Certifying Digital Rights' Expression Languages

by

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Abstract

This is the abstract.

${\bf Acknowledgements}$

I would like to thank all the little people who made this possible.

Dedication

This is dedicated to the one I love.

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Chapter 1

Introduction

Digital Rights Management (DRM) refers to the digital management of rights associated with the access or usage of digital assets. There are various aspects of rights management however. According to the authors of the whitepaper "A digital rights management ecosystem model for the education community," digital rights management systems cover the following four areas: 1) defining rights 2) distributing/acquiring rights 3) enforcing rights and 4) tracking usage [4].

Rights Expression Languages (REL)s, or more precisely when dealing with digital assets, Digital Rights Expression Languages (DREL)s deal with the "rights definition" aspect of the DRM ecosystem. A DREL, allows the expression and definition of digital asset usage rights such that other areas of the DRM ecosystem namely the enforcement mechanism and the usage tracking components can function correctly.

Currently the most popular RELs are the eXtensible rights Markup Language (XrML) [bib], and the Open Digital Rights Language (ODRL)[bib]. Both of these languages are XML based and are considered declarative languages. XrML has been selected to be the REL for MPEG-21 which is an ISO standard for multimedia applications. ODRL is also a standards based REL which has been accepted as part of the W3C community with the mandate of standardizing how rights and policies, related to the usage of digital content on the Open Web Platform, OWP, are expressed [wikipedia]. ODRL 2.0 supports expression of rights and also privacy rules for social media while ODRL 1.0 was only dealing with the mobile ecosystem – ODRL 1.0 was adopted by the Open Mobile Alliance, OMA in 2000.

As popular as both XrML and ODRL are, their adoption and usage is still somewhat limited in practice. Both Apple and Microsoft for example have defined their own lightweight RELs [problem with RELs paper] in Fair Play (Apple) and in PlayReady (Microsoft). The authors of [the problem with RELs] argue that both these RELs and other ones are simply too complex to be used effectively since they try to cover much of the DRM ecosystem.

Another issue with the current batch of RELs are due to their semantics being expressed in a natural language (e.g. English). By necessity natural languages are ambiguous and open to interpretation.

To formalize the semantics of RELs several approaches have been attempted by various authors. The main categories are logic based, operational semantics based interpreters and finally web ontology based (from the Knowledge Representation Field). In this thesis we will focus on the logic based approach to formalizing semantics and will study a specific logic based language that is a translation from a subset of ODRL.

Chapter 2

Middle

2.1 Logic Based Semantics for ODRL

Formal logic can represent the statements and facts we express in a natural language like English. Propositional logic is expressive enough to express simple facts as propositions and uses connectives to allow for the negation, conjunction and disjunction of the facts. However propositional logic is not expressive enough to express policies of the kind used in languages like ODRL and XrML. For example, a simple policy expressed in English like "All who pay 5 dollars can watch the movie Toy Story" cannot be expressed in propositional logic because the concept of variables doesn't exist in propositional logic.

A richer logic called "Predicate Logic" or "First Order Logic" (FOL) is more suitable and has the expressive power to represent policies written in English. Moreover, FOL can be used to capture the meaning of policies in an unambiguous way.

Halpern and Weissman [Using First Order Logic to Reason about Policies] propose a fragment of FOL to represent and reason about policies. The fragment of FOL they arrive at is called *Lithium* which is decidable and allows for efficiently answering interesting queries. Lithium restricts policies to be written based on the concept of "bipolarity" which disallows by construction policies that both permit and deny an action on an object.

2.2 Pucella 2006

Pucella and Weissman [6] specify a predicate logic based based language that represents a subset of ODRL.

2.3 what will I do?

2.3.1 Coq

âĂć Program correctness âĂć Formal verification of software âĂć Certified programs âĂć Proof assistant âĂć Interactive and mechanized theorem proving âĂć Examples of machine assisted proofs: CompCert, four-color theorem proof âĂć Coq is based on a higher-order functional programming language âÅć Dependent Types âÜN Subset types âÜN Easier than writing explicit proofs âAć Write formal specification and proofs that programs comply to their specification (a-short-intro-to-coq) âĂć Automatically extract code from specifications as Ocaml or Haskell (a-short-intro) âÅć Properties, programs and proofs are all formalized in the same language called CIC (Calculus of inductive Constructions). (ashort-intro) â Áć Coq uses a sort called Prop for propositions â Áć Coq art: â Áć Well-formed propositions are assertions one can express about values such as mathematical objects or even programs e.g. 3 < 8 âUN Note that assertions may be true, false or simply conjectures âŬŃ An assertion is only true in general if a proof is provided âŬŃ However hand written proofs are difficult to verify âUN Coq provides an environment for developing proofs including a formal language to express proofs in, the language itself being built using proof theory making it possible to step by step verification of the proofs âUN Mechanized proof verification requires a "proof" that the verification algorithm is correct itself in applying all the formal rules correctly

Chapter 3

ODRL0 Syntax

3.1 Introduction

Authors of [6] use abstract syntax instead of XML (ODRL 2.0 can also be encoded in JSON and RDF/OWL Ontology) to express statements in the ODRL language. Abstract syntax is a more compact representation than XML in which ODRL policies may be written in and furthermore abstract syntax simplifies specifying the semantics as we shall see later. As an example the agreement "If Mary Smith pays five dollars, then she is allowed to print the eBook 'Treasure Island' twice and she is allowed to display it on her computer as many times as she likes" written in ODRL's XML encoding and the equivalent one expressed in the abstract syntax [6].

Listing 3.1: agreement for Mary Smith in XML

```
<agreement>
 <asset> <context> <uid> Treasure Island </uid> </context> 
   asset>
 <permission>
   <display>
    <constraint>
     <cpu> <context> <uid> Mary's computer </uid> </context> <</pre>
        /cpu>
    </constraint>
   </display>
   <print>
    <constraint> <count> 2 </count> </constraint>
   </print>
  <requirement>
   <prepay>
    <payment> <amount currency="AUD"> 5.00</amount> </payment>
   </prepay>
  </requirement>
```

```
</permission>
<party> <context> <name> Mary Smith </name> </context> </
   party>
</agreement>
```

The agreement 3.1 is shown below using the syntax from [6].

Listing 3.2: agreement for Mary Smith as BNF (as used in [6])

```
agreement
for Mary Smith
about Treasure Island
with prePay[5.00] -> and[cpu[Mary's Computer] => display,
count[2] => print].
```

In the following we will cover the *abstract syntax* of a subset of ODRL that we later express using Coq's constructs such as *Inductive Types* and Definitions. We will call this subset *ODRL0* both because it is a variation of Pucella's ODRL language and also because it is missing some constructs from Pucella's ODRL (and hence from ODRL proper).

3.2 ODRL0

In ODRL0, agreements and facts (i.e. environments) will only contain the number of times each policy has been used to justify an action. In ODRL0 agreements and facts will not contain:

- 1. Which payments have been made
- 2. Which acknowledgments have been made

This means Paid and Attributed predicates are not used in ODRL0. Also removed are related constructs prepay and attribution. We also had to remove two other constructs based on prepay and attribution out of ODRL0 in inSeq and anySeq. prepay, attribution, inSeq and anySeq make up what is called requirements in ODRL.

In ODRL a prerequisite is either true, a constraint, a requirement or a condition. true is the prerequisite that always holds. Constraints are facts that are outside of control of users. For example, there is nothing Alice can do to satisfy the constraint "user must be Bob". Requirements are facts that are in users' control. For example, Alice may satisfy the requirement "The user must pay 5 dollars". Finally conditions are constraints that must not hold.

In ODRLO, a prerequisite is either *true*, a *constraint*, or the negation of a *constraint*. So we have removed requirements from the picture and don't have explicit conditions. Conditions are replaced by a category called *NotCons* directly in the production for prerequisites (see 3.12). Note that we have also removed the condition *not*[policySet] from

ODRL since the authors in [6] have shown the semantics of this component are not well-defined and including it leads to intractability results.

We will add the missing pieces as described above (except for not[policySet]) making up what we will call ODRL1 and perhaps ODRL2 (the latter only if needed). We will also describe ODRL0 in a BNF grammar that looks more like Pucella's ODRL grammar. BNF style grammars are less formal as they give some suggestions about the surface syntax of expressions [Pierce1] without getting into lexical analysis and parsing related aspects such as precedence order of operators. The Coq version in contrast is more formal and could be directly used for building compilers and interpreters. We will present both the BNF version and the Coq version for each construct of ODRL0 [Pierce1]. To get started let's see what the listing 3.2 would look like in ODRL0's Coq version.

Listing 3.3: Coq version of agreement for Mary Smith

3.3 Productions

The top level ODRL0 production is the *agreement*. An agreement expresses what actions a set of subjects may perform on an object and under what conditions. Syntactically an agreement is composed of a set of subjects/users called a *principal* or *prin*, an *asset* and a *policySet*.

Listing 3.4: agreement

Principals or prins are composed of *subjects* which are specified based on the application e.g. Alice, Bob, etc for the application we will be using throughout.

```
Listing 3.5: prin

\langle prin \rangle ::= \{ \langle subject_1 \rangle, \ldots, \langle subject_m \rangle \}
```

```
Listing 3.6: subject
```

```
<subject> ::= N
```

Assets are also application specific but similar to subjects we will use specific ones for the DRM application (taken from [6]). ebook, $The\ Report$ and latestJingle are examples of specific subjects we will be using throughout. Syntactically an asset is represented as a natural number (N). Similarly for subjects.

Listing 3.7: asset

```
<asset> ::= N
```

Agreements include policy sets. Each policy set specifies a prerequisite and a policy. In general if the prerequisite holds the policy is taken into consideration. Otherwise the policy will not be looked at. Some policy sets are specified as exclusive. The Primitive Exclusive Policy Sets are exclusive to agreement's users in that only those users may perform the actions specified in the policy set. The implication is that all other users who are not specified in the agreement's principal (prin) are forbidden from performing the specified actions. Finally policy sets could be grouped together in a conjunction allowing a single agreement to be associated with many policy sets.

Listing 3.8: policySet

A policy specifies an action to be performed on an asset, depending of whether the policy's prerequisite holds or not. If the prerequisite holds the agreement's user is permitted to perform the action on the agreement's asset; otherwise permission is denied. Similar to policy sets, policies could also be grouped together in a conjunction. The policy also includes a unique identifier. The policy identifier is added to help the translation (from agreements to formulas) but is optional in ODRL proper.

Listing 3.9: policy

An Action (act) is represented as a natural number. Similar to assets and subjects, actions are application specific. Some example actions taken from [6] are Display and Print.

Listing 3.10: act

```
<act> ::= N
```

A Policy Id (policy Id) is a unique identifier specified as (increasing) positive integers.

```
Listing 3.11: policyId
```

```
<policyId> ::= N
```

In ODRL0 a prerequisite is either true or it is a constraint. The true prerequisite always holds. A constraint is an intrinsic part of a policy and cannot be influenced by agreement's user. Minimum height requirements for popular attractions and rides are examples of we would consider a constraint. The constraint ForEachMember is interesting in its expressive power but has complicated semantics as we shall see in the ?? section. Roughly speaking, ForEachMember takes a prin (a list of subjects) and a list L of constraints. The ForEachConstraint holds if each subject in prin satisfies each constraint in L.NotCons is a negation of a constraint. The set of prerequisites are closed under conjunction (AndPrqs), disjunction (OrPrqs) and exclusive disjunction (XorPrqs).

Listing 3.12: preRequisite

Constraints are either *Principal*, *Count* or *CountByPrin*. Principal constraints basically require matching to specified prins. For example, the user being Alice is a Principal constraint. A count constraint refers to a set of policies P and specifies the number of times the user of an agreement has invoked the policies in P to justify her actions. If the count constraint is part of a policy then the set P is composed of the single policy. In the case that the count constraint is part of a policy set, the set P is the set of policies specified in the policy set.

Listing 3.13: constraint

Chapter 4

ODRLO Syntax In Coq

ODRL0 productions were presented as high level abstract syntax in 3.3. Below we present the corresponding encodings in Coq.

An agreement is a new inductive type in Coq by the same name. The constructor Agreement takes a prin, an asset and a policySet. prin is defined to be a non empty list of subjects (see 4.1).

Types asset, subject, act and policy Id are simply defined as nat which is the datatype of natural numbers defined in coq's library module Coq.Init.Datatypes (nat is itself an inductive datatype). We use Coq constants to refer to specific objects of each type. For example, the subject 'Alice' is defined as DefinitionAlice: subject := 101. and the act 'Play' as DefinitionPlay: act := 301.. For each "nat" type in ODRL0 we have also used constants that play the role of "Null" objects (see "Null Object Pattern" [5]), for example NullSubject. This is needed partly because of the way ODRL0 language elements are defined which corresponds to the need to use nonemptylist exclusively even though at intermediate stages during the various algorithms Coq's list is a better fit because it allows empty lists.

Next we define the *policySet* datatype. Note the close/one-to-one mapping to its counterpart in 3.8. There are three ways a *policySet* can be constructed (see 3.8) corresponding to three constructors: *PrimitivePolicySet*, *PrimitiveExclusivePolicySet* and *AndPolicySet*. Both *PrimitivePolicySet* and *PrimitiveExclusivePolicySet* take a *preRequisite* and a *policy* as parameters. Finally *AndPolicySet* takes a non empty list of *policySet*s.

A policy is defined as a datatype with constructors PrimitivePolicy and AndPolicy (see 3.9). PrimitivePolicy takes a preRequisite, a policyId and a action act. Ignoring the policyId for a moment (it is only added to help the translation otherwise policyIds don't exist in ODRL proper), a primitive policy consists of a prerequisite and an action. If the prerequisite holds the action is allowed to be performed on the asset. The AndPolicy constructor is simply a nonemptylist (see listing 4.2) of policys.

The data type nonemptylist reflects the definition of "policy conjunction" in listing 3.9 in chapter 3 (see definition of nonemptylist in listing 4.2). Essentially nonemptylist

represents a list data structure that has at least one element and it is defined as a new polymorphic inductive type in its own section.

Listing 4.1: Coq version of agreement

```
Inductive agreement : Set :=
    | Agreement : prin → asset → policySet → agreement.

Definition prin := nonemptylist subject.

Definition asset := nat.

Definition subject := nat.

Definition act := nat.

Definition policyId := nat.

Inductive policySet : Set :=
    | PrimitivePolicySet : preRequisite → policy → policySet
    | PrimitiveExclusivePolicySet : preRequisite → policy → policySet
    | AndPolicySet : nonemptylist policySet → policySet.

Inductive policy : Set :=
    | PrimitivePolicy : preRequisite → policyId → act → policy
    | AndPolicy : nonemptylist policy → policy.
```

Listing 4.2: nonemptylist type

```
Section nonemptylist.

Variable X : Set.

Inductive nonemptylist : Set :=
    | Single : X → nonemptylist
    | NewList : X → nonemptylist → nonemptylist.

End nonemptylist.
```

In listing 4.3 preRequisite is defined as a new datatype with constructors TruePrq, Constraint, ForEachMember, NotCons, AndPrqs, OrPrqs and XorPrqs (see 3.12 for the abstract syntax equivalent).

TruePrq represents the always true prerequisite. The Constraint prerequisite is defined as the type constraint so its description is deferred here. Intuitively a constraint is a prerequisite to be satisfied that is outside the control of the user(s). For example, the constraint of being 'Alice' if you are 'Bob' (or 'Alice' for that matter). The constructor ForEachMember is defined to be a prin and a non empty list of constraints.

Intuitively a ForEachMember prerequisite holds if each subject in prin satisfies each constraint in the list of constraints. The constructor NotCons is defined the same way the Constraint" constructor is. This constructor is defined as the type constraint and it is meant to represent the negation of a constraint as we shall see in the translation (see 6.11). The remaining constructors AndPrqs, OrPrqs and XorPrqs take as parameters non empty lists of prerequisites. They represent conjunction, inclusive disjunction and exclusive disjunction of prerequisites respectively.

Finally a *constraint* (see 3.13 for the abstract syntax equivalent) is defined as a new datatype with constructors *Principal*, *Count* and *CountByPrin*.

Principal constraint takes a *prin* to match. For example, the constraint of the user being Bob would be represented as "Principal constraint". The *Count* constructor takes a *nat* which represents the number of times the user of an agreement has invoked the corresponding policies to justify her actions. If the count constraint is part of a policy then the corresponding policies is basically the single policy, whereas in the case that the count constraint is part of a policy set, the corresponding policies would be the set of those policies specified in the policy set. The *CountByPrin* is similar to *Count* but it takes an additional *prin* parameter. In this case the subjects specified in the *prin* parameter override the agreements' user(s).

Listing 4.3: preRequisite

```
Inductive preRequisite : Set :=
    | TruePrq : preRequisite
    | Constraint : constraint → preRequisite
    | ForEachMember : prin → nonemptylist constraint → preRequisite
    | NotCons : constraint → preRequisite
    | AndPrqs : nonemptylist preRequisite → preRequisite
    | OrPrqs : nonemptylist preRequisite → preRequisite
    | XorPrqs : nonemptylist preRequisite → preRequisite.

Inductive constraint : Set :=
    | Principal : prin → constraint
    | Count : nat → constraint
    | CountByPrin : prin → nat → constraint.
```

Chapter 5

ODRLO Semantics

5.1 Introduction

In this section, we describe the semantics of ODRL0 language by a translation from agreements to a subset of many-sorted first-order logic formulas with equality. Note that in the listings in this chapter we use [] (double square brackets) notation as a mapping of ODRL0 syntactic elements to their translations as many-sorted first-order logic formulas. We also use \triangleq between a translation and its corresponding formula to mean the translation is "mapped to" the formula.

The semantics will help answer queries of the form "may subject s perform action act to asset a?". If the answer is yes, we say permission is granted. Otherwise permission is denied.

At a high-level, an agreement is translated into a conjunction of formulas of the form $\forall x (prerequisites(x) \rightarrow P(x))$ where P(x) itself is a conjunction of formulas of the form $prerequisites(x) \rightarrow (\neg) Permitted(x, act, a)$, where Permitted(x, act, a) means the subject x is permitted to perform action act on asset a.

5.2 Agreement Translation

The translation of an agreement returns the translation for a policySet with arguments $prin_u$, the agreement's user and a, the asset.

Listing 5.1: Agreement Translation

 $[agreement \ for \ prin_u \ about \ a \ with \ policySet] \triangleq [policySet]^{prin_u,a}$

5.3 Policy Set Translation

The translation for a policySet ($[policySet]^{prin_u,a}$) is defined by cases, one for each clause of the grammar in listing 3.8. Recall that a policySet is either a PrimitivePolicySet, a PrimitiveExclusivePolicySet or a AndPolicySet. Each of these has its own translation function, which will be defined in the next 3 subsections.

5.3.1 PrimitivePolicySet Translation

Translation of a PrimitivePolicySet ($preRequisite \rightarrow policy$) yields a formula that includes a test on whether the subject is in the set of agreements' users, the translation of the policy and the translation of the prerequisite. Basically if the subject in question is a user of the agreement and the policySet prerequisites hold, then the policy holds. Translation of the policy for a PrimitivePolicySet is called a $positive\ translation$. A positive translation is one where the actions described by the policies are permitted.

Listing 5.2: Policy Set Translation: PrimitivePolicySet

```
[preRequisite \rightarrow policy]^{e,prin_u,a} \triangleq \forall x \ (([prin_u]_x \land [preRequisite]_x^{e,getId(p),prin_u,a}) \rightarrow [policy]_x^{positive,e,prin_u,a})
```

Listing 5.3: Positive Policy Translation: Single policy

```
[preRequisite \Rightarrow_{policyId} act]_{x}^{positive,e,prin_{u},a} \triangleq ([preRequisite]_{x}^{e,policyId,prin_{u}}) \Rightarrow Permitted(x, [act], a)
```

If the policy is a AndPolicy, the translation yields a conjunction of positive translations of each policy in turn.

Listing 5.4: Positive Policy Translation: List of policies

```
 \llbracket and[policy_1,...,policy_m] \rrbracket^{positive,e,prin_u,a} \triangleq \llbracket policy_1 \rrbracket^{positive,e,prin_u,a} \wedge ... \wedge \llbracket policy_m \rrbracket^{positive,e,prin_u,a}
```

5.3.2 PrimitiveExclusivePolicySet Translation

Primitive Exclusive Policy Set (pre Requisite \mapsto policy) yields the conjunction of two implications. The first implication, is the same as one found in the translation of Primitive Policy Set. The second implication however restricts access (to make the policy set exclusive) to only those subjects that are in the agreement's user. Translation of the policy in the second implication is called a negative translation. A negative translation is one where the actions described by the policies are not permitted.

Listing 5.5: Policy Set Translation: PrimitiveExclusivePolicySet

```
[preRequisite \mapsto policy]^{e,prin_u,a} \triangleq \forall x \ (([prin_u]_x \land [preRequisite]_x^{e,getId(p),prin_u,a}) \rightarrow [policy]_x^{positive,e,prin_u,a}) \land \forall x \ (\neg [prin_u]_x \rightarrow [policy]_x^{negative,e,a})
```

Listing 5.6: Negative Policy Translation: Single policy

```
\begin{bmatrix}
preRequisite \Rightarrow_{policyId} act \end{bmatrix}_{x}^{negative,e,prin_{u},a} \triangleq \\
([preRequisite]_{x}^{e,policyId,prin_{u}}) \Rightarrow \neg (Permitted(x, [act], a))
```

If the policy is a *AndPolicy*, the translation yields a conjunction of negative translations of each policy in turn.

Listing 5.7: Negative Policy Translation: List of policies

```
[\![and[policy_1,...,policy_m]\!]\!]^{negative,e,a} \triangleq [\![policy_1]\!]^{negative,e,a} \wedge ... \wedge [\![policy_m]\!]^{negative,e,a}
```

5.3.3 AndPolicySet Translation

And Policy Set translates to conjunctions of the corresponding policy set translations.

```
Listing 5.8: Policy Set Translation : AndPolicySet
```

```
 \llbracket and[policySet_1,...,policySet_m] \rrbracket^{e,prin_u,a} \triangleq \llbracket policySet_1 \rrbracket^{e,prin_u,a} \wedge ... \wedge \llbracket policySet_m \rrbracket^{e,prin_u,a}
```

5.4 Principal Translation

Translation for a $prin([prin]_x)$ is a formula that is true if and only if the subject x is in the prin set. A prin is either a single subject or a list of subjects $(\{subject_1, ..., subject_m\}$ so the translation covers both cases. Each of these has its own translation function, which will be defined in the following 2 subsections.

If the prin is a single subject, the translation is a formula that is true if and only if the subject x is the same as the single subject subject.

5.4.1 Single Subject Translation

Listing 5.9: Prin Translation: Single subject

```
[subject]_x \triangleq x = subject
```

5.4.2 List of Subjects Translation

Translation of a list of subjects is the disjunction of the translations for each subject.

Listing 5.10: Prin Translation: List of subjects

 $[\![\{subject_1,...,subject_m\}]\!]_x \ \triangleq \ [\![subject_1]\!]_x \ \lor \ ... \ \lor \ [\![subject_m]\!]_x$

5.5 Prerequisite Translation

Translation for a prerequisite is a formula $[prerequisite]_x^{[id_1,...,id_m],prin,a}$, where the set of ids refer to identifiers for policies that are implied by the prerequisites, prin is the agreement's user(s) (and to which the prerequisites apply), a is the asset and x is a variable of type subject. The translation for a prerequisite is described by translation formulas for each type of prerequisite. A prerequisite is either always true, a Constraint, a ForEachMember, a NotCons, a AndPrqs, a OrPrqs or a XorPrqs. Each of these has its own translation function, which will be defined in the following subsections.

5.5.1 True Prerequisite Translation

The translation for a TruePrq yields a formula that is always true.

Listing 5.11: Prerequisite Translation: Always True Prerequisite

 $[prerequisite :: true] \triangleq True$

5.5.2 Constraint Prerequisite Translation

The translation for a *Constraint* is handled by a specialized constraint translation function (coverage of which starts at 5.18.

Listing 5.12: Prerequisite Translation: Constraint

 $[\![prerequisite :: constraint]\!]_x^{[id_1, \dots, id_m], prin_u} \triangleq [\![constraint]\!]_x^{[id_1, \dots, id_m], prin_u}$

5.5.3 For Each Member Prerequisite Translation

The translation for a ForEachMember is also is handled by a specialized translation function (covered at 5.21.

Listing 5.13: Prerequisite Translation: For Each Member

5.5.4 NotCons Prerequisite Translation

The translation for a *NotCons* yields a formula that is simply the negation of the translation for a constraint.

Listing 5.14: Prerequisite Translation: Not Constraint

```
[\![not\ prerequisite :: constraint]\!]_x^{[id_1,\dots,id_m],prin_u} \triangleq \neg [\![constraint]\!]_x^{[id_1,\dots,id_m],prin_u}
```

5.5.5 AndPrqs Prerequisite Translation

The translation for a AndPrqs yields a formula that is the conjunction of the translation for each preRequisite.

Listing 5.15: Prerequisite Translation: Conjunction

```
 \llbracket and \ [preRequisite_1,...,preRequisite_k] \rrbracket^{[id_1,...,id_m],prin_u} \ \triangleq \ \llbracket preRequisite_1 \rrbracket^{[id_1,...,id_m],prin_u} \ \land \\ ... \ \land \ \llbracket preRequisite_k \rrbracket^{[id_1,...,id_m],prin_u}
```

5.5.6 OrPrqs Prerequisite Translation

The translation for a OrPrqs yields a formula that is the inclusive disjunction of the translation for each preRequisite.

Listing 5.16: Prerequisite Translation: Inclusive Disjunction

5.5.7 XorPrqs Prerequisite Translation

The translation for a XorPrqs yields a formula that is the exclusive disjunction of the translation for each preRequisite.

Listing 5.17: Prerequisite Translation: Exclusive Disjunction

5.6 Constraint Translation

Translation for a constraint is a formula $[constraint]_x^{[id_1,...,id_m],prin_u,a}$, where the set of ids refer to identifiers for policies that are implied by the constraint, $prin_u$ is the agreement's user(s) (and to which the constraint applies), a is the asset and x is a variable of type subject. The translation for a constraint is described by translation formulas for each type of constraint. A constraint is either a Principal, a Count, or a CountByPrin. Each of these has its own translation function, which will be defined in the following subsections.

5.6.1 Principal Constraint Translation

The translation for a *Principal* is handled by a specialized translation function (covered at 6.5.

Listing 5.18: Constraint Translation: Principal

```
[\![constraint::prin]\!]_x^{[subject_1,\ldots,subject_m]} \triangleq [\![prin]\!]_x^{[subject_1,\ldots,subject_m]}
```

5.6.2 Count Constraint Translation

The translation for a *Count* is handled by a specialized translation function (covered at ??.

Listing 5.19: Constraint Translation: Count

```
 [\![ constraint :: count[N] ]\!]_x^{[id_1, \dots, id_m], prin_u} \triangleq [\![ count[N] ]\!]_x^{[id_1, \dots, id_m], prin_u}
```

5.6.3 CountByPrin Constraint Translation

The translation for a CountByPrin is handled by the same specialized translation function as that for Count. The difference is that CountByPrin overrides the subjects in $prin_u$ by a different set of subjects (covered at ??.

Listing 5.20: Constraint Translation: Count by Principal

5.6.4 for Each Member Translation

Listing 5.21: For Each Member Translation: Count by Principal

5.6.5 "Not Constraint" Translation

The translation for "Not Constraint" was listed in listing 5.14 earlier but we repeat it here to go along the Coq version.

Listing 5.22: Not Constraint Translation

```
[\![not\ constraint]\!]_x^{[id_1,\ldots,id_m],prin_u} \triangleq \neg [\![constraint]\!]_x^{[id_1,\ldots,id_m],prin_u}
```

5.7 Count Translation

Translation for Count or CountByPrin is based on whether the translation is on a single pair and multiple pairs of subject/policyIds. Each of these two cases will be described in the following subsections.

5.7.1 Count Translation For Subject/ID Pair

The translation for Count or CountByPrin for a pair of subject and policy identifier is a formula that is true if the number of times the $subject_1$ has invoked a policy with policy identifier id_1 is smaller than N.

Listing 5.23: Count Translation: subject and policyld pair

```
[[count[N]]]_x^{subject_1,id_1} \triangleq getCount(subject_1,id_1) < N
```

5.7.2 Count Translation For Subject/ID Pairs

The translation for Count or CountByPrin for subject and policy identifier pairs is a formula that is true if the total number of times that a subject has invoked a policy with policy identifier id_i is smaller than N.

Listing 5.24: Count Translation: subject and policyId pairs

Chapter 6

ODRL0 Semantics In Coq

6.1 Introduction

The translation functions plus the auxiliary types and infrastructure, implementing the semantics have been encoded in Coq. Translation functions build Coq terms of type Prop. Well-formed propositions (or Props) are assertions one can express about values such as mathematical objects or even programs (e.g. 3 < 8) in Coq. Assertions may be true, false or simply conjectures, however an assertion is only true in general if a proof is provided ([3]).

Whether a permission is granted or denied depends on the agreements in question but also on the facts recorded in the environment. For ODRL0 those facts revolve around the number of times a policy has been used to justify an action (see section 3.2 for more details on odrl0). We encode this information in an *environment* which is a conjunction of equalities of the form count(s, policyId) = n.

The Coq version of the count equality is a new inductive type called *count_equality*. An environment is defined to be a non-empty list of *count_equality* objects (see listing 6.1).

Listing 6.1: Environments and Counts

```
Inductive count_equality : Set :=
    | CountEquality : subject → policyId → nat → count_equality.

Inductive environment : Set :=
    | SingleEnv : count_equality → environment
    | ConsEnv : count_equality → environment → environment.
```

We also define a getCount function (see listing 6.2) that given a pair consisting of a subject and policy id, looks for a corresponding count in the environment. getCount assumes the given environment is consistent, so it returns the first matched count it sees for a (subject, id) pair. If a count for a (subject, id) pair is not found it returns 0.

Listing 6.2: getCount Function

```
Fixpoint getCount
 (e:environment)(s:subject)(id: policyId): nat :=
 match e with
 \mid SingleEnv f \Rightarrow
    match f with
   | CountEquality s1 id1 n1 \Rightarrow
       if (beq_nat s s1)
       then if (beq_nat id id1) then n1 else 0
       else 0
    end
 \mid ConsEnv f rest \Rightarrow
    match f with
   | CountEquality s1 id1 n1 \Rightarrow
       if (beq_nat s s1)
       then if (beq_nat id id1) then n1 else (getCount rest s id)
       else (getCount rest s id)
    end
 end.
```

6.2 Translations

Translation of the top level *agreement* element proceeds by case analysis on the structure of the agreement. However an agreement can only be built one way; by calling the constructor *Agreement*. The translation proceeds by calling the translation function for the corresponding *policySet* namely the parameter to *Agreement* called *ps*.

Listing 6.3: Translation of agreement

```
Definition trans_agreement (e:environment)(ag:agreement) : Prop := match ag with

| Agreement prin_u a ps ⇒ trans_ps e ps prin_u a end.
```

Translation of a policySet (called $trans_ps$ in listing 6.4), takes as input e, the environment, ps, the policy set, $prin_u$, the agreement's user, and a, the asset, and proceeds by case analysis of different policySet constructors and recursing into translation functions for the composing elements. A policySet is either a PrimitivePolicySet, PrimitiveExclusivePolicySet or a AndPolicySet.

Note that to implement the translation for an AndPolicySet a local function $trans_ps_list$ has been defined where for a single policySet, $trans_ps$ is called, and for a list of policySets, the conjunction of $trans_ps$ are returned.

Listing 6.4: Translation of Policy Set

```
Fixpoint trans_ps
 (e:environment)(ps:policySet)(prin_u:prin)(a:asset){struct ps} : Prop :=
let trans_ps_list := (fix trans_ps_list (ps_list:nonemptylist policySet)(prin_u:prin)
    (a:asset){struct ps_list}:=
 match ps_list with
   | Single ps1 ⇒ trans_ps e ps1 prin_u a
   | NewList ps ps_list' ⇒ ((trans_ps e ps prin_u a) /\ (trans_ps_list ps_list' prin_u a
    ))
 end) in
  match ps with
   | PrimitivePolicySet prq p \Rightarrow \forall x, (((trans_prin x prin_u) / )
                           (trans_preRequisite e x prq (getId p) prin_u)) →
                           (trans_policy_positive e x p prin_u a))
   | PrimitiveExclusivePolicySet prq p \Rightarrow \forall x, (((trans_prin x prin_u) / 
                                   (trans_preRequisite e x prq (getId p) prin_u)) →
                                   (trans_policy_positive e x p prin_u a)) /\
                                  ((\mathtt{not}\ (\mathtt{trans\_prin}\ \mathtt{x}\ \mathtt{prin\_u})) \to (\mathtt{trans\_policy\_negative}\ \mathtt{e})
    x p a)))
   | AndPolicySet ps_list ⇒ trans_ps_list ps_list prin_u a
```

Translation of a prin (called $trans_prin$ in listing 6.5) takes as input x, the subject in question, p, the principal or the prin, and proceeds based on whether p is a single subject or a list of subjects. If p is a single subject, s, the $Prop\ x = s$ is returned. Otherwise the disjunction of the translation of the first subject in p(s) and the rest of the subjects is returned.

Listing 6.5: Translation of a Prin

```
\begin{aligned} & \texttt{Fixpoint trans\_prin} \\ & (\texttt{x:subject})(\texttt{p: prin}) \colon \texttt{Prop} := \\ & \texttt{match p with} \\ & | \texttt{Single s} \Rightarrow (\texttt{x=s}) \\ & | \texttt{NewList s rest} \Rightarrow ((\texttt{x=s}) \setminus / \texttt{trans\_prin x rest}) \\ & \texttt{end.} \end{aligned}
```

A positive translation for a policy (called $trans_policy_positive$ in listing 6.6) takes as input e, the environment, x, the subject, p, the policy to translate, $prin_u$, the agreement's user, and a, the asset and proceeds based on whether we have a PrimitivePolicy or a AndPolicy. If the policy is a PrimitivePolicy an implication is returned which indicates x is permitted to do action to a, if the preRequisite holds.

Permitted is a predicate specified as ParameterPermitted : subject > act > asset - > Prop. So Permitted predicate takes a subject, an act (an action) and an asset and builds a term of type Prop.

Note that to implement the translation for an AndPolicy a local function $trans_p_list$ has been defined where for a single policy, $trans_policy_positive$ is returned, and for a list of policys, the conjunction of $trans_policy_positives$ are returned.

Listing 6.6: Translation of a positive policy

A negative translation for a policy (called $trans_policy_negative$ in listing 6.7) takes as input e, the environment, x, the subject, p, the policy to translate, and a the asset and proceeds based on whether we have a PrimitivePolicy or a AndPolicy. If the policy is a PrimitivePolicy an implication is returned which indicates x is forbidden to do action to a regardless of whether preRequisite holds. Note that the notation (\neg) indicates that Permitted may be negated. As the case for the positive translation, to implement the translation for an AndPolicy a local function $trans_p_list$ has been defined where for a single policy, $trans_policy_negative$ is returned, and for a list of policys, the conjunction of $trans_policy_negative$ s are returned.

Listing 6.7: Translation of a negative policy

The translation of a prerequisite (called $trans_preRequisite$ in listing 6.8) takes as input e, the environment, x, the subject, prq, the preRequisite to translate, IDs, the set of identifiers (of policies implied by the prq), $prin_u$, the agreement's user, and proceeds by case analysis on the structure of the prerequisite. A prerequisite is either a TruePrq, a Constraint, a ForEachMember, a NotCons, a AndPrqs, a OrPrqs or a XorPrqs.

In listing 6.8 the translation for TruePrq is the Prop True, the translations for Constraint, ForEachMember and NotCons simply call respective translation functions for corresponding types constraint and forEachMember (namely $trans_constraint$, $trans_forEachMem$ and $trans_notCons$). Note that the translation for AndPrqs, OrPrqs and XorPrqs have not yet been implemented but based on the their many-sorted-logic formulas' specifications (3.12) they will be conjunctions, disjunctions and exclusive disjunctions of translations for each prerequisite.

Listing 6.8: Translation of a PreRequisite

The translation of a constraint (called trans_constraint in listing 6.9) takes as input e the environment, x the subject, const, the constraint to translate, IDs, the set of identifiers (of policies implied by the parent preRequisite) and $prin_u$, the agreement's user and proceeds by case analysis on the structure of the constraint. A constraint is either a Principal, a Count or a CountByPrin.

In listing 6.9 the translation for *Principal* returns the translation function (namely $trans_prin$) for the prin (the prin that accompanies the const constraint). The translation for Count and CountByPrin return the translation function $trans_count$. For Count the prin used is the agreement's user, whereas the prin used is the one passed to CountByPrin namely prin.

Listing 6.9: Translation of a Constraint

```
Fixpoint trans_constraint
  (e:environment)(x:subject)(const:constraint)(IDs:nonemptylist policyId)
  (prin_u:prin){struct const} : Prop :=
  match const with
   | Principal prn ⇒ trans_prin x prn

  | Count n ⇒ trans_count e n IDs prin_u

  | CountByPrin prn n ⇒ trans_count e n IDs prn

end.
```

The translation of a forEachMember (called $trans_forEachMember$ in listing 6.10) takes as input e the environment, x the subject, principals, the set of subjects that override the agreement's user(s), $const_list$ the set of constraints and IDs, the set of identifiers (of policies implied by the parent preRequisite).

To implement the translation for a forEachMember we start by calling an auxiliary function $process_two_lists$ that effectively returns a new list composed of pairs of members of the first list and the second list (the cross-product of the two input lists). In the case of a forEachMember translation, the call is " $process_two_lists$ principals $const_list$ " which returns a list of pairs of subject and constraint namely $prins_and_constraints$. $prins_and_constraints$ is then passed to a locally defined function $trans_forEachMember_-Aux$ where for a single pair of subject and constraint $trans_constraint$ is called and for a list of pairs of subject and constraints, the conjunction of $trans_constraints$ (for the first pair) and $trans_forEachMember_Auxs$ (for the rest of the pairs) are returned.

Listing 6.10: Translation of forEachMember

```
Fixpoint trans_forEachMember
      (e:environment)(x:subject)(principals: nonemptylist subject)(const_list:
   nonemptylist constraint)
      (IDs:nonemptylist policyId){struct const_list} : Prop :=
let trans_forEachMember_Aux
 := (fix trans_forEachMember_Aux
      (prins_and_constraints: nonemptylist (Twos subject constraint))
      (IDs:nonemptylist policyId){struct prins_and_constraints} : Prop :=
    match prins_and_constraints with
     | Single pair1 \Rightarrow trans_constraint e x (right pair1) IDs (Single (left pair1))
     NewList pair1 rest_pairs ⇒
        (trans_constraint e x (right pair1) IDs (Single (left pair1))) /\
        (trans_forEachMember_Aux rest_pairs IDs)
    end) in
    let prins_and_constraints := process_two_lists principals const_list in
    trans_forEachMember_Aux prins_and_constraints IDs.
```

The translation of a NotCons (called $trans_notCons$ in listing 6.11) takes as input e the environment, x the subject, const, the constraint to translate, IDs, the set of identifiers (of policies implied by the parent preRequisite) and $prin_u$, the agreement's user and proceeds to return the negation of $trans_constraint$ (see listing 6.9).

Listing 6.11: Translation of not cons

```
\label{eq:definition} \begin{split} & \text{Definition trans\_notCons} \\ & (\texttt{e:environment})(\texttt{x:subject})(\texttt{const:constraint})(\texttt{IDs:nonemptylist policyId})(\texttt{prin\_u:prin}) : \\ & \text{Prop} := \\ & \quad \text{(trans\_constraint e x const IDs prin\_u)}. \end{split}
```

The translation of a Count or a CountByPrin (called $trans_count$ in listing 6.12) takes as input e the environment, n the total number of times the subjects mentioned in $prin_u$ (last parameter) may invoke the policies identified by IDs (third parameter).

To implement the translation for a *Count* or a *CountByPrin* we start by calling an auxiliary function $process_two_lists$ that effectively returns a new list composed of pairs of members of the first list and the second list (the cross-product of the two input lists). In the case of $trans_count$, the call is " $process_two_lists\ IDs\ prin_u$ " which returns a list of pairs of policyId and subject namely $ids_and_subjects$. $ids_and_subjects$ is then passed to a locally defined function $trans_count_aux$.

 $trans_count_aux$ returns the current count for a single pair of policyId and subject (the call to getCount which looks up the environment e and returns the current count per

each subject and policyId) and for a list of pairs of policyId and subjects, the addition of get_count (for the first pair) and $trans_count_auxs$ (for the rest of the pairs) is returned.

A local variable $running_total$ has the value returned by $trans_count_aux$. Finally the proposition $running_total < n$ is returned as the translation for a Count or a CountByPrin.

Note that the only difference between translations for a Count and a CountByPrin is the additional prn parameter for CountByPrin which allows for getting counts for subjects not necessarily the same as $prin_u$, the agreement's user(s).

Listing 6.12: Translation of count

Queries

7.1 Introduction

We first mentioned queries in chapter 5 on page 13. Ultimately policy statements describing an agreement will be used to enforce those agreements. To enforce policy agreements, access queries are asked from the policy engine and access is granted or denied based on the answer.

By defining formal semantics for ODRL the authors of [6] were able to prove that answering a query on whether access should be granted or not, is decidable and NP-hard for the full ODRL.

In this chapter we will review our encoding of queries in Coq and Coq representations of other definitions used in [6] which we will use to prove decidability results of our own.

7.2 Queries

Queries are tuples of the form (A, s, action, a, e) in [6]. The tuple corresponds to the question of determining whether a set A of agreements imply that a subject s may perform action action on an asset a given the environment e. The Coq representation is listed in listing 7.1. We distinguish single agreement queries from multiple agreement queries by defining two separate types: $single_query$ and $general_query$.

Listing 7.1: Queries

```
Inductive single_query : Set :=
    | SingletonQuery : agreement → subject → act → asset → environment →
        single_query.
Inductive general_query : Set :=
```

```
| \  \, {\tt GeneralQuery:nonemptylist\ agreement} \to {\tt subject} \to {\tt act} \to {\tt asset} \to {\tt environment} \\ \to {\tt general\_query.}
```

7.3 Answering Queries

Answering a query as defined earlier can lead to one of four outcomes: "Query Inconsistent", "Permission Granted", "Permission Denied" and "Permission Unregulated" ([6]). We will use the equivalent short forms: error (7.2), permitted (7.3), denied (7.4) and "not applicable" (7.5) defined in [7].

Listing 7.2: Answerable Queries: Error

Listing 7.3: Answerable Queries: Permit

Listing 7.4: Answerable Queries: Deny

Listing 7.5: Answerable Queries: Not Applicable

Examples

8.1 Introduction

In this chapter we will take a tour of the syntax and semantics we have so far developed by examining some example agreements. In the following we will start by reviewing some of the examples used in [6].

Ultimately the goal of specifying all the syntax and semantics is to declare some interesting theorems about policy expressions and proving them in Coq however we will start with some specific propositions/theorems about these examples to get a feel for how proofs are done in Coq.

8.2 Agreement 2.1

Consider example 2.1 (from [6]) where the *policySet* is a AndPolicySet with p1 and p2 as the individual policySets. Let p1 be defined as $Count[5] \rightarrow print$ and p2 as $and[Alice, Count[2]] \rightarrow print$.

The agreement is that the asset The Report may be printed a total of five times by either Alice or Bob, and twice more by Alice. So if Alice and Bob have used policy p1 to justify their printing of the asset m_{p1} and n_{p1} times, respectively, then either may do so again if $m_{p1} + n_{p1} < 5$. If they have used p2 to justify their printing of the asset m_{p2} and n_{p2} times, respectively, then only Alice may do so again if $m_{p2} + n_{p2} < 2$. Note that since Bob doesn't meet the prerequisite of being Alice, n_{p2} is effectively 0, so we have $m_{p2} < 2$ as the condition for Alice being able to print again (Alice does meet the prerequisite of being Alice).

Listing 8.1: Agreement 2.1 (as used in [6])

```
agreement
for {Alice, Bob}
about TheReport
with and [p1, p2].
```

The Coq version of the agreement 2.1 (listing 8.1) and its sub-parts is listed below. It is best to start with the agreement itself called A2.1 in the listing and compare to the agreement 2.1 listed in 8.1.

Listing 8.2: Agreement 2.1 in Coq

8.3 Agreement 2.5

Consider example 2.5 (from [6]) where the policySet is a PrimitivePolicySet with a Count constraint as prerequisite and a AndPolicy as the policy. The AndPolicy is the conjunction of two PrimitivePolicys. Both policies have prerequisites of type ForEachMember with actions display and print respectively. The prin component for both ForEachMembers is Alice, Bob, whereas the constraint for the first ForEachMember is Count[5] and for the second is Count[2].

Listing 8.3: Agreement 2.5 (as used in [6])

```
agreement for {Alice, Bob} about ebook with Count [10] \rightarrow and [forEachMember[{Alice, Bob}; Count[5]] \Rightarrow_{id1} display, forEachMember[{Alice, Bob}; Count[1]] \Rightarrow_{id2} print].
```

The Coq version of the agreement 2.5 (listing 8.3) and its sub-parts is listed below. See agreement 2.5 listed in 8.3 for comparison.

The agreement is that the asset *ebook* may be displayed up to five times by Alice and Bob each, and printed once by each. However the total number of actions (either *display* or *print*) justified by the two policies by either Alice and Bob is at most 10.

Listing 8.4: Example 2.5

```
Definition tenCount:preRequisite := (Constraint (Count 10)).
Definition fiveCount:constraint := (Count 5).
Definition oneCount:constraint := (Count 1).

Definition prins2_5 := (NewList Alice (Single Bob)).
Definition forEach_display:preRequisite := ForEachMember prins2_5 (Single fiveCount)

Definition forEach_print:preRequisite := ForEachMember prins2_5 (Single oneCount).

Definition primPolicy1:policy := PrimitivePolicy forEach_display id1 Display.
Definition primPolicy2:policy := PrimitivePolicy forEach_print id2 Print.

Definition policySet2_5:policySet :=
PrimitivePolicySet tenCount (AndPolicy (NewList primPolicy1 (Single primPolicy2))).

Definition A2.5 := Agreement prins2_5 ebook policySet2_5.
```

8.4 Agreement 2.6

Consider example 2.6 (from [6]) where the *policySet* is a *PrimitiveExclusivePolicySet* with a *InSequence requirement* as prerequisite. We will cover requirements type constraints in *ODRL1* since in ODRL0 we have elided their use. We will also describe this example in detail when ODRL1 constructs are added.

Listing 8.5: Example 2.6

```
Definition prins2_6 := prins2_5.

Definition aliceCount10:preRequisite := Constraint (CountByPrin (Single Alice) 10).

Definition primPolicy2_6:policy := PrimitivePolicy aliceCount10 id3 Play.

Definition policySet2_6_modified:= PrimitiveExclusivePolicySet TruePrq

primPolicy2_6.
```

Some Simple Theorems

9.1 Introduction

In this chapter we will declare and prove some very simple theorems about the examples from chapter [8]. This simple introduction is only meant to give us a feel for how theorems are stated in Coq and how proofs are constructed using Coq tactics.

Propositions are types in Coq whose type is the sort Prop. Any term t whose type is a proposition is a proof term or, for short, a proof. A Hypothesis is a local declaration h: P where h is an identifier and P is a proposition. An Axiom is similar to a hypothesis except it is declared at the global scope and so it is always available. A Theorem or Lemma is any identifier whose type is a proposition ([3]). Keywords "Hypothesis", "Axiom" and "Theorem" or "Lemma" are used in each case respectively.

To build a proof in Coq the user states the proposition to prove; this is called a goal to be proved or discharge, along with some hypothesis that makes up the local context. The user then uses commands called tactics to manipulate the local context and to decompose the goal into simpler goals. The goal simplification into sub-goals will continue until all the sub-goals are solved.

In listing 9.1 we have declared a theorem called example 1 and the corresponding proposition forall x : nat, x < x + 1.

Note that the notation P:T is also used to declare program P has type T. This duality of notation is due to Curry-Howard isomorphism which relates the two worlds of type theory and structural logic together. Once the Theorem has been declared Coq displays the proposition to be proved under a horizontal line written ——, and displays the context of local facts and hypothesis, if any, above the horizontal line. At this point one can enter proof mode by using Proof. upon which Coq is ready to accept tactics. Entering tactics that can break the stated goal (under the horizontal line) into one or more sub-goals is how one progresses until no goals left at which point Coq responds with "No more subgoals" ([2]).

Listing 9.1: Proof Example

9.2 Theorem One

In listing 9.3 we define a policySet with a constraint such that if Alice has used the policy with id1 to justify her printing a_1 times, she may do so again if $a_1 < 5$. The agreement AgreeCan simply links the asset TheReport with the subject Alice and the policySet previously defined.

We capture the fact that Alice has used the policy with id1 to justify her printing 2 times in an environment called eA1. Recall that environments are defined to be non-empty lists of $count_equality$ objects (see 6.1).

We also declare a hypothesis H with the proposition that results from the translation of the agreement (see definition of $trans_a greement$ in listing 6.3) and the environment. The proposition can be shown in Coq after some clean-up (e.g. replaced 101 by Alice) and using the form $Eval\ compute$:

Listing 9.2: Hypothesis for Theorem One

```
\forall \; \mathtt{x} : \mathtt{subject}, \, \mathtt{x} = \mathtt{Alice} \; / \backslash \; \mathtt{True} \; 	o \; 2 < 5 \; 	o \; \mathtt{Permitted} \; \mathtt{x} \; \mathtt{Print} \; \mathtt{TheReport}.
```

The theorem *One* that we are going to prove is trivial but nonetheless in English it states that Alice is Permitted to Print TheReport. The proof comes after the command 'Proof.' and ends with 'Qed'.

Listing 9.3: Theorem One

```
Definition psA1:policySet :=
PrimitivePolicySet
TruePrq
(PrimitivePolicy (Constraint (Count 5)) id1 Print).

Definition AgreeCan := Agreement (Single Alice) TheReport psA1.

Definition eA1 : environment :=
(SingleEnv (make_count_equality Alice id1 2)).

Hypothesis H: trans_agreement eA1 AgreeCan.

Theorem One: Permitted Alice Print TheReport.

Proof. simpl in H. apply H. split. reflexivity. auto. omega. Qed.
```

9.3 Theorem Two

In listing 9.5 we define an exclusive policy set *policySet* containing a policy *pol* that allows printing. The agreement AgreeA5 includes the exclusive policy set to express that Bob may print LoveAndPeace. However any subject that is not the agreement's user (e.g. Bob) is forbidden from printing LoveAndPeace.

Notice that due to the fact that environments are defined as non-empty lists, we have added a Null count to it (see eA5). We continue to capture the relevant facts from the environment and the agreement through defining a hypothesis (e.g. H). The hypothesis is shown below:

Listing 9.4: Hypothesis for Theorem Two

Theorem $T1_A5$ states the exclusivity of the policy set, namely that any subject that is not Bob is not permitted to print the asset LoveAndPeace. Theorem $T2_A5$ uses $T1_A5$ to prove Alice is not permitted to print the asset.

Listing 9.5: Theorem Two

```
Definition prin_bob := (Single Bob).
Definition pol:policy := PrimitivePolicy TruePrq id3 Print.
Definition pol_set:policySet := PrimitiveExclusivePolicySet TruePrq pol.
Definition AgreeA5 := Agreement prin_bob LoveAndPeace pol_set.
Definition eA5 : environment := (SingleEnv (make_count_equality NullSubject NullId 0))

.

Hypothesis H: trans_agreement eA5 AgreeA5.

Theorem T1_A5: ∀ x, x<>Bob → Permitted x Print LoveAndPeace.
Proof. simpl in H. apply H. Qed.

Theorem T2_A5: Permitted Alice Print LoveAndPeace.
Proof. simpl in H. apply T1_A5. apply not_eq_S. omega. Qed.

End A5.
```

9.4 Theorem Three

In listing 6.1 we defined environments as non-empty lists of $count_equality$ objects which are in turn defined as counts per each subject, policy-id pair. These count formulas represent how many times each policy has been used to justify an action by a subject wrt a policy (specified by the policy id) and semantically it makes sense that they are unique in time. When and if two $count_equality$ objects with the same subject and policy id refer to different counts, we say we have inconsistent count formulas. The listing 9.6 defines the binary predicate inconsistent.

Listing 9.6: Inconsistent Count Formulas

```
 \begin{array}{l} \textbf{Definition inconsistent (f1 f2 : count\_equality) : Prop :=} \\ \textbf{match f1 with (CountEquality s1 id1 n1)} \Rightarrow \\ \textbf{match f2 with (CountEquality s2 id2 n2)} \Rightarrow \\ \textbf{s1} = \textbf{s2} \rightarrow \textbf{id1} = \textbf{id2} \rightarrow \textbf{n1} <> \textbf{n2} \\ \textbf{end} \\ \textbf{end}. \\ \end{array}
```

Next we would like to expand the notion of inconsistency to more than two count formulas. We first define a predicate over a count formula and an environment as in listing 9.7. If the environment is a singleton then we just compare the two count formulas for inconsistency, else we build the disjunction of the inconsistency between the count formula on one hand and the head of the environment and the rest of the environment, respectively.

Listing 9.7: Inconsistent Count Formula And Environment

```
\begin{tabular}{ll} Fixpoint formula_inconsistent_with_env (f: count_equality) & (e: environment): Prop := & match e with & | SingleEnv g \Rightarrow inconsistent f g & | ConsEnv g rest \Rightarrow (inconsistent f g) \ / (formula_inconsistent_with_env f rest) & end. & \\ \end{tabular}
```

Finally we define a new inductive data type that represents *consistent* environments (see 9.8). An environment is consistent if it is a singleton count formula, if it consists of only two consistent count formulas and finally if the environment consists of a consistent environment and the consistent composition of a count formula and the consistent environment (see constructor *consis_more*.

Listing 9.8: Inconsistent Environment

```
\begin{array}{l} \textbf{Inductive} \  \, \textbf{env\_consistent} : \textbf{environment} \to \textbf{Prop} := \\ | \  \, \textbf{consis\_1} : \forall \  \, \textbf{f}, \  \, \textbf{env\_consistent} \  \, (\textbf{SingleEnv f}) \\ | \  \, \textbf{consis\_2} : \forall \  \, \textbf{f} \  \, \textbf{g}, \  \, \tilde{} (\textbf{inconsistent f} \  \, \textbf{g}) \to \  \, \textbf{env\_consistent} \  \, (\textbf{ConsEnv f} \  \, (\textbf{SingleEnv g})) \\ | \  \, \textbf{consis\_more} : \forall \  \, \textbf{f} \  \, \textbf{e}, \\ | \  \, \textbf{env\_consistent} \  \, \textbf{e} \to \  \, \tilde{} (\textbf{formula\_inconsistent\_with\_env f} \  \, \textbf{e}) \to \  \, \textbf{env\_consistent} \  \, (\textbf{ConsEnv f} \  \, \textbf{e}). \\ | \  \, \textbf{ConsEnv f} \  \, \textbf{e}). \end{array}
```

We will now pose several small theorems about consistency of count formulas and environments and provide proofs for them (see listing 9.9).

Listing 9.9: Inconsistent Environment

```
Theorem f1_and_f2_are_inconsistent: inconsistent f1 f2.
unfold inconsistent. simpl. omega. Qed.
Theorem f1_and___env_of_f2_inconsistent: formula_inconsistent_with_env f1 (
   SingleEnv f2).
Proof.
unfold formula_inconsistent_with_env. apply f1_and_f2_are_inconsistent. Qed.
Theorem two_inconsistent_formulas_imply_env_inconsistent:
\forall f g, inconsistent f g \rightarrow env_consistent (ConsEnv f (SingleEnv g)).
Proof.
intros, unfold not, intros H'.
inversion H'. intuition. intuition. Qed.
Theorem e2_is_inconsistent: ~env_consistent e2.
Proof
apply two_inconsistent_formulas_imply_env_inconsistent.
apply f1_and_f2_are_inconsistent. Qed.
Theorem env_consistent_implies_two_consistent_formulas:
\forall (f g: count_equality),
  env_consistent (ConsEnv f (SingleEnv g))\rightarrow inconsistent f g.
Proof.
intros. inversion H. exact H1. intuition. Qed.
Theorem two_consistent_formulas_imply_env_consistent:
 \forall (f g: count_equality),
   \siminconsistent f g \rightarrow env_consistent (ConsEnv f (SingleEnv g)).
Proof.
intros. apply consis_2. exact H. Qed.
Theorem env_inconsistent_implies_two_inconsistent_formulas:
\forall (f g: count_equality),
   \tilde{\ }env_consistent (ConsEnv f (SingleEnv g))
ightarrow inconsistent f g.
Proof.
induction f.
induction g.
```

```
unfold inconsistent.
intros.
subst.
generalize (dec_eq_nat n n0).
intro h; elim h.
intro; subst.
elim H.
apply consis_2.
unfold inconsistent.
intro.
assert (s0=s0); auto.
assert (p0=p0); auto.
specialize HO with (1:=H1) (2:=H2).
elim HO; auto.
auto.
Qed.
Theorem same_subjects_policyids_different_counts_means_inconsistent : \forall (s1 s2:
    subject),
             \forall (id1 id2: policyId),
             \forall (n1 n2: nat),
 (\mathtt{s1}=\mathtt{s2} \ / \backslash \ \mathtt{id1}=\mathtt{id2} \ / \backslash \ \mathtt{n1} <> \mathtt{n2}) \rightarrow
 inconsistent (CountEquality s1 id1 n1) (CountEquality s2 id2 n2).
Proof.
intros. unfold inconsistent. intros. intuition. Qed.
```

Proposed Future Work

10.1 Introduction

By defining formal semantics for ODRL authors of [6] were able to show some important results. First result is that answering the question of whether a set of ODRL statements imply a permission or a denial (or other possibilities as we will see) is decidable and also that its complexity is NP-hard (see Theorem 4.1 from [6] re-printed here below).

Theorem 4.1 The problem of deciding, for a query q = (A, s, act, a, E), whether f_q^+ is E-valid is decidable but NP-hard. Similarly for f_q^- .

The authors of [6] then prove that by removing the construct not[policySet] from ODRL's syntax makes answering the same query decidable and efficient (polynomial time complexity).

We will prove equivalent results as above starting with the decidability result of answering a query in ODRL0 which does not include not[policySet]. The theorem in listing 10.1 states that for all environments, all single agreements, all subjects, all actions and all assets, either permission is granted, permission is denied, permission is unregulated or query is inconsistent.

Listing 10.1: Environments and Counts

```
Theorem queriesAreDecidable: \( \text{ (e:environment)}, \)
\( \text{ (agr: agreement)}, \)
\( \text{ (s:subject)}, \)
\( \text{ (action:act)}, \)
\( \text{ (a:asset)}, \)
\( (permissionGranted e [agr] s action a) \/ (permissionDenied e [agr] s action a) \/ (queryInconsistent e [agr] s action a) \/ (permissionUnregulated e [agr] s action a).
```

We will state and prove the complexity (expecting a polynomial time complexity) of the decision procedure (see [1] for a detailed example of an algorithm complexity proof).

We will then augment ODRL0 with the constructs we omitted from the full ODRL (resulting in what we have earlier called ODRL1 or ODRL2) including the troublesome construct not[policySet] and prove decidability and NP-hardness as stated in theorem 4.1.

10.2 SELinux

We started out by looking at DRELs and specifically ODRL where rights expressions are used to arbitrate access to assets under conditions. Recall that the main construct in ODRL is the agreement which specifies users, asset(s) and policies (as part of policy sets) whereby controls on users' access to the assets are described. This is reminiscent of how access control conditions are expressed in access control policy languages such as eXtensible Access Control Markup Language (XACML) and Security Enhanced Linux (SELinux).

While XACML is a high-level and platform independent access control system SELinux is platform dependent (e.g. Linux based) and low-level. SELinux enhances the Discretionary access control (DAC) that most unix based systems employ by Mandatory access control (MAC) where designed access control policies are applied throughout the system possibly overriding whatever DAC is in place by the system users.

SELinux uses Linux's extended file attributes to attach a security context to passive entities (e.g. files, directories, sockets) and also to each active entity typically a Linux user space process. Security context is a data structure that is composed of a user, a role and a domain (or type). While users can map directly to ordinary user names they can also be defined separately. Roles are meant to group users and add flexibility when assigning permissions and are the basis for role-based access control (RBAC). Finally domains or types are the basis for defining common access control requirements for both passive and active entities.

The enforcement of SELinux policies are performed by the security server. Whenever a security operation is requested from user space by a system call, the security server is invoked to arbitrate the operation and either allow the operation or to deny it. Each operation is identified by two pieces of information: an object class (e.g. file) and a permission (e.g. read, write). When an operation is requested to be performed on an object, the class and the permissions associated with the object along with security contexts of the source (typically the source entity is a process) and the object are passed to the security server. The security server consults the loaded policy (loaded at boot time) and allows and denies the access request [sarna-starosta]

10.3 SELinux Policy Language

The SELinux policy has four different kinds of statements: declarations, rules, constraints and assertions [archer]. Assertions are compile time checks that the *checkpolicy* tool per-

forms at compile time. The other three kinds of statements however are evaluated at run-time.

Declaration statements declare a user, a role and a type.

Listing 10.2: Declarations

```
user u types Ru;
role r types Tr;
type t, attrib_{1}, ..., attrib_{n};
```

Rule statements define access vector rules and type transition rules. Access vector (AV) rules (see 10.3) specify which operations are allowed and whether to audit (log). Any operation not covered by AV rules are denied and all denied operations are logged. The semantics of the AV rule with avkind allow is: processes with type sourcetype are allowed to perform operations in perm on objects with class obj-class and type targettype. auditallow means to allow and audit, dontaudit means to never audit and finally neverallow provides a mechanism to override allow rules. When a process changes security context, the role may change, assuming a role transition rule exists relating the old and the new roles (??).

Listing 10.3: AV rule

```
avkind sourcetype targettype:object-class perm
avkind=allow, auditallow, dontaudit, neverallow
```

Constraints are additional conditions that must hold for an attempted operation to be allowed. Constrains relate all of their arguments (the security contexts) to the server (see 10.4. Whenever a permission is requested on an obj-class, the security server checks that the two security contexts are related by a constrain statement.

Listing 10.4: Constrain rule

```
constrain classes, perms, sourcetype, sourcerole, sourceuser, targettype, targetrole,
    targetuser
```

10.4 Authorization Relation

The authorization elation characterizes the operations allowed by a policy. auth(C, P, T1, R1, U1, T2, R2, U2) holds if the security context defined by the tuple <T1, R1, U1> and <T2, R2, U2> are consistent and an entity with type T1 has permission P for targets with class C and type T2, and the constraint defined by constrain holds. A security context

type <T, R, U> (type, role and user) is consistent with respect to a given policy if (1) either there exists a types such that a role-type relation role(R, types) holds and T is in types, or R is $object_r$ (note that since the notion of a role is irrelevant for files, all files are assigned the object_r role) and T is not a process type, and (2) either there exists roles such that user(U, roles) holds and R is in roles, or R is object_r.

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