Modeling & Control of Human Acuated Systems

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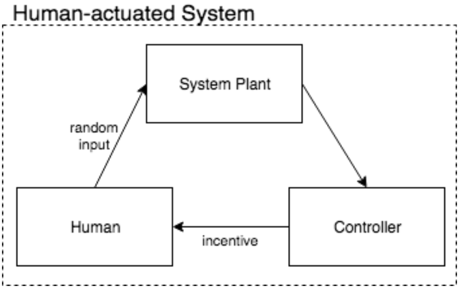
Human Actuated Systems (HAS)?



Dynamical systems where
the system inputs are induced by human behaviours.

In such system...

- we cannot directly command human behavior
- still, their behaviours can be "encouraged" with (price) incentives.

Human Actuated Systems (HAS)?

Motivation and Objective

Motivation:

- Large-scale infrastructure has stochastic human decision-makers in the loop (e.g. smart grids, smart cities)
- We can collect data to better understand human behavior
- Behavioral economics and control theory are disparate disciplines

Objective of this study

Construct a control-theoretic framework to optimize performance of human-actuated systems.

Specifically, we are trying to answer:

1. How to model human behaviours with dynamical systems?
2. How to incentivize human actuators to make desired behaviors for a system-wide benefit?

Overview

1. Literature
2. System Model
3. Inventory Control Problem
4. Conclusion
5. Appendix: Reference Tracking Problem
6. Appendix: formulation of SQP problem

Overview

Existing Literature

In the existing controls literature,

- Human behaviors are addressed as *noises/disturbances*, [1-3]
- Human behaviors are *to improve system performance* [4]
- Finding *optimal behaviors* that mimic or improve upon human behavior [5]

In our previous study,

- Added a new perspective: *Desired human behaviors are encouraged by incentives* [6]
- Convex optimization with convexity constraints.
  - Restricted solution domain, allowing just few alternatives.

In this study:

- Generalized model of human actuated system and control schemes, without restricting domain and number of alternatives.

Overview

System Modeling Framework

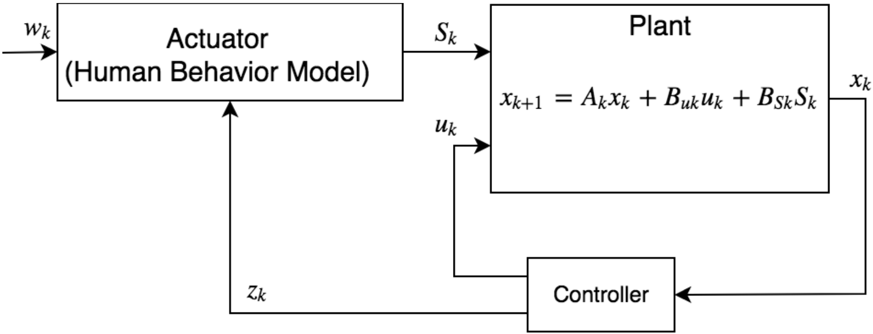
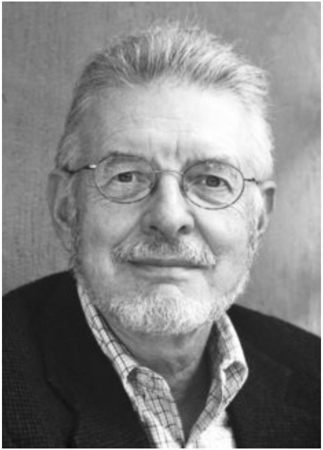


Figure: Block diagram of the Human-actuated system

Discrete Choice Model (DCM)



Professor Daniel Mcfadden

2000 Nobel Prize Winner in Economics

Berkeley, MIT, USC

Focus Area: Discrete Choice Theory

Discrete Choice Model (DCM)

Example of discrete choices:

If these were your only options, which of the following laptops would you choose to purchase?

	Alt1	Alt2	Alt3	
Brand	Microsoft	Apple	Google	
Operating System	Windows 10	OS X	Chrome OS	
Screen Size	13.5”	13”	12”	
Battery Life	12 hours	10 hours	12 hours	
Front Camera	Yes	Yes	No	
Rear Camera	Yes	No	No	
Stylus	Yes	No	No	I would not choose any of these.
Removable Keyboard	Yes	No	No	
Price	$1,499	$1,299	$999	
	○	○	○	○

Attributes

Discrete Choice Model (DCM)

$$U_j \doteq \beta_j^\top z_j + \gamma_j^\top w_j + \beta_{0j} + \epsilon_j, \quad (1)$$

where

U_j : Utility of j-th alternative, $j \in \{1, ..., J\}$

β_j: Parameters of controlled attributes

z_j: Controlled attributes

γ_j: Parameters of uncontrolled attributes

w_j: Uncontrolled attributes

β_{0j}: Alternative specific constant

ϵ_j: Undefined errors

Probability of Alternatives in Discrete Choice Model

Logit model

Assuming that the undefined errors, ϵ_j, have *iid* Extreme Value distribution, the probability of choosing j-th alternative is [7]:

$$\Pr(\text{alternative } j \text{ is selected}) = \Pr\left[\bigcap_{j \neq i} (U_j > U_i)\right] = \frac{e^{V_j}}{\sum_{i=1}^{J} e^{V_i}}, \tag{2}$$

where $V_j \doteq \beta_j^\top z_k + \gamma_j^\top w_k + \beta_{0j}$.

Probability of Alternatives in Discrete Choice Model

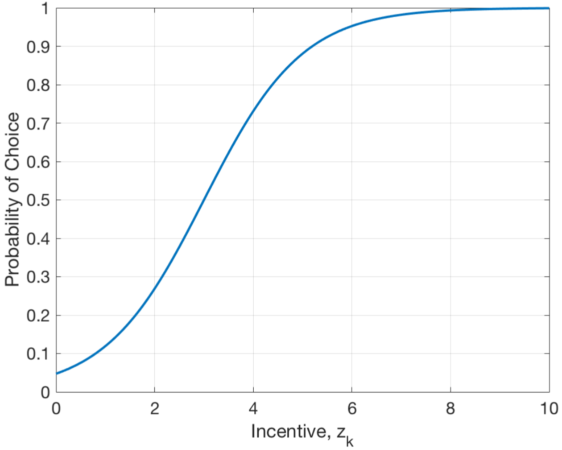


Figure: Binary Logit model example

Mathematical Formulation of Human-actuated System

With a linear system plant,

$$x_{k+1} = A_k x_k + B_{uk} u_k + B_{Sk} S_k, \tag{3}$$

where

- x_k is state
- A_k is the system matrix
- B_{uk} is the direct input matrix
- B_{Sk} is the human input matrix
- u_k is the direct input
- S_k is the random human input to the system.
  - $S_k \in \{0, 1\}^J$
  - $[S_k]_j = 1$ if alternative j is chosen and 0 otherwise
  - $\sum_{j \in \mathcal{J}} [S_k]_j = 1$

Mathematical Formulation of Human-actuated System

Consider mean dynamics,

$$\bar{x}_{k+1} = A_k \bar{x}_k + B_{uk} u_k + B_{Sk} g(z_k, w_k), \tag{4}$$

where

- $\bar{x}_k \doteq \mathbb{E}[x_k]$
- $g(z_k, w_k) \doteq \begin{bmatrix} \mathbb{E}\left[[S_k]_1\right] & \cdots & \mathbb{E}[[S_k]_J] \end{bmatrix}^\top$
- $\mathbb{E}[[S_k]_j] = g_j(z_k, w_k) = \frac{e^{V_{jk}([z_k]_j, [w_k]_j)}}{\sum_{i=1}^{J} e^{V_{ik}([z_k]_i, [w_k]_i)}}, \quad \forall j \in \{1, \ldots, J\}$
  - $\equiv \Pr([S_k]_j = 1)$

Overview

Inventory Control Problem

Objective

Maximizing revenue by giving *price discount*, z_k, and *restocking inventories*, u_k.

Inventory Control Problem

System dynamics,

$$x_{k+1} = x_k + u_k - B_S S_k, \tag{5}$$

where

- x_k is the stock level of the item
- u_k is the number of items ordered from supplier
- $S_k \in \{0, 1\}^J$ is the vector of indicators
- $B_S = [0, 1, ..., J-1]^\top$; number of items purchased by a customer

(Mean dynamics is developed as previously mentioned.)

Formulation of Optimization Problem

$$\text{minimize}_{u_k, z_k} \sum_{k=0}^{N-1} \left[c_u u_k - r B_S g(z_k, w_k) + \sum_{j=1}^{J} [c_z]_j [z_k]_j g_j(z_k, w_k) \right] \tag{6}$$

$$\text{subject to: } \bar{x}_{k+1} = \bar{x}_k + u_k - B_S g(z_k, w_k), \tag{7}$$

$$0 \leq \bar{x}_k \leq \bar{x}_{\max} \tag{8}$$

$$0 \leq u_k \leq u_{\max} \tag{9}$$

$$0 \leq z_k \leq z_{\max}, \tag{10}$$

where

- c_u is the cost-per-unit from the supplier
- c_z is the cost-per-incentive from the store manager
- r is the revenue-per-unit from customers

Solving Optimization with Dynamic Programming (DP)

The Bellman equation is

$$J_k(\bar{x}_k) = \min_{u_k, z_k} \left\{ c_u u_k - r B_S g(z_k, w_k) + \sum_{j=1}^{J} [c_z]_j [z_k]_j g_j(z_k, w_k) + J_{k+1}(\bar{x}_{k+1}) \right\}, \tag{11}$$

where $\bar{x}_{k+1} = \bar{x}_k + u_k - B_S g(z_k, w_k)$ for $k \in \{0, \dots, N-1\}$.

Simulation Overview

Consider:

- 30 customers
- Three alternatives:
  - buy nothing
  - buy one
  - buy two
- customers buy less items without discount
- customers are less sensitive to price discount for two items

Simulation Result

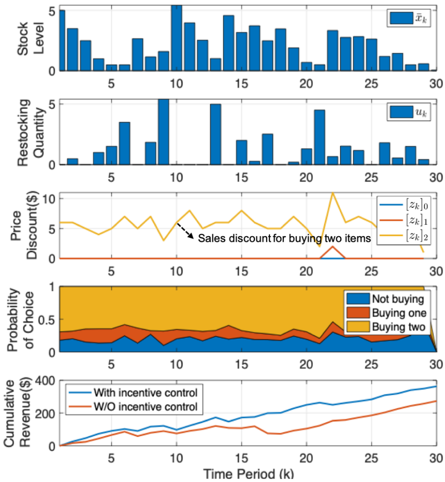


Figure: A simulation result for an inventory control problem.

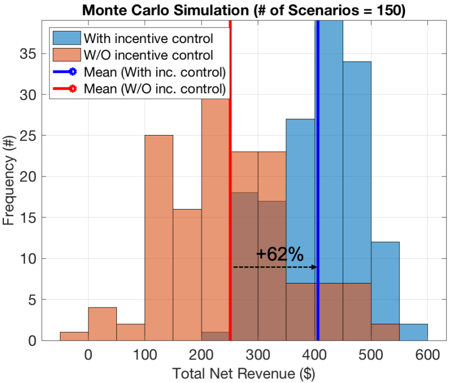


Figure: Monte Carlo simulation results for over 150 randomized scenarios.

Overview

Conclusion

Contributions:

- Proposed mathematical framework to model human-actuated dynamical systems
- Bridged decision-making in behavioral economics with control theory
- Various applications
  - e.g., Energy system operation; demand response program, EV charging station management, etcs.

On-Going work:

- Demand Response incentive control, with Smart-Grid-Smart-City dataset from Australia.
- Controller design with higher order moments, replacing simplified mean dynamics.

Reference

[1] Arnold, Ludwig. Random dynamical systems. Springer Science & Business Media, 2013.
[2] Gray, Robert M., and Lee D. Davisson. An introduction to statistical signal processing. Cambridge University Press, 2004.
[3] Maruyama, Gisiro. "Continuous Markov processes and stochastic equations." Rendiconti del Circolo Matematico di Palermo 4.1 (1955): 48.
[4] Leeper, Adam Eric, et al. "Strategies for human-in-the-loop robotic grasping." Proceedings of the seventh annual ACM/IEEE international conference on Human-Robot Interaction. ACM, 2012.
[5] Mnih, Volodymyr, et al. "Human-level control through deep reinforcement learning." Nature 518.7540 (2015): 529.
[6] Bae, Sangjae, Sang Min Han, and Scott Moura. "System Analysis and Optimization of Human-Actuated Dynamical Systems." 2018 Annual American Control Conference (ACC). IEEE, 2018.
[7] Train, Kenneth E. Discrete choice methods with simulation. Cambridge university press, 2009.

Overview

Objective

Minimizing a deviation of the system plant's expected state trajectory $\bar{x}$ from a reference trajectory x^{ref}.

with

$$f(\bar{x}_k, u_k, z_k, w_k) = A\bar{x}_k + B_u u_k + B_S g(z_k, w_k), \quad k \in \{0, \dots, N-1\}. \tag{12}$$

The optimization problem

$$\min_{\bar{x}_k, u_k, z_k} \quad F = \sum_{k=0}^{N-1} \Big[(\bar{x}_k - x_k^{\mathrm{ref}})^\top Q_k (\bar{x}_k - x_k^{\mathrm{ref}}) + (u_k - u_k^{\mathrm{ref}})^\top R_{uk} (u_k - u_k^{\mathrm{ref}}) + (z_k - z_k^{\mathrm{ref}})^\top R_{zk} (z_k - z_k^{\mathrm{ref}}) \Big] + (\bar{x}_N - x_N^{\mathrm{ref}})^\top Q_N (\bar{x}_N - x_N^{\mathrm{ref}}) \tag{13}$$

subject to: (14)

$$\bar{x}_0 = \bar{x}_{\mathrm{init}}, \tag{15}$$

$$\bar{x}_{k+1} = f(\bar{x}_k, u_k, z_k, w_k), \tag{16}$$

$$z_k \geq 0, \tag{17}$$

Solving with Sequential Quadratic Program (SQP)

Sequential Quadratic Programming (SQP) iteratively finds sub-optimal control polices by solving quadratic approximations to the original problem at every iteration.

(Details of reformulating it to a SQP problem is in Appendix.)

Simulation Overview

Consider a third order system

$$A = \begin{bmatrix} 0.1 & 1 & -1 \\ 1 & 0.1 & 1 \\ 1 & 0 & 0.5 \end{bmatrix}, \tag{18}$$

- the open-loop system is *unstable*, yet *controllable*
- the human's choice $\in \{-5, 0, +5\}$
- the same probability over alternatives without incentive
- the human is equally sensitive to all incentives for choosing each alternative.

Simulation Result

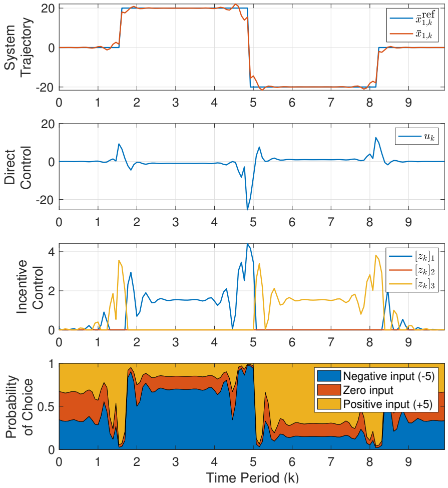


Figure: Simulated reference tracking results solved via SQP.

Findings:

(i) For $k \in [2, 4]$ and $k \in [5.5, 7.5]$, $u_k \approx 0$, yet $([z_k]_1 \neq 0, [z_k]_3 \neq 0)$

$\rightarrow$ *Optimal strategy: is to leverage human actuation z_k instead of direct control u_k.*

(ii) the control effort cost with incentive control (349.1) is lower than that without incentive control (472.13).

$\rightarrow$ *Strong potential for cost savings in practical CPHS control problems*

Linearization

SQP essentially replaces the nonlinear constraint (16) with a linear approximation. Using Taylor's theorem, we can approximate the system as[1]

$$\tilde{x}_{k+1} = \tilde{A}_k \tilde{x}_k + \tilde{B}_{uk} \tilde{u}_k + \tilde{B}_{zk} \tilde{z}_k, \tag{19}$$

where $\tilde{x}_k \doteq \bar{x}_k - \bar{x}_k^{\mathrm{ref}}$, $\tilde{u}_k \doteq u_k - u_k^{\mathrm{ref}}$, $\tilde{z}_k \doteq z_k - z_k^{\mathrm{ref}}$, and

$$\tilde{A}_k \doteq \nabla_{\bar{x}} f_k \left(\bar{x}_k^{\mathrm{ref}}, u_k^{\mathrm{ref}}, z_k^{\mathrm{ref}}, w_k \right)^\top \tag{20}$$

$$\tilde{B}_{uk} \doteq \nabla_{u} f_k \left(\bar{x}_k^{\mathrm{ref}}, u_k^{\mathrm{ref}}, z_k^{\mathrm{ref}}, w_k \right)^\top \tag{21}$$

$$\tilde{B}_{zk} \doteq \nabla_{z} f_k \left(\bar{x}_k^{\mathrm{ref}}, u_k^{\mathrm{ref}}, z_k^{\mathrm{ref}}, w_k \right)^\top . \tag{22}$$

[1] The derivation takes only one vector of controlled incentive $z_k \in \mathbb{R}^J$, but can be easily extended to multiple incentive controls by extending the dimension of z_k, i.e. $z_k \in \mathbb{R}^{J \times s}$, where $s > 1$ is the number of incentive controls. This results in $s - 1$ additional columns to $\tilde{B}_{zk}$, which corresponds to the partial derivatives of $f_k(\bar{x}_k, u_k, z_k, w_k)$ with respect to each additional incentive control.

Linearization (Cont'd)

Note that $\tilde{A}_k = A$ and $\tilde{B}_{uk} = B_u$ since $f_k(\bar{x}_k, u_k, z_k, w_k)$ is linear with respect to $\bar{x}_k$ and u_k. Denote by $\tilde{B}_{zk,m}^\top$ the m-th row of $\tilde{B}_{zk}$ for $m = 1, \ldots, n$. Then,

$$\tilde{B}_{zk,m}^\top = \begin{bmatrix} \frac{\beta_{k1} \exp\{V_{k1}([z_k^{\mathrm{ref}}]_1, [w_k]_1)\}}{\left(\sum_{i=1}^J \exp\{V_{ki}([z_k^{\mathrm{ref}}]_i, [w_k]_i)\}\right)^2} \psi_{k,m}(z_k^{\mathrm{ref}}, w_k, 1) \\ \vdots \\ \frac{\beta_{jk} \exp\{V_{kJ}([z_k^{\mathrm{ref}}]_J, [w_k]_J)\}}{\left(\sum_{i=1}^J \exp\{V_{ki}([z_k^{\mathrm{ref}}]_i, [w_k]_i)\}\right)^2} \psi_{k,m}(z_k^{\mathrm{ref}}, w_k, J) \end{bmatrix}^\top \tag{23}$$

where for $j = 1, \ldots, J$, the function $\psi_{k,m}$ is defined

$$\begin{aligned} &\psi_{k,m}(z_k^{\mathrm{ref}}, w_k, j) \\ &\doteq \sum_{i=1}^{J} \left([B_{zk,m}^\top]_j - [B_{zk,m}^\top]_i \right) \exp\{V_{ik}([z_k^{\mathrm{ref}}]_i, [w_k]_i)\}. \end{aligned} \tag{24}$$

Reformulation with stacked variables

To apply the SQP framework to the aforementioned optimization problem, we first rewrite the optimization problem with respect to stacked variables v and v^{ref}

$$\text{minimize}_v \quad (v - v^{\mathrm{ref}})^\top H (v - v^{\mathrm{ref}}) \tag{25}$$

subject to:

$$\begin{bmatrix} \bar{x}_{\mathrm{init}} - \{v\}_{\bar{x}_0} \\ A\{v\}_{\bar{x}_0} + B_u\{v\}_{u_0} + B_S\{v\}_{z_0} - \{v\}_{\bar{x}_1} \\ \vdots \\ A\{v\}_{\bar{x}_{N-1}} + B_u\{v\}_{u_{N-1}} + B_S\{v\}_{z_{N-1}} - \{v\}_{\bar{x}_N} \end{bmatrix} = 0, \tag{26}$$

$$[\{v\}_{z_0}, \dots, \{v\}_{z_{N-1}}] \geq 0, \tag{27}$$

Reformulation with stacked variables (Cont'd)

where

$$v = [\bar{x}_0, u_0, z_0, ..., \bar{x}_{N-1}, u_{N-1}, z_{N-1}, \bar{x}_N], \tag{28}$$

$$v^{\mathrm{ref}} = [\bar{x}_0^{\mathrm{ref}}, u_0^{\mathrm{ref}}, z_0^{\mathrm{ref}}, ..., \bar{x}_{N-1}^{\mathrm{ref}}, u_{N-1}^{\mathrm{ref}}, z_{N-1}^{\mathrm{ref}}, \bar{x}_N^{\mathrm{ref}}], \tag{29}$$

$$H = \mathrm{diag}\{Q_0, R_{u0}, R_{z0}, ..., Q_{N-1}, R_{u(N-1)}, R_{z(N-1)}, Q_N\}, \tag{30}$$

and $\{v\}_{(*)}$ denotes the elements of the stacked variable v that correspond to $(*)$.

We then take a second-order approximation of the Lagrangian function $\mathcal{L}$ of (25)-(27) with respect to an optimal solution $\hat{v}^{(i)}$ obtained at iteration i. We also linearize the equality constraints (26) with respect to $\hat{v}^{(i)}$

Formulation of SQP optimization problem

We eventually formulate the SQP optimization problem

$$\text{minimize}_p \;\; F^{(i)} + \nabla F^{(i)\top} p + \frac{1}{2} p^\top \nabla^2_{vv} \mathcal{L}^{(i)} p \tag{31}$$

subject to:

$$(\mathrm{diag}\{\begin{bmatrix} A & B_u & \tilde{B}_{zk} \end{bmatrix}\}_{k=0}^{N-1})^\top p + \begin{bmatrix} \bar{x}_{\text{init}} - \{\hat{v}^{(i)}\}_{\bar{x}_0} \\ A\{\hat{v}^{(i)}\}_{\bar{x}_0} + B_u\{\hat{v}^{(i)}\}_{u_0} + B_S\{\hat{v}^{(i)}\}_{z_0} - \{\hat{v}^{(i)}\}_{\bar{x}_1} \\ \vdots \\ \Big(A\{\hat{v}^{(i)}\}_{\bar{x}_{N-1}} + B_u\{\hat{v}^{(i)}\}_{u_{N-1}} \\ + B_S\{\hat{v}^{(i)}\}_{z_{N-1}} - \{\hat{v}^{(i)}\}_{\bar{x}_N}\Big) \end{bmatrix} = 0, \tag{32}$$

$$\nabla c(\hat{v}^{(i)})^\top p + c(\hat{v}^{(i)}) \geq 0, \tag{33}$$

where p is the SQP decision variable, and $\tilde{B}_k$ is defined in (22). Gradient $\nabla c(\hat{v}^{(i)})$ has a value of 1 in the place of elements that correspond to the incentive control z_k.

Update algorithm and convergence

Once we obtain p^* from the aforementioned SQP, the optimal control $\hat{v}^{(i)}$ at iteration i is updated according to

$$\hat{v}^{(i+1)} = \hat{v}^{(i)} + \alpha^{(i)} p^*. \tag{34}$$

Then, the algorithm iterates until a set of convergence criteria are satisfied, e.g. $p^* < \epsilon$ for some $\epsilon > 0$.

SQP inherits the convergence rate of the quasi-Newton method, and convergence can be guaranteed by an appropriate choice of $\alpha^{(i)}$ assuming appropriate regularity conditions hold and unlikely edge cases (e.g. infeasibility of the problem) do not arise.