# Transition to a Green Economy: Policy Competition and Cooperation*

**Job Market Paper**

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**Abstract**

What does a country gain or lose by free-riding off the climate benefits by other countries' carbon abatement efforts versus taking action to hasten a conversion to green energy sources at home? By incorporating industry lifecycle theory and the negative externalities from greenhouse gas emissions into an open-economy macroeconomic model, I analyze the welfare effects of industrial policies that subsidize production of capital goods (like solar panels or wind turbines) used to produce green energy. The model predicts that a production subsidy for the green capital goods industry is desirable for the home country, as it accelerates innovation in the industry and consequently green energy adoption. This acceleration at home delays innovation abroad, generating a beggar-thy-neighbor effect, despite the environmental benefits from home innovation. Thus, in a Nash equilibrium, both nations competitively raise production subsidies, improving welfare in both countries by reducing distortions created by the subsidy and greenhouse gas emissions. A cooperative equilibrium still yields a Pareto improvement, given the incomplete resolution of the free-riding problem in the Nash equilibrium. To quantitatively analyze the welfare and environmental effects of policies implemented by the US and the EU, I estimate the innovation timing elasticity, showing for the first time that the pace of innovation increases with the number of firms operating in an industry. The estimate is sufficiently high to shift the optimal national policy from free-riding to subsidizing green capital goods production in the quantitative analysis.

**Keywords:** industrial policy, green economy, innovation, competition, cooperation

**JEL Codes:** F12, F41, F42, F43, L52

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# 1 Introduction

Reducing carbon dioxide emissions is an urgent necessity, given that carbon dioxide is the main contributor to greenhouse gas emissions and a key factor in global warming and climate change (Ozturk and Acaravci (2010)). Consequently, many countries are promoting green energy-related industries such as solar panels, wind turbines, and electric vehicle batteries. For instance, the United States (US) passed the Inflation Reduction Act (IRA), which provides subsidies to US businesses, households, and sub-national governments for investments leading to reduced greenhouse gas emissions. In response to the IRA, the European Union (EU) unveiled its Green Industrial Plan (GIP), followed by a newly announced subsidy scheme for solar panels, batteries, wind turbines, electrolyzers, and heat pumps.

Given that environmental issues inherently involve free-riding problems and the costs associated with transitioning to production systems using green energy sources have been considered high, the recent competitive implementation of policies supporting a transition to a green economy is noteworthy. Many studies, such as Cline (1992), Nordhaus and Yang (1996), Carraro and Egenhofer (2007), Yang (2008), Nordhaus (2010), and Weitzman (2014), explain that an individual country entirely bears the costs of abating greenhouse gas emissions but the avoidance of climate damage is a worldwide public good, which leads to strong incentives for free-riding. In addition, Shwom et al. (2010) and Nordhaus (2010) show a common belief that environmental policies will be costly and thus have adverse effects on the economy and employment. In this context, this paper aims to examine why many countries are competitively promoting the green industry and whether one country's policy triggers reactions from others. To answer these questions, this paper investigates the welfare effects of recent industrial policies on both the implementing country and its counterparts.

For this analysis, I incorporate industry lifecycle theory into an open economy macroeconomic model from Corsetti et al. (2007), as in Baek (2023), for two reasons. First, industries that are essential for transitioning to a green economy can reasonably be considered to be in the early stages of their lifecycle and to have high-growth potential. This is because renewable energy usage remains low in most countries, despite an evident trend toward increasing green energy consumption.[1] This view is also supported by governments who state their expectations for the high growth potential and growing market size of those industries as one of the major reasons to support them, in addition to environmental urgency. Second, based on industry lifecycle theories such as Abernathy and Utterback (1978) and Klepper (1996), it is important to note that industries in their early stages, although possessing high growth potential, do not experience rapid growth immediately. Rather, their productivity or quality improves gradually at first and then escalates dramatically following radical innovations within the industry. This suggests that the benefits of industrial policies aimed at supporting nascent industries may take time to materialize. Consequently, it is essential to consider the welfare effects that occur during the 'transition' from one steady state to another, as induced by a policy, when evaluating the overall welfare effects of industrial policy.

Taking into account environmental problems and industry lifecycle factors, the model in this paper exhibits distinct features compared to other canonical models such as Lashkaripour and Lugovskyy (2022), Bartelme et al. (2021) and Bai et al. (2023). First, the model in this paper incorporates two externalities. One is the negative externality caused by greenhouse gas emissions from production using conventional energy. The other is knowledge spillover in the targeted industry, which can accelerate the transition from the early stage to the high-growth stage. These two externalities are similar to the 'double externality' concept in Nordhaus (2021). I will show that the knowledge spillover acts as a catalyst helping the world overcome the free-riding problem related to the reduction of greenhouse gas emissions. Further, the objective of industrial policy in this paper differs from those in other studies. In this paper, governments aim to accelerate innovation in the targeted industry to leverage the knowledge spillover and hasten the abatement of damaging emissions. Because the innovation leads to productivity growth, this goal is ultimately linked to economic growth. This stands in contrast to the objective of reallocating resources to maximize gains from economies of scale in a static context, as pursued in industrial policies in other papers. Third, given this different policy objective, the welfare effects of industrial subsidies outlined in other canonical models still

[1] The renewable energy usage rate was 12% in 2020 in the US and 18% in 2020 in the EU according to the Energy Information Administration (EIA) and Eurostat.

apply in this model. However, they are considered either as costs or as supplementary benefits incurred to accelerate innovation in the targeted industry.

With these novel features, this paper contributes to the literature in three ways. First, the model formally explains why both the home and counterpart countries are competitively supporting the targeted industry under these circumstances. Even though each country can benefit from the other's innovation through (i) the reduction of greenhouse gas emissions and (ii) terms of trade gains from accessing cheaper products from the other country, the benefits of accelerating domestic innovation are greater due to the large economies of scale achieved by accelerated productivity growth and a consequent expansion in domestic green energy adoption. Therefore, supporting the green capital industry is desirable for the home country. In contrast, the home production subsidy delays foreign innovation and has a high probability of incurring a beggar-thy-neighbor effect on the foreign country. Thus, both countries are naturally engaged in a competitive game where each country sets its production subsidy to maximize its own welfare, given the other country's subsidy level. In the Nash equilibrium, both countries end up setting production subsidies higher than the optimal rate they would choose if the other country did not react. Yet, ironically, each country's welfare level increases in the Nash equilibrium compared to the level under the optimal subsidy given no countering subsidy abroad, due to efforts to take the lead in green technology. This result contrasts with the conventional prisoner's dilemma that characterizes many environmental conservation issues.

Second, this paper estimates an innovation timing elasticity, which denotes the responsiveness of the timing of innovation to an increase in the number of firms operating in the industry. This elasticity provides a novel interpretation regarding the benefits of industrial policy. Even though this elasticity is closely related to knowledge spillover, learning-by-doing, or economies of scale within an industry, it captures such concepts in a distinct way from existing literature. Many studies, such as Caballero and Lyons (1992), Irwin and Klenow (1994), Bartelme et al. (2021), and Lashkaripour and Lugovskyy (2022), estimate a parameter that determines the degree of within-industry spillover. The parameter in those prior studies is constant, regardless of the industry's stage in its lifecycle. Moreover, the related effects are either realized simultaneously or just one period after resources have been allocated to the industry. In contrast, the elasticity estimated in this paper is specific to the early stage of an industry's lifecycle. The impact of within-industry spillover is not immediate; rather, it serves to shorten the time required for radical innovation in the industry. In this way, even though the policy's role is limited only to accelerating innovation, it can be significant when the targeted industry is in its early stage of the lifecycle.

Third, this paper provides a quantitative analysis of recent industrial policies conducted by the US and the EU. The results are consistent with the model's predictions that the welfare effects of one-sided industrial policies benefit the country implementing the policy but worsen the welfare of the counterpart. Accordingly, both countries set higher production subsidies in the Nash equilibrium. In this equilibrium, the welfare of both countries increases, and global greenhouse emissions decrease further compared to the case where only one country supports the green industry. In addition, the analysis shows that there is still a possibility for Pareto improvement and further reduction in greenhouse gas emissions through cooperative policy.

**Related Literature** This paper contributes to the literature on industrial policy, innovation, and growth (Redding, 1999; Melitz, 2005; Rodrik, 2006; Aghion et al., 2015; Atkeson and Burstein, 2019; Choi and Levchenko, 2021; Lane, 2022; Bai et al., 2023). Among these, Bai et al. (2023) is the closest to this paper in two aspects: it characterizes optimal innovation and trade policy in a 'dynamic setting' and focuses on the environment when 'new technology' emerges. Bartelme et al. (2021) and Lashkaripour and Lugovskyy (2022) are also closely related to this paper in that the welfare effects based on economies of scale, which arise during the reallocation of resources to the targeted industry, operate similarly in the model presented in this paper. However, this paper differs from the existing literature in that it incorporates the additional negative externality arising from greenhouse gas emissions, which is the focus of the environmental literature (Cline, 1992; Nordhaus and Yang, 1996; Stern, 2007; Carraro and Egenhofer, 2007; Garnaut, 2008; Yang, 2008; Nordhaus, 2010), and analyzes the interaction between the innovation-related externality and the environmental externality.

This paper is also related to the literature on the estimation of knowledge spillovers and learning-by-doing within industries (Caballero and Lyons, 1992; Irwin and Klenow, 1994; Lieberman, 1987; Lindström, 2000; Thornton and Thompson, 2001; Lieberman, 1987; Bartelme et al., 2021). Bartelme et al., 2021 is most relevant in this respect as it estimates an elasticity related to external economies of scale (known as a scale elasticity) and quantitatively analyzes welfare gains from optimal industrial policy based on the estimates. This scale elasticity is concerned with an immediate change from one steady state to another in a static setting. In contrast, the innovation timing elasticity estimated in this paper pertains to the 'transition path' to reach a designated new steady state (high-growth stage). Thus, this study focuses on 'how much faster' the transition is completed, offering a novel rationale for industrial policy, as opposed to focusing on 'how much higher' the level of innovation is.

This paper contributes to the literature that studies the stylized patterns of industry dynamics (Vernon, 1966; Abernathy and Utterback, 1978; Gort and Klepper, 1982; Jovanovic and MacDonald, 1994; Klepper, 1996; Antràs, 2005; Eriksson et al., 2021). Baek (2023) provides a general framework which incorporates the theory on how innovation in an industry changes along its life cycle into a canonical open macroeconomic model. This paper is an application of Baek (2023) to the green industrial policy, also extending it by providing a model-based empirical specification to estimate the sensitivity of the timing of innovation to industry lifecycle dynamics.

Lastly, this paper is related to the literature that studies the international transmission of a home country's productivity increase to a foreign country (Ghironi and Melitz, 2005; Matsuyama, 2007; Corsetti et al., 2007). The mechanisms in Ghironi and Melitz (2005) and Corsetti et al. (2007) operate similarly in this model, making home innovation beneficial to the foreign country from a static perspective.

The paper is structured as follows. Section 2 reviews the related literature. Section 3 presents the model setup, and Section 4 analyzes the welfare effects of industrial policy. Section 5 presents the estimation of innovation timing elasticity and provides a quantitative analysis of recent industrial policies by the US and the EU. Section 6 concludes.

# 2 The Model

In the model, the world economy consists of two countries, home and foreign. Each country comprises households, final goods firms, green capital goods firms, and a government. The size of the households is denoted by $L$ in the home country and $L^*$ in the foreign country.

The green capital goods market is crucial in the model because innovation takes place in this market, and the government aims to accelerate this innovation within its jurisdiction. I nest the model of the industry lifecycle within the model of open economy spillovers by Corsetti et al. (2007) to analyze the welfare effects of an industrial policy by taking the industry lifecycle into account.

Throughout the paper, I set the home country wage as the numeraire for convenience.

## 2.1 Households

The utility function of the representative consumer in the home country at time t has the following form which is separable in consumption and pollution as in Keeler et al. (1971), Cabo et al. (2015), and Hassler et al. (2016).

$$U_t = C_t e^{-l_t - \kappa\left(\int_{I_t}^1 Y_{i,t}^e di + \int_{I_t^*}^1 Y_{i,t}^{e*} di\right)} \tag{1}$$

In equation (1), $C_t$ represents the aggregate consumption of the final good at time $t$, and $l_t$ is the labor supply by the representative consumer at time $t$. The exponent $-\kappa\left(\int_{I_t}^1 Y_{i,t}^e di + \int_{I_t^*}^1 Y_{i,t}^{e*} di\right)$ captures the negative externalities arising from the production activities of manufacturing sectors that utilize conventional energy, such as fossil fuels, in both domestic and foreign countries.[2] In this term, $Y_{i,t}^e$ denotes production from the manufacturing sector $i$ using conventional energy, $\kappa$ represents the degree of negative externality from the production, and $I_t$ and $I_t^*$ represent the share of manufacturing sectors that use green energy in the home and foreign countries, respectively. Details on production and the share of final goods producers who adopt green capital will be provided in a later section.

[2] Since environmental issues caused by greenhouse gases are global, I assume that the degree of negative impact from production using conventional energy is the same, whether the production takes place in the home or foreign countries.

The aggregate final good comprises agricultural and manufacturing final goods. The agricultural good, denoted by $C_t^A$, is a final good and indentical across countries of origin. I include the agricultural good in this benchmark model to facilitate tractability for analytical results, but I remove it in numerical simulations later. In contrast, manufactured final goods, denoted by $C_t^M$, consist of final goods from a continuum of sectors. The home good differs from the foreign good even though products in a specific sector from the same origin are identical. Consumers allocate their consumption between these two types of final goods with consumption shares of $\iota$ and $1-\iota$, as shown in the following equation.

$$C_t = \left(C_t^A\right)^{\iota} \left(C_t^M\right)^{1-\iota} \tag{2}$$

The representative consumer allocates a share of expenditure $\beta$ to domestic composite manufacturing final goods and $1-\beta$ to foreign composite manufacturing final goods, as represented by:

$$C_t^M = \left(C_{h,t}^M\right)^{\beta} \left(C_{f,t}^M\right)^{1-\beta} \tag{3}$$

where $C_{h,t}^M$ and $C_{f,t}^M$ denote the domestic and foreign composite manufacturing final goods, respectively.

The domestic composite manufacturing final goods are aggregated through a Cobb-Douglas function, using a continuum of final goods in sector $i$ that range from 0 to 1 and have a mass of one, as follows:

$$C_{h,t}^M = e^{\int_0^1 \ln C_{hi,t}^M di} \tag{4}$$

where $C_{hi,t}^M$ is the consumption of the domestic final goods from sector $i$.

Households supply labor in a competitive market, serving both fixed-cost-related activities and production activities. I will elaborate on this in the next section. The labor supply is determined endogenously within the model.

Households own green capital goods firms located in the home country, which operate under monopolistic competition, in their own country. Each household receives an equal share of the profits from all firms in the green capital goods sector in their country:[3]

$$\Pi_t \equiv \int_0^{n_t} \Pi_t^g(\omega) d\omega \tag{5}$$

where $n_t$ denotes the number of firms in the domestic green capital goods market at time $t$, and $\Pi_t^g(\omega)$ represents the profit of a domestic green capital goods firm producing variety $\omega$.

The representative consumer maximizes its utility (1) in time $t$ subject to the following budget constraint[4]:

$$P_t C_t = w_t l_t + \Pi_t - \frac{T_t}{L} \tag{6}$$

where $T_t$ is a lump-sum tax used to provide subsidies to the green capital goods industry. I will explain the subsidy in detail in a later section.

Similar expressions hold in the foreign country.

## 2.2 Final Goods Production

Final goods markets are perfectly competitive. The agricultural final good is produced with one unit of labor.

$$Y_t^A = L_t^A \tag{7}$$

For manufacturing final goods, firms in all sectors use labor and energy for production. Firms in each sector can choose to use either green or conventional energy. If a firm decides to use green energy, it must employ green capital, denoted by $Z^g$, to generate the energy itself. When a firm opts to use conventional energy, denoted by $E$, it can purchase this energy from the market at a price of $\psi$. It is assumed that productivity from using green capital varies across sectors.[5] Specifically, productivity

[3]Profits from all firms will be zero in equilibrium because firms' productivities are identical in the green capital goods markets, and a zero-profit condition will be imposed.

[4]Since consumers do not save in the model, they maximize their current utility in each period by only considering their current budget constraint.

[5]For example, the iron and steel manufacturing industry is one of the most energy- and carbon-intensive industries. Thus, it is more difficult for such industries to use renewable energy for their production compared to other industries.

from green capital in sector $i$, denoted by $\lambda_i$, decreases as $i$ increases.[6] The production function for final goods in sector $i$ varies depending on the type of capital used, as follows:

$$\begin{aligned} Y^g_{i,t} &= A_{i,t} \left(L^g_{i,t}\right)^{\alpha} \left(\lambda_i Z^g_{i,t}\right)^{1-\alpha} \\ Y^e_{i,t} &= A_{i,t} \left(L^e_{i,t}\right)^{\alpha} \left(E_{i,t}\right)^{1-\alpha} \end{aligned} \tag{8}$$

where $A_{i,t}$ represents the productivity of sector $i$ at time $t$, $L^k_{i,t}$ indicates the amount of labor employed in sector $i$ at time $t$, depending on its energy type. $E_{i,t}$ and $Z^g_{i,t}$ are the amounts of conventional energy and green capital used in sector $i$ at time $t$, respectively. For simplicity, I assume that capital fully depreciates in each period. Firms in each sector choose whether they will use conventional or green energy depending on their costs relative to productivity. The index for the marginal adopting sector, $I_t$, serves as the green energy adoption ratio in the manufacturing sectors. The green energy adoption ratio is determined so that the marginal adopting sector ($i = I_t$) has a productivity for green capital, $\lambda_{I_t}$, satisfying the following equation:

$$\frac{P^g_t}{\lambda_{I_t}} = \psi_t \tag{9}$$

where $P^g_t$ represents the price of aggregated green capital goods in the home country at time $t$ and $\psi_t$ represents the price of conventional energy. It is worth mentioning that this mechanism is similar to that in Helpman and Trajtenberg (1998), which studies the welfare effects of the diffusion of a new general purpose technology. In this regard, the increase in green energy adoption can be interpreted as the diffusion of a general purpose technology.

## 2.3 Green Capital Goods Production

The green capital goods sector is based on the model presented in Corsetti et al. (2007). The green capital market operates under monopolistic competition, where varieties are aggregated using the CES function.

[6] This implies the production cost of using green capital increases as $i$ increases.

$$Z_t^g = \left( \int_0^{n_t} z_t^g(\omega)^{\frac{\gamma-1}{\gamma}} d\omega + \int_0^{n_t^*} z_t^g(\omega_f)^{\frac{\gamma-1}{\gamma}} d\omega_f \right)^{\frac{\gamma}{\gamma-1}} \tag{10}$$

where $\gamma$ represents the elasticity of substitution across varieties in the green capital goods market, and $n_t$ and $n_t^*$[7] indicate the mass of firms in the home and foreign green capital goods market at time $t$, respectively. Here, $\omega$ and $\omega_f$ denote the home and foreign varieties, respectively.

Firms in the green capital goods market only use labor for production, and the production function is as follows:

$$y_t^g(\omega) = a_t^g(\omega) l_t^g(\omega) \tag{11}$$

where $a_t^g(\omega)$ is the productivity of the firm producing variety $\omega$ at time $t$ and $l_t^g(\omega)$ is the labor employed by the firm. I assume that $a_t^g(\omega)$ is the same for all firms in each country but can be different across countries for simplicity.

A firm also needs to hire $\frac{1}{v_t}$ units of labor each period, regardless of its amount of production. Firms allocate this fixed cost to different activities depending on their industry's stage along the lifecycle. As suggested in Abernathy and Utterback (1978) and Klepper (1996), firms in the early stage of the lifecycle need to invest in R&D for innovation. A 'dominant design,' defined as a product design widely accepted by consumers, has not yet emerged at this stage. Firms forgo alternative opportunities and operate in the early industry with low profit for some time to secure future higher profits by inventing a dominant design. In this context, without a certain amount of R&D investment, they are unlikely to innovate, and there is no reason to operate in the industry.[8]

For simplicity, it is assumed that this fixed cost remains constant over time and is the same for all firms in each country. Given wage $w_t$, the fixed cost $q_t^g(\omega)$ is then,

$$q_t^g(\omega) = \frac{w_t}{v_t} = \frac{1}{v} \tag{12}$$

[7] As in Corsetti et al. (2007), I assume that all foreign firms serve the home market, while all domestic firms serve foreign markets.

[8] Empirical regularity, as suggested by Akcigit and Kerr (2018), supports this assumption. It indicates a substantial decline in innovation-intensity, defined as the number of patents per employee, with increasing firm size among innovative firms.

The operating profit, which is revenue minus cost, of the firm producing variety $\omega$ at time $t$ is

$$\Pi_t^g(\omega) = p_t^g(\omega) z_t^g(\omega) + e_t p_t^{g*}(\omega) z_t^{g*}(\omega) - l_t^g(\omega) \tag{13}$$

where $p_t^g(\omega)$ denotes the price for home variety $\omega$ at time t in the home market which is denominated in the home currency, and $p_t^{g*}(\omega)$ denotes the price for home variety $\omega$ at time t in the foreign market which is denominated in the foreign currency. $e_t \equiv \frac{w_t^*}{w_t}$ represents the exchange rate defined as the relative value of the foreign wage in terms of the home wage.

The overall model structure is illustrated in Figure 1.

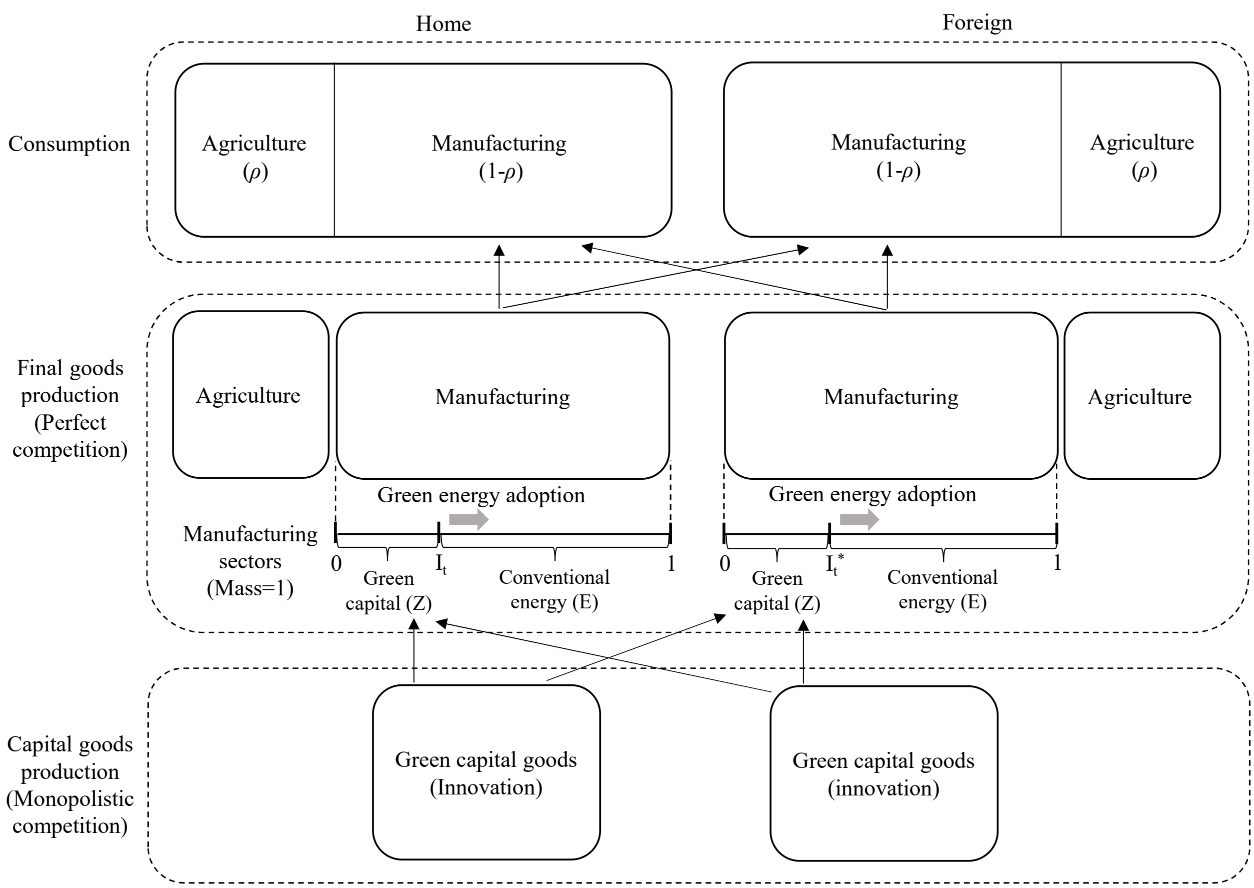


Figure 1: Model Structure

### 2.3.1 Innovation and Productivity

Given that the ratio of global renewable energy usage to total primary energy consumption was approximately 15 percent in 2020 according to the EIA, and considering that many countries and international organizations, such as the US and the EU, are targeting net-zero greenhouse gas emissions—defined as balancing the amount of emitted greenhouse gases with the amount removed from the atmosphere—it is reasonable to expect that the green capital industry is still in the early stages of its lifecycle.

As in Baek (2023), I assume that the productivity of the green capital industry endogenously changes as predicted by the industry lifecycle theory such as Jovanovic and MacDonald (1994) and Abernathy and Utterback (1978): Productivity grows slowly for some time after the industry's inception; then it increases rapidly following radical innovations; and finally, it gradually declines and stabilizes at a low level. This pattern of productivity change is assumed as follows.

**Assumption 1** *The productivity of a firm producing variety $\omega$ remains constant at the current level $a_t^g(\omega)$ until its accumulated knowledge stock, denoted by $k_t(\omega)$, exceeds a certain threshold, $\bar{K}$ at time $t^r(\omega)$. After reaching $t^r(\omega)$, the firm's productivity jumps to $\bar{a}_H$ with probability $p^r$. For simplicity, I assume that $p^r = 1$.*

In this context, a key objective of industrial policy is to expedite the accumulation of knowledge in the targeted industry, thereby hastening its transition to the high-growth stage. The accumulation of a firm's knowledge stock, denoted by $k_t(\omega)$, is assumed to depend not only on the firm's own R&D efforts but also on industry-wide R&D activities (or learning-by-doing). There are three reasons why I assume that knowledge diffusion primarily occurs domestically. First, numerous countries endeavor to restrict technology diffusion in high-growth potential industries. For instance, in recent years, the United States has implemented several restrictions on technology sales to Chinese firms, invoking national security concerns. Central industries affected include telecommunications (Huawei) and semiconductor manufacturing. Second, according to industry lifecycle theories as presented in Vernon (1966) and Klepper (1996), firms are small during the initial stages of the industry lifecycle, and the production of new products largely takes place in the innovator's home country. Consequently, major channels for international knowledge diffusion, such as trade (Eaton and Kortum, 2006; Buera and Oberfield, 2020; Cai et al., 2022)[9], multinational production firms (Lind and Ramondo, 2023), FDI (Javorcik, 2004; Fons-Rosen et al., 2017), or migration (Bahar and Rapoport, 2018) are less likely to be in effect. Third, in a scenario where the degree of domestic knowledge diffusion is stronger than international knowledge diffusion (Branstetter, 2001), results of this paper do not qualitatively change. Accordingly, the following assumption is made.

**Assumption 2** $k_t(\omega) = k\left(\int_0^t q_j^g(\omega)dj, \int_0^t Q_j^g dj\right)$ *where* $\int_0^t q_j^g(\omega)dj$ *represents the cumulative R&D expenditure by firm* $\omega$*, and* $\int_0^t Q_j^g dj$ *represents the cumulative R&D expenditure by all firms in the green capital goods industry.* $k(\cdot)$ *is an increasing function with respect to both arguments.*

Under the assumption that the fixed costs are allocated to R&D investment in the early stage, the cumulative R&D expenditure for both an individual firm and the green capital goods industry can be expressed as follows:

$$\int_0^t q_j^g(\omega)dj = \int_0^t \frac{1}{v}dj, \ \int_0^t Q_j^g dj = \int_0^t \frac{1}{v}n_j dj \tag{14}$$

Furthermore, following Irwin and Klenow (1994), I assume that a firm's knowledge stock function takes the following form:

$$k_t(\omega) = \left(\int_0^t q_j^g(\omega)dj\right)^{\zeta_1}\left(\int_0^t Q_j^g dj\right)^{\zeta_2} \tag{15}$$

Utilizing Assumption 1[10] and equations (14) and (15), the timing of innovation is given by:

$$t^r = t^r(\omega) = v\bar{K}^{\frac{1}{\zeta_1+\zeta_2}} n_{i,t}^{-\frac{\zeta_2}{\zeta_1+\zeta_2}} \tag{16}$$

I define the *innovation timing elasticity* as the elasticity of the timing of innovation with respect to the industry's total cumulative R&D expenditure (or production), denoted by $\epsilon_{t^r}$. The elasticity is

[9]According to the 2012 United States benchmark input-output table (BEA), approximately 78% of domestic usage of products associated with energy-related capital goods originates from domestic sources.

[10]Based on Assumption 1, the industry's timing of innovation aligns with that of an individual firm ($t^r = t^r(\omega)$).

given as follows:

$$\epsilon_{t^r} \equiv \frac{d \ln t^r}{d \ln \int_0^t Q_j dj} = \frac{d \ln t^r}{d \ln n_t} = -\frac{\zeta_2}{\zeta_1 + \zeta_2} \tag{17}$$

It is worth mentioning that the innovation timing elasticity is determined by comparing the relative contributions of the industry's total cumulative R&D expenditure (or production) to that of each firm's own cumulative R&D expenditure (or production) in constructing the firm's knowledge stock. This elasticity is the most important parameter for successful industrial policy in the model. I will estimate this elasticity for quantitative analysis in a later section.

## 2.4 Industrial Policy

As mentioned earlier, this paper focuses on the role of industrial policy in fostering growth in targeted industries. Both domestic and foreign governments offer production subsidies to firms in the green capital industry at rates denoted by $s$ and $s^*$, respectively. These subsidies aim to facilitate either a significant decrease in the price of green capital goods or a notable improvement in their quality through innovation. Such innovation leads to a reduction in the cost of utilizing green energy, encouraging more final goods sectors to adopt production processes that leverage green energy. The subsidy continues either until an innovation occurs in the targeted industry or until the other country discontinues its provision. In this paper, I examine the impact of production subsidies as a representative example of industrial policy.[11]

In addition, I assume the government levies a lump-sum tax from consumers to fund the production subsidy. The total subsidy amount at time $t$, denoted as $S_t$, is given by

$$S_t = \int_0^{n_t} s p_t^g(\omega) y_t^g(\omega) d\omega = s n_t p_t^g(\omega) y_t^g(\omega) \tag{18}$$

[11]Refer to Baek (2023) for analysis with an R&D subsidy using a similar model to this paper.

Thus, the government's budget constraint is as follows:

$$sn_t p_t^g(\omega) y_t^g(\omega) = T_t \tag{19}$$

## 2.5 Equilibrium

For convenience, I establish the following assumptions. These will be employed in Section 3 to derive closed-form analytical results from the model. However, all these assumptions will be relaxed in Section 4 where I move to numerical simulations.

**Assumption 3** *(symmetry)* $v = v^* = L = L^* = A_{i,t} = A^*_{i,t} = 1$ *and* $\psi_t = \psi_t^* = \psi$.

**Assumption 4** *Trading of final goods does not incur any trade costs, while green capital goods are subject to them.*

**Assumption 5** $\iota > (1-\iota)(1-\alpha)I_t$ *and* $\iota > (1-\iota)(1-\alpha)I_t^*$

**Assumption 6** $\beta = \frac{1}{1+\phi}$

**Assumption 7** *(Innovation) Before innovation occurs in a country's green capital market,* $a_t^g = a_t^{g*} = 1$ *and* $I_t = I_t^* = \underline{I}$*. Once innovation takes place,* $a_t^g$ *and* $a_t^{g*}$ *increase to* $\overline{a}_H$ *and* $I_t$ *and* $I_t^*$ *increase to* $\overline{I}$*.*

Assumption 5 implies that the size of the agricultural final goods market is larger than that of the green capital goods market in each country. This assumption, combined with Assumption 4, ensures that the relative wage between the home and foreign countries remains constant at 1 ($w_t^* = 1$, $e_t = 1$).[12].

Assumption 6 implies that the degree of home bias for the final goods is the same as that of the capital goods. This assumption simplifies the analytic solutions in the welfare analysis.

[12] When $\rho > (1-\rho)(1-\alpha)I_t$ and $\rho > (1-\rho)(1-\alpha)I_t^*$ are met, assume that no firm operates in the green capital market in a given country due to a lack of competitiveness, leading the final goods firms in that country to import all their green capital from the other country. Even under this scenario, with $e_t = 1$, a positive amount of agricultural final goods are produced in both countries given the larger size of the agricultural final goods market compared to the green capital market in both nations. Therefore, $e_t$ remains at 1.

Assumption 7 provides a simplified representation of the innovation dynamics discussed in Section 2.3.1. While $I_t$ and $I_t^*$ could be endogenously determined by the productivity structure characterized by $\lambda_i$, even in the short run, employing Assumption 7 significantly simplifies the welfare analysis. It also captures the difficulty associated with altering production methods in the short term.

Similar to Corsetti et al. (2007), the system of equations for equilibrium can be simplified to a system of two zero-profit conditions, (20) and (21), involving two endogenous variables, $n_t$ and $n_t^*$. Solving for these variables enables us to determine all other variables. See Appendix A for the formal derivation.

$$\Pi_t^g(\omega) = \frac{(1+s)(1-\iota)(1-\alpha)}{\gamma}\left[\left(\frac{p_t^g(\omega)}{P_t^g}\right)^{1-\gamma} I_t + \phi\left(\frac{p_t^g(\omega)}{P_t^{g*}}\right)^{1-\gamma} I_t^*\right] = 1 \tag{20}$$

$$\Pi_t^{g*}(\omega_f) = \frac{(1+s^*)(1-\iota)(1-\alpha)}{\gamma}\left[\left(\frac{p_t^{g*}(\omega_f)}{P_t^{g*}}\right)^{1-\gamma} I_t^* + \phi\left(\frac{p_t^{g*}(\omega_f)}{P_t^{g}}\right)^{1-\gamma} I_t\right] = 1 \tag{21}$$

where $\phi \equiv (1+\tau)^{1-\gamma}$ and $\tau$ represents the trade costs, and the prices are as follows.

$$p_t^g(\omega) = \frac{1}{1+s}\frac{\gamma}{\gamma-1}\frac{1}{a_t^g} \tag{22}$$

$$p_t^{g*}(\omega_f) = \frac{1}{1+s^*}\frac{\gamma}{\gamma-1}\frac{1}{a_t^{g*}} \tag{23}$$

$$P_t^g = \left(n_t\left(p_t^g(\omega)\right)^{1-\gamma} + \phi n_t^*(p_t^{g*}(\omega_f))^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{24}$$

$$P_t^{g*} = \left(n_t^*\left(p_t^{g*}(\omega_f)\right)^{1-\gamma} + \phi n_t\left(p_t^{g}(\omega)\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{25}$$

The equilibrium is summarized in Table 1.

# 3 Welfare Analysis

In this section, using the equilibrium obtained in Section 2.5, I analyze the welfare effects of home and foreign production subsidies on their own and the counterpart country's welfare.

Table 1: Summary of Equilibrium

| Home | Foreign |
|---|---|
| $n_t = \frac{(1+s)(1-\iota)(1-\alpha)}{\gamma}\frac{I_t+\phi^2 I_t^* - S^{-\gamma}A^{1-\gamma}\phi(I_t+I_t^*)}{1+\phi^2-(S^{\gamma}A^{\gamma-1}+S^{-\gamma}A^{1-\gamma})\phi}$ | $n_t^* = \frac{(1+s^*)(1-\iota)(1-\alpha)}{\gamma}\frac{I_t^*+\phi^2 I_t - S^{\gamma}A^{\gamma-1}\phi(I_t+I_t^*)}{1+\phi^2-(S^{\gamma}A^{\gamma-1}+S^{-\gamma}A^{1-\gamma})\phi}$ |
| $p_t^g(\omega) = \frac{1}{1+s}\frac{\gamma}{\gamma-1}\frac{1}{a_t^g}$ | $p_t^{g*}(\omega_f) = \frac{1}{1+s^*}\frac{\gamma}{\gamma-1}\frac{1}{a_t^{g*}}$ |
| $P_t^g = \left(n_t\left(p_t^g(\omega)\right)^{1-\gamma} + \phi n_t^*(p_t^{g*}(\omega_f))^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$ | $P_t^{g*} = \left(n_t^*\left(p_t^{g*}(\omega_f)\right)^{1-\gamma} + \phi n_t\left(p_t^g(\omega)\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$ |
| $0 \leq i \leq I_t,\ P_{hi,t}^M = \alpha^{-\alpha}(1-\alpha)^{-\alpha+1}\lambda_i^{\alpha-1}P_t^{g\,1-\alpha}$ | $0 \leq i \leq I_t^*,\ P_{fi,t}^{M*} = \alpha^{-\alpha}(1-\alpha)^{-\alpha+1}\lambda_i^{\alpha-1}P_t^{g*\,1-\alpha}$ |
| $I_t < i,\ P_{hi,t}^M = \alpha^{-\alpha}(1-\alpha)^{-\alpha+1}{\psi_t}^{1-\alpha}$ | $I_t^* < i,\ P_{fi,t}^{M*} = \alpha^{-\alpha}(1-\alpha)^{-\alpha+1}{\psi_t^*}^{1-\alpha}$ |
| $P_{h,t}^M = e^{\int_0^1 \ln P_{hi,t}^M di}$ | $P_{f,t}^{M*} = e^{\int_0^1 \ln P_{fi,t}^{M*} di}$ |
| $P_t^M = \beta^{-\beta}(1-\beta)^{-\beta+1}\left(P_{h,t}^M\right)^{\beta}\left(P_{f,t}^{M*}\right)^{1-\beta}$ | $P_t^{M*} = \beta^{-\beta}(1-\beta)^{-\beta+1}\left(P_{f,t}^{M*}\right)^{\beta}\left(P_{h,t}^M\right)^{1-\beta}$ |
| $P_t = \iota^{-\iota}(1-\iota)^{-\iota+1}\left(P_t^M\right)^{1-\iota}$ | $P_t^* = \iota^{-\iota}(1-\iota)^{-\iota+1}\left(P_t^{M*}\right)^{1-\iota}$ |
| $E_{i,t} = \frac{(1-\alpha)(1-\iota)}{\psi}$ | $E_{i,t}^* = \frac{(1-\alpha)(1-\iota)}{\psi}$ |
| $Z_{i,t}^g = \frac{(1-\alpha)(1-\iota)}{P_t^g}$ | $Z_{i,t}^{g*} = \frac{(1-\alpha)(1-\iota)}{P_t^{g*}}$ |
| $z_t^g(\omega) = \left(\frac{p_t^g}{P_t^g}\right)^{-\gamma}\int_0^{I_t} Z_{i,t}^g di$ | $z_t^{g*}(\omega) = \left(\frac{(1+\tau)p_t^g}{P_t^{g*}}\right)^{-\gamma}\int_0^{I_t^*} Z_{i,t}^{g*} di$ |
| $z_t^g(\omega_f) = \left(\frac{(1+\tau)p_t^{g*}}{P_t^g}\right)^{-\gamma}\int_0^{I_t} Z_{i,t}^g di$ | $z_t^{g*}(\omega_f) = \left(\frac{p_t^{g*}}{P_t^{g*}}\right)^{-\gamma}\int_0^{I_t^*} Z_{i,t}^{g*} di$ |
| $C_t^A = \iota$ | $C_t^{A*} = \iota$ |
| $C_{hi,t}^M = \frac{(1-\iota)\beta}{P_{hi,t}^M},\ C_{hi,t}^{M*} = \frac{(1-\iota)(1-\beta)}{P_{hi,t}^M}$ | $C_{fi,t}^{M*} = \frac{(1-\iota)\beta}{P_{fi,t}^{M*}},\ C_{fi,t}^M = \frac{(1-\iota)(1-\beta)}{P_{fi,t}^{M*}}$ |
| $C_t = P_t^{-1}$ | $C_t = P_t^{*-1}$ |
| $y_t^g(\omega) = z_t^g(\omega) + (1+\tau)z_t^{g*}(\omega)$ | $y_t^{g*}(\omega_f) = z_t^{g*}(\omega_f) + (1+\tau)z_t^g$ |
| $Y_{i,t}^k = C_{hi,t}^M + C_{hi,t}^{M*},\ k \in \{g,e\}$ | $Y_{i,t}^{k*} = C_{fi,t}^M + C_{fi,t}^{M*},\ k \in \{g,e\}$ |
| $Y_t^A = l_t - (1-\iota)\alpha - \gamma n_t$ | $Y_t^{A*} = l_t^* - (1-\iota)\alpha - \gamma n_t^*$ |
| $T_t = \frac{s\gamma n_t}{1+s}$ | $T_t^* = \frac{s^*\gamma n_t^*}{1+s^*}$ |
| $l_t = 1 + T_t$ | $l_t^* = 1 + T_t^*$ |
| $0 \leq t \leq t^r,\ I_t = \underline{I}$ | $0 \leq t \leq t^{r*},\ I_t^* = \underline{I}$ |
| $t^r < t,\ I_t = \overline{I}$ | $t^{r*} < t,\ I_t^* = \overline{I}$ |
| $S \equiv \frac{1+s}{1+s^*},\ A \equiv \frac{a_t^g}{a_t^{g*}}$ | |

The welfare functions for the home and foreign country are defined as follows:

$$\ln W = \int_0^{\bar{T}} e^{-\rho t}\ln U_t dt \tag{26}$$

$$\ln W^* = \int_0^{\bar{T}} e^{-\rho t} \ln U_t^* dt \tag{27}$$

where $\rho$ denotes the discount rate. I define $\bar{T}$ as the last period for the welfare analysis to prevent the welfare measure from approaching infinity, under the assumption of $\rho = 0$, which will be explained next. The time $\bar{T}$ can be interpreted as marking the end of one cycle of dominant technology. With an infinite time horizon, an industry is likely to experience several cycles of changing dominant technologies. I set the discount factor to zero ($\rho = 0$) for two reasons. First, much of the existing literature (Cline, 1992; Stern, 2007; Garnaut, 2008) sets the discount factor at zero for environmental issues, as global environmental changes inevitably affect the welfare of future generations. This literature is based on the idea that all generations should be treated equally. Second, zero discounting raises fewer concerns in the welfare analysis in this paper, given that the time horizon is restricted to one lifecycle of dominant technology. It is also assumed that $s = s^* = 0$ in the initial conditions.

In what follows, I first examine the case where only the home country provides a production subsidy to the green capital industry and assess its welfare effects. Then, I analyze how the welfare effects change when both the home and foreign countries provide production subsidies to maximize their own welfare.

## 3.1 Industrial Policy in the Absence of a Foreign Response

In this section, only the home country offers a production subsidy to the green capital industry ($s \geq 0$, $s^* = 0$). Given the results in the comparative statics in Table 8, the following proposition is derived.

**Proposition 1** *In the initial conditions of $s = s^* = 0$, the home production subsidy accelerates home innovation ($\frac{dt^r(s,s^*)}{ds} < 0$) and delays foreign innovation ($\frac{dt^{r*}(s,s^*)}{ds} > 0$).*

**Proof.** Based on equation (122) and (123), and $\epsilon_{t^r} < 0$, we can prove the following: $\frac{dt^r(s,s^*)}{ds} = \frac{dt^r(s,s^*)}{d\ln n_t}\frac{d\ln n_t}{ds} = \epsilon_{t^r}\frac{d\ln n_t}{ds} < 0$ and $\frac{dt^{r*}(s,s^*)}{ds} = \frac{dt^{r*}(s,s^*)}{d\ln n_t^*}\frac{d\ln n_t^*}{ds} = \epsilon_{t^r}\frac{d\ln n_t^*}{ds} > 0$ ■

This conflict regarding innovation timing serves as the central mechanism triggering policy competition between the home and foreign countries in this model, a point that will be explained in this section. Based on Proposition 1, I introduce three stages as follows:

**Definition 1**

- *Early State ($0 < t < t^r(s, s^*)$): Neither the domestic nor the foreign green capital industries have experienced innovation.*
- *Leading Stage ($t^r(s, s^*) \leq t < t^{r*}(s, s^*)$): Innovation has occurred in the domestic green capital industry but not yet in the foreign industry.*
- *Mature Stage ($t^{r*}(s, s^*) \leq t$): Both the domestic and foreign green capital industries have innovated.*

Following this definition, the welfare functions of the home and foreign countries can be expressed as:

$$\ln W = \int_0^{t^r(s,s^*)} \ln U_E dt + \int_{t^r(s,s^*)}^{t^{r*}(s,s^*)} \ln U_L dt + \int_{t^{r*}(s,s^*)}^{T} \ln U_M dt \tag{28}$$

$$\ln W^* = \int_0^{t^r(s,s^*)} \ln U_E^* dt + \int_{t^r(s,s^*)}^{t^{r*}(s,s^*)} \ln U_L^* dt + \int_{t^{r*}(s,s^*)}^{T} \ln U_M^* dt \tag{29}$$

where $U_S$ and $U_S^*$ denote the the utility level of the home and foreign countries in stage $S$, respectively. Thus, the welfare effect of the home production subsidy can be decomposed into three parts as follows:

$$\frac{d\ln W}{ds} = \underbrace{\int_0^{t^r(s,s^*)} \frac{d\ln U_E}{ds} dt}_{\substack{\text{Short-run resource} \\ \text{reallocation effect}}} \underbrace{-(\ln U_L - \ln U_E)\frac{dt^r(s,s^*)}{ds}}_{\text{Earlier domestic innovation effect}} \underbrace{-(\ln U_M - \ln U_L)\frac{dt^{r*}(s,s^*)}{ds}}_{\text{Delayed foreign innovation effect}} \tag{30}$$

$$\frac{d\ln W^*}{ds} = \underbrace{\int_0^{t_g^r(s,s^*)} \frac{d\ln U_E^*}{ds} dt}_{\substack{\text{Short-run resource} \\ \text{reallocation effect}}} \underbrace{-(\ln U_L^* - \ln U_E^*)\frac{dt_g^r(s,s^*)}{ds}}_{\text{Earlier domestic innovation effect}} \underbrace{-(\ln U_M^* - \ln U_L^*)\frac{dt_g^{r*}(s,s^*)}{ds}}_{\text{Delayed foreign innovation effect}} \tag{31}$$

I will analyze each effect in detail in this section.

**Short-run Resource Reallocation Effect** In the initial condition, the welfare effects of the home subsidy for both the home and foreign countries are derived as follows. See Appendix C.1 for the derivation and determination of the sign of each term.

$$\int_0^{t_g^r(0,0)} \frac{d\ln U_E}{ds} dt = t_g^r(0,0) \left[ \underbrace{-\frac{d\ln P_E}{ds}}_{\text{CPI: (+)}} \underbrace{-(1-\iota)(1-\alpha)I_E}_{\text{Tax: (-)}} \right] = t_g^r(0,0) \frac{(1-\iota)(1-\alpha)I_E}{\gamma-1} > 0 \tag{32}$$

$$\int_0^{t_g^r(0,0)} \frac{d\ln U_E^*}{ds} dt = t_g^r(0,0) \underbrace{\left( -\frac{d\ln P_E^*}{ds} \right)}_{\text{CPI: net zero effect}} = 0 \tag{33}$$

Equation (32) and (33) show that the home production subsidy causes home welfare to increase but has a net zero effect on foreign welfare in the early stage. For the home country, the second term in equation (32), $-(1-\iota)(1-\alpha)I_E$, indicates the loss from collecting tax. To finance the tax, the representative household must supply more labor, resulting in a loss of utility. Despite this cost, the economy initially benefits more from the decrease in the utility-based CPI, which occurs for two reasons: first, the subsidy directly reduces the price of green capital, which in turn lowers the price of final goods produced using green capital; second, it increases the mass of the green capital industry, further reducing the utility-based CPI. This latter mechanism aligns with the channel described in Lashkaripour and Lugovskyy (2022), where an economy benefits from a production subsidy by exploiting economies of scale through an increasing number of firms in the targeted industry.[13]

In contrast, the foreign utility-based CPI does not change due to the tension between terms of trade gain and loss from economies of scale. The foreign country benefits as foreign consumers and final goods firms can purchase cheaper final goods and green capital from the home country. However, as seen in equation (123), the home subsidy leads to a decrease in the number of foreign green capital goods firms, resulting in a loss from economies of scale. Overall, the net effect is zero.

[13] This is well represented by the term, $\frac{(1-\iota)(1-\alpha)I_E}{\gamma-1}$, in the home short-run resource reallocation effect. The component $(1-\iota)(1-\alpha)I_E$ represents the size of the targeted industry, while $\frac{1}{\gamma-1}$ is the scale elasticity, as cited in Lashkaripour and Lugovskyy (2022). Consequently, this term indicates that the positive effect in the early stage amplifies as the product of scale elasticity and sector size increases.

**Earlier Home Innovation Effect** As seen in Proposition 1, the home production subsidy increases the mass of firms in the home green capital industry, thereby hastening innovation in the industry. The welfare effect caused by earlier home innovation is derived as follows. See Appendix C.2 for the derivation and determination of the sign of each term.

$$-(\ln U_L - \ln U_E)\frac{dt^r(s,s^*)}{ds} = t^r(0,0)\left[\underbrace{-\ln P_L + \ln P_E}_{\text{CPI: }(+)} + \underbrace{\kappa(\overline{I} - \underline{I})Y_i^e}_{\text{Less emission:}(+)}\right]\underbrace{\left(-\epsilon_{t^r}\frac{d\ln n_E}{ds}\right)}_{\substack{\text{Earlier home}\\ \text{innovation: }(+)}} > 0 \tag{34}$$

$$-(\ln U_L^* - \ln U_E^*)\frac{dt^r(s,s^*)}{ds} = t^r(0,0)\left[\underbrace{-\ln P_L^* + \ln P_E^*}_{\text{CPI: }(+)} + \underbrace{\kappa(\overline{I} - \underline{I})Y_i^e}_{\text{Less emission:}(+)}\right]\underbrace{\left(-\epsilon_{t^r}\frac{d\ln n_E}{ds}\right)}_{\substack{\text{Earlier home}\\ \text{innovation: }(+)}} > 0 \tag{35}$$

For the home country, gains follow innovation due to a reduction in greenhouse gas emissions. This is attributed to an increased use of green capital in the home country ($\underline{I} \to \overline{I}$), leading to the production of fewer final goods utilizing conventional energy sources. Moreover, the home country experiences a decline in the utility-based CPI, a change driven unambiguously by two primary factors: first, enhanced productivity in green capital production directly reduces the price of home green capital, thereby decreasing the prices of final goods made with green capital; and second, a surge in the number of domestic green capital goods firms—resulting from increased competitiveness against foreign counterparts and an expanding green capital market—enhances economies of scale, further driving down the prices of goods produced using green capital.

The foreign country shares the emission reduction gains observed in the home country, even though its own greenhouse gas emissions remain unchanged. It also benefits from improved terms of trade. A decline in competitiveness leads to a reduction in the number of foreign green capital firms, resulting in losses from economies of scale. However, the advantage of accessing more affordable final goods and green capital from the home country outweighs the negative effects of reduced economies of scale. This outcome regarding international welfare spillover from home productivity improvement aligns with the findings presented in Ghironi and Melitz (2005) and Corsetti et al. (2007). Taking into account these effects, the foreign country also benefits from earlier home innovation.

**Delayed Foreign Innovation Effect** The home production subsidy causes the number of operating firms in the foreign green capital industry to decrease, and this delays innovation in the foreign green capital industry. The welfare effect related to this is represented in the equations below. See Appendix C.3 for for their derivation and determination of the sign of each term.

$$-(\ln U_M - \ln U_L)\frac{dt^{r*}(s,s^*)}{ds} = t^r(0,0)\left[\underbrace{-\ln P_M + \ln P_L}_{\text{CPI: (-)}} + \underbrace{\kappa(\overline{I} - \underline{I})Y_i^e}_{\text{Less emission: (+)}}\right]\underbrace{\left(-\epsilon_{t^r}\frac{d\ln n_E^*}{ds}\right)}_{\substack{\text{Delayed foreign} \\ \text{innovation: (-)}}} < 0 \quad (36)$$

$$-(\ln U_M^* - \ln U_L^*)\frac{dt_g^r(s,s^*)}{ds} = t^r(0,0)\left[\underbrace{-\ln P_M^* + \ln P_L^*}_{\text{CPI: (+)}} + \underbrace{\kappa(\overline{I} - \underline{I})Y_i^e}_{\text{Less emission: (+)}}\right]\underbrace{\left(-\epsilon_{t^r}\frac{d\ln n_E^*}{ds}\right)}_{\substack{\text{Delayed foreign} \\ \text{innovation: (-)}}} < 0 \quad (37)$$

Foreign innovation has two conflicting effects on home welfare. First, the loss of comparative advantage decreases home welfare. When a foreign country succeeds in innovation and catches up to home productivity, the home country loses its comparative advantages in the green capital goods market. Since the home green capital firms already possess high productivity, the negative effects from a loss in global market share and a consequent decrease in economies of scale dominate the terms of trade gains stemming from the foreign innovation. This is notable as it contrasts with the impacts of earlier home innovation on foreign welfare. Furthermore, this effect aligns with the findings in Bai et al. (2023). The model in Bai et al. (2023) suggests imposing higher tariffs on the sectors where the home country has comparative advantages. This aims to discourage innovation efforts in foreign sectors and delay their catch-up. Second, the home country benefits from the reduction of greenhouse gas emissions by the foreign manufacturing sectors. Thus, the overall effect of foreign innovation depends on the relative magnitudes of these two conflicting effects. Finally, the welfare effects from delayed foreign innovation are opposite because the net impacts of the previously explained two effects are delayed.

Foreign innovation unambiguously boosts foreign welfare, as the foreign country benefits not only from reduced greenhouse gas emissions but also from a decline in the utility-based CPI due to enhanced competitiveness of its green capital goods firms. However, since these advantages materialize later, the effect of delayed foreign innovation diminishes foreign welfare.

### 3.1.1 Overall Welfare Effect

By summing the above explained three effects, the overall welfare effect of home production subsidy for the home and foreign country is determined. The following proposition shows that in the initial condition, the home production subsidy unambiguously increases home welfare.

**Proposition 2** *(The necessity of industrial policy) In the initial condition of $s = s^* = 0$, $\frac{d \ln W}{ds} > 0$.*

**Proof.** See Appendix B.1 ■

Intuitively, this result occurs because 1) the gain from its own innovation is greater than the welfare effect of foreign innovation ($\ln U_L - \ln U_E > \ln U_M - \ln U_L$) and 2) the degree to which the home production subsidy accelerates home innovation is greater than that of delaying foreign innovation ($-\frac{d \ln t_r}{ds} > \frac{d \ln t_r^*}{ds}$).

In contrast, a home production subsidy has the potential to decrease foreign welfare, as demonstrated in the following proposition.

**Proposition 3** *(Beggar-thy-neighbor) When $s = s^* = 0$, if the following condition is satisfied, $\frac{d \ln W^*}{ds} < 0$.*

$$\underbrace{(-\ln P_M^* + \ln P_L^*)\epsilon_{t^r} \frac{d \ln n_E^*}{ds}}_{\substack{\textit{The absolute value of loss in CPI} \\ \textit{in the mature stage}}} > \underbrace{(-\ln P_L^* + \ln P_E^*)\left(-\epsilon_{t^r} \frac{d \ln n_E}{ds}\right)}_{\substack{\textit{Gain in CPI} \\ \textit{in the leading stage}}} + \underbrace{-\epsilon_{t^r}\left(\kappa(\overline{I} - \underline{I})Y_i^e\right)}_{\substack{\textit{Net gain} \\ \textit{from less emission}}} \tag{38}$$

**Proof.** See Appendix B.2 ■

As Proposition 3 demonstrates, in the initial condition, the direction of the welfare effect of a home production subsidy on the foreign country is not definite. The left-hand side of the above inequality (38) represents the absolute value of the foreign country's loss experienced through the utility-based CPI by its delayed innovation.

Meanwhile, the terms on the right-hand side illustrate the gains that the foreign country derives from earlier home innovation. As elucidated above, this gain encompasses reductions in the utility-based CPI. Furthermore, even though the home and foreign countries experience an equal degree of gain and loss in terms of greenhouse gas emissions ($\kappa(\overline{I} - \underline{I})Y_i^e$) from earlier home innovation and delayed foreign innovation, the degree to which the home production subsidy accelerates home innovation is greater than that by which it delays foreign innovation ($-\frac{d\ln tr}{ds} > \frac{d\ln t_r^*}{ds}$). Consequently, there exists a net gain from an environmental aspect.

Accordingly, the overall welfare effect of a home production subsidy on foreign welfare is contingent upon the balance of these losses and gains.

Table 2: Summary of Welfare Effects of One-Sided Home Production Subsidy

| Effect | Home | | | Foreign | | |
|---|---|---|---|---|---|---|
| | (+) | (-) | Sum | (+) | (-) | Sum |
| Short-run resource allocation | CPI | Tax | (+) | | | |
| Earlier home innovation | Pollution, CPI | | (+) | Pollution CPI | | (+) |
| Delayed foreign innovation | CPI | Pollution | (+)/(-) | | Pollution, CPI | (-) |
| Overall effect | | | (+) | | | (+)/(-) |

Table 2 summarizes the welfare effects of a home production subsidy with no foreign reaction. It is worth discussing why the benefit derived from its own innovation is substantially larger than the welfare gain from the other country's innovation in this model. First, green capital goods function as a 'general purpose technology'. After innovation, the adoption ratio of green capital goods in the home country rises from $\underline{I}$ to $\overline{I}$. Since energy use is essential for production, an increase in productivity of this general purpose technology, coupled with its expanded usage, significantly impacts the overall manufacturing sectors through the input-output linkage. This mechanism is akin to that described in Helpman and Trajtenberg (1998). Second, 'home bias' plays an important role. As the domestic market size for green capital goods expands ($(1-\iota)(1-\alpha)\underline{I} \to (1-\iota)(1-\alpha)\overline{I}$) due to innovation, domestic firms in the targeted industry benefit exclusively from the growth in the home market, owing to home bias. This dynamic is evident as an increase in $I_t$ results in a surge in the number of domestic green capital goods firms while concurrently reducing the number of foreign green capital goods. Consequently, home bias enhances the gains from economies of scale.

It is also important to mention the importance of taking into account the industrial lifecycle and the innovation timing elasticity when designing industrial policies. The model suggests that the success of the industrial policy hinges crucially on the speed at which the policy fosters innovation in the country, especially when the targeted industry is in its nascent stage. This can be formally demonstrated in the model by the fact that the net growth effect, which is the sum of earlier home innovation and delayed foreign innovation effects, is proportional to the innovation timing elasticity. This also indicates that the greater the innovation timing elasticity, the more aggressively the home country should offer production subsidies, and the more vigorously the foreign country should adopt counterbalancing policies or even more actively support its industry.

In this context, I will introduce a game situation in the next section where both the home and foreign countries provide a production subsidy to maximize their own welfare.

## 3.2 Policy Competition and Cooperation

**Policy Competition** When each country provides a production subsidy to its green capital goods firms, it affects the welfare of the other country by affecting resource allocation and thus innovation timing in the industry. Thus, I will find a Nash equilibrium in this game situation where each country maximizes its welfare by choosing a production subsidy rate, given the other country's production subsidy rate. I will again focus on the symmetric condition, $s = s^* \geq 0$. The following proposition and corollary show the Nash equilibrium of the game.

**Proposition 4** *(Policy competition) Under the condition of symmetric production subsidy ($s = s^*$),* *there exists a unique positive value $\bar{s}$ satisfying $\frac{d\ln W}{ds} = 0$. The value of $\bar{s}$ is given by*

$$\bar{s} = \frac{\frac{1}{t^r(0,0)} \frac{d\ln W}{ds} \mid_{s=s^*=0}}{(1-\iota)(1-\alpha)\underline{I}\frac{d\ln n_E}{ds} \mid_{s=s^*=0}} \tag{39}$$

*where $\frac{d\ln W}{ds} \mid_{s=s^*=0}$ represents the welfare effect under the initial condition $s = s^* = 0$.*

**Proof.** See Appendix B.3 ■

**Corollary 1** *When $s = s^* = \bar{s}$, $\frac{d\ln W}{ds} = \frac{d\ln W^*}{ds^*} = 0$, which means that these subsidy rates satisfy Nash equilibrium.*

Proposition 4 indicates that, up to $\bar{s}$, there is an incentive for each country to increase its production subsidy more than the other country does. This result is interesting in the context that, even though there is an incentive for free-riding in the game—a situation that arises because each country not only benefits equally from reducing greenhouse gas emissions regardless of where the reduction occurs, but also experiences gains from terms of trade based on the other country's innovation without bearing any cost—every country ends up striving to reduce greenhouse gas emissions. This is because the benefits of earlier innovation, securing a larger share of the global market, and exploiting economies of scale[14] derived from an increase in the mass of operating firms outweigh the benefits of free-riding.

**Policy Cooperation** I analyze how welfare effects change when the home and foreign countries cooperatively adjust their production subsidies to the same degree. The following proposition and corollaries demonstrate that the cooperative equilibrium is Pareto-superior to the non-cooperative equilibrium.

**Proposition 5** *(Policy cooperation) When the home and foreign country cooperatively change subsidy rates ($s_c = s = s^*$ and $ds_c = ds = ds^*$), $\frac{d\ln W}{ds_c} = \frac{d\ln W^*}{ds_c^*} > 0$ at $s = s^* = \bar{s}$.*

**Proof.** See Appendix B.4 ■

[14] These benefits also indicate the high risk of falling behind in the essential industry.

**Corollary 2** *The optimal cooperative subsidy rate is greater than the optimal independent subsidy rate ($\bar{s} < \bar{s}_c$).*

**Corollary 3** *The welfare under the cooperative equilibrium is Pareto improved compared to the welfare under the Nash equilibrium.*

When each country sets its production subsidy independently, the marginal cost of the policy incrementally increases as the subsidy rate increases. This is because, when a country aims to make its firms more competitive through higher subsidies, despite these firms having the same productivity as those in the foreign country, additional distortion is created. This distortion is eliminated when the two countries coordinate their production subsidies. Proposition 5 implies that, when the two countries set their production subsidies cooperatively, innovation occurs marginally later compared to a scenario where independent policies are set at the same subsidy rate level. However, both countries can benefit more by eliminating both the additional distortion and the loss caused by delaying the other country's innovation.

Moreover, since the benefit of the cooperative policy outweighs its cost for each country at the subsidy rate in the Nash equilibrium, $\bar{s}$, they are incentivized to extend their support for the green energy industry. Consequently, innovation occurs more swiftly in both countries compared to the Nash equilibrium.

# 4 Quantitative Analysis

In this section, I quantitatively analyze how recent industrial policies supporting the green capital industry affect both home and foreign countries. I use the US and the EU as representatives of home and foreign countries, respectively, given that these regions are actively supporting industries related to the transition to a green economy.

The innovation timing elasticity is the most important parameter in determining the welfare effects of industrial policy in the model. Thus, I first estimate the parameter based on the model. Then, I use this estimate to quantify welfare gains from unilateral versus cooperative action. Finally, I rerun these counterfactuals with several adjustments: I make the relative wage fully endogenous by removing the agricultural good that anchors it, introduce the production of capital goods used for conventional energy, and generalize consumption preferences from Cobb-Douglas to generalized CES.

## 4.1 Estimation of Innovation Timing Elasticity

### 4.1.1 Data

For the estimation of innovation timing elasticity, I require industry-country data for each year on the following variables: the share of the industry's export value relative to the total export value in that industry, the number of operating firms in the industry, and the wage level in the industry. To obtain this data, I use two data sources: UN Comtrade Database and the UNIDO Industrial Statistics Database, which I explain in detail.

**UN Comtrade** I obtain data on the share of export value for an industry in a country relative to the total export value for that industry from the UN Comtrade Database. To maintain consistency in the share over time, I initially fix the set of countries under consideration to prevent fluctuations in the share due to changes in the countries included in the data. Based on value-added data from the UNIDO Industrial Statistics Database, I select 67 countries that account for 98.7% of the total value added in the dataset. The list of selected countries is provided in Appendix F. Accordingly, I only use data on transactions occurring between these countries from 1962 to 2020.

Since both the importing and exporting countries report transactions, theoretically, there are two data entries for a single transaction. Following the approach of Feenstra et al. (2005), I prioritize using the importing partners' reported data whenever possible. If this data is unavailable, I use the export data reported by the exporting country if available.

For industry classification, I employ the SITC classification, as it allows for analysis over the longest time period. To merge this data with the UNIDO Industrial Statistics Database—which utilizes the 2-digit ISIC Rev 3 classification for industries—I match the 4-digit SITC industry codes to the 2-digit ISIC codes using a matching program provided by Liao et al. (2020). Subsequently, using the 2-digit ISIC classification, I calculate the share of export value for an industry in a country relative to the total export value for that industry for all the years under consideration.

**UNIDO Industrial Statistics Database** The UNIDO Industrial Statistics Database provides data from 1963 to 2020 for 174 countries, including annual output, value-added, gross fixed capital formation, employment, wages, and the number of establishments across 23 manufacturing industries. The dataset adheres to the 2-digit ISIC Rev 3 system for industry classification. For the estimation, I utilize the data on the number of establishments and wages. Given that the raw data are reported in U.S. dollars, there is no need for currency conversion, facilitating direct usage of the data.

### Step 1: Estimate Industry-Country Innovation Indicator

I estimate the innovation timing elasticity in three steps. In the first step, I estimate the innovation indicator, which captures both the quality and productivity levels, for an industry in a specific country within a given year. I refer to Khandelwal et al. (2013) for this estimation, who utilize trade data to estimate the quality of products in various industries. Their approach hinges on the intuition that, conditional on price, a product with a higher quantity is perceived to be of higher quality.

I adopt their intuition with some variations. Firstly, I aim to capture not only quality but also productivity improvements. Therefore, rather than controlling for price, I control for production cost, using wages as a proxy. Secondly, I rely on the export share in the global export market, data derived from the UN Comtrade Dataset, instead of quantity. This is because the latter is often missing or inconsistently reported, making it difficult to construct reliable and sufficiently long continuous time series. Thirdly, I control for the number of operating firms to better capture an (average) firm's innovation capacity. Thus, the underlying principle of the methodology in this paper is that a higher share in the export market, conditional on production cost and the number of establishments, indicates higher quality or productivity of an average firm.

Building on the intuition described above, I derive the equation for estimating the innovation indicator. Similar to Khandelwal et al. (2013), assume that products from industry $k$ are globally aggregated in the export market in the following way.

$$Q_{k,t} = \left[ \sum_{i=1}^{N} (\xi_{ik,t} Q_{ik,t})^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}} \tag{40}$$

where $Q_{k,t}$ denotes the composite product of industry $k$ in the global export market at time $t$. $Q_{ik,t}$ denotes the aggregated products produced in industry $k$ in country $i$ at time $t$, and $\xi_{ik,t}$ represents the average quality of products produced in industry $k$ in country $i$ at time $t$. $N$ stands for the number of countries in the global export market. Then, the innovation indicator for industry $k$ in country $i$ can be derived from the share of exports from industry $k$ in country $i$ out of the total global export value in the industry, $s_{ik,t}$, as follows:

$$\begin{aligned} s_{ik,t} \equiv \frac{P_{ik,t} Q_{ik,t}}{P_{k,t} Q_{k,t}} &= \xi_{ik,t}^{\gamma-1} \left( \frac{P_{ik,t}}{P_{k,t}} \right)^{1-\gamma} = \xi_{ik,t}^{\gamma-1} \left( \frac{\frac{\gamma}{\gamma-1} n_{ik,t}^{\frac{1}{1-\gamma}} \frac{w_{ik,t}}{a_{ik,t}}}{P_{k,t}} \right)^{1-\gamma} \\ &= \left( \frac{\gamma}{\gamma-1} \right)^{1-\gamma} \left( \underbrace{\xi_{ik,t} a_{ik,t}}_{\substack{\text{Innovation} \\ \text{indicator}}} \right)^{\gamma-1} n_{ik,t} w_{ik,t}^{1-\gamma} P_{k,t}^{\gamma-1} \end{aligned} \tag{41}$$

where $P_{k,t}$ and $P_{ik,t}$ represent the price for the industry $k$-composite product in the global export market and the price for the aggregated products produced in industry $k$ in country $i$ at time $t$, respectively. Additionally, $n_{ik,t}$, $w_{ik,t}$, and $a_{ik,t}$ denote the number of firms, the wage level, and the productivity level in industry $k$ in country $i$ at time $t$. It is worth mentioning that identifying the innovation indicator is identical to identifying productivity in the model in Section 2. Despite the model not accounting for quality differences, as demonstrated in equation (41), the change in the innovation indicator has the same welfare effect as a change in productivity.

Taking the log of equation (41) yields the following expression:

$$\ln s_{ik,t} + (\gamma - 1) \ln w_{ik,t} - \ln n_{ik,t} = (1 - \gamma) \ln \frac{\gamma}{\gamma - 1} + (\gamma - 1) \ln \xi_{ik,t} a_{ik,t} + (\gamma - 1) \ln P_{k,t} \tag{42}$$

In equation 42, $(\gamma - 1)\ln P_{k,t}$ is replaced with the industry-year fixed effect as it is common to all countries in industry $k$ at time $t$. Additional country-year fixed effects are incorporated to control for other nationwide shocks. Consequently, the innovation indicator can be estimated as the residual from the following OLS regression, assuming a specific value for $\gamma$:

$$\ln s_{ik,t} + (\gamma - 1)\ln w_{ik,t} - \ln n_{ik,t} = \delta_{k,t} + \delta_{i,t} + e_{ik,t} \tag{43}$$

where $\delta_{k,t}$ represents industry-year fixed effects, and $\delta_{i,t}$ represents country-year fixed effects.

Based on Lashkaripour and Lugovskyy (2022), I set $\gamma = 4.71$. This value represents the average of the estimates for the manufacturing sector's elasticity of substitution across countries in Lashkaripour and Lugovskyy (2022). After the OLS regression, the logarithm of the innovation indicator is derived as follows:

$$\ln \widehat{\xi_{ik,t} a_{ik,t}} = \frac{\hat{e}_{ik,t}}{\gamma - 1} \tag{44}$$

In Appendix G, I provide Figure 4, which shows the innovation indicators for industries in the United States, for reference.

**Step 2: Identify Industry-Country Innovation Timing**

With the log difference of the estimated industry-country innovation indicators, denoted as $d \ln \widehat{\xi_{ik,t} a_{ik,t}}$[15], I calculate two separate metrics. The first metric, $LI_{ik,t}$, represents the average growth rate of the indicator over the last 10 years, spanning from $t-10$ to $t-1$, and the second metric, $FI_{ik,t}$, illustrates the average growth rate over the subsequent 10 years, from $t$ to $t+9$. The formulas are as follows:

$$LI_{ik,t} = \frac{\sum_{t-10}^{t-1} d \ln \widehat{\xi_{ik,s} a_{ik,s}}}{10}, \quad FI_{ik,t} = \frac{\sum_{t}^{t+9} d \ln \widehat{\xi_{ik,s} a_{ik,s}}}{10} \tag{45}$$

Then, the innovation timing is identified as the year when the difference in the average growth

[15] This indicates the growth rate of innovation indicators.

rates between the last and the following 10 years, denoted as $FI_{ik,t} - LI_{ik,t}$, is largest for each industry-country time series. Given that the average length of the time series for the industry-country innovation indicator is about 27 years, it is reasonable to assume that there is only one lifecycle within each series.

To ensure robust identification of innovation timing, I have implemented several measures. First, I exclude industry-country observations where the $FI_{ik,t}$ value is negative at the identified innovation timing, as this cannot be considered indicative of innovation. Second, I omit the identified innovation timings for industry-country pairs if their $FI_{ik,t} - LI_{ik,t}$ value at the identified innovation timing[16] falls within the lowest 10 percent of all such values across industry-country pairs. This decision is based on the fact that the cutoff for the lowest 10 percent of the differences in the average growth rate between the last and the following 10 years is about 1.7 percent. Therefore, if the increase in the average growth rate during the 10 years after the identified innovation timing is smaller than this cutoff, such cases are not considered indicative of innovation. For the robustness check, I modify this cutoff and examine how the elasticity of innovation timing changes. As indicated in Table 3, the elasticity does not vary significantly with different cutoff values.

**Step 3: Estimate the Innovation Timing Elasticity**

From the above steps, I obtain the estimated industry-country innovation timing, denoted as $t^r_{ik}$. This is used to estimate the innovation timing elasticity. Building on equation (16) the following equation is derived:

$$\ln t^r_{ik} = \ln v_k + \frac{1}{\zeta_1 + \zeta_2} \ln \bar{K}_{ik} + \frac{\zeta_2}{\zeta_1 + \zeta_2} \ln n_{ik} \tag{46}$$

To apply the above equation in regression analysis, I need data reflecting the knowledge stock threshold for innovation for each industry-country pair, denoted as $\bar{K}_{ik}$. I assume that $\bar{K}_{ik}$ is determined as follows:

[16] As explained, this value is the largest for each industry-country time series.

$$\ln \bar{K}_{ik} = \theta_k + \theta_i + \zeta_3 \ln \xi_{ik,t_0} a_{ik,t_0} \tag{47}$$

where $\lambda_{ik,t_0} a_{ik,t_0}$ represents the initial level of the innovation indicator, and $\theta_k$ and $\theta_i$ represent industry and country fixed effects, respectively.

Applying equation (47) to equation (46), equation (46) can be reformulated as:

$$\begin{aligned} &\ln t_{ik}^r = \ln v_k + \frac{\theta_k}{\zeta_1 + \zeta_2} + \frac{\theta_i}{\zeta_1 + \zeta_2} + \frac{\zeta_3}{\zeta_1 + \zeta_2} \ln \xi_{ik,t_0} a_{ik,t_0} + \frac{\zeta_2}{\zeta_1 + \zeta_2} \ln n_{ik} \\ &\Rightarrow \ln t_{ik}^r = \delta_k + \delta_i + \beta_1 \ln \widehat{\xi_{ik,t_0} a_{ik,t_0}} + \beta_2 \ln n_{ik} + e_{ik} \end{aligned} \tag{48}$$

where $\delta_k$ represents industry fixed effects, and $\delta_i$ represents country fixed effects.

To proceed to the final step of the estimation, I once again compile data for $n_{ik}$ from the UNIDO Industrial Statistics Database. Given my focus on knowledge spillover within industries, I choose to use 'establishment share'—defined as the ratio of the number of establishments in industry $k$ in country $i$ at time $t$ to the total number of establishments in country $i$ at time $t$—rather than using the absolute number of establishments. This choice is based on the possibility that a smaller country with a more concentrated industry could potentially experience greater spillover effects despite having fewer establishments compared to a larger country. To reflect the cumulative knowledge stock and avoid pinpointing the innovation time based on shocks specific to a given year, I calculate a 10-year average of this establishment share, encompassing the period from $t^r - 10$ to $t^r - 1$, to use for $n_{ik}$.

Additionally, I select industries that correspond to the green capital goods sector in the model. For this purpose, I use energy-relevant trade data provided by the International Trade Administration. First, I collect all ten-digit HS codes under categories including renewables, thermal power, and the battery supply chain. By matching these HS codes to 2-digit ISIC Rev 3 codes, I identify the sectors of Chemicals, Fabricated Metals, Machinery and Equipment, Computers and Electronics, Electrical Machinery, Motor Vehicles, and Other Transport Equipment as the green capital goods sectors.

Using this dataset and the estimation equation (48), I obtain the estimation results, which are presented in Table 3.

Table 3: Estimation Results for Innovation Timing Elasticity

| | log of innovation time | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | $\gamma = 4.71$ | $\gamma = 2.5$ | $\gamma = 7.5$ | $\gamma = 10$ | $\gamma = 4.71$ | $\gamma = 4.71$ |
| Cutoff for innovation | lowest 10% | lowest 10% | lowest 10% | lowest 10% | lowest 5% | lowest 25% |
| log of initial innovation indicator | -0.410 | -0.363$^{*}$ | -0.356 | -0.168 | -0.588$^{*}$ | -0.223 |
| | (0.292) | (0.146) | (0.356) | (0.405) | (0.279) | (0.375) |
| log of establishment share | -0.303$^{***}$ | -0.334$^{***}$ | -0.225$^{*}$ | -0.235$^{*}$ | -0.309$^{***}$ | -0.258$^{*}$ |
| | (0.085) | (0.082) | (0.092) | (0.093) | (0.083) | (0.106) |
| Observations | 215 | 211 | 203 | 206 | 226 | 173 |
| $R^2$ | 0.461 | 0.493 | 0.468 | 0.444 | 0.446 | 0.512 |

Notes: Standard errors in parentheses. $^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$. The estimation includes industry fixed effects and country fixed effects. For the results presented in columns (1), (2), (3), and (4), the CES parameter $\gamma$ varies while the cutoff for innovation is set at the lowest 10% of $FI_{ik,t} - LI_{ik,t}$ which is calculated based on Equation (45). For the results in columns (5) and (6), the innovation timing data consist of values above the lowest 5% and 25% of $FI_{ik,t} - LI_{ik,t}$, respectively, while the value of the CES parameter $\gamma$ is fixed at 4.71.

In the baseline case represented in column (1) in Table 3, as mentioned earlier, I use $\gamma = 4.71$ and set the cutoff value for the differences in the average growth rate between the following and the last 10 years, which is considered indicative of innovation, at the lowest 10%. For a robustness check, I alter these values and examine how the estimate of innovation timing elasticity varies. As shown in Table 3, the estimates are significant across all specifications and do not change significantly depending on the values of $\gamma$ and the cutoff for innovation, ranging from -0.23 to -0.33.

Based on equation (16), the innovation timing elasticity reflects the relative contribution of industry knowledge spillover from the industry's overall R&D efforts to the construction of a firm's knowledge stock; however, it does not provide information about the return to scale of R&D expenditure. According to the estimation result in baseline case, it can be interpreted that about 30% of a firm's knowledge stock is constructed from industry knowledge spillovers, making the firm's own R&D expenditure about 2.3 times more important than industry knowledge spillover. This implies that while knowledge spillover plays an important role in the accumulation of a firm's knowledge stock, the firm's own R&D effort is essential for innovation.

The results for the initial innovation indicator, $\hat{\beta}_1$, indicate that this estimate is not statistically significant in many specifications, including the baseline specification. This suggests that while industries with higher initial levels of productivity or quality appear to innovate faster, the time required for industry-wide radical innovation is not significantly influenced by these initial levels of productivity or quality.

## 4.2 Model Generalization

In this section, I generalize the baseline model in Section 2 by aligning it more closely with other canonical models, such as Lashkaripour and Lugovskyy (2022) and Bartelme et al. (2021), and by relaxing all the simplifying assumptions made in Section 2.5.

For this purpose, first, I introduce another capital goods sector that produces capital goods for conventional energy sources as in Hötte (2020). In the baseline model, the economy experiences only benefits from increasing economies of scale in the green capital sector as it transitions to a high-growth stage. By introducing a conventional capital goods sector, the gains from innovation are reduced compared to the baseline model, as the economy also experiences losses from decreasing economies of scale in the conventional capital sector. It is noteworthy that the elasticity of substitution across varieties in both the green and conventional capital goods sectors is set to the same value, a detail I will explain in the calibration section. Consequently, there is no incentive to reallocate resources for maximizing economies of scale, as described in the mechanisms of Lashkaripour and Lugovskyy (2022) and Bartelme et al. (2021). In this context, the policy environment is more conservative in the static aspect compared to those in the referenced papers.

Second, I ensure that the green energy usage ratios, $I_t$ and $I_t^*$, are endogenously determined at all stages. To account for this, I introduce the following productivity structure for green energy adoption:

$$\lambda_i = \bar{\lambda}(1 - i)^{\sigma} \tag{49}$$

where $\bar{\lambda}$ is a constant, and $\sigma$ is the parameter governing the degree of increasing difficulty in green energy adoption as the industry index $i$ increases.

Third, I relax the assumption of constant relative wages by eliminating agricultural final goods. The relative wage is now determined by the ensuing balance-of-payment equilibrium condition.

Lastly, I endogenously determine the consumption shares of domestic and foreign manufacturing goods by introducing the following CES structure.

$$C_t^M = \left( {C_{h,t}^M}^{\frac{\eta-1}{\eta}} + {C_{f,t}^M}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{50}$$

Detailed equations for the generalized model are provided in Appendix H.

## 4.3 Calibration

Using the estimates from the previous section, I set the innovation timing elasticity, $\epsilon_{t^r}$, to -0.30. Regarding the degree of productivity increase after innovation, a projection of a 30% growth rate post-innovation in the green capital industry ($a^g = 1.3$) aligns with the median growth rate observed over 10 years following a radical innovation, based on the estimation results for the innovation indicator.[17] For $\sigma$, the parameter determining the difficulty of green energy adoption, it is reasonable to assume that green energy adoption does not significantly change due solely to production subsidies before innovation, but becomes more flexible after innovation. To reflect this, I set $\sigma = 6.0$ in the early stage and $\sigma = 1.5$[18] in the leading and mature stages.

Apart from the aforementioned parameters, I choose parameter values based on standard values found in the literature. Based on Lashkaripour and Lugovskyy (2022), the elasticity of substitution between varieties in the green and conventional capital sectors is set to $\gamma = 6.40$. This value represents the average elasticity of substitution across varieties for the seven sectors: Chemicals, Fabricated Metals, Machinery and Equipment, Computers and Electronics, Electrical Machinery, Motor Vehicles,

[17]The estimation results for the innovation indicator reveal that the median annual increase in productivity over the subsequent 10 years post-innovation is 5%. This implies that productivity grows by an additional 5% each year for 10 years, compared to the previous 10 years before the innovation. Cumulatively, productivity becomes about 1.6 times higher after 10 years than it was in the identified year of innovation. This aligns an immediate increase in productivity of approximately 30%, which remains constant for 10 years, as assumed in the model.

[18]Together with a 30% productivity growth after innovation, setting $\sigma$ at 1.5 implies that the green energy adoption ratio in the mature stage is 0.36, which is close to the EU's renewable energy usage ration target by 2030, 0.425.

and Other Transport Equipment. For the elasticity of substitution across countries in manufacturing final goods sectors, I set $\eta = 4.71$, also based on Lashkaripour and Lugovskyy (2022). This value is the average elasticity of substitution across countries for manufacturing sectors. Guided by Mickovic and Wouters (2020), I set the energy cost share at $1 - \alpha = 0.25$. Data from the Energy Information Administration (EIA) and Eurostat indicate that, in 2020, the renewable energy share of total energy production was 0.12 in the United States and 0.18 in EU countries. Consequently, I set the current green energy adoption ratio in production at $\underline{I} = 0.15$. Drawing upon findings in Lanzi et al. (2016), I set the initial degree of negative externality from greenhouse gas emissions, reflected in $N \equiv e^{-\kappa(2-2\underline{I})Y_i^e}$, to reduce the representative consumer's utility by 5%.[19] Given the focus on the 2030 targets by the US and the EU[20], I set the initial required time for innovation to be $t^r(0,0) = 10$. The end period for this quantitative analysis is set to $\bar{T} = 20$.

**Industrial Linkage Structure** For counterfactual analysis, information on the industrial linkage structure between the US and the EU is necessary. This includes several key shares: the share of home-produced final goods in total final good consumption; the share of domestic sales in total sales of home-produced final goods; the share of home-produced capital goods in total capital formation; the share of sales to the domestic market in total sales of home-produced capital goods[21]; and the share of export value for each type of capital goods and final goods in total exports.

To get data on these shares, I use OECD Inter-Country Input-Output (ICIO) Tables for the year 2018. Since I focus on the welfare effects between the US and the EU, I only use the input-output linkage between the US and the EU. To obtain shares related to final goods, I use data on final consumption expenditure of households, non-profit institutions serving households, and general government from domestically produced products and products imported from the counterpart country. To obtain shares related to capital goods, I use gross fixed capital formation data from domestically

[19]Since greenhouse gas emission decreases the representative consumer's utility by 5%, and $N \equiv e^{-\kappa(2-2\underline{I})Y_i^e}$ is multiplied to the other terms in the utility function, N is set at 0.95.

[20]The White House outlined a goal for 80% renewable energy generation by 2030, followed by 100% carbon-free electricity five years later. The European Commission established a target of renewable energy to constitute 42.5% of total consumption by 2030.

[21]I assume that for both green and conventional capital goods, the share of home-produced capital goods in total capital formation and the share of sales to the domestic market in total home-produced capital goods are the same.

produced products and products imported from the counterpart country.

Information on parameters and industrial linkage structure data used for the quantitative analysis is summarized in Table 4.

Table 4: Parameters and Economics Linkage Structure

| Parameters | Description | Value | | Source |
|---|---|---|---|---|
| $\epsilon_{t^r}$ | Innovation timing elasticity | -0.30 | | Estimation |
| $a_g$ | Productivity growth by innovation | 1.3 | | Estimation |
| $\sigma$ | Green energy adoption | 6.0 / 1.5 | | Match green energy adoption target |
| $\gamma$ | CES parameter for capital goods | 5.79 | | Lashkaripour and Lugovskyy (2022) |
| $\eta$ | CES parameter for final goods | 4.71 | | Lashkaripour and Lugovskyy (2022) |
| $1-\alpha$ | Energy cost share | 0.25 | | Mickovic and Wouters (2020) |
| $I_0$ | Initial green energy adoption ratio | 0.15 | | EIA, Eurostat |
| $t^r(0,0)$ | Initial innovation timing | 10 | | Governments, EU Commission |
| $N$ | Negative environmental externality | 0.95 | | Lanzi et al. (2016) |
| Share | Description | US | EU | Source |
| $s_{hh}^{fc}$ ($s_{ff}^{fc}$) | Final goods domestic demand | 0.93 | 0.97 | OECD ICIO |
| $s_{hh}^{fp}$ ($s_{ff}^{fp}$) | Final goods domestic sales | 0.96 | 0.94 | OECD ICIO |
| $s_{hh}^{zc}$ ($s_{ff}^{zc}$) | Capital goods domestic demand | 0.92 | 0.97 | OECD ICIO |
| $s_{hh}^{zp}$ ($s_{ff}^{zp}$) | Capital goods domestic sales | 0.97 | 0.93 | OECD ICIO |
| $s_{h}^{fe}$ ($s_{f}^{fe}$) | Final goods export | 0.37 | 0.66 | OECD ICIO |
| $s_{h}^{ge}$ ($s_{f}^{ge}$) | Green capital goods export | 0.02 | 0.05 | OECD ICIO |
| $s_{h}^{ee}$ ($s_{f}^{ee}$) | Conventional capital goods export | 0.11 | 0.29 | OECD ICIO |

Notes: For the parameter governing the difficulty of green energy adoption ($\sigma$), the first value is applied in the early stage and the second value is applied in the leading and mature stages.

## 4.4 Counterfactual Analysis

For the counterfactual analysis, I employ the method presented by Dekle et al. (2008). As in Bartelme et al. (2021), it is assumed in the equilibrium observed in the ICIO dataset for the year 2018 that both the US and the EU do not provide production subsidies. The detailed system of equations that determines counterfactual changes is provided in Appendix I.

Figure 2 and Table 5 show several equilibria under various scenarios and the welfare changes in each scenario. The results offer a compelling explanation for why the US and the EU are actively and competitively supporting essential industries for a transition to a green economy. If each country conducts industrial policy without any reaction from the other, the optimal subsidy rates would be 0.21 and 0.19 for the US and the EU, respectively. However, when each country attempts to set its subsidy rate to maximize its own welfare, while taking into account the other country's subsidy, there are multiple Nash equilibria. In these equilibria, both the US and the EU set a higher subsidy rate than in the scenario where each implements a production subsidy alone. The possible equilibrium subsidy rates for both the US and the EU range from 0.24 to 0.33. This outcome arises because the welfare gain from a country's own innovation significantly outweighs the welfare gain derived from free-riding on the other country's innovation. The gains from choosing free-riding include those from the other country's carbon abatement and terms of trade gains resulting from the other country's productivity increase. This can be demonstrated by examining the welfare effects in scenarios with optimal subsidies and no reaction from the other country. In those scenarios, the welfare gain from advancing its own innovation stands at 3.73% and 3.93% for the US and the EU, respectively, while the welfare gains from the other country's earlier innovation are only 0.94% and 0.81%.

Table 5: Welfare of the US and the EU in Each Equilibrium

| Stage | US only | | EU only | | Nash | | Cooperative | |
|---|---|---|---|---|---|---|---|---|
| | US | EU | US | EU | US | EU | US | EU |
| Early | +0.18% | +0.95% | +0.31% | +0.18% | +1.69% $\sim$ +1.16% | +1.69% $\sim$ +1.16% | +0.88% | +0.88% |
| Leading | +3.73% | +0.94% | +0.81% | +3.93% | +0.00% | +0.00% | +0.00% | +0.00% |
| Mature | -0.48% | -2.10% | -3.97% | -0.67% | +3.82% $\sim$ +4.85% | +3.82% $\sim$ +4.85% | +5.17% | +5.17% |
| Overall | +3.41% | -0.25% | -2.89% | +3.41% | +5.57% $\sim$ +6.07 | +5.57% $\sim$ +6.07 | +6.09% | +6.09% |

Notes: This table illustrates the variations in the welfare of the US and the EU at each scenario compared to the initial equilibrium under the baseline model.

The results also indicate that if each country implements an optimal subsidy and the other country does not respond to the policy, the policy has a beggar-thy-neighbor effect. The US's optimal subsidy decreases the EU's welfare by 0.25%, while the EU's optimal subsidy decreases the US's welfare by 2.89%. This is because the loss from delayed innovation is large for both countries, amounting to -3.97% for the US and -2.10% for the EU, respectively. Interestingly, the EU is much less damaged

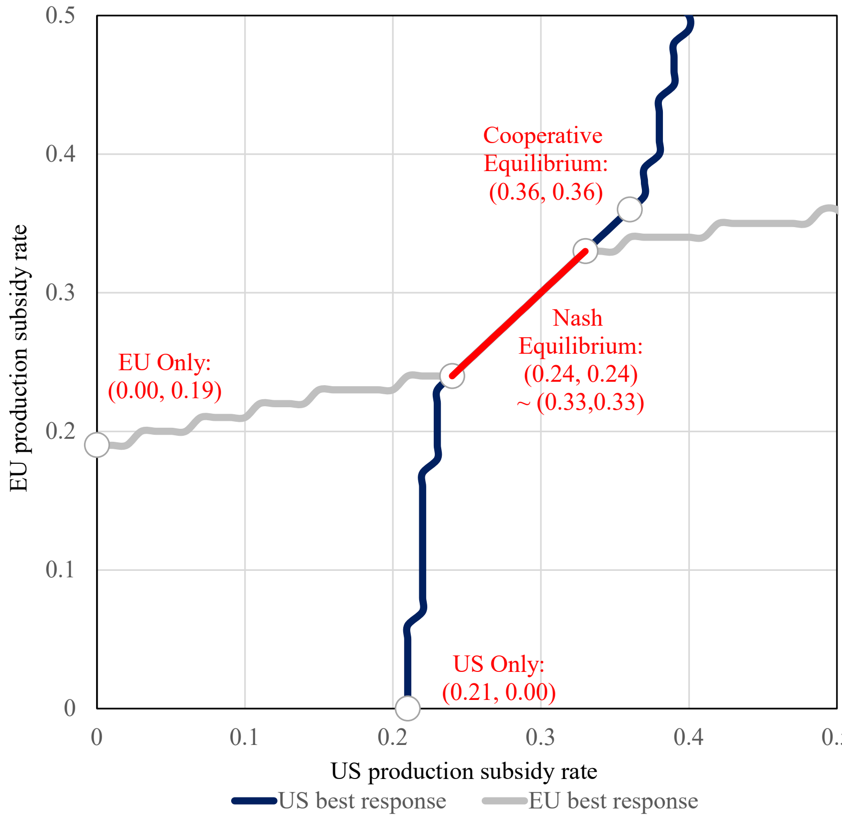


Figure 2: Equilibrium in Various Scenarios ($\epsilon_{t^r} = -0.3$)

Notes: Figure 2 presents the US and the EU's best functions under the generalized model, which show each country's optimal production subsidy rate given any subsidy rate chosen by the other country.

than the US by the other country's subsidy. This result suggests that the loss from delayed innovation is larger for the country without a comparative advantage. Industrial linkage data in Table 4 indicate that the EU has a comparative advantage in both the capital goods and manufacturing goods sectors. This interpretation is supported by the observation that the innovation timing in the green capital goods sector in the US is delayed by about 2 years (from year 10 to year 11.7) due to the EU's policy, while the timing in the EU is delayed by less than 1 year (from year 10 to year 10.9).

Additionally, it is notable that the cooperative equilibrium leads to a Pareto improvement, increasing the welfare gain for both the US and the EU from about 5.82%[22] to 6.09%. This finding aligns with equilibria observed in games concerning other environmental issues. In many cases, en-

[22]This value represents the midpoint between the two end points in multiple Nash equilibria, which are 5.57% and 6.07%, respectively.

vironmental issues are likely to give rise to a free-riding problem, preventing any entity from taking appropriate measures to resolve the environmental problems even though they could improve their utility by addressing the issue cooperatively. Thus, a cooperative policy is desirable even though such cooperation is hard to maintain (Nordhaus and Yang 1996; Carraro and Egenhofer 2007; Yang 2008). Similarly, since the free-riding problem is not entirely resolved in the Nash equilibrium, there exists a potential for Pareto improvement through cooperative policy. However, it is also important to note that the scope for improvement through cooperative policy in this case is limited, as both countries have already established high production subsidies as a result of their competition.

Lastly, this analysis sheds light on the environmental effects of industrial policy, in addition to the welfare effects. The numbers in Table 6 represent the reduction in greenhouse gas emissions as a proportion of total emissions over 10 years ($t^r(0,0) = 10$) in the initial equilibrium.[23] Since it is desirable for each country to support the green capital industry, greenhouse gas emissions decrease in any equilibrium. However, this reduction is not substantial when only one country supports the green industry; although such a policy accelerates a country's own innovation, it also delays that of the other country. The former leads to a decrease in emissions, while the latter causes an increase, yielding a net effect that is not significantly large. As shown in Table 6, greenhouse gas emissions decrease much more significantly in both the Nash and cooperative equilibria.

Table 6: Change in Greenhouse Gas Emission in Each Equilibrium

| | US only | EU only | Nash | Cooperative |
|---|---|---|---|---|
| Greenhouse gas emission | -2.45% | -1.72% | -7.15% $\sim$ -9.58% | -10.17% |

Notes: This table illustrates the changes in the global greenhouse gas emission at each equilibrium compared to the initial equilibrium under the baseline model.

#### 4.4.1 Shift from Free-riding to Active Subsidizing

The most significant difference between the results of the quantitative analysis in this paper and those found in the existing literature is that each country actively provides support to the green capital

[23] This can also be interpreted as a yearly decrease in global greenhouse gas emissions by that amount over 10 years.

goods sector, even without any arrangement for cooperation. This international policy dynamic leads to a faster decrease in carbon emissions, contrasting with a situation where each country opts for free-riding on another country's efforts to resolve environmental problems. This difference is primarily due to the existence of knowledge spillover and the potential for faster innovation. Consequently, the extent of each country's efforts depends on the magnitude of this externality, which can be characterized by the value of the innovation timing elasticity. In this context, I will examine the role of the innovation timing elasticity in-depth.

For this purpose, I first investigate how each country's optimal policy changes when there is no externality to hasten innovation timing in the green capital goods sector, meaning $\epsilon_{t^r} = 0$. Figure 3 presents the results for equilibria under various scenarios with the condition of $\epsilon_{t^r} = 0$.

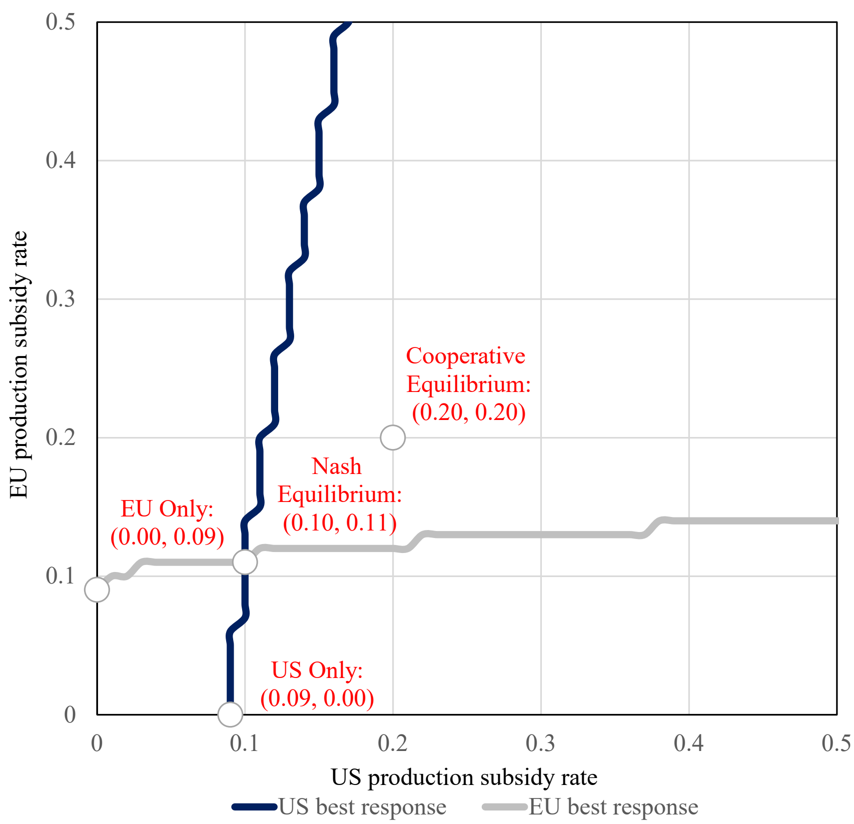


Figure 3: Equilibrium under Various Scenarios ($\epsilon_{t^r} = 0$)

Notes: Figure 3 displays the best-response functions for the US and the EU under the generalized model with $\epsilon_{t^r} = 0$. These functions indicate each country's optimal production subsidy rate, given any subsidy rate selected by the other country.

The best response function of the US and the EU is clearly different from that in Figure 2. First, each country chooses a much smaller subsidy rate across all scenarios. Second, each country does not respond significantly to the other country's increase in production subsidy. This is evidenced by the observation that the subsidy rate only increases from 0.09 to 0.10 for the US and from 0.09 to 0.11 for the EU in the unique Nash equilibrium. In contrast, in Figure 2, within the range of multiple Nash equilibria, if one country increases its subsidy rate, the other country responds by increasing its subsidy rate by the same amount.

To derive intuition, I also construct a simple competitive game where the US and the EU choose between two options: free-riding ($s = 0$) versus active subsidizing ($s = 0.33$)[24]. Table 7 shows the payoff matrices of this game under various values of $\epsilon_{t^r}$.

Table 7: Competitive Game between the US and the EU under Various $\epsilon_{t^r}$

(a) $\epsilon_{t^r} = 0$

| | | EU | |
|---|---|---|---|
| | | $s^* = 0.00$ | $s^* = 0.33$ |
| US | $s = 0.00$ | $\boxed{(0\%, 0\%)}$ | (7.83%, -7.23%) |
| | $s = 0.33$ | (-7.67%, 9.15%) | (7.68%, 7.68%) |

(b) $\epsilon_{t^r} = -0.1$

| | | EU | |
|---|---|---|---|
| | | $s^* = 0.00$ | $s^* = 0.33$ |
| US | $s = 0.00$ | $\boxed{(0\%, 0\%)}$ | (-2.06%, -2.34%) |
| | $s = 0.33$ | (-0.77%, 1.27%) | $\boxed{(3.02\%, 3.02\%)}$ |

(c) $\epsilon_{t^r} = -0.3$

| | | EU | |
|---|---|---|---|
| | | $s^* = 0.00$ | $s^* = 0.33$ |
| US | $s = 0.00$ | (0%, 0%) | (-7.59%, 1.16%) |
| | $s = 0.33$ | (2.38%, -0.76%) | $\boxed{(6.07\%, 6.07\%)}$ |

Notes: Subtables (a), (b), and (c) represent the payoff matrices of the US and the EU under the conditions of $\epsilon_{t^r} = 0$, $\epsilon_{t^r} = -0.1$, and $\epsilon_{t^r} = -0.3$, respectively. In each cell, the first value is the payoff of the US, and the second value is the payoff of the EU. The bordered cells represent the Nash equilibrium in each game.

In the case of $\epsilon_{t^r} = 0$, since there is no externality related to innovation, free-riding becomes the dominant strategy for both the US and the EU. Thus, choosing free-riding by both countries

[24] 0.33 is the highest subsidy rate in the Nash equilibria in counterfactual analysis.

is the unique Nash equilibrium. This result aligns with the documented difficulties in overcoming free-riding problems in the setting where only environmental externality exists (Nordhaus and Yang 1996; Carraro and Egenhofer 2007; Yang 2008). In the case of $\epsilon_{t^r} = -0.1$, due to the externality from knowledge spillover, free-riding is no longer a dominant strategy, and there are two Nash equilibria: either both countries free-ride or both actively subsidize. In the case of $\epsilon_{t^r} = -0.3$, the timing elasticity is high enough such that active subsidizing becomes the dominant strategy for both countries, and choosing active subsidizing by both countries is the unique Nash equilibrium. These results demonstrate that the estimated innovation timing elasticity is sufficiently high to shift national optimal policy from free-riding to active subsidizing. Additionally, considering the externality related to growth potential, which varies depending on the lifecycle stage of an industry, is important for designing industrial policy.

# 5 Conclusion

This paper aims to explain the reasons behind an interesting phenomenon: many countries are competitively supporting industries essential for a transition to a green economy, despite the policy costs and the possibility of free-riding. This phenomenon indicates that each country has strong organic incentives to implement industrial policy—incentives that outweigh the costs stemming from the policy and benefits of free-riding. As many governments emphasize, these incentives stem from the high-growth potential and growing market size of the targeted industries. Industries may possess such high-growth potential exclusively in the early stages of their lifecycle. For this reason, industry lifecycle theory needs to be incorporated into the general equilibrium models of industrial policy focused on the green energy transition.

The policy aimed at fostering a transition to a green economy is inevitably linked to another source of externality: greenhouse gas emissions. Accordingly, the model in this study incorporates two sources of externality: knowledge spillovers within the industry and environmental externalities. Among these two types of externalities, the former plays a crucial role in triggering policy competition between countries. The central mechanism behind this competition is that a domestic production subsidy accelerates the timing of innovation in the home country while delaying it in the foreign country. This dynamic aligns with Bai et al. (2023), where a country aims to enhance its own welfare by influencing the innovation efforts of other countries. Furthermore, this dynamic highlights the importance of analyzing the welfare effects of domestic policies on other countries, a topic that has received less emphasis in most of the existing literature but is a focal point in this paper. This emphasis is particularly persuasive, given that it is unrealistic to assume that foreign countries will remain passive in response to attempts by the home country to stifle R&D efforts in promising sectors.

This paper also sheds light on the importance of welfare analysis during the transition from one steady state to another. In the model, even though the outcome in the steady state is the same regardless of policy intervention, the welfare effect from policy is substantial. This is somewhat surprising, given that the innovation timing elasticity is not large—estimated to be around -0.30—and firms play a significantly larger role in accumulating knowledge stock. This occurs because if innovation takes place earlier in an industry in one country, it negatively impacts the same industry in the counterpart country. Thus, even though the foreign industry eventually innovates, the transition period—when the foreign country has yet to innovate—becomes longer, and the benefits of taking the lead first become larger. In this regard, the innovation timing elasticity can provide important information for designing industrial policy.

The model in this paper can be used to study more general issues. One example is the welfare effects arising from the evolution of new technologies, such as Artificial Intelligence. The mechanism for endogenous technology adoption, similar to Helpman and Trajtenberg (1998), can capture the welfare effects of emerging new technologies within the context of an open economy. I hope this model can be applied to analyze many other interesting topics.

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# A Equilibrium

**Prices** Prices for green capital goods are set by imposing a constant markup over marginal costs as follows:

$$p_t^g(\omega) = \frac{1}{1+s}\frac{\gamma}{\gamma-1}\frac{1}{a_t^g}, \quad e_t p_t^{g*}(\omega) = p_t^{g*}(\omega) = (1+\tau)\frac{1}{1+s}\frac{\gamma}{\gamma-1}\frac{1}{a_t^g} = (1+\tau)p_t^g(\omega) \tag{51}$$

$$p_t^{g*}(\omega_f) = \frac{1}{1+s^*}\frac{\gamma}{\gamma-1}\frac{1}{a_t^{g*}}, \quad \frac{p_t^g(\omega_f)}{e_t} = p_t^g(\omega_f) = (1+\tau)\frac{1}{1+s^*}\frac{\gamma}{\gamma-1}\frac{1}{a_t^{g*}} = (1+\tau)p_t^{g*}(\omega_f) \tag{52}$$

where $\tau$ represents trade cost, and prices with an asterisk are denominated in foreign currency.

The price of aggregated green capital goods for each country is as follows:

$$P_t^g = \left(n_t\left(p_t^g(\omega)\right)^{1-\gamma} + \phi n_t^*(e_t p_t^{g*}(\omega_f))^{1-\gamma}\right)^{\frac{1}{1-\gamma}} = \left(n_t\left(p_t^g(\omega)\right)^{1-\gamma} + \phi n_t^*(p_t^{g*}(\omega_f))^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{53}$$

$$P_t^{g*} = \left(n_t^*\left(p_t^{g*}(\omega_f)\right)^{1-\gamma} + \phi n_t\left(\frac{p_t^g(\omega)}{e_t}\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} = \left(n_t^*\left(p_t^{g*}(\omega_f)\right)^{1-\gamma} + \phi n_t\left(p_t^g(\omega)\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \tag{54}$$

where $\phi \equiv (1+\tau)^{1-\gamma}$

Prices for agricultural final goods are 1 in both the home and foreign countries ($P_t^A = 1, P_t^{A*} = 1$). By solving the manufacturing final goods firms' cost minimization problem, their prices are determined for the home and foreign countries by

$$\begin{aligned} P_{hi,t}^M &= \frac{1}{A_{i,t}}\alpha^{-\alpha}(1-\alpha)^{-\alpha+1}\lambda_i^{\alpha-1}P_t^{g\,1-\alpha} = \alpha^{-\alpha}(1-\alpha)^{-\alpha+1}\lambda_i^{\alpha-1}P_t^{g\,1-\alpha}, \quad 0 \le i < I_t \\ P_{hi,t}^M &= \frac{1}{A_{i,t}}\alpha^{-\alpha}(1-\alpha)^{-\alpha+1}{\psi_t}^{1-\alpha} = \alpha^{-\alpha}(1-\alpha)^{-\alpha+1}{\psi_t}^{1-\alpha}, \quad I_t \le i \end{aligned} \tag{55}$$

$$\begin{aligned} P_{fi,t}^{M*} &= \frac{1}{A_{i,t}^*}\alpha^{-\alpha}(1-\alpha)^{-\alpha+1}\lambda_i^{\alpha-1}P_t^{g*1-\alpha} = \alpha^{-\alpha}(1-\alpha)^{-\alpha+1}\lambda_i^{\alpha-1}P_t^{g*1-\alpha}, \quad 0 \le i < I_t^* \\ P_{fi,t}^{M*} &= \frac{1}{A_{i,t}^*}\alpha^{-\alpha}(1-\alpha)^{-\alpha+1}\psi_t^{*1-\alpha} = \alpha^{-\alpha}(1-\alpha)^{-\alpha+1}\psi_t^{*1-\alpha}, \quad I_t^* \le i \end{aligned} \tag{56}$$

The home and foreign aggregated manufacturing final goods price are

$$P_{h,t}^M = e^{\int_0^1 \ln P_{hi,t}^M di} \tag{57}$$

$$P_{f,t}^{M*} = e^{\int_0^1 \ln P_{fi,t}^{M*} di} \tag{58}$$

The composite manufacturing final goods price in the home and foreign country are

$$P_t^M = \beta^{-\beta}(1-\beta)^{-\beta+1}\left(P_{h,t}^M\right)^{\beta}\left(e_t P_{f,t}^{M*}\right)^{1-\beta} = \beta^{-\beta}(1-\beta)^{-\beta+1}\left(P_{h,t}^M\right)^{\beta}\left(P_{f,t}^{M*}\right)^{1-\beta} \tag{59}$$

$$P_t^{M*} = \beta^{-\beta}(1-\beta)^{-\beta+1}\left(P_{f,t}^{M*}\right)^{\beta}\left(\frac{P_{h,t}^{M}}{e_t}\right)^{1-\beta} = \beta^{-\beta}(1-\beta)^{-\beta+1}\left(P_{f,t}^{M*}\right)^{\beta}\left(P_{h,t}^{M}\right)^{1-\beta} \tag{60}$$

The utility-based CPI is given by

$$P_t = \iota^{-\iota}(1-\iota)^{-\iota+1}\left(P_t^{M}\right)^{1-\iota} \tag{61}$$

$$P_t^* = \iota^{-\iota}(1-\iota)^{-\iota+1}\left(P_t^{M*}\right)^{1-\iota} \tag{62}$$

**Final goods Markets** By solving the representative consumer's utility maximization problem, we derive the following final goods demands:

$$\text{Aggregated final goods: } C_t = P_t^{-1},\ C_t^* = P_t^{*-1} \tag{63}$$

$$\text{Agricultural final goods: } C_t^A = \iota,\ C_t^{A*} = \iota \tag{64}$$

$$\text{Home manufacturing sector } i \text{ goods: } C_{hi,t}^{M} = \frac{(1-\iota)\beta}{P_{hi,t}^{M}},\ C_{hi,t}^{M*} = \frac{(1-\iota)(1-\beta)e_t}{P_{hi,t}^{M}} = \frac{(1-\iota)(1-\beta)}{P_{hi,t}^{M}} \tag{65}$$

$$\text{Foreign manufacturing sector } i \text{ goods: } C_{fi,t}^{M*} = \frac{(1-\iota)\beta}{P_{fi,t}^{M*}},\ C_{fi,t}^{M} = \frac{(1-\iota)(1-\beta)}{e_t P_{fi,t}^{M*}} = \frac{(1-\iota)(1-\beta)}{P_{fi,t}^{M*}} \tag{66}$$

Accordingly, the equilibrium conditions for the final goods markets are as follows:

$$\text{Agricultural final goods: } Y_t^A + Y_t^{A*} = C_t^A + C_t^{A*} \tag{67}$$

$$\text{Home manufacturing sector } i \text{ goods: } Y_{i,t}^{k} = C_{hi,t}^{M} + C_{hi,t}^{M*},\ k \in \{g, e\} \tag{68}$$

$$\text{Foreign manufacturing sector } i \text{ goods: } Y_{i,t}^{k*} = C_{fi,t}^{M*} + C_{fi,t}^{M},\ k \in \{g, e\} \tag{69}$$

**Green Capital Goods and Conventional Energy Market** By solving the final goods firm's cost minimization problem with green capital as its energy input, the following demand for green capital goods varieties $\omega$ and $\omega_f$ is derived.

$$z_t^g(\omega) = \left(\frac{p_t^g}{P_t^g}\right)^{-\gamma}\int_0^{I_t} Z_{i,t}^g di,\quad z_t^{g*}(\omega) = \left(\frac{(1+\tau)p_t^g}{e_t P_t^{g*}}\right)^{-\gamma}\int_0^{I_t^*} Z_{i,t}^{g*} di = \left(\frac{(1+\tau)p_t^g}{P_t^{g*}}\right)^{-\gamma}\int_0^{I_t^*} Z_{i,t}^{g*} di \tag{70}$$

$$z_t^{g*}(\omega_f) = \left(\frac{p_t^{g*}}{P_t^{g*}}\right)^{-\gamma}\int_0^{I_t^*} Z_{i,t}^{g*} di,\quad z_t^{g}(\omega_f) = \left(\frac{e_t(1+\tau)p_t^{g*}}{P_t^{g}}\right)^{-\gamma}\int_0^{I_t} Z_{i,t}^{g} di = \left(\frac{(1+\tau)p_t^{g*}}{P_t^{g}}\right)^{-\gamma}\int_0^{I_t} Z_{i,t}^{g} di \tag{71}$$

The demand for aggregated green capital from sector $i$ in the home and foreign countries, denoted by $Z^g_{i,t}$ and $Z^{g*}_{i,t}$, is given as follows:

$$Z^g_{i,t} = \frac{(1-\alpha)(1-\iota)\beta}{P^g_t} + \frac{(1-\alpha)(1-\iota)(1-\beta)e_t}{P^g_t} = \frac{(1-\alpha)(1-\iota)}{P^g_t} \tag{72}$$

$$Z^{g*}_{i,t} = \frac{(1-\alpha)(1-\iota)\beta}{P^{g*}_t} + \frac{(1-\alpha)(1-\iota)(1-\beta)}{e_t P^{g*}_t} = \frac{(1-\alpha)(1-\iota)}{P^{g*}_t} \tag{73}$$

Thus, the equilibrium conditions for the green capital goods markets are:

$$y^g_t(\omega) = z^g_t(\omega) + (1+\tau)z^{g*}_t(\omega) \tag{74}$$

$$y^{g*}_t(\omega_f) = z^{g*}_t(\omega_f) + (1+\tau)z^g_t(\omega_f) \tag{75}$$

For the conventional energy market, where it is assumed the supply is perfectly elastic at prices $\psi_t$ and $\psi^*_t$ in the home and foreign countries, equilibrium is determined by the demand side. The demand for conventional energy from sector $i$ that decides to use conventional energy is as follows:

$$E_{i,t} = \frac{(1-\alpha)(1-\iota)}{\psi_t} \tag{76}$$

$$E^*_{i,t} = \frac{(1-\alpha)(1-\iota)}{\psi^*_t} \tag{77}$$

**Zero-Profit Conditions** Free entry into the green capital market implies that a firm's profit will be zero in equilibrium. Thus, a firm's operating profit in each industry should equal the fixed cost in both the home and foreign countries, as follows:

$$\begin{aligned}
\Pi^g_t(\omega) &= \frac{(1+s)p^g_t(\omega)y^g_t(\omega)}{\gamma} \\
&= \frac{(1+s)p^g_t(\omega)}{\gamma}\left[z^g_t(\omega) + (1+\tau)z^{g*}_t(\omega)\right] \\
&= \frac{(1+s)(1-\iota)(1-\alpha)}{\gamma}\left[\left(\frac{p^g_t(\omega)}{P^g_t}\right)^{1-\gamma} I_t + \phi\left(\frac{p^g_t(\omega)}{P^{g*}_t}\right)^{1-\gamma} I^*_t\right] = 1
\end{aligned} \tag{78}$$

$$\begin{aligned}
\Pi^{g*}_t(\omega_f) &= \frac{(1+s^*)p^{g*}_t(\omega_f)y^{g*}_t(\omega_f)}{\gamma} \\
&= \frac{(1+s^*)p^{g*}_t(\omega_f)}{\gamma}\left[z^{g*}_t(\omega_f) + (1+\tau)z^g_t(\omega_f)\right] \\
&= \frac{(1+s^*)(1-\iota)(1-\alpha)}{\gamma}\left[\left(\frac{p^{g*}_t(\omega_f)}{P^{g*}_t}\right)^{1-\gamma} I^*_t + \phi\left(\frac{p^{g*}_t(\omega_f)}{P^g_t}\right)^{1-\gamma} I_t\right] = 1
\end{aligned} \tag{79}$$

**Balance of Payment Equilibrium Condition** I assume balanced trade in the model, whereby the value of a country's imports equals the value of its exports. Thus, the following equation holds at equilibrium.

$$\begin{aligned} & M_t^{A*} + (1-\iota)(1-\beta) + (1+\tau)p_t^g z_t^{g*}(\omega)n_t = M_t^A + (1-\iota)(1-\beta) + (1+\tau)p_t^{g*} z_t^g(\omega_f)n_t^* \\ & \Rightarrow M_t^{A*} + \frac{\phi n_t p_t^g(\omega)^{1-\gamma}}{(P^{g*})^{1-\gamma}}(1-\iota)(1-\alpha)I_t^* = M_t^A + \frac{\phi n_t p_t^{g*}(\omega_f)^{1-\gamma}}{(P^{g*})^{1-\gamma}}(1-\iota)(1-\alpha)I_t \end{aligned} \tag{80}$$

where $M_t^A(= max[0, C_t^A - Y_t^A])$ and $M_t^{A*}(= max[0, C_t^{A*} - Y_t^{A*}])$ represent the quantity of agricultural final goods imported in the home and foreign countries, respectively.

**Green Energy Adoption** The green energy adoption ratios in the home and foreign countries, denoted by $I_t$ and $I_t^*$, are determined as follows: $I_t$ and $I_t^*$ satisfy the following equations.

$$\frac{P_t^g}{\lambda_{I_t}} \leq \psi_t \text{ for } i \leq I_t, \quad \psi_t < \frac{P_t^g}{\lambda_{I_t}} \text{ for } I_t < i \tag{81}$$

$$\frac{P_t^{g*}}{\lambda_{I_t^*}^*} \leq \psi_t^* \text{ for } i \leq I_t^*, \quad \psi_t^* < \frac{P_t^{g*}}{\lambda_{I_t^*}^*} \text{ for } I_t^* < i \tag{82}$$

**Resource Constraints** For the home and foreign countries, the resource constraints are given as follows, respectively:

$$\begin{aligned} & l_t \geq Y_{i,t}^A + \int_0^{I_t} \alpha P_{hi,t}^M Y_{i,t}^g di + \int_{I_t}^1 \alpha P_{hi,t}^M Y_{i,t}^e di + \int_0^{n_t} \frac{y_t^g(\omega)}{a_t^g} d\omega + \int_0^{n_t} q_t^g(\omega) d\omega \\ & \Rightarrow l_t \geq Y_{i,t}^A + (1-\iota)\alpha + \gamma n_t \end{aligned} \tag{83}$$

$$\begin{aligned} & l_t^* \geq Y_{i,t}^{A*} + \int_0^{I_t^*} \alpha P_{fi,t}^{M*} Y_{i,t}^{g*} di + \int_{I_t^*}^1 \alpha P_{fi,t}^{M*} Y_{i,t}^{e*} di + \int_0^{n_t^*} \frac{y_t^{g*}(\omega_f)}{a_t^{g*}} d\omega_f + \int_0^{n_t^*} q_t^{g*}(\omega_f) d\omega_f \\ & \Rightarrow l_t^* \geq Y_{i,t}^{A*} + (1-\iota)\alpha + \gamma n_t^* \end{aligned} \tag{84}$$

**Tax** From Section 2.4, the tax used to finance the production subsidy is determined in the home and foreign countries, respectively, as follows:

$$T_t = s n_t p_t^g(\omega) y_t^g(\omega) \tag{85}$$

$$T_t^* = s^* n_t^* p_t^{g*}(\omega_f) y_t^{g*}(\omega_f) \tag{86}$$

**Definition of Equilibrium** A general equilibrium with the home and foreign production subsidies, $\{s, s^*\}$, consists of the home and foreign outputs, $\{Y_t^A, Y_{i,t}^k, y_t^g(\omega)\}$ and $\{Y_t^{A*}, Y_{i,t}^{k*}, y_t^{g*}(\omega_f)\}$, the home and foreign labor supply, $\{l_t, l_t^*\}$, and the home and foreign final goods demands, $\{C_t, C_t^A, C_{hi,t}^M, C_{fi,t}^M\}$ and $\{C_t^*, C_t^{A*}, C_{hi,t}^{M*}, C_{fi,t}^{M*}\}$, and the home and foreign green capital goods demands, $\{z_t^g(\omega), z_t^g(\omega_f), Z_{i,t}^g\}$ and $\{z_t^{g*}(\omega_f), z_t^{g*}(\omega), Z_{i,t}^{g*}\}$, and the home and foreign conventional energy demands, $E_{i,t}$ and $E_{i,t}^*$, and the mass of the home and foreign capital goods firms, $n_t$ and $n_t^*$, and the home and foreign green capital usage ratio, $I_t$ and $I_t^*$, and the home and foreign prices, $\{P_t, P_t^M, P_{h,t}^M, P_{hi,t}^k, P_t^g, p_t^g(\omega)\}$ and $\{P_t^*, P_t^{M*}, P_{f,t}^{M*}, P_{fi,t}^{k*}, P_t^{g*}, p_t^{g*}(\omega_f)\}$, and the home and foreign tax, $T_t$ and $T_t^*$, such that equation (51)-(86) hold.[25]

Similar to the approach taken in Corsetti et al. (2007), the equilibrium system of equations defined above can be simplified to two key equations: (78) and (79). These equations involve two endogenous variables, $n_t$ and $n_t^*$. Solving for these variables allows us to determine all other variables using the equations (51)-(84). By solving equations (78) and (79), we derive the following solution.

$$n_t = \frac{(1+s)(1-\rho)(1-\alpha)}{\gamma} \frac{I_t + \phi^2 I_t^* - S^{-\gamma} A^{1-\gamma} \phi (I_t + I_t^*)}{1 + \phi^2 - (S^\gamma A^{\gamma-1} + S^{-\gamma} A^{1-\gamma})\phi} \tag{87}$$

$$n_t^* = \frac{(1+s^*)(1-\rho)(1-\alpha)}{\gamma} \frac{I_t^* + \phi^2 I_t - S^{\gamma} A^{\gamma-1} \phi (I_t + I_t^*)}{1 + \phi^2 - (S^\gamma A^{\gamma-1} + S^{-\gamma} A^{1-\gamma})\phi} \tag{88}$$

where $S \equiv \frac{1+s}{1+s^*}$ and $A \equiv \frac{a_t^g}{a_t^{g*}}$.

# B Proofs

## B.1 Proposition 2

Based on the results of comparative statics in Table 8, $Y_{i,t}^e$ is fixed in each stage in each country. Thus, the total global greenhouse gas emission in each stage is given by $\int_{I_t}^1 Y_{i,t}^e di + \int_{I_t^*}^1 Y_{i,t}^{e*} di = (1-I_t)Y_i^e + (1-I_t^*)Y_i^e$, where $Y_i^e$ denotes the fixed amount of production using conventional energy.

When $s = s^* = 0$, based on equations (32), (34), and (36), the welfare effect of a home production subsidy on the home country can be expressed as follows:

[25] There are 42 endogenous variables and 42 equations since the following equations each contribute two: (63), (64), (65), (66), (70), (71).

$$
\begin{aligned}
\frac{d\ln W}{ds} &= t^r(0,0)\Bigg[-\frac{d\ln P_E}{ds} - (1-\iota)(1-\alpha)I_E + \left(-\ln P_L + \ln P_E + \kappa(\overline{I}-\underline{I})Y_i^e\right)\left(-\epsilon_{t^r}\frac{d\ln n_E}{ds}\right) \\
&\quad + \left(-\ln P_L + \ln P_E + \kappa(\overline{I}-\underline{I})Y_i^e\right)\left(-\epsilon_{t^r}\frac{d\ln n_E^*}{ds}\right)\Bigg] \\
&= t^r(0,0)\Bigg[\frac{(1-\iota)(1-\alpha)I_E}{(\gamma-1)} + \left(-\ln P_L + \ln P_E + \kappa(\overline{I}-\underline{I})Y_i^e\right)\left(-\epsilon_{t^r}\frac{d\ln n_E}{ds}\right) \\
&\quad + \left(-\ln P_M + \ln P_L + \kappa(\overline{I}-\underline{I})Y_i^e\right)\left(-\epsilon_{t^r}\frac{d\ln n_E^*}{ds}\right)\Bigg]
\end{aligned}
\tag{89}
$$

Based on the comparative statics in Table 8, the welfare can be reexpressed as follows:

$$
\begin{aligned}
\frac{d\ln W}{ds} = t^r(0,0)\Bigg[&\frac{(1-\iota)(1-\alpha)I_E}{(\gamma-1)} + (-\ln P_L + \ln P_E)\left(-\epsilon_{t^r}\frac{d\ln n_E}{ds}\right) \\
&+ (-\ln P_M + \ln P_L)\left(-\epsilon_{t^r}\frac{d\ln n_E^*}{ds}\right) + \kappa(\overline{I}-\underline{I})Y_i^e\Bigg] > 0
\end{aligned}
\tag{90}
$$

Based on the results presented in Appendix C.2 and C.3, we have $-\ln P_L + \ln P_E > 0$ and $-\ln P_M + \ln P_L < 0$. Therefore, we can conclude that the welfare effect of a home production subsidy on the home country near the initial equilibrium is positive. ■

## B.2 Proposition 3

When $s = s^* = 0$, based on equations (33), (35), and (37), the welfare effect of a home production subsidy on the foreign country can be expressed as follows:

$$
\begin{aligned}
\frac{d\ln W^*}{ds} =& t^r(0,0)\Bigg[\left(-\ln P_L^* + \ln P_E^* + \kappa(\overline{I}-\underline{I})Y_i^e\right)\left(-\epsilon_{t^r}\frac{d\ln n_E}{ds}\right) \\
&+ \left(-\ln P_M^* + \ln P_L^* + \kappa(\overline{I}-\underline{I})Y_i^e\right)\left(-\epsilon_{t^r}\frac{d\ln n_E^*}{ds}\right)\Bigg]
\end{aligned}
\tag{91}
$$

Additionally, using the results in Table 8, we can find the condition under which a home production subsidy worsens foreign welfare as follows:

$$
\begin{aligned}
&\frac{d\ln W^*}{ds} < 0 \Leftrightarrow \\
&-\epsilon_{t^r}\left(\kappa(\overline{I}-\underline{I})Y_i^e\right) - (-\ln P_L^* + \ln P_E^*)\epsilon_{t^r}\frac{d\ln n_E}{ds} - (-\ln P_M^* + \ln P_L^*)\epsilon_{t^r}\frac{d\ln n_E^*}{ds} < 0 \Leftrightarrow \\
&(-\ln P_M^* + \ln P_L^*)\epsilon_{t^r}\frac{d\ln n_E^*}{ds} > -(-\ln P_L^* + \ln P_E^*)\epsilon_{t^r}\frac{d\ln n_E}{ds} - \epsilon_{t^r}\left(\kappa(\overline{I}-\underline{I})Y_i^e\right)
\end{aligned}
\tag{92}
$$

■

## B.3 Proposition 4

Based on equations (32), (34), (36), and the results in Table 8, the welfare effect of increasing the subsidy rate under the symmetric condition ($s = s^*$) on the conducting country can be rewritten as follows.

$$
\begin{aligned}
\frac{d\ln W}{ds} &= t^r(s,s^*)\left(-\frac{d\ln P_E}{ds} - (1-\iota)(1-\alpha)I_E - \frac{s}{1+s}\frac{2\gamma\phi(1-\iota)(1-\alpha)I_E}{(1-\phi)^2}\right) \\
&- t^r(s,s^*)\epsilon_{tr}\left(-\ln P_L + \ln P_E + \kappa(\overline{I}-\underline{I})Y_i^e\right)\frac{d\ln n_E}{ds} \\
&- t^r(s,s^*)\epsilon_{tr}\left(-\ln P_M + \ln P_L + \kappa(\overline{I}-\underline{I})Y_i^e\right)\frac{d\ln n_E^*}{ds} \\
&= t^r(s,s^*)\Bigg[\left(\frac{1}{1+s}\frac{\gamma(1-\iota)(1-\alpha)I_E}{(\gamma-1)} - (1-\iota)(1-\alpha)I_E - \frac{s}{1+s}\frac{2\gamma\phi(1-\iota)(1-\alpha)I_E}{(1-\phi)^2}\right) \\
&- \frac{1}{1+s}\epsilon_{tr}\left(-\ln P_L + \ln P_E + \kappa(\overline{I}-\underline{I})Y_i^e\right)\left(1+\frac{2\gamma\phi}{(1-\phi)^2}\right) \\
&+ \frac{1}{1+s}\epsilon_{tr}\left(-\ln P_M + \ln P_L + \kappa(\overline{I}-\underline{I})Y_i^e\right)\frac{2\gamma\phi}{(1-\phi)^2}\Bigg] \\
&= t^r(s,s^*)\left[\frac{1}{1+s}\frac{1}{t^r(0,0)}\left.\frac{d\ln W}{ds}\right|_{s=s^*=0} - \frac{s}{1+s}(1-\iota)(1-\alpha)I_E - \frac{s}{1+s}\frac{2\gamma\phi(1-\iota)(1-\alpha)I_E}{(1-\phi)^2}\right] \\
&= t^r(s,s^*)\left[\frac{1}{1+s}\frac{1}{t^r(0,0)}\left.\frac{d\ln W}{ds}\right|_{s=s^*=0} - \frac{s}{1+s}\left(1+\frac{2\gamma\phi}{(1-\phi)^2}\right)(1-\iota)(1-\alpha)I_E\right]
\end{aligned}
\tag{93}
$$

To see how this welfare effect changes while increasing $s$, I take the derivative of $\frac{d\ln W}{ds}$ with respect to $s$. Proposition 2 shows $\frac{d\ln W}{ds}\ |_{s=s^*=0}> 0$. Thus, the following can be proven.

$$
\frac{d\frac{d\ln W}{ds}}{ds} = t^r(s,s^*)\left[-\frac{1}{(1+s)^2}\frac{1}{t^r(0,0)}\left.\frac{d\ln W}{ds}\right|_{s=s^*=0} - \frac{1}{(1+s)^2}\left(1+\frac{2\gamma\phi}{(1-\phi)^2}\right)(1-\iota)(1-\alpha)I_E\right] < 0
\tag{94}
$$

Given that $\frac{d\ln W}{ds}\ |_{s=s^*=0}> 0$ and $\frac{d\ln W}{ds}$ decreases as the subsidy rate increases, there uniquely exists a positive value $\bar{s}$. From equation (93), $\bar{s}$ is solved as follows.

$$
\begin{aligned}
&t^r(s,s^*)\left[\frac{1}{1+s}\frac{1}{t^r(0,0)}\frac{d\ln W}{ds}\bigg|_{s=s^*=0}-\frac{s}{1+s}\left(1+\frac{2\gamma\phi}{(1-\phi)^2}\right)(1-\iota)(1-\alpha)I_E\right]=0\\
&\Rightarrow s\left(1+\frac{2\gamma\phi}{(1-\phi)^2}\right)(1-\iota)(1-\alpha)I_E=\frac{1}{t^r(0,0)}\frac{d\ln W}{ds}\bigg|_{s=s^*=0}\\
&\Rightarrow \bar{s}=\frac{\frac{1}{t^r(0,0)}\frac{d\ln W}{ds}\ |_{s=s^*=0}}{\left(1+\frac{2\gamma\phi}{(1-\phi)^2}\right)(1-\iota)(1-\alpha)I_E}=\frac{\frac{1}{t^r(0,0)}\frac{d\ln W}{ds}\ |_{s=s^*=0}}{(1-\iota)(1-\alpha)I_E\frac{d\ln n_E}{ds}\ |_{s=s^*=0}}
\end{aligned}
\tag{95}
$$

■

## B.4 Proposition 5

Based on the comparative statics in Table 9, the welfare effect of a cooperative production subsidy change at $\bar{s}$ is as follows.

$$
\begin{aligned}
\frac{d\ln W}{ds^c}\bigg|_{s=s^*=\bar{s}}&=t^r(\bar{s},\bar{s})\left(-\frac{d\ln P_E}{ds^c}-\frac{dl_E}{ds^c}\right)-t^r(\bar{s},\bar{s})\epsilon_{t^r}(-\ln P_M+\ln P_E+2\kappa(\overline{I}-\underline{I})Y_i^e)\frac{d\ln n_E}{ds^c}\\
&=t^r(\bar{s},\bar{s})\left[\left(\frac{1}{1+\bar{s}}\frac{\gamma(1-\iota)(1-\alpha)I_E}{\gamma-1}-(1-\iota)(1-\alpha)I_E\right)\right.\\
&\left.-\epsilon_{t^r}\frac{1}{1+\bar{s}}\left(\frac{1}{\gamma-1}(\ln\overline{I}-\ln\underline{I})+\ln\bar{a}_H+2\kappa(\overline{I}-\underline{I})Y_i^e\right)\right]
\end{aligned}
\tag{96}
$$

Using equilibrium $\bar{s}$ at equation (95), the above equation can be expressed as follows.

$$
\begin{aligned}
\left.\frac{d\ln W}{ds^c}\right|_{s=s^*=\bar{s}} &= t^r(\bar{s},\bar{s})\frac{1}{1+\bar{s}}\Bigg[\left(\frac{\gamma(1-\iota)(1-\alpha)I_E}{\gamma-1}-(1+\bar{s})(1-\iota)(1-\alpha)I_E\right) \\
&-\epsilon_{t^r}\left(\frac{1}{\gamma-1}(\ln\overline{I}-\ln\underline{I})+\ln\bar{a}_H+2\kappa(\overline{I}-\underline{I})Y_i^e\right)\Bigg] \\
&= t^r(\bar{s},\bar{s})\frac{1}{1+\bar{s}}\Bigg[\left(\frac{(1-\iota)(1-\alpha)I_E}{\gamma-1}-\bar{s}(1-\iota)(1-\alpha)I_E\right) \\
&-\epsilon_{t^r}\left(-\ln P_L+\ln P_E+\kappa(\overline{I}-\underline{I})Y_i^e\right)-\epsilon_{t^r}\left(-\ln P_M+\ln P_L+\kappa(\overline{I}-\underline{I})Y_i^e\right)\Bigg] \\
&= t^r(\bar{s},\bar{s})\frac{1}{1+\bar{s}}\Bigg[\left(\frac{(1-\iota)(1-\alpha)I_E}{\gamma-1}-\frac{\frac{\gamma(1-\iota)(1-\alpha)I_E}{(\gamma-1)}-(1-\iota)(1-\alpha)I_E}{1+\frac{2\gamma\phi}{(1-\phi)^2}}\right) \\
&-\epsilon_{t^r}\left(-\ln P_M+\ln P_L+\kappa(\overline{I}-\underline{I})Y_i^e\right)\left(1+\frac{\frac{2\gamma\phi}{(1-\phi)^2}}{1+\frac{2\gamma\phi}{(1-\phi)^2}}\right)\Bigg] \\
&= t^r(\bar{s},\bar{s})\frac{1}{1+\bar{s}}\Bigg[\left(\frac{1}{\gamma-1}-\frac{\frac{\gamma}{\gamma-1}-1}{1+\frac{2\gamma\phi}{(1-\phi)^2}}\right)(1-\iota)(1-\alpha)I_E \\
&-\epsilon_{t^r}\left(-\ln P_M+\ln P_L+\kappa(\overline{I}-\underline{I})Y_i^e\right)\left(1+\frac{\frac{2\gamma\phi}{(1-\phi)^2}}{1+\frac{2\gamma\phi}{(1-\phi)^2}}\right)\Bigg] \\
&= t^r(\bar{s},\bar{s})\frac{1}{1+\bar{s}}\Bigg[\left(\frac{\frac{2\gamma\phi}{(1-\phi)^2}}{(\gamma-1)\left(1+\frac{2\gamma\phi}{(1-\phi)^2}\right)}\right)(1-\iota)(1-\alpha)I_E \\
&-\epsilon_{t^r}\left(-\ln P_M+\ln P_L+\kappa(\overline{I}-\underline{I})Y_i^e\right)\left(1+\frac{\frac{2\gamma\phi}{(1-\phi)^2}}{1+\frac{2\gamma\phi}{(1-\phi)^2}}\right)\Bigg]>0
\end{aligned}
\tag{97}
$$

Since both the first term and the second term in the equation above are positive, we can prove that $\frac{d\ln W}{ds_c}=\frac{d\ln W^*}{ds_c^*}>0$ at $s=s^*=\bar{s}$. ■

# C Welfare Effect Decomposition

## C.1 Short-run Resource Reallocation Effect

By taking the derivative of the short-run resource reallocation effect with respect to $s$ for both the home and foreign countries in the initial equilibrium, we obtain the following two equations.

$$
\begin{aligned}
\int_0^{t^r(0,0)} \frac{d\ln U_E}{ds} dt &= t^r(0,0)\frac{d\ln U_E}{ds} \\
&= t^r(0,0)\left[-\frac{d\ln P_E}{ds} - \frac{dl_E}{ds} - \kappa\int_{\underline{I}}^1 \left(\frac{dY^e_{i,E}}{ds} + \frac{dY^{e*}_{i,E}}{ds}\right) di\right] \\
&= t^r(0,0)\left[-\frac{d\ln P_E}{ds} - (1-\iota)(1-\alpha)I_E\right] \\
&= t^r(0,0)\frac{(1-\iota)(1-\alpha)I_E}{(\gamma-1)} > 0
\end{aligned} \tag{98}
$$

$$
\begin{aligned}
\int_0^{t^r(0,0)} \frac{d\ln U^*_E}{ds} dt &= t^r(0,0)\frac{d\ln U^*_E}{ds} \\
&= t^r(0,0)\left[-\frac{d\ln P^*_E}{ds} - \frac{dl^*_E}{ds} - \kappa\int_{\underline{I}}^1 \left(\frac{dY^e_{i,E}}{ds} + \frac{dY^{e*}_{i,E}}{ds}\right) di\right] \\
&= -t^r(0,0)\frac{d\ln P^*_E}{ds} = 0
\end{aligned} \tag{99}
$$

Based on the results from the comparative statics in Table 8, we can determine that $\frac{d\ln U_E}{ds} > 0$ and $\frac{d\ln U^*_E}{ds} = 0$.

## C.2 Earlier Home Innovation Effect

First, the earlier home innovation effect for the home and foreign country can be expressed as follows:

$$
\begin{aligned}
-(\ln U_L - \ln U_E)\frac{dt^r(0,0)}{ds} &= -t^r(0,0)\epsilon_{t^r}(\ln U_L - \ln U_E)\frac{d\ln n_E}{ds} \\
&= -t^r(0,0)\epsilon_{t^r}\left[-\ln P_L + \ln P_E - \kappa(2-\overline{I}-\underline{I})Y^e_i + \kappa(2-2\underline{I})Y^e_i\right]\frac{d\ln n_E}{ds} \\
&= -t^r(0,0)\epsilon_{t^r}\left[-\ln P_L + \ln P_E + \kappa(\overline{I}-\underline{I})Y^e_i\right]\frac{d\ln n_E}{ds}
\end{aligned} \tag{100}
$$

$$
\begin{aligned}
-(\ln U^*_L - \ln U^*_E)\frac{dt^r(s,s^*)}{ds} &= -t^r(0,0)\epsilon_{t^r}(\ln U^*_L - \ln U^*_E)\frac{d\ln n_E}{ds} \\
&= -t^r(0,0)\epsilon_{t^r}\left[-\ln P^*_L + \ln P^*_E - \kappa(1-I_L)Y^e_{i,L} + \kappa(1-I_E)Y^e_{i,E}\right]\frac{d\ln n_E}{ds} \\
&= -t^r(0,0)\epsilon_{t^r}\left[-\ln P^*_L + \ln P^*_E + \kappa(\overline{I}-\underline{I})Y^e_i\right]\frac{d\ln n_E}{ds}
\end{aligned} \tag{101}
$$

To determine the sign of the welfare effect, we must inspect the sign of $-\ln P_L + \ln P_E$ and $-\ln P^*_L + \ln P^*_E$. Using the solutions in Table 1, we can derive the following closed-form solutions for the price of aggregated green capital goods in the early and leading stages for each country:

$$
\ln P^g_E = \ln P^{g*}_E = \frac{1}{1-\gamma}\left(\ln\frac{(1-\iota)(1-\alpha)}{\gamma} + \ln(1+\phi)\underline{I}\right) \tag{102}
$$

$$\ln P_L^g = \frac{1}{1-\gamma}\left(\ln\frac{(1-\iota)(1-\alpha)}{\gamma} + \ln\frac{(\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2)\overline{I}}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}\right) \tag{103}$$

$$\ln P_L^{g*} = \frac{1}{1-\gamma}\left(\ln\frac{(1-\iota)(1-\alpha)}{\gamma} + \ln\frac{(1-\overline{a}_H{}^{\gamma-1}\phi+\overline{a}_H{}^{\gamma-1}\phi^3-\phi^2)\underline{I}}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}\right) \tag{104}$$

where $\overline{a}_H$ represents the productivity level after innovation.

By applying the above closed-form solutions for the price of aggregated green capital goods, $-\ln P_L + \ln P_E$ can be expressed as follows:

$$\begin{aligned}
-\ln P_L + \ln P_E =& \frac{(1-\iota)(1-\alpha)\underline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2)\overline{I}}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi} - \ln(1+\phi)\underline{I}\right)\\
&+ \frac{(1-\iota)(1-\alpha)(\overline{I}-\underline{I})}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2)\overline{I}}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi} - \ln(1+\phi)\underline{I}\right)\\
&+ \frac{(1-\iota)(1-\alpha)(\overline{I}-\underline{I})}{1+\phi}\frac{1}{\gamma-1}\left(\ln\bar{\lambda} - \ln\lambda_{\underline{I}}\right)\\
&+ \frac{(1-\iota)(1-\alpha)\phi\underline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(1-\overline{a}_H{}^{\gamma-1}\phi+\overline{a}_H{}^{\gamma-1}\phi^3-\phi^2)\underline{I}}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi} - \ln(1+\phi)\underline{I}\right)\\
=& \frac{(1-\iota)(1-\alpha)\underline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2)}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi} - \ln(1+\phi)\right)\\
&+ \frac{(1-\iota)(1-\alpha)(\overline{I}-\underline{I})}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2)}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi} - \ln(1+\phi)\right)\\
&+ \frac{(1-\iota)(1-\alpha)(\overline{I}-\underline{I})}{1+\phi}\frac{1}{\gamma-1}\left(\ln\bar{\lambda} - \ln\lambda_{\underline{I}}\right) + \frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\overline{I} - \ln\underline{I}\right)\\
&+ \frac{(1-\iota)(1-\alpha)\phi\underline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(1-\overline{a}_H{}^{\gamma-1}\phi+\overline{a}_H{}^{\gamma-1}\phi^3-\phi^2)}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi} - \ln(1+\phi)\right) > 0
\end{aligned} \tag{105}$$

where $\bar{\lambda} \equiv \frac{\int_{\underline{I}}^{\overline{I}}\lambda_i di}{\overline{I}-\underline{I}}$ represents the average productivity of using green capital for the manufacturing sectors located between $\underline{I}$ and $\overline{I}$.

Given that $\ln\frac{(\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2)}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi} - \ln(1+\phi) > 0$, $\ln\frac{(1-\overline{a}_H{}^{\gamma-1}\phi+\overline{a}_H{}^{\gamma-1}\phi^3-\phi^2)}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi} - \ln(1+\phi) < 0$ and $\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2 > 1-\overline{a}_H{}^{\gamma-1}\phi+\overline{a}_H{}^{\gamma-1}\phi^3-\phi^2$, it can be concluded that the sign of $-\ln P_L + \ln P_E$ is positive.

By using equations (102), (103) and (104), $-\ln P_L^* + \ln P_E^*$ can be written as follows:

$$
\begin{aligned}
-\ln P_L^* + \ln P_E^* &= \frac{(1-\iota)(1-\alpha)\underline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(1-\overline{a}_H{}^{\gamma-1}\phi+\overline{a}_H{}^{\gamma-1}\phi^3-\phi^2)}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}-\ln(1+\phi)\right)\\
&+\frac{(1-\iota)(1-\alpha)\phi\underline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2)}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}-\ln(1+\phi)\right)\\
&+\frac{(1-\iota)(1-\alpha)\phi(\overline{I}-\underline{I})}{1+\phi}\frac{1}{\gamma-1}\left(\ln\frac{(\overline{a}_H{}^{\gamma-1}-\phi+\phi^3-\overline{a}_H{}^{\gamma-1}\phi^2)}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}-\ln(1+\phi)\right)\\
&+\frac{(1-\iota)(1-\alpha)\phi(\overline{I}-\underline{I})}{1+\phi}\frac{1}{\gamma-1}\left(\ln\bar{\lambda}-\ln\lambda_{\underline{I}}\right)+\frac{(1-\iota)(1-\alpha)\phi\overline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\overline{I}-\ln\underline{I}\right)
\end{aligned}
\tag{106}
$$

I take the derivative of $-\ln P_L^* + \ln P_E^*$ with respect to $\overline{a}_H$, and its sign is shown to be positive, as follows.

$$
\begin{aligned}
\frac{d(-\ln P_L^* + \ln P_E^*)}{d\overline{a}_H} &= \frac{(1-\iota)(1-\alpha)\underline{I}}{1+\phi}\left(\frac{-\overline{a}_H{}^{\gamma-2}\phi}{1-\overline{a}_H{}^{\gamma-1}\phi}+\frac{\overline{a}_H{}^{\gamma-2}\phi}{\overline{a}_H{}^{\gamma-1}-\phi}\right)\\
&+\frac{(1-\iota)(1-\alpha)(1+\phi)\underline{I}}{1+\phi}\left(\frac{(\overline{a}_H{}^{\gamma-2}-\overline{a}_H{}^{-\gamma})\phi}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}\right)\\
&+\frac{(1-\iota)(1-\alpha)\phi(\overline{I}-\underline{I})}{1+\phi}\left(\frac{\overline{a}_H{}^{\gamma-2}\phi}{\overline{a}_H{}^{\gamma-1}-\phi}+\frac{(\overline{a}_H{}^{\gamma-2}-\overline{a}_H{}^{-\gamma})\phi}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}\right)\\
&=\frac{(1-\iota)(1-\alpha)(1+\phi)\underline{I}}{1+\phi}\left(\frac{-\overline{a}_H{}^{\gamma-1}\phi(\overline{a}_H{}^{\gamma-1}-1)}{\overline{a}_H(1-\overline{a}_H{}^{\gamma-1}\phi)(\overline{a}_H{}^{\gamma-1}-\phi)}+\frac{(\overline{a}_H{}^{\gamma-1}-\overline{a}_H{}^{1-\gamma})\phi}{\overline{a}_H(1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi)}\right)\\
&+\frac{(1-\iota)(1-\alpha)\phi(\overline{I}-\underline{I})}{1+\phi}\left(\frac{\overline{a}_H{}^{\gamma-2}\phi}{\overline{a}_H{}^{\gamma-1}-\phi}+\frac{(\overline{a}_H{}^{\gamma-2}-\overline{a}_H{}^{-\gamma})\phi}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}\right)\\
&=\frac{(1-\iota)(1-\alpha)\phi(1+\phi)\underline{I}}{1+\phi}\left(\frac{\overline{a}_H{}^{\gamma-1}+1}{\overline{a}_H(1-\overline{a}_H{}^{\gamma-1}\phi)(\overline{a}_H{}^{\gamma-1}-\phi)}\right)\\
&+\frac{(1-\iota)(1-\alpha)\phi(\overline{I}-\underline{I})}{1+\phi}\left(\frac{\overline{a}_H{}^{\gamma-2}\phi}{\overline{a}_H{}^{\gamma-1}-\phi}+\frac{(\overline{a}_H{}^{\gamma-2}-\overline{a}_H{}^{-\gamma})\phi}{1+\phi^2-(\overline{a}_H{}^{\gamma-1}+\overline{a}_H{}^{1-\gamma})\phi}\right)>0
\end{aligned}
\tag{107}
$$

It is easy to demonstrate that $-\ln P_L^* + \ln P_E^* = 0$ when $\overline{a}_H = 1$. Consequently, we can confirm that $-\ln P_L^* + \ln P_E^* > 0$.

Taking into account all the results above, we can demonstrate that the early home innovation effect is positive for both the home and foreign countries.

## C.3 Delayed Foreign Innovation Effect

The delayed foreign innovation effect for the home and foreign country can be expressed as follows:

$$
\begin{aligned}
-(\ln U_M - \ln U_L)\frac{dt^{r*}(s,s^*)}{ds} &= -t^r(0,0)\epsilon_{t^r}(\ln U_M - \ln U_L)\frac{d\ln n_E^*}{ds} \\
&= -t^r(0,0)\epsilon_{t^r}\left[-\ln P_M + \ln P_L - \kappa(2-2\overline{I})Y_i^e + \kappa(2-\overline{I}-\underline{I})Y_i^e\right]\frac{d\ln n_E^*}{ds} \\
&= -t^r(0,0)\epsilon_{t^r}\left[-\ln P_M + \ln P_L + \kappa(\overline{I}-\underline{I})Y_i^e\right]\frac{d\ln n_E^*}{ds}
\end{aligned} \tag{108}
$$

$$
\begin{aligned}
-(\ln U_M^* - \ln U_L^*)\frac{dt^{r}(s,s^*)}{ds} &= -t^r(0,0)\epsilon_{t^r}(\ln U_M^* - \ln U_L^*)\frac{d\ln n_E^*}{ds} \\
&= -t^r(0,0)\epsilon_{t^r}\left[-\ln P_M^* + \ln P_L^* - \kappa(2-2\overline{I})Y_i^e + \kappa(2-\overline{I}-\underline{I})Y_i^e\right]\frac{d\ln n_E^*}{ds} \\
&= -t^r(0,0)\epsilon_{t^r}\left[-\ln P_M^* + \ln P_L^* + \kappa(\overline{I}-\underline{I})Y_i^e\right]\frac{d\ln n_E^*}{ds}
\end{aligned} \tag{109}
$$

Again, to determine the sign of the delayed foreign innovation effect, we need to inspect the signs of $-\ln P_M + \ln P_L$ and $-\ln P_M^* + \ln P_L^*$. Using the solutions from Table 1, we can derive the following closed-form solutions for the prices of aggregated green capital goods in the leading and mature stages for each country:

$$
\ln P_L^g = \frac{1}{1-\gamma}\left(\ln\frac{(1-\iota)(1-\alpha)}{\gamma} + \ln\frac{(\overline{a}_H{}^{\gamma-1} - \phi + \phi^3 - \overline{a}_H{}^{\gamma-1}\phi^2)\overline{I}}{1+\phi^2 - (\overline{a}_H{}^{\gamma-1} + \overline{a}_H{}^{1-\gamma})\phi}\right) \tag{110}
$$

$$
\ln P_L^{g*} = \frac{1}{1-\gamma}\left(\ln\frac{(1-\iota)(1-\alpha)}{\gamma} + \ln\frac{(1 - \overline{a}_H{}^{\gamma-1}\phi + \overline{a}_H{}^{\gamma-1}\phi^3 - \phi^2)\underline{I}}{1+\phi^2 - (\overline{a}_H{}^{\gamma-1} + \overline{a}_H{}^{1-\gamma})\phi}\right) \tag{111}
$$

$$
\ln P_M^g = \ln P_M^{g*} = \frac{1}{1-\gamma}\left(\ln\frac{(1-\iota)(1-\alpha)}{\gamma} + \ln(1+\phi)\overline{a}_H{}^{\gamma-1}\overline{I}\right) \tag{112}
$$

By applying the above closed-form solutions for the price of aggregated green capital goods, $-\ln P_M + \ln P_L$ can be expressed as follows:

$$
\begin{aligned}
-\ln P_M + \ln P_L = &\frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln(1+\phi)\overline{a}_H{}^{\gamma-1}\overline{I} - \ln\frac{(\overline{a}_H{}^{\gamma-1} - \phi + \phi^3 - \overline{a}_H{}^{\gamma-1}\phi^2)\overline{I}}{1+\phi^2 - (\overline{a}_H{}^{\gamma-1} + \overline{a}_H{}^{1-\gamma})\phi}\right) \\
&+ \frac{(1-\iota)(1-\alpha)\phi\overline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln(1+\phi)\overline{a}_H{}^{\gamma-1}\overline{I} - \ln\frac{(1 - \overline{a}_H{}^{\gamma-1}\phi + \overline{a}_H{}^{\gamma-1}\phi^3 - \phi^2)\underline{I}}{1+\phi^2 - (\overline{a}_H{}^{\gamma-1} + \overline{a}_H{}^{1-\gamma})\phi}\right) \\
&+ \frac{(1-\iota)(1-\alpha)\phi(\overline{I}-\underline{I})}{1+\phi}\left(\ln\bar{\lambda} - \ln\lambda_{\underline{I}}\right)
\end{aligned} \tag{113}
$$

I take the derivative of $-\ln P_M + \ln P_L$ with respect to $\overline{a}_H$, and its sign is shown to be negative, as follows.

$$\begin{aligned}
\frac{d(-\ln P_M+\ln P_L)}{d\overline{a}_H} &= (1-\iota)(1-\alpha)\overline{I}\left(\frac{1}{\overline{a}_H}-\frac{({\overline{a}_H}^{\gamma-2}-{\overline{a}_H}^{-\gamma})\phi}{1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi}\right)\\
&+\frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\left(-\frac{{\overline{a}_H}^{\gamma-2}}{{\overline{a}_H}^{\gamma-1}-\phi}+\frac{{\overline{a}_H}^{\gamma-2}\phi^2}{1-{\overline{a}_H}^{\gamma-1}\phi}\right)\\
&=\frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\left(\frac{(1+\phi)(1+\phi^2-2{\overline{a}_H}^{\gamma-1}\phi)}{\overline{a}_H(1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi)}\right.\\
&\left.-\frac{{\overline{a}_H}^{\gamma-2}(1-{\overline{a}_H}^{\gamma-1}\phi-({\overline{a}_H}^{\gamma-1}-\phi)\phi^2)}{({\overline{a}_H}^{\gamma-1}-\phi)(1-{\overline{a}_H}^{\gamma-1}\phi)}\right)\\
&=\frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\left(\frac{(1+\phi)(1-{\overline{a}_H}^{\gamma-1}\phi-({\overline{a}_H}^{\gamma-1}-\phi)\phi)}{\overline{a}_H(1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi)}\right.\\
&\left.-\frac{{\overline{a}_H}^{\gamma-1}(1-{\overline{a}_H}^{\gamma-1}\phi-({\overline{a}_H}^{\gamma-1}-\phi)\phi^2)}{\overline{a}_H({\overline{a}_H}^{\gamma-1}-\phi)(1-{\overline{a}_H}^{\gamma-1}\phi)}\right)\\
&=\frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\left(\frac{(1+\phi)(1-{\overline{a}_H}^{\gamma-1}\phi-({\overline{a}_H}^{\gamma-1}-\phi)\phi)}{\overline{a}_H(1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi)}\right.\\
&\left.-\frac{{\overline{a}_H}^{\gamma-1}(1-{\overline{a}_H}^{\gamma-1}\phi-({\overline{a}_H}^{\gamma-1}-\phi)\phi)}{\overline{a}_H({\overline{a}_H}^{\gamma-1}-\phi)(1-{\overline{a}_H}^{\gamma-1}\phi)}-\frac{{\overline{a}_H}^{\gamma-1}(\phi-\phi^2)}{\overline{a}_H(1-{\overline{a}_H}^{\gamma-1}\phi)}\right)\\
&=\frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\left(-\frac{(1-{\overline{a}_H}^{\gamma-1}\phi-({\overline{a}_H}^{\gamma-1}-\phi)\phi)(1+\phi)\phi^2({\overline{a}_H}^{\gamma-1}-1)^2}{\overline{a}_H(1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi)({\overline{a}_H}^{\gamma-1}-\phi)(1-{\overline{a}_H}^{\gamma-1}\phi)}\right.\\
&\left.+\frac{{\overline{a}_H}^{\gamma-1}\phi(1-\phi)^2(1-{\overline{a}_H}^{\gamma-1}\phi-({\overline{a}_H}^{\gamma-1}-\phi)\phi)}{\overline{a}_H(1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi)({\overline{a}_H}^{\gamma-1}-\phi)(1-{\overline{a}_H}^{\gamma-1}\phi)}-\frac{{\overline{a}_H}^{\gamma-1}(\phi-\phi^2)}{\overline{a}_H(1-{\overline{a}_H}^{\gamma-1}\phi)}\right)<0
\end{aligned} \tag{114}$$

It is easy to demonstrate that $-\ln P_M+\ln P_L=0$ when $\overline{a}_H=1$. Consequently, we can confirm that $-\ln P_M+\ln P_L<0$.

By using equations (103), (104) and (112), $-\ln P_L^*+\ln P_E^*$ can be written as follows:

$$\begin{aligned}
-\ln P_M^*+\ln P_L^* &= \frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln{(1+\phi)}{\overline{a}_H}^{\gamma-1}-\ln\frac{1-{\overline{a}_H}^{\gamma-1}\phi+{\overline{a}_H}^{\gamma-1}\phi^3-\phi^2}{1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi}\right)\\
&+\frac{(1-\iota)(1-\alpha)(\overline{I}-\underline{I})}{1+\phi}\left(\ln\bar{\lambda}-\ln\lambda_{\underline{I}}\right)+\frac{(1-\iota)(1-\alpha)\overline{I}}{1+\phi}\frac{1}{\gamma-1}\left(\ln\overline{I}-\ln\underline{I}\right)\\
&+\frac{(1-\iota)(1-\alpha)\phi\underline{I}}{1+\phi}\frac{1}{1-\gamma}\left(\ln{(1+\phi)}{\overline{a}_H}^{\gamma-1}-\ln\frac{{\overline{a}_H}^{\gamma-1}-\phi+\phi^3-{\overline{a}_H}^{\gamma-1}\phi^2}{1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi}\right)>0
\end{aligned} \tag{115}$$

Given that $\ln{(1+\phi)}{\overline{a}_H}^{\gamma-1}-\ln\frac{1-{\overline{a}_H}^{\gamma-1}\phi+{\overline{a}_H}^{\gamma-1}\phi^3-\phi^2}{1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi}>0$, $\ln{(1+\phi)}{\overline{a}_H}^{\gamma-1}-\ln\frac{{\overline{a}_H}^{\gamma-1}-\phi+\phi^3-{\overline{a}_H}^{\gamma-1}\phi^2}{1+\phi^2-({\overline{a}_H}^{\gamma-1}+{\overline{a}_H}^{1-\gamma})\phi}<0$ and ${\overline{a}_H}^{\gamma-1}-\phi+\phi^3-{\overline{a}_H}^{\gamma-1}\phi^2>1-{\overline{a}_H}^{\gamma-1}\phi+{\overline{a}_H}^{\gamma-1}\phi^3-\phi^2$, it can be concluded that the sign of $-\ln P_M^*+\ln P_L^*$ is positive.

Overall, the delayed foreign innovation effect is ambiguous for the home country since it benefits from reduced greenhouse gas emissions from the foreign manufacturing sectors, but it suffers losses from the Utility-based CPI. In contrast, the delayed foreign innovation effect reduces foreign welfare.

# D Comparative Statics for Non-cooperative Change in $s$ under $s = s^*$

As shown in Appendix A, an equilibrium with a production subsidy at each period is represented by the set $\{n_t, n_{1,t}^*\}$, which satisfies the zero-profit condition for both the domestic and foreign green capital goods firms.

I assume a symmetric condition where $v = v^* = L = L^* = a_t(\omega) = a_t^*(\omega) = 1$, $I_t = I_t^* = I_E$ and $s = s^*$. Given these conditions, there is a symmetric equilibrium such that $n_t = n_t^*$, $l_t = l_t^* = \gamma n_t = \gamma n_t^* = 1$. I then take a first-order approximation of this model in the neighborhood of this symmetric equilibrium and analyze the local effects of production subsidy in the early stage.

For computational convenience, I extend the system of equations describing equilibrium to include 6 equations with 6 endogenous variables. These variables include the mass of firms in both the domestic and foreign green capital goods sectors, $\{n_t, n_t^*\}$, the price of aggregated green capital goods, $\{P_t^z, P_t^{z*}\}$, and the utility-based CPI, $\{P_t, P_t^*\}$. I take a first-order approximation of the system with respect to $s$ and obtain the following equations.

$$(78):\ \frac{\gamma(1+\phi)}{1+s} + (\gamma-1)\frac{d\ln P_E^g}{ds} + (\gamma-1)\phi\frac{d\ln P_E^{g*}}{ds} = 0 \tag{116}$$

$$(79):\ (\gamma-1)\frac{d\ln P_E^{g*}}{ds} + (\gamma-1)\phi\frac{d\ln P_E^g}{ds} = 0 \tag{117}$$

$$(53):\ (1-\gamma)(1+\phi)\frac{d\ln P_E^g}{ds} = \frac{d\ln n_E}{ds} - \frac{1-\gamma}{1+s} + \phi\frac{d\ln n_E^*}{ds} \tag{118}$$

$$(54):\ (1-\gamma)(1+\phi)\frac{d\ln P_E^{g*}}{ds} = \frac{d\ln n_E^*}{ds} + \phi\frac{d\ln n_E}{ds} - \frac{(1-\gamma)\phi}{1+s} \tag{119}$$

$$(61):\ \frac{d\ln P_E}{ds} = (1-\iota)(1-\alpha)\beta I_E\frac{d\ln P_E^g}{ds} + (1-\iota)(1-\alpha)(1-\beta)I_E\frac{d\ln P_E^{g*}}{ds} \tag{120}$$

$$(62):\ \frac{d\ln P_E^*}{ds} = (1-\iota)(1-\alpha)\beta I_E\frac{d\ln P_E^{g*}}{ds} + (1-\iota)(1-\alpha)(1-\beta)I_E\frac{d\ln P_E^g}{ds} \tag{121}$$

By solving the aforementioned six equations, I obtain the solutions presented in Table 8.

Table 8: Comparative Statics for Non-cooperative Change in $s$ under $s = s^*$

$$\frac{d\ln n_E}{ds} = \frac{1}{1+s}\left[1 + \frac{2\gamma\phi}{(1-\phi)^2}\right] > 0 \tag{122}$$

$$\frac{d\ln n_E^*}{ds} = -\frac{1}{1+s}\frac{2\gamma\phi}{(1-\phi)^2} < 0 \tag{123}$$

$$\frac{d\ln P_E^g}{ds} = -\frac{1}{1+s}\frac{\gamma}{(\gamma-1)(1-\phi)} < 0 \tag{124}$$

$$\frac{d\ln P_E^{g*}}{ds} = \frac{1}{1+s}\frac{\gamma\phi}{(\gamma-1)(1-\phi)} > 0 \tag{125}$$

$$\frac{d\ln P_E}{ds} = -\frac{1}{1+s}\frac{\gamma(1-\iota)(1-\alpha)I_E}{(\gamma-1)} < 0 \tag{126}$$

$$\frac{d\ln P_E^*}{ds} = 0 \tag{127}$$

$$\frac{d\ln Y_{i,E}^g}{ds} = -(1-\alpha)\frac{d\ln P_E^g}{ds} = \frac{1}{1+s}\frac{\gamma(1-\alpha)}{(\gamma-1)(1-\phi)} > 0 \tag{128}$$

$$\frac{d\ln Y_{i,E}^e}{ds} = -(1-\alpha)\frac{d\ln \psi_E}{ds} = 0 \tag{129}$$

$$\frac{d\ln Y_{i,E}^{g*}}{ds} = -(1-\alpha)\frac{d\ln P_E^{g*}}{ds} = -\frac{1}{1+s}\frac{\gamma\phi(1-\alpha)}{(\gamma-1)(1-\phi)} < 0 \tag{130}$$

$$\frac{d\ln Y_{i,E}^{e*}}{ds} = -(1-\alpha)\frac{d\ln \psi_E^*}{ds} = 0 \tag{131}$$

# E Comparative Statics for Cooperative Change in $s$ under $s = s^*$

In this Appendix, I analyze the local effects of a production subsidy in the early stage when both the home and foreign countries cooperatively adjust their production subsidies to maximize joint welfare ($s = s^* = \bar{s}$). In this scenario, the system for the first-order approximation is modified as follows.

$$(78)\text{:}\ \frac{\gamma(1+\phi)}{1+s}+(\gamma-1)\frac{d\ln P_E^g}{d\bar{s}}+(\gamma-1)\phi\frac{d\ln P_E^{g*}}{d\bar{s}}=0 \tag{132}$$

$$(79)\text{:}\ \frac{\gamma(1+\phi)}{1+s}+(\gamma-1)\frac{d\ln P_E^{g*}}{d\bar{s}}+(\gamma-1)\phi\frac{d\ln P_E^{g}}{d\bar{s}}=0 \tag{133}$$

$$(53)\text{:}\ (1-\gamma)(1+\phi)\frac{d\ln P_E^g}{d\bar{s}}=\frac{d\ln n_E}{d\bar{s}}+\phi\frac{d\ln n_E^*}{d\bar{s}}-\frac{(1-\gamma)(1+\phi)}{1+s} \tag{134}$$

$$(54)\text{:}\ (1-\gamma)(1+\phi)\frac{d\ln P_E^{g*}}{d\bar{s}}=\frac{d\ln n_E^*}{d\bar{s}}+\phi\frac{d\ln n_E}{d\bar{s}}-\frac{(1-\gamma)(1+\phi)}{1+s} \tag{135}$$

$$(61)\text{:}\ \frac{d\ln P_E}{d\bar{s}}=(1-\iota)(1-\alpha)\beta I_E\frac{d\ln P_E^g}{d\bar{s}}+(1-\iota)(1-\alpha)(1-\beta)I_E\frac{d\ln P_E^{g*}}{d\bar{s}} \tag{136}$$

$$(62)\text{:}\ \frac{d\ln P_E^*}{d\bar{s}}=(1-\iota)(1-\alpha)\beta I_E\frac{d\ln P_E^{g*}}{d\bar{s}}+(1-\iota)(1-\alpha)(1-\beta)I_E\frac{d\ln P_E^{g}}{d\bar{s}} \tag{137}$$

By solving the aforementioned six equations, I obtain the solutions presented in Table 9.

Table 9: Comparative Statics for Cooperative Change in $s$ under $s = s^*$

$$\frac{d\ln n_E}{ds^c}=\frac{d\ln n_E^*}{ds^c}=\frac{1}{1+s}>0 \tag{138}$$

$$\frac{d\ln P_E^g}{ds}=\frac{d\ln P_E^{g*}}{ds}=-\frac{1}{1+s}\frac{\gamma}{\gamma-1}<0 \tag{139}$$

$$\frac{d\ln P_E}{ds}=\frac{d\ln P_E^*}{ds}=-\frac{1}{1+s}\frac{\gamma(1-\iota)(1-\alpha)I_E}{\gamma-1}<0 \tag{140}$$

$$\frac{d\ln Y_{i,E}^g}{ds}=\frac{d\ln Y_{i,E}^{g*}}{ds}=\frac{1}{1+s}\frac{\gamma(1-\alpha)}{\gamma-1}>0 \tag{141}$$

$$\frac{d\ln Y_{i,E}^e}{ds}=\frac{d\ln Y_{i,E}^{e*}}{ds}=0 \tag{142}$$

# F Country List Used for Innovation Timing Elasticity Estimation

I provide a list of countries used for estimating innovation timing elasticity in Table 10. These countries account for 98.7% of the total value added in the UNIDO Industrial Statistics Database for the year 2020.

Table 10: Country List Used for Innovation Timing Elasticity Estimation

| | | | |
|---|---|---|---|
| Algeria | Argentina | Australia | Austria |
| Bangladesh | Belarus | Belgium | Brazil |
| Canada | Chile | China | Hong Kong |
| Taiwan | Colombia | Croatia | Czechia |
| Denmark | Egypt | Estonia | Finland |
| France | Germany | Ghana | Greece |
| Hungary | Iceland | India | Indonesia |
| Iran (Islamic Republic of) | Ireland | Israel | Italy |
| Japan | Kazakhstan | Lithuania | Luxembourg |
| Malaysia | Mexico | Morocco | Netherlands |
| New Zealand | Norway | Pakistan | Peru |
| Philippines | Poland | Portugal | Puerto Rico |
| Qatar | Republic of Korea | Romania | Russian Federation |
| Saudi Arabia | Singapore | Slovakia | Slovenia |
| South Africa | Spain | Sweden | Switzerland |
| Türkiye | Thailand | Ukraine | United Arab Emirates |
| United Kingdom | United States of America | Viet Nam | |

# G Innovation Indicators

I present innovation indicators for the U.S. manufacturing industries in Figure 4. Data for other countries are available upon request.

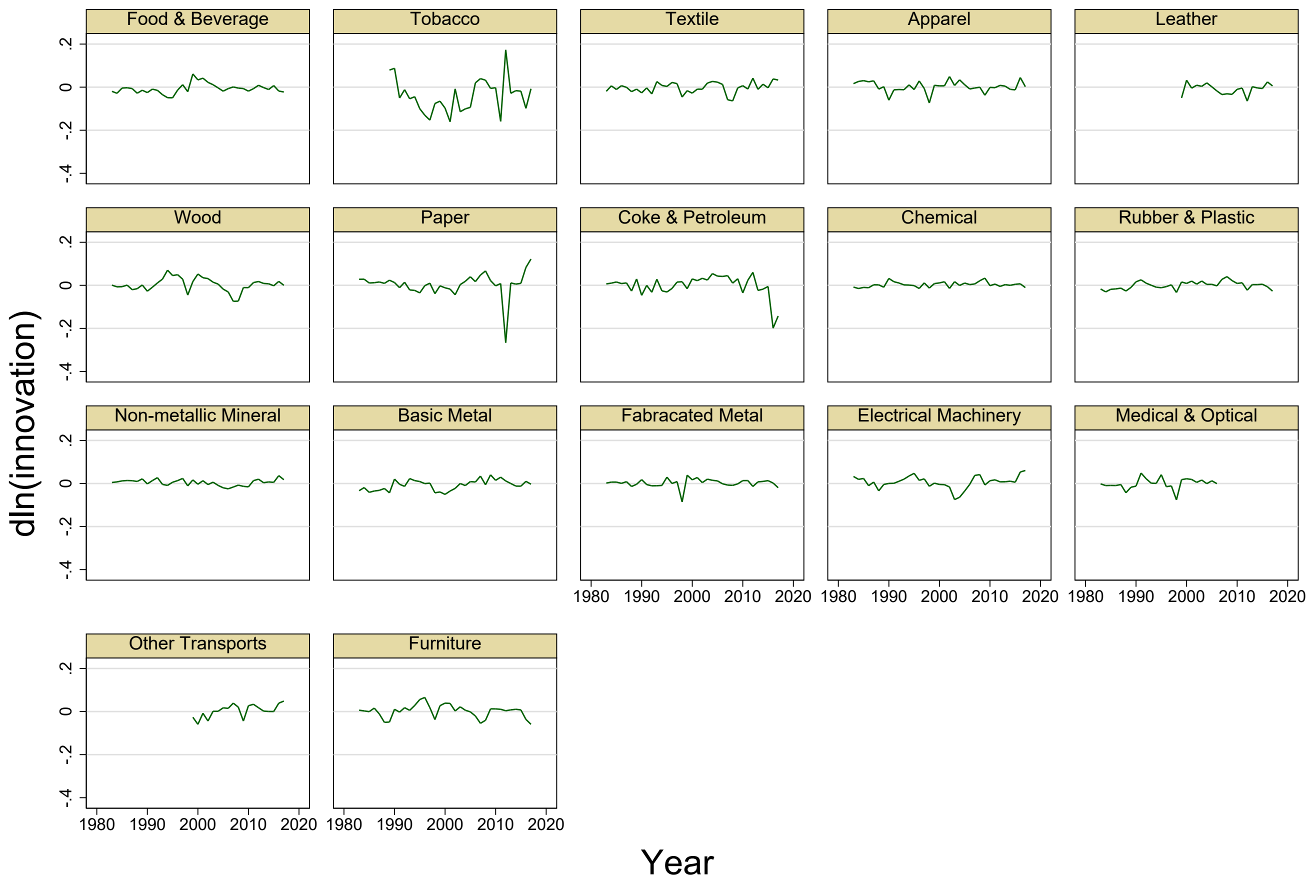


Figure 4: Innovation Indicators for the US Manufacturing Industries

# H Generalized Model

In the generalized model, I make four changes. First, I introduce a conventional capital goods sector in which firms produce capital goods that are used with conventional energy. Similar to the green capital goods sector, the production function of a firm producing a variety $\omega^e$ in this sector takes the following form.

$$y_t^e(\omega^e) = a_t^e(\omega^e) l_t(\omega^e) \tag{143}$$

Conventional capital goods are aggregated using a CES aggregator, similar to the green capital goods sector, as follows.

$$Z_t^e = \left( \int_0^{n_t^e} z_t^e(\omega^e)^{\frac{\gamma-1}{\gamma}} d\omega^e + \int_0^{n_t^{e*}} z_t^e(\omega_f^e)^{\frac{\gamma-1}{\gamma}} d\omega_f^e \right)^{\frac{\gamma}{\gamma-1}} \tag{144}$$

$$Z_t^{e*} = \left( \int_0^{n_t^{e*}} z_t^{e*}(\omega_f^e)^{\frac{\gamma-1}{\gamma}} d\omega_f^e + \int_0^{n_t^e} z_t^{e*}(\omega^e)^{\frac{\gamma-1}{\gamma}} d\omega^e \right)^{\frac{\gamma}{\gamma-1}} \tag{145}$$

where $\gamma$ represents the elasticity of substitution between varieties. I set $\gamma$ to the same value as that in the green capital goods sector.

As in Hötte (2020), it can be assumed that if a final goods firm uses conventional capital, it should also use a natural resource as an additional input. Assuming that $\psi$ units of labor are required to obtain a unit of natural resource, $N_{i,t}$, and given the production function $Y_{i,t}^e = A_{i,t} {L_{i,t}^e}^{\alpha\xi} N_{i,t}^{\alpha(1-\xi)} {Z_{i,t}^e}^{1-\alpha}$, the function can be rewritten as $Y_{i,t}^e = A_{i,t} {L_{i,t}^e}^{\alpha\xi} \left(\frac{L_{i,t}^e}{\psi}\right)^{\alpha(1-\xi)} {Z_{i,t}^e}^{1-\alpha} = \frac{A_{i,t}}{\psi^{\alpha(1-\xi)}} {L_{i,t}^e}^{\alpha} {Z_{i,t}^e}^{1-\alpha}$. Thus, the production function with conventional capital retains a similar form as that with green capital, as follows.

$$Y_{i,t}^e = A_{i,t} \left(L_{i,t}^e\right)^{\alpha} \left(Z_{i,t}^e\right)^{1-\alpha} \tag{146}$$

Second, I ensure that the green energy usage ratios, $I_t$ and $I_t^*$, are endogenously determined at all stages. To account for this, I introduce the following productivity structure for green energy adoption:

$$\lambda_i = \bar{\lambda}(1-i)^{\sigma} \tag{147}$$

where $\bar{\lambda}$ is a constant, and $\sigma$ is the parameter governing the degree of increasing difficulty in green energy adoption as the industry index $i$ increases.

Third, I exclude agricultural final goods so that the relative wage varies due to production subsidies or productivity increases. Accordingly, the balance of payment equilibrium condition changes as follows:

$$\begin{aligned} &P_{h,t}^M C_{h,t}^{M*} + (1+\tau)p_t^g z_t^*(\omega^g)n_t^g + (1+\tau)p_t^e z_t^*(\omega^e)n_t^e + D \\ &= e_t\left[P_{f,t}^{M*} C_{f,t}^M + (1+\tau)p_t^{g*} z_t(\omega_f^g)n_t^{g*} + (1+\tau)p_t^{e*} z_t(\omega_f^e)n_t^{e*}\right] \end{aligned} \tag{148}$$

where D represents the trade deficit of the home country. As in Bartelme et al. (2021), I assume the trade deficit does not change as we move to the counterfactual equilibrium.

Lastly, manufacturing final goods are aggregated using a CES function instead of a Cobb-Douglas function, as follows:

$$C_t^M = \left( {C_{h,t}^M}^{\frac{\eta-1}{\eta}} + {C_{f,t}^M}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{149}$$

where $\eta$ represents the elasticity of substitution between the home and foreign aggregated manufacturing final goods.

# I Exact Hat Algebra for Counterfactual Analysis

The relevant equations for the welfare change caused by production subsidies or productivity changes are as follows: the zero-profit conditions for home and foreign green capital firms (equations (78) and (79)); the zero-profit conditions for home and foreign conventional capital firms; the aggregated green capital goods prices in the home and foreign countries (equations (53) and (54)); the aggregated conventional capital goods prices in the home and foreign countries (equations (144) and (145)); tax in the home and foreign country (equations (85) and (86)); the utility-based Consumer Price Index (CPI) in the home and foreign countries (equations (61) and (62)); the equations for green energy usage determination in both the home and foreign countries; and the welfare function for both the home and foreign countries. Accordingly, the following equations are used for counterfactual changes in the early stage.

$$(1+s)^{\gamma}\left[(\hat{P^g})^{\gamma-1}(\hat{P^M})^{1-\eta}(\hat{P})^{\eta-1}\hat{I}s^{zp}_{hh}+\hat{e}^{\gamma}(\hat{P^g})^{\gamma-1}(\hat{P^{M*}})^{1-\eta}(\hat{P^*})^{\eta-1}\hat{I}^*(1-s^{zp}_{hh})\right]=1 \tag{150}$$

$$(\hat{P^e})^{\gamma-1}(\hat{P^M})^{\eta-1}(\hat{P})^{\eta-1}\left(\frac{1-\hat{I}I_0}{1-I_0}\right)s^{zp}_{hh}+\hat{e}^{\gamma}(\hat{P^e})^{\gamma-1}(\hat{P^{M*}})^{\gamma-1}(\hat{P^*})^{\gamma-1}\left(\frac{1-\hat{I}^*I_0^*}{1-I_0^*}\right)(1-s^{zp}_{hh})=1 \tag{151}$$

$$(1+s^*)^{\gamma}\left[(\hat{P^{g*}})^{\gamma-1}(\hat{P^{M*}})^{1-\eta}(\hat{P^*})^{\eta-1}\hat{I}^*s^{zp}_{ff}+\hat{e}^{-\gamma}(\hat{P^{g*}})^{\gamma-1}(\hat{P^M})^{1-\eta}(\hat{P})^{\eta-1}\hat{I}(1-s^{zp}_{ff})\right]=1 \tag{152}$$

$$(\hat{P^{e*}})^{\gamma-1}(\hat{P^{M*}})^{1-\eta}(\hat{P^*})^{\eta-1}\left(\frac{1-\hat{I}^*I_0^*}{1-I_0^*}\right)s^{zp}_{ff}+\hat{e}^{-\gamma}(\hat{P^{e*}})^{\gamma-1}(\hat{P^M})^{1-\eta}(\hat{P})^{\eta-1}\left(\frac{1-\hat{I}I_0}{1-I_0}\right)(1-s^{zp}_{ff})=1 \tag{153}$$

$$(\hat{P^g})^{1-\gamma}=(1+s)^{\gamma-1}\hat{n^g}s^{zc}_{hh}+(1+s^*)^{\gamma-1}\hat{e}^{1-\gamma}\hat{n^{g*}}(1-s^{zc}_{hh}) \tag{154}$$

$$(\hat{P^e})^{1-\gamma}=\hat{n^e}s^{zc}_{hh}+\hat{e}^{1-\gamma}\hat{n^{e*}}(1-s^{zc}_{hh}) \tag{155}$$

$$(\hat{P^{g*}})^{1-\gamma}=(1+s^*)^{\gamma-1}\hat{n^{g*}}s^{zc}_{ff}+(1+s)^{\gamma-1}\hat{e}^{\gamma-1}\hat{n^g}(1-s^{zc}_{ff}) \tag{156}$$

$$(\hat{P^{e*}})^{1-\gamma}=\hat{n^{e*}}s^{zc}_{ff}+\hat{e}^{\gamma-1}\hat{n^e}(1-s^{zc}_{ff}) \tag{157}$$

$$\hat{C}=\frac{(1-I_0\hat{I})\left[1-\left(\frac{1-I_0\hat{I}}{1-I_0}\right)^{\sigma-1}\right]}{(\sigma-1)I_0(\hat{I}-1)}\hat{P^e} \tag{158}$$

$$\hat{C^*}=\frac{(1-I_0^*\hat{I}^*)\left[1-\left(\frac{1-I_0^*\hat{I}^*}{1-I_0^*}\right)^{\sigma-1}\right]}{(\sigma-1)I_0^*(\hat{I}^*-1)}\hat{P^{e*}} \tag{159}$$

$$\hat{P^M_h}=(\hat{P^g})^{(1-\alpha)I_0}(\hat{C})^{(1-\alpha)(\hat{I}-1)I_0}(\hat{P^e})^{(1-\alpha)(1-I_0\hat{I})} \tag{160}$$

$$\hat{P_f^{M*}} = (\hat{P^{g*}})^{(1-\alpha)I_0^*}(\hat{C^*})^{(1-\alpha)(\hat{I^*}-1)I_0^*}(\hat{P^{e*}})^{(1-\alpha)(1-I_0\hat{I^*})} \tag{161}$$

$$\hat{P}^{1-\eta} = \hat{P_h^M}^{1-\eta} s_{hh}^{fc} + \hat{e}^{1-\eta}\hat{P_f^{M*}}^{1-\eta}(1-s_{hh}^{fc}) \tag{162}$$

$$\hat{P^*}^{1-\eta} = \hat{P_f^{M*}}^{1-\eta} s_{ff}^{fc} + \hat{e}^{\eta-1}\hat{P_h^M}^{1-\eta}(1-s_{ff}^{fc}) \tag{163}$$

$$\hat{P^g} = \left(\frac{1-\hat{I}I_0}{1-I_0}\right)^{\sigma}\hat{P^e} \tag{164}$$

$$\hat{P^{g*}} = \left(\frac{1-\hat{I^*}I_0^*}{1-I_0^*}\right)^{\sigma}\hat{P^{e*}} \tag{165}$$

$$\begin{aligned}
&\hat{e}^{\gamma}(\hat{P^g})^{\gamma-1}(\hat{P_f^{M*}})^{1-\eta}(\hat{P^*})^{\eta-1}\hat{I^*}s_h^{ge} + \hat{e}^{\gamma}(\hat{P^e})^{\gamma-1}(\hat{P_f^{M*}})^{1-\eta}(\hat{P^*})^{\eta-1}\left(\frac{1-\hat{I^*}I_0^*}{1-I_0^*}\right)s_h^{ee}+ \\
&+\hat{e}^{\eta}(\hat{P_h^M})^{1-\eta}(\hat{P^*})^{\eta-1}s_h^{fe} + (1-s_h^{ge}-s_h^{ee}-s_h^{fe}) \\
&= \hat{e}^{1-\gamma}(\hat{P^{g*}})^{\gamma-1}(\hat{P_h^M})^{1-\eta}(\hat{P})^{\eta-1}\hat{I}s_f^{ge} + \hat{e}^{1-\gamma}(\hat{P^{e*}})^{\gamma-1}(\hat{P_h^M})^{1-\eta}(\hat{P})^{\eta-1}\left(\frac{1-\hat{I}I_0}{1-I_0}\right)s_f^{ee} \\
&+\hat{e}^{1-\eta}(\hat{P_f^{M*}})^{1-\eta}(\hat{P})^{\eta-1}(1-s_f^{ge}-s_f^{ee})
\end{aligned} \tag{166}$$

$$T_E = \frac{s}{1+s}(1-\alpha)I_0\hat{n} \tag{167}$$

$$T_E^* = \frac{s^*}{1+s^*}(1-\alpha)I_0^*\hat{n^*} \tag{168}$$

$$\hat{U_E} = \hat{P}^{-1}e^{-T_E}N^{\frac{2-\hat{I}I_0-\hat{I^*}I_0^*}{2-I_0-I_0^*}-1} \tag{169}$$

$$\hat{U_E^*} = \hat{P^*}^{-1}e^{-T_E^*}N^{\frac{2-\hat{I}I_0-\hat{I^*}I_0^*}{2-I_0-I_0^*}-1} \tag{170}$$

where $\hat{C}$ and $\hat{C^*}$ represent the average change in price in sectors that use conventional capital in the initial equilibrium but change their production method because using green capital became cheaper, in the home and foreign country, respectively. In addition, $s_h^{ge}$, $s_h^{ee}$ and $s_h^{fe}$ denote the share of export value from the green capital goods, the conventional capital goods, and the manufacturing final goods out of total export value in the home country, respectively. Accordingly, $1-s_h^{ge}-s_h^{ee}-s_h^{fe}$ represents the share of the trade deficit out of the total export value in the home country.

For the assessment of welfare changes in both the leading and mature stages relative to the initial welfare level, four equations are modified. Specifically, equations (150), (152), (154), and (156) change as follows to calculate welfare changes in the leading and mature stages compared to the welfare level in the initial equilibrium.

$$a_g^{\gamma-1}\left[(\hat{P^g})^{\gamma-1}(\hat{P^M})^{1-\eta}(\hat{P})^{\eta-1}\hat{I}s_{hh}^{zp} + \hat{e}^{\gamma}(\hat{P^g})^{\gamma-1}(\hat{P^{M*}})^{1-\eta}(\hat{P^*})^{\eta-1}\hat{I^*}(1-s_{hh}^{zp})\right] = 1 \tag{171}$$

$$a_g^{*\gamma-1}\left[(\hat{P^{g*}})^{\gamma-1}(\hat{P^{M*}})^{1-\eta}(\hat{P^*})^{\eta-1}\hat{I^*}s_{ff}^{zp} + \hat{e}^{-\gamma}(\hat{P^{g*}})^{\gamma-1}(\hat{P^M})^{1-\eta}(\hat{P})^{\eta-1}\hat{I}(1-s_{ff}^{zp})\right] = 1 \tag{172}$$

$$(\hat{P^g})^{1-\gamma} = a_g^{\gamma-1}\hat{n^{g*}}s_{ff}^{zc} + a_g^{*\gamma-1}\hat{e}^{1-\gamma}\hat{n^g}(1-s_{ff}^{zc}) \tag{173}$$

$$(\hat{P^{g*}})^{1-\gamma} = a_g^{*\gamma-1}\hat{n^{g*}}s_{ff}^{zc} + a_g^{\gamma-1}\hat{e}^{\gamma-1}\hat{n^g}(1-s_{ff}^{zc}) \tag{174}$$

Additionally, since the government no longer provides a production subsidy at these stages, equations (167) and (168) are not used.

Lastly, to compute the welfare change from the leading stage to the mature stage, I divide the welfare change from the initial equilibrium to the mature stage by the welfare change from the initial equilibrium to the leading stage ($\frac{\hat{U_M}}{\hat{U_L}}$).