hold in the environment.

9.7 Queries in SELinux

The decision problem in SELinux access control is whether an entity with security context $\langle T1, R1, U1 \rangle$ may perform action $P1$ to entity with object class $C1$ with security context $\langle T2, R2, U2 \rangle$.

To answer such queries we use the authorization relation $auth(C, P, T1, R1, U1, T2, R2, U2)$ which is equivalent to the *Permitted* predicate we saw earlier for ODRL.

Listing 9.8: f_q^+ for SELinux

$$\begin{aligned} allow(T1, T2, C, P) \wedge (E \text{ consistent wrt } \langle T1, R1, U1 \rangle \wedge \langle T2, R2, U2 \rangle) \wedge \\ (((C, P) == (process, transition)) \implies allow(R1, R2)) \\ \implies auth(C, P, T1, R1, U1, T2, R2, U2) \end{aligned}$$

Listing 9.9: f_q^- for SELinux

$$\begin{aligned} deny(T1, T2, C, P) \vee \neg(E \text{ consistent wrt } \langle T1, R1, U1 \rangle \wedge \langle T2, R2, U2 \rangle) \vee \\ (((C, P) == (process, transition)) \implies deny(R1, R2)) \\ \implies \neg auth(C, P, T1, R1, U1, T2, R2, U2) \end{aligned}$$

9.8 Decidability of Queries in SELinux

In this thesis we will be investigating the question of decidability for answering queries in SELinux policies based on the same four outcomes we encountered earlier in 6.3 namely error, permitted, denied and “not applicable”. We will first state a decidability theorem similar to the theorem in listing 9.1 (minor adjustments may be needed to allow for differences with SELinux policy language) and present a proof for it in Coq. The literature in the SELinux implies only two outcomes are possible: permitted or denied. We will next attempt to prove this conjecture in Coq. Finally we will add constraint relations (see listing 9.10 and listing 9.11) to SELinux policies (which we have not included so far) and prove the same decidability results again for the augmented policy.

Listing 9.10: f_q^+ for SELinux

$$\begin{aligned} allow(T1, T2, C, P) \wedge constrain(C, P, T1, R1, U1, T2, R2, U2) \wedge (E \text{ consistent wrt } \langle T1, R1, U1 \rangle \wedge \langle T2, R2, U2 \rangle) \wedge (((C, P) == (process, transition)) \implies \\ allow(R1, R2)) \implies auth(C, P, T1, R1, U1, T2, R2, U2) \end{aligned}$$

Listing 9.11: f_q^- for SELinux

$$\begin{aligned} deny(T1, T2, C, P) \vee \neg constrain(C, P, T1, R1, U1, T2, R2, U2) \vee \neg(E \text{ consistent wrt } \langle T1, R1, U1 \rangle \wedge \langle T2, R2, U2 \rangle) \vee (((C, P) == (process, transition)) \implies \\ deny(R1, R2)) \implies \neg auth(C, P, T1, R1, U1, T2, R2, U2) \end{aligned}$$

References

- [ALP03] Myla Archer, Elizabeth I. Leonard, and Matteo Pradella. Analyzing security-enhanced linux policy specifications. In *4th IEEE International Workshop on Policies for Distributed Systems and Networks (POLICY 2003)*, 4-6 June 2003, Lake Como, Italy, page 158. IEEE Computer Society, 2003.
- [ANP⁺03] Anne Anderson, Anthony Nadalin, B Parducci, D Engovatov, H Lockhart, M Kudo, P Humenn, S Godik, S Anderson, S Crocker, et al. eXtensible access control markup language (xacml) version 1.0. *OASIS*, 2003.
- [ASLZ08] Masoom Alam, Jean-Pierre Seifert, Qi Li, and Xinwen Zhang. Usage control platformization via trustworthy SELinux. In *Proceedings of the 2008 ACM symposium on Information, computer and communications security*, pages 245–248. ACM, 2008.
- [BC04] Yves Bertot and Pierre Castéran. *Interactive Theorem Proving and Program Development. Coq’Art: The Calculus of Inductive Constructions*. Springer Science & Business Media, 2004. <http://www.labri.fr/perso/casteran/CoqArt/index.html>.
- [Ber10] Yves Bertot. Coq in a hurry. Technical report, May 2010. <http://cel.archives-ouvertes.fr/inria-00001173/PDF/coq-hurry.pdf>.
- [CPR04] Geoff Collier, Harry Piccariello, and Robby Robson. A digital rights management ecosystem model for the education community. *ECAR (forthcoming)*, May, 2004.
- [HKS04] Markus Holzer, Stefan Katzenbeisser, and Christian Schallhart. Towards formal semantics for ODRL. In Renato Iannella and Susanne Guth, editors, *Proceedings of the First International Workshop on the Open Digital Rights Language (ODRL)*, Vienna, Austria, April 22-23, 2004, pages 137–148, 2004.
- [Hue92] Gérard P. Huet. The gallina specification language: A case study. In R. K. Shyamasundar, editor, *Foundations of Software Technology and Theoretical Computer Science, 12th Conference, New Delhi, India, December 18-20, 1992, Proceedings*, volume 652 of *Lecture Notes in Computer Science*, pages 229–240. Springer, 1992.

- [HW08] Joseph Y. Halpern and Vicky Weissman. Using First-Order Logic to Reason about Policies. *ACM Transactions on Information and System Security*, 11(4), 2008.
- [Ian02] R. Iannella. Open digital rights language (ODRL) version 1.1. 2002. [Online; accessed 09-February-2015].
- [JHM06] Pramod A. Jamkhedkar, Gregory L. Heileman, and Iván Martínez-Ortiz. The problem with rights expression languages. In Moti Yung, Kaoru Kurosawa, and Reihaneh Safavi-Naini, editors, *Proceedings of the Sixth ACM Workshop on Digital Rights Management, Alexandria, VA, USA, October 30, 2006*, pages 59–68. ACM, 2006.
- [KG10] Andreas Kasten and Ralf Jäger. Making the semantics of ODRL and URM explicit using web ontologies. In *The 8th International Workshop for Technical, Economic and Legal Aspects of Business Models for Virtual Goods*, pages 77–91, Namur, Belgium, 2010.
- [MPT12] Massimiliano Masi, Rosario Pugliese, and Francesco Tiezzi. Formalisation and implementation of the XACML access control mechanism. In Gilles Barthe, Benjamin Livshits, and Riccardo Scandariato, editors, *Engineering Secure Software and Systems - 4th International Symposium, ESSoS 2012, Eindhoven, The Netherlands, February, 16-17, 2012. Proceedings*, volume 7159 of *Lecture Notes in Computer Science*, pages 60–74. Springer, 2012.
- [MRB98] Robert Martin, Dirk Riehle, and Frank Buschmann. *Pattern Languages of Program Design 3*, volume 256. Addison-Wesley, 1998.
- [MRD09] Xavier Maroñas, Eva Rodríguez, and Jaime Delgado. An architecture for the interoperability between rights expression languages based on XACML. In *Proceedings of the 7th International Workshop for technical, economic and legal aspects of business models for virtual goods, Virtual Goods*, 2009.
- [NIS09] NIST. A SURVEY OF ACCESS CONTROL MODELS. http://csrc.nist.gov/news_events/privilege-management-workshop/PvM-Model-Survey-Aug26-2009.pdf, 2009. [Online; accessed 16-August-2015].
- [PCG⁺11] Benjamin C. Pierce, Chris Casinghino, Michael Greenberg, Vilhelm Sjöberg, and Brent Yorgey. *Software Foundations*. University of Pennsylvania, 2011. <http://www.cis.upenn.edu/~bcpierce/sf/>.
- [Pie02] Benjamin C. Pierce. *Types and Programming Languages*. MIT Press, Cambridge, MA, USA, 2002.
- [PRD05] Jose Prados, Eva Rodríguez, and Jaime Delgado. Interoperability between different rights expression languages and protection mechanisms. In *Automated Production of Cross Media Content for Multi-Channel Distribution*,

2005. *AXMEDIS 2005. First International Conference on*, pages 8–pp. IEEE, 2005.
- [PW06] Riccardo Pucella and Vicky Weissman. A formal foundation for ODRL. *CoRR*, abs/cs/0601085, 2006.
 - [SM12] Michel St-Martin. *A verified algorithm for detecting conflicts in XACML access control rules*. PhD thesis, Carleton University, 2012.
 - [SMF13] Michel St-Martin and Amy Felty. A verified algorithm for detecting conflicts in XACML access control rules. In *Informal Proceedings of the First International Workshop on Automated Reasoning in Security and Software*, pages 4–11, 2013. <http://www.site.uottawa.ca/~afelty/dist/arsec13.pdf>.
 - [SS09] Nicholas Paul Sheppard and Reihaneh Safavi-Naini. On the operational semantics of rights expression languages. In Ehab Al-Shaer, Hongxia Jin, and Gregory L. Heileman, editors, *Proceedings of the 9th ACM Workshop on Digital Rights Management, Chicago, Illinois, USA, November 9, 2009*, pages 17–28. ACM, 2009.
 - [SSS04] Beata Sarna-Starosta and Scott D. Stoller. Policy analysis for security-enhanced linux. In *Proceedings of the 2004 Workshop on Issues in the Theory of Security (WITS)*, pages 1–12. Citeseer, 2004.
 - [SVLS02] Stephen Smalley, Chris Vance, NAI Labs, and Wayne Salamon. Implementing SELinux as a Linux security module. In *USENIX Technical Conference*, 2002. [Online; accessed 16-February-2015].
 - [TK06] Michael Carl Tschantz and Shriram Krishnamurthi. Towards reasonability properties for access-control policy languages. In David F. Ferraiolo and Indrakshi Ray, editors, *SACMAT 2006, 11th ACM Symposium on Access Control Models and Technologies, Lake Tahoe, California, USA, June 7-9, 2006, Proceedings*, pages 160–169. ACM, 2006.
 - [Wik15a] Wikipedia. FairPlay — Wikipedia, The Free Encyclopedia. <http://en.wikipedia.org/wiki/FairPlay>, 2015. [Online; accessed 16-February-2015].
 - [Wik15b] Wikipedia. Open Web Platform — Wikipedia, The Free Encyclopedia. http://en.wikipedia.org/wiki/Open_Web_Platform, 2015. [Online; accessed 16-February-2015].
 - [Wik15c] Wikipedia. PlayReady — Wikipedia, The Free Encyclopedia. <http://en.wikipedia.org/wiki/PlayReady>, 2015. [Online; accessed 16-February-2015].
 - [WLD⁺02] Xin Wang, Guillermo Lao, Thomas DeMartini, Hari Reddy, Mai Nguyen, and Edgar Valenzuela. XrML - eXtensible rights markup language. In Michiharu Kudo, editor, *Proceedings of the 2002 ACM Workshop on XML Security, Fairfax, VA, USA, November 22, 2002*, pages 71–79. ACM, 2002.