Industrial Policy in the context of Industry Lifecycle: Catch-Up versus Frontier Technology Races

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Abstract

The welfare effects of industrial policy can vary depending where the targeted industry happens to be in its lifecycle. In this study, I develop an open economy macroeconomic model incorporating industry lifecycle theory to investigate how the timing of industrial policy affects innovation and welfare in both the home and foreign countries. The model provides distinct welfare implications in two scenarios: *catch-up* and *frontier technology races*. In the former scenario, the targeted industry is nascent with high growth potential at home, but mature abroad. In contrast, in the latter scenario, both the home and foreign industries have high growth potential and are in competition with each other. For the home country, a production subsidy accelerates innovation in the targeted industry and thus can enhance welfare in both scenarios, despite a trade-off between short-term losses and long-term gains. For the foreign country, in the catch-up scenario, a home production subsidy unambiguously increases foreign welfare. Conversely, in the scenario of frontier technology races, it may induce a beggar-thy-neighbor effect by delaying innovation abroad. In such circumstances, the foreign country responds by implementing aggressive countervailing policies to mitigate the negative spillover effects. If both countries instead cooperatively support the industry, the welfare outcome is a Pareto improvement compared to the Nash equilibrium.

Keywords: industrial policy, industry lifecycle, innovation, policy competition and cooperation

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1 Introduction

China's economy has grown rapidly during the last three decades, with annual real GDP growth exceeding 9 percent from 1991 to 2021 (IMF 2022). The Chinese government has conducted a range of assertive industrial policies for manufacturing sectors during this period, which many studies such as Rodrik (2006), Gabriele (2010), and Felipe et al. (2013) argue has played significant role in attaining such rapid growth.

The government continues trying to upgrade China's industrial structure further using tools of industrial policy. For example, Made in China 2025 is a national plan to develop China's manufacturing industry, implementing a transition from labor-intensive workshops into a technology-intensive powerhouse. In response to such efforts by China to become a global leader in high tech industries, many politicians and scholars in advanced economies have begun calling for implementation of new industrial policy to foster high tech industries or preserve aging manufacturing sectors.

In this context, a central question emerges: How does industrial policy affect the welfare of both the domestic economy and its counterpart countries? This paper shows that the answer depends on where the targeted industry happens to be in its lifecycle in the home country and abroad. Especially, the welfare analysis demonstrates that the welfare effects are very different for the foreign country in two cases: *catch-up* vs *frontier technology races*. Subsidies that aid catch-up have positive spillovers for trading partners, while frontier races targeting firms trying to advance a technological frontier are more likely to have a beggar-thy-neighbor effect.

This paper suggests a theoretical link between the growth-boosting effect of industrial policy and industry lifecycle theory. Regarding how industrial policy affects economic growth, Rodrik (2006) presents some stylized facts. First, he presents empirical evidence supporting the idea that growth accelerations are associated with the increase in the share of manufacturing industries in an economy. He emphasizes that industrial policies played an important role in successfully achieving such structural changes in some countries such as China and India. Second, he argues that the level of productivity with which a good is produced does converge to that of advanced countries. Moreover, the paper argues that the lower the unit value of goods a country initially produces, the greater is the growth the country will experience. These stylized facts suggest that industrial policy is more likely to succeed in boosting economic growth and eventually improving a country's welfare when it targets innovative, young industries.

That implication can be theoretically supported by industry lifecycle theory such as Abernathy and Utterback (1978) and Jovanovic and MacDonald (1994). The theory predicts that productivity in an industry remains low in the early stage of the industry lifecycle but then increases rapidly following a radical innovation. In this context, if industrial policy encourages major innovations in a young industry to occur earlier, the industry can accelerate the transition to the high-productivity stage, thereby boosting aggregate economic growth. On the other hand, theory suggests that productivity organically tapers off in an industry in the late stage of its lifecycle. Thus, if industrial policy supports a mature industry, such a policy would not effectively stimulate economic growth. My model is built on this theoretical background.

Another important aspect of the research question concerns how growth-stimulating industrial policy affects the welfare of a foreign country. Existing literature studying the international spillover from home productivity improvement (Ghironi and Melitz (2005), Matsuyama (2007) and Corsetti et al. (2007)) suggests that the welfare of the foreign country is primarily affected by changes in the terms of trade. Those studies mostly consider two conflicting effects for the foreign welfare. First is a "direct effect", whereby domestic productivity improvement lowers the prices of home products, benefiting foreign consumers. Second is a "indirect effect" in which the domestic wage increases relative to the foreign wage due to increased labor demand, causing the foreign countries' terms of trade to deteriorate. While my model encompasses these mechanisms for the international spillover of domestic productivity increase, it introduces a novel channel through which a change in the mass of firms in an industry influences the timing of innovation in that industry. Based on this channel, the model suggests the possibility of beggar-thy-neighbor effects stemming from the home country's growth-stimulating industrial policy.

I also analyze how the foreign country responds to the domestic industrial policy. Policymakers should consider this seriously because if an industrial policy diminishes the welfare of its counterpart countries, those countries may respond by implementing measures to offset the effects of the policy. I introduce a game situation where the home and foreign country compete to foster the same industry. The Nash equilibrium in this game demonstrates the consequent policy competition in which both countries respond to each other's policy decisions by more aggressively supporting the targeted industry. The analysis suggests if two countries cooperatively support the industry, the welfare outcome is a Pareto improvement compared to the Nash equilibrium.

The paper is structured as follow. Section 2 reviews related literature. Section 3 presents model setup and Section 4 analyzes the welfare effect of industrial policy under the influence of the industry lifecycle. Section 5 derives theoretical implications through model simulation. Section 6 examines Korea's industrial policy in 1970s to check whether the outcomes are consistent with the central mechanism of my model. Section 7 compares the welfare effects of a production and R&D subsidy. Section 8 concludes.

2 Literature Review

2.1 Industrial Policy and Welfare

Long-run Growth This paper builds on and contributes to the literature that studies how industrial policy affects productivity and welfare of the home country under open economy in dynamic view, which includes Redding (1999) and Melitz (2005). The literature that emphasizes innovation as the engine of growth (Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992)) is also closely related in that the main source of welfare improvement in the model is innovation and the consequential increase in productivity. A novel feature of the model in this paper is that the type of innovation and the effect of innovation on aggregate productivity in an industry are endogenously determined depending on where the industry places along its lifecycle while other literature assumes that the type of innovation or the parameter which determines the degree of innovation in an industry are time invariant. The model in this paper also supplements Melitz (2005) by suggesting a plausible shape of learning curve. He suggests whether industrial policy to protect an infant industry increases home welfare or not depends on the shape of learning curve in the industry and the degree of substitutability between home and foreign goods, but does not present how the learning curve usually looks.

This paper is related to the literature such as Rodrik (2006), Aghion et al. (2015), Atkeson and Burstein (2019), Choi and Levchenko (2021), and Lane (2022) which studies the role of industrial policy in economic growth. Especially, since empirical results of Rodrik (2006) and Aghion et al. (2015) suggest that policies that support younger and high growth potential sectors tend to increase the targeted industry's productivity more, this paper complements their results. Rodrik (2013) also provides empirical evidence that younger manufacturing sectors exhibit more rapid labor productivity growth. However, based on industry lifecycle theory and the empirical finding from Korea's Heavy and Chemical industry drive in Section 6, this paper is different in that actual productivity increase in the targeted industry is realized with a lag after policy intervention.

From the feature of the lagged realization of productivity increase based on industry lifecycle, this paper contributes to literature which suggests empirical evidence on short-run and long-run welfare trade off from industrial policy (Kim et al. (2021) and Choi and Levchenko (2021)). This paper provides a theoretical foundation on the trade off between short-run distortion and long-run gain from growth and emphasizes the importance of welfare analysis of industrial policy in dynamic view.

Scale Economies Another related literature is the one that studies the welfare effects of policy considering scale economises. Bartelme et al. (2021) and Lashkaripour and Lugovskyy (2021) suggest optimal policy to maximize the welfare by exploiting the differences in scale economies across industries and improving terms of trade in static view. Their theoretical frameworks are built on the assumption that scale economies for industries do not change over time. Even though there are no static scale economies at any stage along the industry lifecycle in my model, an industry can move faster from the early-stage to the high-productivity stage by using more labor in the industry. This is "dynamic scale economy", where the realization of cost-reduction occurs earlier with the increase in the scale of cumulative output produced in the industry. Since the static scale economies used in Lashkaripour and Lugovskyy (2021) also works in the short-run in my model, there can be complementarity or trade-off between static and dynamic scale economies in the model.[1]

2.2 Industry Lifecycle and Innovation

This paper is related to the literature which studies the stylized pattern of industry dynamics (Vernon (1966), Abernathy and Utterback (1978), Gort and Klepper (1982), Jovanovic and MacDonald (1994), Klepper (1996), Antràs (2005), and Eriksson et al. (2021)). This paper contributes to the literature by incorporating the theory on how innovation in an industry changes along its lifecycle into a canonical open macroeconomic model to analyze its implications on the aggregate economy, while most of the literature focuses on regularities along the lifecycle with partial equilibrium analysis.

In this aspect, the literature which develops a growth or trade model that incorporates multiple types of innovations with a consideration of industry dynamics (Krugman (1979), Klette and Kortum (2004), Atkeson and Burstein (2010), Akcigit and Kerr (2018), and Hsieh et al. (2021)) is also closely related to this paper. This paper contributes to the literature in two aspects. First, this paper derives policy implications based on welfare analysis. The implications are novel in that it provides a criteria regarding the right "timing" of industrial policy. Second, it studies how industrial policy affects the foreign innovation and welfare as well as those for the home country. While Krugman (1979) studies the effects of innovation in North (developed country) on the welfare for South (developing country), his framework is appropriate for analyzing only an asymmetric case where a developing country tries to catch up a developed country by imitation of technology. This paper also analyzes the case where two symmetric countries compete to take the leadership in an industry using a frontier technology.

[1] Suppose the high growth potential industry has a lower elasticity of substitution across varieties within the industry (micro elasticity of substitution) than other industries. In this case, if the government supports the industry to hasten innovation (dynamic scale economy), more labor will be allocated to the industry from other industries in response to the policy. In static view, based on Lashkaripour and Lugovskyy (2021), it means resources are re-allocated from low scale economy industry to high scale economy industry. This is an example of complementarity between static and dynamic scale economy. In contrast, if the micro elasticity of substitution of the targeted industry is higher than others, there is trade-off between static and dynamic scale economy.

2.3 Growth and International Welfare Spillover

My work contributes to literature which studies international spillovers from productivity increases in the home country (Ghironi and Melitz (2005), Matsuyama (2007) and Corsetti et al. (2007)). This paper makes two contributions to the existing literature. First, it provides a theoretical framework to analyze the effects of industrial policy targeting a specific industry by extending the model of Corsetti et al. (2007) to a multi-industry model. Second, it suggests a novel channel through which a productivity improvement in the home country affects foreign welfare. In my model, if there is a productivity increase in an industry in the home country, it causes a decrease in the mass of firms in the same industry in the foreign country. Due to shrinking of firm activity in the foreign industry, innovation will be delayed in the industry and net foreign welfare accordingly decreases in this dynamic setting. This channel provides the possibility of "beggar-thy-neighbor" while the other effects from the existing literature (e.g. effects based on terms of trade and love for variety) still apply in the model.

3 The Model

I nest the model of industry lifecycle in the model of open economy spillovers by Corsetti et al. (2007) to analyze welfare effects of an industrial policy by taking the industry lifecycle into account.

In the model, the world economy consists of two countries, home and foreign. In each country, there are households, firms, and a government. The size of households is L in the home country and L^* in the foreign country.

Since industrial policy supports targeted industries by design, I extend the model in Corsetti et al. (2007) to a model with two industries. Industry 1 in the home country is a young industry with high growth potential. In contrast, industry 2 in the home country is a mature industry where innovation has tapered off. An example of an industry like industry 2 in the model could be agriculture.

In Case 1, both foreign industry 1 and 2 are matured. The home and foreign country can be thought of as a developing and developed country respectively in this case. Thus, home industry 1 is trying to catch up to the technological frontier in industry 1 in this case. I will relax this assumption later and consider another case where the developmental states of the home and foreign country are similar and those countries are competing with each other.

Throughout the paper, I set the home and foreign country wage as numeraire in the home and foreign country respectively for convenience.

3.1 Firms

There is a continuum of firms with mass $n_{i,t}$ in industry i in the home country at time t. Similarly, $n^*_{i,t}$ denotes the mass of firms in industry i in the foreign country at time t. The mass of firms in each industry in each country is endogenously determined within the model.

There is only one factor of production, labor, which is mobile across industries. The productivity is assumed to be the same for all firms in each industry in each country for simplicity. However, there is a difference in productivity between industries and between countries. Under these assumptions, the firm producing variety h in industry i in the home country has the following linear constant returns to scale production function.

$$y_{i,t}(h) = a_{i,t} l_{i,t}(h) \tag{1}$$

where $l_{i,t}(h)$ is labor used in the production for variety h in industry i.

A firm also needs to hire $\frac{1}{v_{i,t}}$ units of labor each period regardless of its amount of production. Firms allocate this fixed cost to different activities, depending on their industry's stage along the lifecycle. As suggested in Abernathy and Utterback (1978) and Klepper (1996), firms in the early stage of the lifecycle need to invest in R&D for product innovation. At this stage, a "dominant design," defined as a product design widely accepted by consumers, has not yet emerged. Consequently, numerous firms producing different varieties enter the industry and focus on product R&D to establish a dominant design. Once a dominant design is established, the industry transitions to a high-productivity stage where the production process becomes highly standardized, and manufacturing productivity rapidly increases. Thus, the fixed cost at this stage is primarily used for process R&D or for the construction and maintenance of large facilities. For simplicity, it is assumed that this fixed cost remains constant over time and is the same for all firms across all industries in each country. Given wage w_h, the fixed cost $q_{i,t}(h)$ is then,

$$q_{i,t}(h) = \frac{w_h}{v_{i,t}} = \frac{1}{v} \tag{2}$$

All firms in each country sell their product to home and foreign consumers. When a firm sells its product abroad, it entails iceberg cost, τ. Therefore, the resource constraint for home variety h in industry i at time t is as follows.

$$y_{i,t}(h) \geq Lc_{i,t}(h) + (1+\tau)L^* c^*_{i,t}(h) \tag{3}$$

where $c_{i,t}(h)$ and $c_{i,t}(h)$ represent the consumption of the home variety h in industry i at time t by the home and foreign representative consumers, respectively. L and L^* denote the number of people in the home and foreign countries, respectively.

Let $p_{i,t}(h)$ denote the price for home variety h in industry i at time t in home market which is expressed in terms of home wages, and $p^*_{i,t}(h)$ denote the price for home variety h in industry i at time t in the foreign market which is expressed in terms of foreign wages. ϵ_t is the exchange rate which is defined as the relative price of foreign labor in terms of home labor units[2]. The operating profit in domestic labor units, which is revenue minus variable cost, of a home firm producing variety h in industry i at time t is:

$$\Pi_{i,t}(h) = p_{i,t}(h)c_{i,t}(h)L + \epsilon_t p^*_{i,t}(h)c^*_{i,t}(h)L^* - l_{i,t}(h) \tag{4}$$

Similar expressions hold for the firms in foreign country.

[2]For example, if the foreign wage is twice as expensive as the home wage when compared in a common currency, the exchange rate, ϵ_t, is 2. In a model where the home wage is the only numeraire and the world uses one common currency (meaning the exchange rate is not introduced in the model), ϵ_t is equivalent to the foreign wage.

3.1.1 Innovation and Productivity

A novel feature of the model in this paper is that the productivity of firms in the industry with high-growth potential endogenously increases. As we have seen in the literature regarding industry lifecycle, it is reasonable to assume that the productivity of firms in an industry changes depending on where the industry is in its lifecycle as follows: Productivity grows slowly for some time after the inception of the industry. However, it increases rapidly after major or radical innovations occur. Then, the growth rate gradually declines and stabilizes at a low level. To capture these productivity dynamics, I refer to Jovanovic and MacDonald (1994) regarding how innovations happen along the industry lifecycle.

Let $\overline{a_{i,low}}$ and $\overline{a_{i,high}}$ denote low and high productivity in industry i. To build intuition for analyzing welfare effects of the growth-boosting industrial policy in the context of industry lifecycle, I define two stages along the cycle, "the early stage" and "high-productivity stage", as follows.

Definition 1 *Industry i is in "the early stage" of an industry lifecycle, which means*

i) The productivity of firms in the industry stays at the lower bound, $\overline{a_{i,low}}$, in this stage.

ii) Firms in the industry at this stage have growth potential, as their productivity can increase to $\overline{a_{i,high}}$ following the occurrence of a refinement.

Definition 2 *When a refinement occurs in industry i, the industry enters "the high-productivity stage" of the lifecycle, which means*

i) Upon the occurrence of a refinement in the industry, all firms immediately achieve high productivity, $\overline{a_{i,high}}$.[3]

ii) Productivity of firms in the industry does not grow any more within the lifecycle.

Based on Jovanovic and MacDonald (1994) and Abernathy and Utterback (1978), refinement is defined as a major technological innovation which creates a dominant design in the industry. After

[3]As in Jovanovic and MacDonald (1994), it is more realistic to assume that firms get high productivity with a probability after the refinement. However, since results of the model do not change qualitatively with the assumption in Definition 2, I use the simplified assumption for convenience.

the emergence of the dominant design, firms which successfully create or follow the dominant design have a broad consumer base but the other firms which fail to do it exit. The surviving firms in the industry need to produce a large quantity of their goods and start to focus their R&D on improving manufacturing productivity. Accordingly, these process R&D efforts lead a significant increase in productivity within the industry.

How does the refinement occur? Jovanovic and MacDonald (1994) argue that the technology is refined from outside the industry. Thus, the refinement is assumed to occur exogenously in their paper. However, in most cases, a huge innovation from outside the industry cannot be applied immediately within the industry. Thus, one might reasonably assume that the timing of refinement in industry i depends on accumulated knowledge stock in the industry, $Q_{i,t}$. Formally, the relevant refinement is assumed to occur as follows.

Assumption 1 *Refinement occurs at time t_i^r when the accumulated knowledge stock in industry i, $Q_{i,t}$, exceeds a threshold, $\bar{Q}_i$, for the first time: $Q_{i,t_i^r} \geq \bar{Q}_i$ and $Q_{i,t_i^r-j} < \bar{Q}_i$ for $0 < j < t_i^r$*

Additionally, I assume that knowledge stock in an industry is accumulated by firms' activities in the industry and thus it is increasing with the accumulated number of operating firms in the industry as follows.

Assumption 2 $Q_{i,t} = Q_i\left(\int_0^t n_{i,j}dj\right)$ *where $Q_i()$ is an increasing function*

Assumption 2 is critical to later results and is based on three motivations. First, knowledge stock in an industry is accumulated as R&D efforts of firms in the industry are accumulated. This is in line with Grossman and Helpman (1991) in that the commercial exploitation of technology requires a substantial investment from firms. As mentioned in Section 3.1, each firm devotes its fixed cost to product R&D before the emergence of a dominant design. Since every firm spends the same fixed cost in the model, the total accumulated R&D efforts of firms is proportional to the accumulated number of operating firms ($\int_0^t \int_0^{n_{i,j}} q_{i,j}(h)dhdj = \frac{1}{v}\int_0^t n_{i,j}dj$). In the early stage of an industry lifecycle, firms continuously experiment to design a new consumer product. Even though one firm cannot directly see another firm's R&D, it can still learn from consumers' response to the firm's product. Second, the timing of refinement also depends on the cumulative production in the industry, which is related with the learning-by-doing effect in Melitz (2005). In the model, the productivity of all firms in each industry is assumed to be the same and thus the amount of production by each firm in an industry is also the same in equilibrium ($y_{i,t}(h) = y_i$). Accordingly, the cumulative production in the industry is proportional to the accumulated number of operating firms ($\int_0^t \int_0^{n_{i,j}} y_{i,j}(h)dhdj = \int_0^t \int_0^{n_{i,j}} y_i dhdj = y_i \int_0^t n_{i,j}dj$). Therefore, here, cumulative industry production is positively correlated with the probability of the emergence of a refinement or dominant design that enables standardization in production as in Jovanovic and MacDonald (1994), reflected in a higher industry productivity parameter, $a_{i,t}$. Lastly, having more firms operate generates more competition. Aghion et al. (2015) argues that more competition in an industry causes firms to increase their efforts to innovate, boosting productivity growth.

Based on Assumption 1 and 2, the timing of refinement in industry i, t_r, is determined as follows

$$t_i^r = \frac{Q_i^{-1}(\bar{Q}_i)}{n_{i,s1}} \tag{5}$$

where $n_{i,s1}$ is the mass of firms in industry i in a period in the early stage.

3.2 Households

The utility function of the representative consumer in the home country at time t has the following form.

$$U_t = \frac{C_t^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} - l_t \tag{6}$$

where C_t is a comprehensive consumption index at time t, ψ is the intertemporal elasticity of substitution, and l_t is labor supply by the representative consumer at time t. The comprehensive consumption index aggregates industry-level consumption indexes, $C_{1,t}$ and $C_{2,t}$, in the following way:

$$C_t = \left[C_{1,t}^{1-\frac{1}{\sigma}} + C_{2,t}^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{7}$$

where σ is the elasticity of substitution across industries. The industry i consumption index, $C_{i,t}$, is a composite of varieties produced in industry i in each country, $C_{hi,t}$ and $C_{fi,t}$, as follows.

$$C_{i,t} = \left[C_{hi,t}^{1-\frac{1}{\eta}} + C_{fi,t}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{8}$$

where η is the elasticity of substitution between the origin-specific industry-level consumption aggregates, which is referred to "macro" elasticity of substitution in Feenstra et al. (2018). Lastly, the home and foreign industry-level consumption aggregates, $C_{hi,t}$ and $C_{fi,t}$, are assumed to be

$$C_{hi,t} = \left[\int_0^{n_{i,t}} c_{i,t}(h)^{1-\frac{1}{\gamma}} dh \right]^{\frac{\gamma}{\gamma-1}}, \ C_{fi,t} = \left[\int_0^{n_{i,t}^*} c_{i,t}(f)^{1-\frac{1}{\gamma}} dh \right]^{\frac{\gamma}{\gamma-1}} \tag{9}$$

where γ is the elasticity of substitution across varieties of an industry in each country, which I call as "micro" elasticity of substitution as in Feenstra et al. (2018).[4] It is worth to mention that the micro elasticity of substitution is the same regardless of industries and countries in the model. As in Lashkaripour and Lugovskyy (2021), if the micro elasticity of substitution differs across industries, then each industry has different static scale economy due to the difference. Thus, when a resource allocation changes in an economy due to industrial policy, the difference in static scale economies across industries generate an additional welfare effect. Since this paper focuses on welfare effects caused by hastening the occurrence of innovation (a dynamic scale economy), I abstract from policy incentives based on differences in the micro elasticity of substitution (a static scale economy) by making γ the same across industries. I also assume $\gamma > \eta$ which means that varieties produced in the same country are more substitutable than varieties produced in different countries (i.e., consumers view Ford and Chevrolet as closer substitutes than Ford and Fiat.)

[4] The model in Corsetti et al. (2007) assumes that micro elasticity of substitution in an industry is the same at macro elasticity of substitution. In that case, when γ is large, which is not unusual, industrial structure changes too radically by productivity shocks. For this reason, I set an assumption that micro elasticity of substitution can be different from macro elasticity of substitution. The more generalized model has an advantage that industrial structure changes gradually by external shocks.

Households own all firms in their own country. Each household receives an equal share of profits of all firms in their country:

$$\Pi_t \equiv \int_0^{n_{1,t}} \Pi_{1,t}(h)dh + \int_0^{n_{2,t}} \Pi_{2,t}(h)dh \tag{10}$$

The representative consumer maximizes its utility (6) in time t subject to the following budget constraint[5]:

$$P_t C_t = l_t + \Pi_t - \frac{T_t}{L} \tag{11}$$

where T_t is a lump-sum tax which I will explain in detail in the next section.

By solving the representative consumer's utility maximization problem, the representative consumer's demand for $C_{i,t}$, $C_{hi,t}$, $C_{fi,t}$, $c_{i,t}(h)$, $c_{i,t}(f)$, and labor supply, l, satisfies the first-order conditions:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\sigma} C_t \tag{12}$$

$$C_{hi,t} = \left(\frac{P_{hi,t}}{P_{i,t}}\right)^{-\eta} C_{i,t},\ C_{fi,t} = \left(\frac{P_{fi,t}}{P_{i,t}}\right)^{-\eta} C_{i,t} \tag{13}$$

$$c_{i,t}(h) = \left(\frac{p_{i,t}(h)}{P_{hi,t}}\right)^{-\gamma} C_{hi,t},\ c_{i,t}(f) = \left(\frac{p_{i,t}(f)}{P_{fi,t}}\right)^{-\gamma} C_{fi,t} \tag{14}$$

$$P_t C_t^{\frac{1}{\psi}} = w_t = 1 \tag{15}$$

where P_t, $P_{i,t}$, and $P_{ji,t}$ are the utility-based consumer price index (CPI), the industry i composite price index, and the country j specific industry i composite price index, which are defined as the minimum expenditure required to purchase one unit of the comprehensive consumption index, C_t, the industry i composite consumption index, $C_{i,t}$, and country j specific industry i composite aggregate, $C_{ji,t}$ respectively. P_t, $P_{i,t}$, and $P_{ji,t}$ can be expressed as:

[5]Since consumers do not save in the model, they maximize their current utility in each period by only considering their current budget constraint.

$$P_t = \left[P_{1,t}^{1-\sigma} + P_{2,t}^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{16}$$

$$P_{i,t} = \left[P_{hi,t}^{1-\eta} + P_{fi,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{17}$$

$$P_{hi,t} = \left[\int_0^{n_{i,t}} p_{i,t}(h)^{1-\gamma} dh\right]^{\frac{1}{1-\gamma}}, \; P_{fi,t} = \left[\int_0^{n_{i,t}^*} p_{i,t}(f)^{1-\gamma} df\right]^{\frac{1}{1-\gamma}} \tag{18}$$

Households provide labor in a competitive market both for fixed-cost related and production activities and thus the resource constraint for labor is:

$$Ll_t \geq \int_0^{n_{1,t}} \frac{y_{1,t}(h)}{a_{1,t}} dh + \int_0^{n_{2,t}} \frac{y_{2,t}(h)}{a_{2,t}} dh + \int_0^{n_{1,t}} q_{1,t}(h) dh + \int_0^{n_{2,t}} q_{2,t}(h) dh \tag{19}$$

Again, similar expressions hold in the foreign country.

3.3 Government and Industrial Policy

In this paper, as the definition of industrial policy in Pack and Saggi (2006)[6], I assume *ex ante* that the government wants to promote industries with high growth potential and uses industrial policy to achieve its goal. In this section, I first analyze the impact of a production subsidy as an example of one type of industrial policy. I will also examine the effect of an R&D subsidy in a later section.

Assume that the government provides a production subsidy to firms in industry 1 until refinement occurs in the industry. The production subsidy rate for industry i at time t is $s_{i,t}$. I assume the government levies a lump sum tax from the consumers to fund the production subsidy. The amount of the total subsidy at time t, S_t, is:

$$S_t = \int_0^{n_{1,t}} s_{1,t} p_{1,t}(h) y_{1,t}(h) dh + \int_0^{n_{2,t}} s_{2,t} p_{2,t}(h) y_{2,t}(h) dh = s_{1,t} n_{1,t} p_{1,t} y_{1,t} + s_{2,t} n_{2,t} p_{2,t} y_{2,t} \tag{20}$$

[6]According to Pack and Saggi (2006), industrial policy is defined as "any type of selective intervention or government policy that attempts to alter the structure of production toward sectors that are expected to offer better prospects for economic growth than would occur in the absence of such intervention."

Thus government budget constraint is as follows:

$$sn_{1,t}p_{1,t}y_{1,t} = T_t \tag{21}$$

3.4 Static Equilibrium

I will describe the equilibrium without any production subsidy first and then explain how equilibrium conditions change by introducing the production subsidy.

3.4.1 Equilibrium without Production Subsidy

Prices Firms face monopolistic competition where a firm sets its price with constant markups over marginal costs as follows:

$$p_{i,t}(h) = \frac{\gamma}{\gamma-1}\frac{1}{a_{i,t}} \equiv p_{i,t},\ \epsilon_t p^*_{i,t}(h) = (1+\tau)\frac{\gamma}{\gamma-1}\frac{1}{a_{i,t}} = (1+\tau)p_{i,t} \tag{22}$$

$$p^*_{i,t}(f) = \frac{\gamma}{\gamma-1}\frac{1}{a^*_{i,t}} \equiv p^*_{i,t},\ \frac{p_{i,t}(f)}{\epsilon_t} = (1+\tau)\frac{\gamma}{\gamma-1}\frac{1}{a^*_{i,t}} = (1+\tau)p^*_{i,t} \tag{23}$$

where τ represents the trade cost.

Using (22) and (23), the country j specific industry i composite price index in the home and foreign country, $P_{ji,t}$ and $P^*_{ji,t}$, can be expressed respectively:

$$P_{hi,t} = p_{i,t}n_{i,t}^{\frac{1}{1-\gamma}},\ P_{fi,t} = (1+\tau)\epsilon_t p^*_{i,t}{n^*_{i,t}}^{\frac{1}{1-\gamma}} \tag{24}$$

$$P^*_{hi,t} = (1+\tau)\frac{p_{i,t}}{\epsilon_t}n_{i,t}^{\frac{1}{1-\gamma}},\ P^*_{fi,t} = p^*_{i,t}{n^*_{i,t}}^{\frac{1}{1-\gamma}} \tag{25}$$

and industry i composite price index in the home and foreign country, $P_{i,t}$ and $P^*_{i,t}$, are:

$$P_{i,t} = \left(p_{i,t}^{1-\eta}{n_{i,t}}^{\frac{1-\eta}{1-\gamma}} + \phi(\epsilon_t p^*_{i,t})^{1-\eta}{n^*_{i,t}}^{\frac{1-\eta}{1-\gamma}}\right)^{\frac{1}{1-\eta}} \tag{26}$$

$$P^*_{i,t} = \left({p^*_{i,t}}^{1-\eta}{n^*_{i,t}}^{\frac{1-\eta}{1-\gamma}} + \phi(\frac{p_{i,t}}{\epsilon_t})^{1-\eta}n_{i,t}^{\frac{1-\eta}{1-\gamma}}\right)^{\frac{1}{1-\eta}} \tag{27}$$

where $\phi \equiv (1+\tau)^{1-\eta}$.

Based on (16), (22), (23), (26) and (27), we can see that the utility based CPI in home and foreign country, P_t and P_t^*, are determined by five endogenous variables, $n_{1,t}$, $n_{2,t}$, $n_{1,t}^*$, $n_{2,t}^*$, and ϵ_t.

Zero-Profit Conditions Free entry implies that profit is zero in equilibrium. Thus, a firm's operating profit in each industry should equal the fixed cost in both the home and foreign country, as follows:

$$\begin{aligned}
\Pi_{i,t}(h) &= \frac{p_{i,t}(h)y_{i,t}(h)}{\gamma} \\
&= \frac{p_{i,t}(h)}{\gamma}\left[c_{i,t}(h)L + (1+\tau)c_{i,t}^*(h)L^*\right] \\
&= \frac{1}{\gamma}\left(\frac{p_{i,t}(h)}{P_{hi,t}}\right)^{1-\gamma}\left[\left(\frac{P_{hi,t}}{P_{i,t}}\right)^{1-\eta}\left(\frac{P_{i,t}}{P_t}\right)^{1-\sigma}P_t^{1-\psi}L + \phi\epsilon_t{}^{\eta}\left(\frac{P_{hi,t}}{P_{i,t}^*}\right)^{1-\eta}\left(\frac{P_{i,t}^*}{P_t^*}\right)^{1-\sigma}P_t^{*1-\psi}L^*\right] = \frac{1}{v}
\end{aligned} \tag{28}$$

$$\begin{aligned}
\Pi_{i,t}^*(f) &= \frac{p_{i,t}^*(f)y_{i,t}^*(f)}{\gamma} \\
&= \frac{p_{i,t}^*(f)}{\gamma}\left[c_{i,t}^*(f)L^* + (1+\tau)c_{i,t}(f)L\right] \\
&= \frac{1}{\gamma}\left(\frac{p_{i,t}^*(f)}{P_{fi,t}^*}\right)^{1-\gamma}\left[\left(\frac{P_{fi,t}^*}{P_{i,t}^*}\right)^{1-\eta}\left(\frac{P_{i,t}^*}{P_t^*}\right)^{1-\sigma}P_t^{*1-\psi}L^* + \phi\epsilon_t{}^{-\eta}\left(\frac{P_{fi,t}^*}{P_{i,t}}\right)^{1-\eta}\left(\frac{P_{i,t}}{P_t}\right)^{1-\sigma}P_t^{1-\psi}L\right] = \frac{1}{v^*}
\end{aligned} \tag{29}$$

Balance of Payment Equilibrium Condition I assume balanced trade in the model where the value of a country's imports is the same as the value of its exports. Thus, the following equation holds in the equilibrium.

$$(1+\tau)L^*\left[p_{1,t}(h)c_{1,t}^*(h)n_{1,t} + p_{2,t}(h)c_{2,t}^*(h)n_{2,t}\right] = \epsilon_t(1+\tau)L\left[p_{1,t}^*(f)c_{1,t}(f)n_{1,t}^* + p_{2,t}^*(f)c_{2,t}(f)n_{2,t}^*\right] \tag{30}$$

Firm Size and Labor From (22), (23), (28), and (29), the size of a firm from industry i in the home or foreign country, respectively, is determined as follows:

$$y_{i,t}(h) = \frac{(\gamma-1)a_{i,t}}{v},\ y^*_{i,t}(f) = \frac{(\gamma-1)a^*_{i,t}}{v^*} \tag{31}$$

Based on (31), the amount of labor hired for production by a firm in industry j in the home and foreign country is:

$$l_{i,t}(h) = \frac{\gamma-1}{v},\ l^*_{i,t}(f) = \frac{\gamma-1}{v^*} \tag{32}$$

Since every firm also hires $\frac{1}{v}$ and $\frac{1}{v^*}$ unit of labor for fixed cost activities at home and abroad respectively, the aggregate labor demand in each economy (L_t and L^*_t) is given by[7]

$$L_t = \frac{\gamma(n_{1,t}+n_{2,t})}{v},\ L^*_t = \frac{\gamma(n^*_{1,t}+n^*_{2,t})}{v^*} \tag{33}$$

From the aggregate labor demand (33) and the representative consumer's budget constraint ($P_t^{1-\psi} = l_t,\ P_t^{*1-\psi} = l^*_t$), labor supply from the representative consumer in the home and foreign country is determined as follow

$$l_t = \frac{\gamma(n_{1,t}+n_{2,t})}{Lv} = P_t^{1-\psi},\ l^*_t = \frac{\gamma(n^*_{1,t}+n^*_{2,t})}{L^*v^*} = P_t^{*1-\psi} \tag{34}$$

Definition of Equilibrium The system of five equations, which are two zero-profit conditions (one for each industry) in the home country from (28), two zero-profit conditions in the foreign country from (29), and the balance of payment equilibrium (30), determines the five endogenous variables, $n_{1,t}$, $n_{2,t}$, $n^*_{1,t}$, $n^*_{2,t}$, and ϵ_t as functions of exogenous variables, v, v^*, L, L^*, $a_{1,t}$, $a_{2,t}$, $a^*_{1,t}$ and $a^*_{2,t}$.

This equilibrium will be used in the welfare analysis for the period when the home government stops giving subsidies following the emergence of the refinement.

[7]Total labor demand from a firm is $\frac{\gamma}{v}$, which the sum of the labor for production ($\frac{\gamma-1}{v}$) and fixed cost activities ($\frac{1}{v}$), in both industries in the home country. Similarly, total labor demand from a firm in both industries is $\frac{\gamma}{v^*}$ abroad.

3.4.2 Equilibrium with Production Subsidy

When the home government gives a subsidy to firms in industry 1, the representative firm in the home industry 1 changes its price to:

$$p_{1,t}(h) = \frac{\gamma}{\gamma - 1}\frac{1}{(1+s)a_{1,t}} = p_{1,t}, \ \epsilon p^*_{1,t}(h) = (1+\tau)\frac{\gamma}{\gamma - 1}\frac{1}{(1+s)a_{1,t}} = (1+\tau)p_{1,t} \tag{35}$$

Also, the profit function of firms in industry 1 changes from (28) as follows:

$$\begin{aligned}
\Pi_{1,t}(h) &= \frac{(1+s)p_{1,t}(h)y_{1,t}(h)}{\gamma} \\
&= \frac{(1+s)p_{1,t}(h)}{\gamma}\left[c_{1,t}(h)L + (1+\tau)c^*_{1,t}(h)L^*\right] \\
&= \frac{1+s}{\gamma}\left(\frac{p_{i,t}(h)}{P_{hi,t}}\right)^{1-\gamma}\left[\left(\frac{P_{hi,t}}{P_{i,t}}\right)^{1-\eta}\left(\frac{P_{i,t}}{P_t}\right)^{1-\sigma}P_t^{1-\psi}L + \phi{\epsilon_t}^{\eta}\left(\frac{P_{hi,t}}{P^*_{i,t}}\right)^{1-\eta}\left(\frac{P^*_{i,t}}{P^*_t}\right)^{1-\sigma}{P^*_t}^{1-\psi}L^*\right] = \frac{1}{v}
\end{aligned} \tag{36}$$

The equations expressing the relationship between the size of a firm and the amount of labor it hires for production are still given by (31) and (32).

The expression for the labor supply is also same as that in the equilibrium without a production subsidy, but the representative consumer's budget constraint in the home country changes to $P_t^{1-\psi} + \frac{T_t}{L} = l_t$:

$$l_t = \frac{\gamma(n_{1,t} + n_{2,t})}{Lv} = P_t^{1-\psi} + \frac{T_t}{L} \tag{37}$$

Now, the system of equilibrium equations which determines the five endogenous variables, $n_{1,t}$, $n_{2,t}$, $n^*_{1,t}$, $n^*_{2,t}$, and ϵ_t, as functions of exogenous variables, v, v^*, L, L^*, $a_{1,t}$, $a_{2,t}$, $a^*_{1,t}$ and $a^*_{2,t}$, consists of (36), the zero profit condition for industry 2 in the home country (28), two zero profit conditions for each industry in the foreign country (29) and balance of payments equation (30).

This equilibrium will be employed in the welfare analysis for the period during which the home government provides subsidies to firms in Industry 1, prior to the occurrence of refinement.

4 Welfare Analysis

In the model, the central mechanisms through which industrial policy affects the welfare of the home and foreign country is a change in the timing of innovation in the two countries. For this reason, a dynamic model is necessary to analyze the welfare effects of industrial policy. I assume that the model is in continuous time and the welfare of the representative consumer is defined as following:

$$\ln W = \int_0^{\infty} e^{-\rho t} \ln U_t dt \tag{38}$$

where ρ denotes discount rate.

Since the equations in the system of equilibrium are non-linear, it is hard to provide a general closed form solution. Thus, as in Corsetti et al. (2007), I set a symmetric initial condition where $v = v^* = L = L^* = a_{1,t} = a_{2,t} = a^*_{1,t} = a^*_{2,t} = 1$ and $s = s^* = 0$. With this initial condition, there is a symmetric equilibrium such that $\epsilon_t = 1$, $n_{1,t} = n_{2,t} = n^*_{1,t} = n^*_{2,t}$, $l_t = l^*_t = P_t^{1-\psi} = P_t^{*1-\psi} = \gamma(n_{1,t} + n_{2,t}) = \gamma(n^*_{1,t} + n^*_{2,t})$.

I impose restrictions on ψ such that $\psi < 1$. This restriction is necessary to reflect a stylized fact documented in industry lifecycle theory by Jovanovic and MacDonald (1994) and Klepper (1996) that the mass of firms decreases after significant productivity improvements.[8] Additionally, this condition, in conjunction with $\gamma > 1$, satisfies $\gamma > \psi$. This implies that labor supply is not excessively elastic in comparison to the elasticity of demand for goods, and it ensures that entry of new firms exerts downward pressure on profits.

[8] In the model, when $a_{1,t}$ increases, there are three effects: income, relative wage, and substitution effect which I will define in detail in the next section. The direction of substitution effect is unambiguous for each industry. The productivity increase in industry 1 makes the industry 1 composite price index decrease relative to the industry 2 composite price index. This leads the demand for varieties and the mass of firms in industry 1 to increase, and affects industry 2 in the opposite way. However, the direction of the income and relative wage effect depends on ψ. The productivity increase leads to an increase in the real wage which creates the income effect. In addition, it causes firms in the home country to demand more labor, which makes the home wage to increase relative to the foreign wage. The direction of these two effect is the same for both industries. If $\psi > 1$, an increase in the real wage leads to an increase in the total expenditure of the representative consumer ($P_t C_t = P_t^{1-\psi}$) and it leads to an increase in the mass of firms in both industries. In contrast, If $\psi < 1$, the income effect makes the mass of firms in both industries decrease. Overall, in order that the mass of firms decreases in industry 1 after a productivity increase, ψ must be smaller than 1. Thus, considering all three effects, when $\psi > 1$, the productivity increase in the home industry 1 causes the mass of firms in in the industry to increase always. Since this is contrast to the stylized fact, I exclude the case of $\psi \geq 1$.

4.1 Case 1: Catch-Up

I assume that industry 1 in the home country is young and has high growth potential. Productivity of firms in industry 1 in the home country is initially low, so $a_{1,t} = \overline{a_{1,low}} = 1$. After a refinement occurs at time $t_r(s)$, the productivity of firms in the home industry 1 jumps to $a_{1,t} = \overline{a_{1,high}} = k > 1$. It is important to note that the timing of refinement, $t_r(s)$, depends on the production subsidy because the mass of operating firms in industry 1 varies by the subsidy, which changes the speed of accumulation of the knowledge stock in the industry. In contrast to the home industry 1, I start with an assumption that the home industry 2 and both industry 1 and 2 in the foreign country are mature, which means that innovations rarely occur in those industries and thus productivity of those industries does not change over time: $a_{2,t} = a^*_{1,t} = a^*_{2,t} = 1$ for all t. This case is appropriate for analyzing the situation where the home and foreign country are a developing and developed country respectively, and the home industry 1 is trying to **catch up** to the technological frontier in the foreign industry 1. In Case 2, I will assume instead that the foreign industry 1 is also still young, creating a **frontier technology race**.

Under the above mentioned assumptions, there are two stages over time in Case 1. The stage 1 and 2 are defined:

Definition 3

- *Stage 1 ($0 < t < t_r(s)$) is a time period when a refinement has not yet occurred in the home industry 1 ($a_{1,t} = 1$)*
- *Stage 2 ($t \geq t_r(s)$) is a time period after an occurrence of a refinement in the home industry 1 ($a_{1,t} = k > 1$)*

Based on Definition 3, the welfare of the home and foreign representative consumer can be rewritten as follows.

$$\ln W = \int_0^{t_r(s)} e^{-\rho t} \ln U_1 dt + \int_{t_r(s)}^{\infty} e^{-\rho t} \ln U_2 dt \tag{39}$$

$$\ln W^* = \int_0^{t_r(s)} e^{-\rho t} \ln U_1^* dt + \int_{t_r(s)}^{\infty} e^{-\rho t} \ln U_2^* dt \tag{40}$$

where U_1 and U_2 (U_1^* and U_2^*) denote the home (foreign) representative consumer's utility in stage 1 and 2 respectively. In what follows, I take a first-order approximation of this model in the neighborhood of the initial symmetric equilibrium mentioned above and analyze the local effects of industrial policy:

$$\frac{d \ln W}{ds} = \underbrace{\int_0^{t_r(0)} e^{-\rho t} \frac{d \ln U_1}{ds} dt}_{\text{Short-run resource reallocation effect}} \underbrace{-e^{-\rho t_r(0)} (\ln U_2 - \ln U_1) \frac{dt_r(s)}{ds}}_{\text{Long-run gain from speeding up innovation}} \tag{41}$$

$$\frac{d \ln W^*}{ds} = \underbrace{\int_0^{t_r(0)} e^{-\rho t} \frac{d \ln U_1^*}{ds} dt}_{\text{Short-run resource reallocation effect}} \underbrace{-e^{-\rho t_r(0)} (\ln U_2^* - \ln U_1^*) \frac{dt_r(s)}{ds}}_{\text{Long-run gain from speeding up innovation}} \tag{42}$$

As it can be seen from (41) and (42), the production subsidy has two effects in terms of welfare at home and abroad: a short-run resource reallocation effect ($\int_0^{t_r(s)} e^{-\rho t} \frac{d \ln U_1}{ds} dt$ and $\int_0^{t_r(s)} e^{-\rho t} \frac{d \ln U_1^*}{ds} dt$ respectively) and a long-run gain from speeding up innovation ($-e^{-\rho t_r(0)} (\ln U_2 - \ln U_1) \frac{dt_r(s)}{ds}$ and $-e^{-\rho t_r(0)} (\ln U_2^* - \ln U_1^*) \frac{dt_r(s)}{ds}$ respectively). I will look into each effect in detail.

4.1.1 Short-run Resource Reallocation Effect

When the home government subsidizes firms in industry 1, the policy affects resource allocation in both the home and foreign countries. Since a refinement hasn't occurred yet, there is no growth effect in this stage. In the welfare aspect, the short-run resource reallocation effect for the home and foreign country is reflected in the first term in equation (41) and (42) ($\int_0^{t_r(s)} e^{-\rho t} \frac{d \ln U_1}{ds} dt$ and $\int_0^{t_r(s)} e^{-\rho t} \frac{d \ln U_1^*}{ds} dt$ respectively). I analyze how the production subsidy affects the mass of firms, prices and the relative wage and how such changes eventually affect the instantaneous welfare in Stage 1 at home and abroad. The detailed method and results are presented in Appendix A.

Resource Reallocation Change in the mass of firms in the home and foreign industry 1 in Stage 1 plays a central role in determining overall welfare effects of the production subsidy in the model.

Thus, I first explain how the mass of firms in the home and foreign industry 1 changes by the subsidy. Regarding this, there are three effects in the model: an income effect, a relative wage effect and a substitution effect. The sign and size of the three effects for each industry are specified as follows:

- The income effect, which corresponds to the first term in (92), (93), (94) and (95) for each industry, is $\frac{(\gamma-1)\psi}{4(\gamma-\psi)}$ for every industry.

- The relative wage effect, which corresponds to the second term in (92), (93), (94) and (95) for each industry, is $\frac{(\gamma-1)\psi}{4(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right)$ for the both home industries and $-\frac{(\gamma-1)\psi}{4(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right)$ for the both foreign industries.

- The substitution effect, which corresponds to the third term in (92), (93), (94) and (95) for each industry, is $\frac{\gamma-1}{\gamma-\sigma}\left(\sigma+\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)$ and $-\frac{\gamma-1}{\gamma-\sigma}\left(\sigma+\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)$ for the home industry 1 and 2, $-\frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)$ and $\frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)$ for the foreign industry 1 and 2.

The following lemma shows that the income effect is derived from the world market clearing condition (43) where the left hand side is the world consumption value and the right hand side is the world production value in the home currency.

$$P_t^{1-\psi}+\epsilon_t P_t^{*1-\psi}=p_{1,t}y_{1,t}n_{1,t}+p_{2,t}y_{2,t}n_{2,t}+\epsilon_t(p_{1,t}^*y_{1,t}^*n_{1,t}^*+p_{2,t}^*y_{2,t}^*n_{2,t}^*) \tag{43}$$

Lemma 1 *In the first-order approximation of the world market clearing condition* (43) *with respect to s, the relative wage effect cancels out between countries, and the substitution effect cancels out between sectors. Consequently, the income effect solely solves the first-order approximation equation.*

I add an intuitive explanation for this effect. When the home government provides a 1 percent production subsidy to firms in industry 1 ($s=0.01$), firms in the home industry 1 reduce their prices by 1 percent, as seen in (35). The 1 percent drop in $p_{1,t}$ directly decreases the left hand side in (43) by $\frac{1}{4}(\psi-1)$ percent. This is because i) a 1 percent decrease in $p_{1,t}$ leads to a $\frac{1}{2}$ percent decrease in the home utility based CPI, P_t, and ii) a $\frac{1}{2}$ percent decrease in P_t causes the home demand, $P_t^{1-\psi}$, to change by $\frac{1}{2}(\psi - 1)$ percent, and iii) the home demand accounts for $\frac{1}{2}$ of the world demand. On the other hand, the 1 percent drop in p_1 directly causes the world revenue, the right hand side in (43), to decrease by $\frac{1}{4}$ percent. This is because i) the revenue of the home industry 1, $p_{1,t}y_{1,t}n_{1,t}$, decreases by 1 percent, and ii) the revenue of the home industry 1 accounts for $\frac{1}{4}$ of the world revenue. Overall, the world experiences an excess demand of $\frac{\psi}{4}$ percent. To eliminate this excess demand, the mass of firms in all industries in both the home and foreign country needs to change uniformly by $\frac{(\gamma-1)\psi}{4(\gamma-\psi)}$. This is because a 1 percent increase in the mass of firms in all industries in both the home and foreign country causes the world demand to change by $\frac{1-\psi}{1-\gamma}$ percent and the world revenue to change by 1 percent, thereby reducing the world excess demand by $\frac{\gamma-\psi}{\gamma-1}$.

The relative wage effect is attributable to the change in the relative wage, ϵ. The following lemma demonstrates that the relative wage effect is derived from the balance of payment equilibrium condition (30).

Lemma 2 *In the first-order approximation of the balance of payment equilibrium condition* (30) *with respect to s, the income effect cancels out between countries, and the substitution effect cancels out between sectors. Consequently, the relative wage effect solely solves the first-order approximation equation.*

Due to the production subsidy, more firms enter industry 1 in the home country and hire additional labor. This results in an increase in the relative wage of domestic labor. In other words, it leads to a decrease in ϵ_t, as indicated in (96). The relative wage effect impacts both the domestic and foreign industries in the opposite direction but with equal magnitude. In addition to the income and substitution effects, ϵ_t decreases until the balance of payment equilibrium (30) is restored.

The substitution effect arises as firms enter and exit each industry to satisfy the zero profit condition again, as stated in (44) and (45).

$$
\begin{aligned}
&\frac{1+s}{\gamma}\left(\frac{p_{1,t}(h)}{P_{h1,t}}\right)^{1-\gamma}\left[\left(\frac{P_{h1,t}}{P_{1,t}}\right)^{1-\eta}\left(\frac{P_{1,t}}{P_t}\right)^{1-\sigma}P_t^{1-\psi}+\phi\epsilon_t{}^{\eta}\left(\frac{P_{h1,t}}{P^*_{1,t}}\right)^{1-\eta}\left(\frac{P^*_{1,t}}{P^*_t}\right)^{1-\sigma}P_t^{*1-\psi}\right] \\
&=\frac{1}{\gamma}\left(\frac{p_{2,t}(h)}{P_{h2,t}}\right)^{1-\gamma}\left[\left(\frac{P_{h2,t}}{P_{2,t}}\right)^{1-\eta}\left(\frac{P_{2,t}}{P_t}\right)^{1-\sigma}P_t^{1-\psi}+\phi\epsilon_t{}^{\eta}\left(\frac{P_{h2,t}}{P^*_{2,t}}\right)^{1-\eta}\left(\frac{P^*_{2,t}}{P^*_t}\right)^{1-\sigma}P_t^{*1-\psi}\right]
\end{aligned} \tag{44}
$$

$$
\begin{aligned}
&\frac{1}{\gamma}\left(\frac{p^*_{1,t}(f)}{P^*_{f1,t}}\right)^{1-\gamma}\left[\left(\frac{P^*_{f1,t}}{P^*_{1,t}}\right)^{1-\eta}\left(\frac{P^*_{1,t}}{P^*_t}\right)^{1-\sigma}P_t^{*1-\psi}+\phi\epsilon_t{}^{-\eta}\left(\frac{P^*_{f1,t}}{P_{1,t}}\right)^{1-\eta}\left(\frac{P_{1,t}}{P_t}\right)^{1-\sigma}P_t^{1-\psi}\right] \\
&=\frac{1}{\gamma}\left(\frac{p^*_{2,t}(f)}{P^*_{f2,t}}\right)^{1-\gamma}\left[\left(\frac{P^*_{f2,t}}{P^*_{2,t}}\right)^{1-\eta}\left(\frac{P^*_{2,t}}{P^*_t}\right)^{1-\sigma}P_t^{*1-\psi}+\phi\epsilon_t{}^{-\eta}\left(\frac{P^*_{f2,t}}{P_{2,t}}\right)^{1-\eta}\left(\frac{P_{2,t}}{P_t}\right)^{1-\sigma}P_t^{1-\psi}\right]
\end{aligned} \tag{45}
$$

Lemma 3 *In the first-order approximation of the zero profit conditions* (44) *and* (45) *with respect to s, the income and relative wage effects cancel out between sectors. Consequently, the substitution effect solely solves the first-order approximation equation.*

Since the production subsidy makes home varieties in industry 1 relatively cheaper, the home and foreign consumers substitute industry 1-foreign varieties with industry 1-home varieties. This leads to an increase in the mass of firms in home industry 1 and a decrease in that in foreign industry 1. In contrast, foreign varieties become relatively cheaper in industry 2 because of the increase in relative wage of home labor. It leads to a decrease in the mass of firms in home industry 2 and an increase in that in foreign industry 2. This substitution continues until the profit of firms in industry 1 and 2 becomes the same in both home and foreign country.

Overall, by combining the three effects, the mass of firms in each industry changes as follows.

Proposition 1 *A production subsidy causes the mass of home industry 1, $n_{1,t}$, to unambiguously increase and that of foreign industry 1, $n^*_{1,t}$, to unambiguously decreases. Changes in $n_{2,t}$ and $n^*_{2,t}$ are ambiguous.*

Proof. See Appendix A ■

The direction of the substitution effect is opposite from the two other effects for $n_{2,t}$ and the direction of relative wage effect is opposite from the two other effects for $n^*_{2,t}$. Thus, the overall effect of a production subsidy on $n_{2,t}$ and $n^*_{2,t}$ depends on the relative sizes of three effects. It is important to note that any increase in $n_{1,t}$ caused by the production subsidy leads to faster accumulation of knowledge stock and more learning by doing in home industry 1, which eventually makes a refinement occur faster in home industry 1.

Welfare Effect In Stage 1, the production subsidy's welfare effect on the home country is summarized in the next proposition.

Proposition 2 *The welfare effect of production subsidy for the home country in Stage 1 is ambiguous and it depends on the values of parameters as seen in* (100)*. For example, if $\gamma \to \infty$, the production subsidy decreases home welfare in the neighborhood of the initial equilibrium. In contrast, if $\phi = 0$ or $\gamma = \eta$, the welfare effect of production subsidy is positive.*

Proof. See Appendix A ■

Since firms have a monopolistic power, the production subsidy makes firms' prices in the targeted industries closer to their marginal costs, which generates an efficiency gain in the home country. In addition, home consumers experience a relative wage gain from the policy. However, at the same time, they have to pay tax which the government charges to finance the subsidy. Overall, in Stage 1, the welfare effect of the production subsidy for the home country depends on the degree of those gains and losses. For example, when markets are highly competitive ($\gamma \to \infty$), the welfare gain from making the prices closer to the marginal costs vanishes. However, the relative wage, which is mostly affected by the macro elasticity substitution (η) does not decrease a lot. Thus, the welfare of the home country decreases by $\frac{\psi\phi}{2\eta-1+2\psi\phi-\phi}$. Also, it is worth to note that the welfare effect is always positive under autarky ($\phi = 0$) unlike that in the open economy since home consumers do not pay the portion of the tax which subsidies the consumption of foreign consumers under autarky.

The following proposition summarizes the production subsidy's welfare effect on the foreign country in Stage 1.

Proposition 3 *A production subsidy unambiguously increases the foreign country's welfare in Stage 1 in the open economy ($\phi > 0$).*

Proof. See Appendix A ■

For foreign country's welfare, the home country's production subsidy decreases foreign country's utility based CPI as shown in (98). However, it also decreases the foreign wage relative to the home wage, which deteriorates foreign country's term of trade. Overall, if trade is not entirely restricted ($\phi > 0$), the former effect is greater than the latter one. Thus, in an open economy, home's subsidy increases the welfare of the foreign country.

4.1.2 Long-run Gain from Speeding up Innovation

After a refinement occurs, the productivity of firms in home industry 1 jumps to k from 1. The refinement occurs earlier in the home industry 1 by the production subsidy provided to the industry since the subsidy causes industry-level R&D efforts and learning-by-doing effects to be accumulated faster, and accelerates competition in the industry.

Proposition 4 *A production subsidy causes the home industry 1 to move from the early stage (Stage 1) to the high-productivity stage (Stage 2) faster.*

Proof.

$$\frac{dt(s)}{ds} = -\frac{Q_1^{-1}(\bar{Q}_1)}{n_{1,s1}(s)^2}\frac{dn_{1,s1}(s)}{ds} = -t(s)\frac{d\ln n_{1,s1}(s)}{ds} < 0 \quad \left(\because \frac{d\ln n_{1,s1}(s)}{ds} > 0 \text{ as shown in (92)}\right) \tag{46}$$

where $n_{1,s1}(s)$ means the mass of the home industry 1 in Stage 1. ■

The following corollaries explains the direction and size of the early innovation effect.

Corollary 1 *The early innovation in the home industry 1 by the home production subsidy increases the welfare of the home and foreign country.*

Corollary 2 *The size of gain from early innovation in the home industry 1 depends on (i) how much the instantaneous utility of the home and foreign representative consumer increases after home innovation ($\ln U_2 - \ln U_1$ and $\ln U_2^* - \ln U_1^*$ respectively), (ii) how much sooner the refinement occurs* *in the home industry 1 because of the home subsidy* ($\frac{dt_r(s)}{ds}$) *and (iii) how long it takes to occur the refinement without the home subsidy in the home industry 1* ($t_r(0)$)*.*

To build intuition for the condition (i) in Corollary 2, even though productivity radically increases after innovation, I look into the local effects of a productivity increase by taking a first-order approximation of the model in the neighborhood of the initial symmetric equilibrium. Table 8 in Appendix B shows the results.

Innovation in the home industry 1 increases both home and foreign country's welfare to the first order as seen in (109) and (111). Since $a_{1,t}$ jumps a lot in Stage 2 by nature, it is necessary to see whether the results in Table 8 are maintained even when $a_{1,t}$ radically goes up. I present, through model simulations under a reasonable parameterization, how mass of firms, relative wage, utility based CPI and instantaneous utility at home and abroad change while $a_{1,t}$ increases in Section 5. The result shows that the more $a_{1,t}$ rises, the more the both the home and foreign utility in Stage 2 increase relative to Stage 1.

Condition (ii) means that the earlier the production subsidy makes a refinement arrive in the targeted industry, the more successful the policy will be. Equation (5) ($t_r(s) = \frac{Q_1^{-1}(\bar{Q}_1)}{n_{1,s1}(s)}$) tells that how much earlier an industrial policy make a refinement occur depends on how much the policy increases the mass of firms in the targeted industry in the early stage.

The intuition from condition (iii) is that the timing that the targeted industry transits to the high growth stage without policy supports, $t_r(0)$, should not be too far away from now. In other words, the home targeted industry should not be too far from the frontier in order for the policy to be successful. If it takes too long time for the targeted industry to achieve high productivity, the cost by the production subsidy becomes larger and the benefit from innovation is discounted a lot.

4.1.3 Overall Welfare Effect

Table 1 summarizes the overall welfare effect of production subsidy in Case 1. This result has two main implications for the home country. First, it suggests that if industrial policy supports a young industry with high growth potential, it can increase productivity growth and welfare in the long-run.

This is in line with the argument of Aghion et al. (2015). Second, despite the long-run benefit, it provides an explanation on why industrial policy is not desirable under certain circumstances even for promoting catch-up in developing countries. The policy is likely to cause welfare loss while subsidising targeted industry, and it takes time before productivity actually increases. If the short-run welfare loss dominates the long-run gain from growth, the policy will eventually decrease the home welfare. This supports the importance of dynamic analysis for evaluating industrial policy.

In contrast to the home country, the industrial policy unambiguously increases the welfare of the foreign country. This suggests that developing countries' industrial policy fostering catching up in industries which are already matured in developed countries is beneficial to developed countries.

Table 1: Summary of welfare effect in Case 1 (Catch-up)

Effect	Home	Foreign
Short-run resource reallocation effect	(+)/(-)	(+)
Long-run gain from speeding up innovation	(+)	(+)
Overall effect	(+)/(-)	(+)

4.2 Case 2: Frontier Technology Races

Now, I assume that the foreign industry 1 is also young and has high growth potential like the home industry 1. Since the industry is operating at the technological frontier in both countries, this sets up a race to take leadership in the industry through innovation. The effects of the subsidy are distinct from the catch-up situation in Case 1. What follows explains how.

Under the symmetric initial condition, without the home country's industrial policy, a refinement occurs at the same time in the home and foreign industry 1 ($t_r(0) = t_r^*(0)$). However, if the home government gives a production subsidy to firms in industry 1, resources will be re-allocated in the home and foreign country in response to the policy. Before a refinement occurs in one of the home and foreign country, equilibrium is the same as that in Stage 1 (short-run) in Case 1, implying that the short-run *resource reallocation effect* in Case 2 is also the same as that in Case 1.

As it can be seen in (92) and (94) in Appendix A, the production subsidy causes the mass of firms in the home industry 1, $n_{1,t}$, to increase and the mass of firms in foreign industry 1, $n^*_{1,t}$, to decrease in Stage 1. This leads the innovation to occur earlier in the home industry 1 and to be delayed in the foreign industry 1 (i.e. if $s > 0$, $t_r(s) < t^*_r(s)$). The following proposition formally shows the home country's production subsidy causes the innovation in the home industry 1 to occur earlier but that in the foreign country to be delayed in Case 2.

Proposition 5 *Production subsidy causes the home industry 1 to move from the early stage to the high-productivity stage but it delays the transition of the foreign industry 1.*

Proof.

$$\frac{dt(s)}{ds} = -\frac{Q_1^{-1}(\bar{Q}_1)}{n_{1,s1}(s)^2}\frac{dn_{1,s1}(s)}{ds} = -t(s)\frac{d\ln n_{1,s1}(s)}{ds} < 0 \tag{47}$$

$$\frac{dt^*_r(s)}{ds} = -\frac{dt_r(s)}{ds}\left(\frac{n^*_{1,s1}(s)}{n^*_{1,s2}} - 1\right) - t_r(0)\frac{n^*_{1,s1}(s)}{n^*_{1,s2}}\frac{d\ln n^*_{1,s1}(s)}{ds} > 0 \tag{48}$$

where $n_{1,sj}(s)$ and $n^*_{1,sj}(s)$ are the mass of the home and foreign industry 1 in Stage j. ■

Based on Proposition 5, three stages can be defined over time as follows.

Definition 4

- *Stage 1* ($0 < t < t_r(s)$)*: A refinement has not occurred yet in both the home and foreign industry 1* ($a_{1,t} = 1$, $a^*_{1,t} = 1$)
- *Stage 2* ($t_r(s) \leq t < t^*_r(s)$)*: A refinement occurred in the home industry 1 but has not occurred yet in the foreign industry 1* ($a_{1,t} = k$, $a^*_{1,t} = 1$)
- *Stage 3* ($t^*_r(s) \leq t$)*: A refinement occurred in both the home and foreign industry 1* ($a_{1,t} = k$, $a^*_{1,t} = k$)

Given the above definition, the welfare of the home and foreign representative consumer can be re-expressed as:

$$\ln W = \int_0^{t_r(s)} e^{-\rho t}\ln U_1 dt + \int_{t_r(s)}^{t^*_r(s)} e^{-\rho t}\ln U_2 dt + \int_{t^*_r(s)}^{\infty} e^{-\rho t}\ln U_3 dt \tag{49}$$

$$\ln W^{*}=\int_{0}^{t_r(s)} e^{-\rho t}\ln U_1^{*}dt+\int_{t_r(s)}^{t_r^{*}(s)} e^{-\rho t}\ln U_2^{*}dt+\int_{t_r^{*}(s)}^{\infty} e^{-\rho t}\ln U_3^{*}dt \tag{50}$$

where U_j (U_j^{*}) denotes the home (foreign) representative consumer's utility in stage j. The local effect of production subsidy on the home and foreign welfare in the neighborhood of the initial equilibrium is respectively:

$$\frac{d\ln W}{ds} = \underbrace{\int_{0}^{t_r(0)} e^{-\rho t}\frac{d\ln U_1}{ds}dt}_{\text{Short-run resource reallocation effect}} \underbrace{-e^{-\rho t_r(0)}(\ln U_2-\ln U_1)\frac{dt_r(s)}{ds}}_{\text{Gain from early home innovation}} \underbrace{-e^{-\rho t_r^{*}(0)}(\ln U_3-\ln U_2)\frac{dt_r^{*}(s)}{ds}}_{\text{Loss from delayed foreign innovation}} \tag{51}$$

$$\frac{d\ln W^{*}}{ds} = \underbrace{\int_{0}^{t_r(0)} e^{-\rho t}\frac{d\ln U_1^{*}}{ds}dt}_{\text{Short-run resource reallocation effect}} \underbrace{-e^{-\rho t_r(0)}(\ln U_2^{*}-\ln U_1^{*})\frac{dt_r(s)}{ds}}_{\text{Gain from early home innovation}} \underbrace{-e^{-\rho t_r^{*}(0)}(\ln U_3^{*}-\ln U_2^{*})\frac{dt_r^{*}(s)}{ds}}_{\text{Loss from delayed foreign innovation}} \tag{52}$$

4.2.1 Loss from Delaying Foreign Innovation

In the equation (51) and (52), the analysis for the first two terms, the short-run resource allocation and gain for early home innovation effect, are the same as Case 1. Thus, in this section, I only add explanation on welfare analysis for the third term ($-e^{-\rho t_r^{*}(0)}(\ln U_3-\ln U_2)\frac{dt_r^{*}(s)}{ds}$ and $-e^{-\rho t_r^{*}(0)}(\ln U_3^{*}-\ln U_2^{*})\frac{dt_r^{*}(s)}{ds}$, respectively) from delayed innovation in the foreign country.

Corollary 3 *The delayed innovation in the foreign industry 1 by the home production subsidy decreases the welfare of the home and foreign country.*

Corollary 4 *The size of loss from delayed foreign innovation depends on (i) how much the instantaneous utility of the home and foreign representative consumer increase after foreign innovation ($\ln U_3-\ln U_2$ and $\ln U_3^{*}-\ln U_2^{*}$ respectively), (ii) the length of the delay before the refinement occurs in the foreign industry 1 attributable to the home subsidy ($\frac{dt_r^{*}(s)}{ds}$) and (iii) how long the refinement takes to occur the refinement initially without the home subsidy in the foreign industry 1 ($t_r^{*}(0)$).*

4.2.2 Overall Welfare Effect

Table 2 summarizes the effects of the home production subsidy on the home and foreign welfare. Because of the loss from delayed foreign innovation, the production subsidy is less likely to increase home welfare compared with Case 1. However, based on the result from in Figure 2a, since the degree of positive spillover from the counterpart country's productivity increase is relatively small, this additional negative effect does not change much policy implications for the home country.

In contrast, the delayed innovation in the foreign country can affect quite negatively foreign welfare. It is interesting in Case 2 that the negative effect from delayed innovation can dominate the other two positive effects and consequently the home production subsidy has a beggar-thy-neighbor effect. The higher the growth potential of the targeted industry, the more a home production subsidy increases the home welfare but decreases the foreign welfare. In such circumstances, the foreign government will respond to the home policy by conducting countervailing policy to offset the beggar-thy-neighbor effect, which means the home and foreign country are in a game situation.

In the next section, I study under what conditions home production subsidy increases or decreases the home and foreign welfare through model simulation for Case 1 and 2.

Table 2: Summary of welfare effect in Case 2

Effect	Home	Foreign
Short-run resource reallocation effect	(+)/(-)	(+)
Gain from speeding up home innovation	(+)	(+)
Loss from delaying foreign innovation	(-)	(-)
Overall effect	(+)/(-)	(+)/(-)

5 Model Simulation

In this section, I study how production subsidy affects the welfare[9] at home and abroad with a reasonable parameterizations.

[9]To define the welfare in equation (38), the utility in (6) should be positive. However, under the assumption of $\psi < 1$, the utility is negative in the initial equilibrium. To address this issue, I employ a different utility function, $U_t' = -\frac{1}{U_t}$, for model simulation. Since this new utility function is a monotonic increasing transformation of the original utility function, it does not affect the analysis presented in the previous sections.

5.1 Catch-Up Revisited

As shown in the previous section, since production subsidy in Case 1 unambiguously beneficial for the foreign country in the model, I will focus on under what conditions production subsidy increases home welfare in this section.

Calibration I set parameter values which are taken from standard values in the literature. The elasticity of substitution between domestic varieties within an industry is set at $\gamma = 4$ and the elasticity of substitution between the home- and foreign-specific industry-level aggregate is set at $\eta = 2.5$ based on Feenstra et al. (2018). The elasticity of substitution between industries is set at $\sigma = 1.36$ following Redding et al. (2021). The trade cost is set at $\tau = 0.25$ following Obstfeld and Rogoff (2001). Since intertemporal elasticity of substitution is usually set between 1/2 and 1, it is set at $\psi = 0.75$. The discount rate is set at $\rho = 0.042$.

I use an initial condition, $v = v^* = 1$ and $L = L^* = 10$. Initially, I assume that productivity of each industry in the home and foreign country is set at $a_{1,t} = 6.67$ and $a_{2,t} = a^*_{1,t} = a^*_{2,t} = 10$, which reflects a situation where the home industry 1 is currently less productive than the foreign industry 1 but is trying to catch up. I assume that productivity increases 50 percent[10] after the refinement ($a_{1,t} = 10$ if $t \geq t_r$).

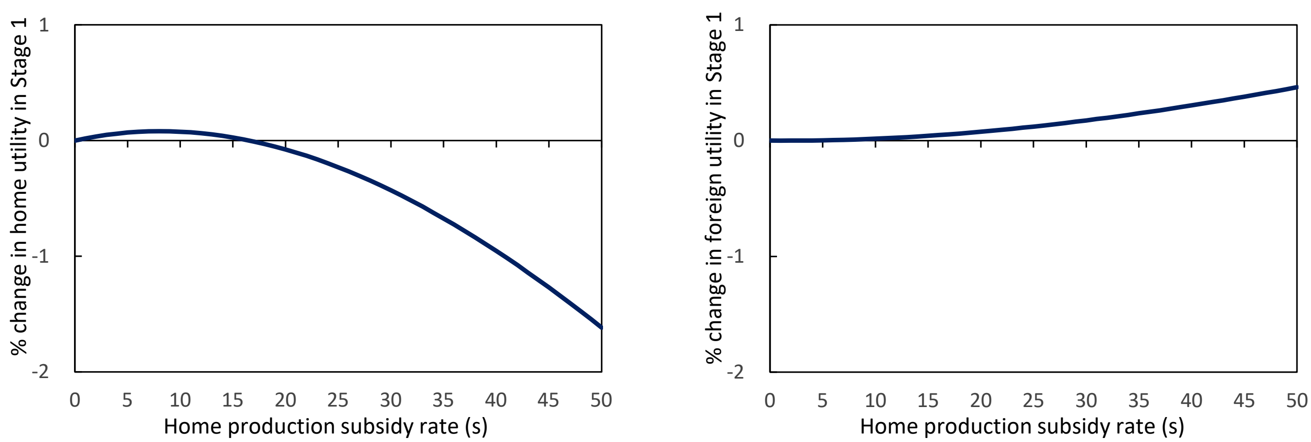
I assume a function for knowledge stock reflecting decreasing marginal accumulation from firms' activity as follows

$$Q_{1,t} = \sqrt{\int_0^t n_{1,j} dj} \tag{53}$$

The threshold for the level of knowledge stock which realizes refinement in the home industry 1 is set at $\bar{Q}_1 = 1.8$. This makes the refinement occur around $t = 10$ without a policy support in the model. I will use these values of parameters as a benchmark.

[10] The 50 percent growth matches the long-run treatment effect of Korean government's HCI drive on the productivity of those industries

Short-run Resource Reallocation Effect As Table 1 shows, the short-run welfare effect of production subsidy is important regarding whether the subsidy is beneficial for the home country or not. Thus, I conduct an experiment to discern how home utility in Stage 1 changes as the production subsidy rate to industry 1 varies. The simulation results using the benchmark parameters show that the production subsidy increases home utility in Stage 1 (pre-refinement) until $s = 0.08$. However, as seen from Figure 1a, the increase in the home utility, which is 0.1% at $s = 0.08$, is negligible in size. Also, when the production subsidy rate exceeds 0.08 ($s > 0.08$), home utility starts to fall increasingly rapidly. If $s > 0.16$, the home instantaneous utility becomes even less than that without the production subsidy. In contrast, the foreign utility in Stage 1 is increasing further while s increases.



(a) Home instantaneous utility in Stage 1 (b) Foreign instantaneous utility in Stage 1

Figure 1: Instantaneous utility in Stage 1 (Pre-refinement) while changing production subsidy rate s

Notes: Figure 1a and 1b show how the home and foreign instantaneous utility in Stage 1 change while s increases.

Long-run Gain from Speeding up Innovation As seen in Corollary 1, both the home and foreign country unambiguously profit from innovation in the home industry 1 being hastened by a production subsidy. Here, I focus on which factors affect the size of long-run gain from earlier home innovation.

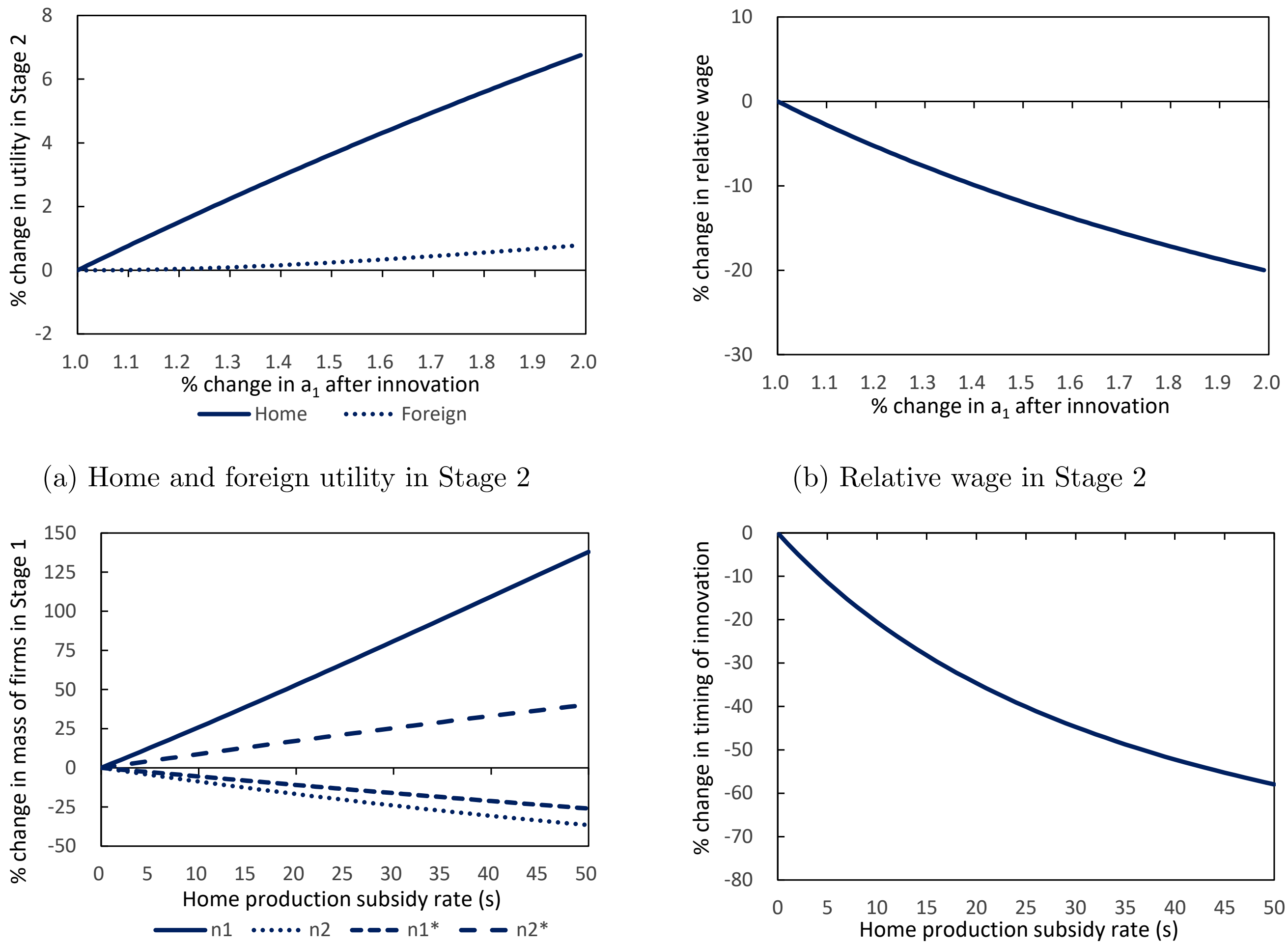
Condition (i) in Corollary 2 means that, for both the home and foreign country, the more utility increases in Stage 2 compared with Stage 1, the more a home production subsidy increases both countries' welfare. As Figure 2a shows, the degree to which utility increases in Stage 2 is determined by the degree of productivity improvement in the targeted industry after innovation. I note two implications from the result in Figure 2a. First, if the growth potential of the targeted industry is high enough, the size of the utility increase in Stage 2 (post-refinement) is much larger than the size of utility losses from the tax in Stage 1 as seen in Figure 1a and 2a.[11] Second, the direct utility gains in the home country from home innovation is much larger than its spillover effect in the foreign country since a productivity increase in the home country causes the home (foreign) country's terms of trade improvement (deterioration) as seen in Figure 2b.

Regarding Condition (ii) in Corollary 2, a higher subsidy induces earlier innovation as seen in Figure 2c and 2d. It is worth noting that this clearly presents the trade off between short-run loss and long-run gain by the policy. As Figure 1a and 2d show, a higher subsidy rate leads to larger short-term losses due to increased taxation in Stage 1 but a larger long-run gain from earlier innovation in targeted industry.

Overall Welfare Effect Figure 3 shows how the overall home and foreign welfare changes as the production subsidy rate s increases. Using the benchmark parameters, the optimal production subsidy rate for the home country is 0.37. Under the optimal subsidy rate, the home and foreign welfare increase 10.1% and 2.1%, respectively. In this case, a home production subsidy increases both home and foreign welfare and thus the foreign country does not need to respond to the home policy.

The simulation results for Case 1 suggest that if industrial policy does not have a sufficiently large growth-boosting effect in the targeted industry, the policy can only increase home welfare modestly at best. This is why the timing of industrial policy is important for success of the policy. If industrial policy supports an already-matured industry, it is hard to expect a growth-boosting effect in the industry. This is consistent with theoretical and empirical analysis of Bartelme et al. (2021) and Lashkaripour and Lugovskyy (2021). When considering additional policy costs such as resource

[11]For example, the home utility in Stage 1 decreases by 1.6% with $s = 0.5$ which is a high subsidy rate. In contrast, the home utility in Stage 2 increases by 3.7% if productivity in the targeted industry rises 50% after innovation.



(a) Home and foreign utility in Stage 2

(b) Relative wage in Stage 2

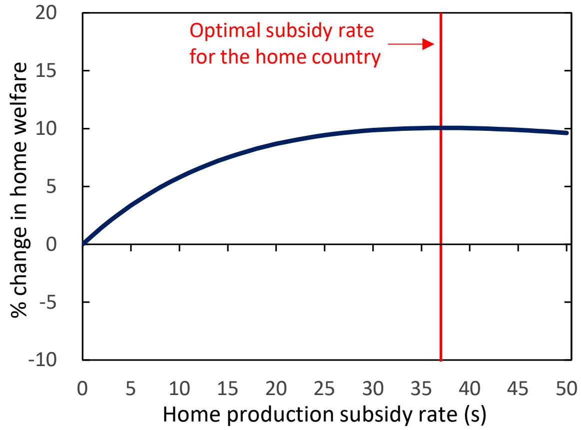
(c) Mass of firms in Stage 1

(d) Timing of innovation in home industry 1

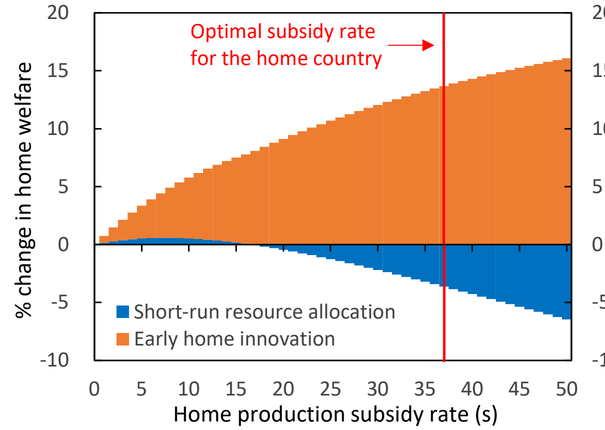
Figure 2: Factors affecting long-run gain in Stage 2 (Post-refinement)

Notes: Figure 2a and 2b show how the home and foreign utility, and the relative wage in Stage 2 change while the degree of productivity improvement in Stage 2 in the home industry 1, which is percentage change in $a_{1,t}$ in Stage 2, varies. Figure 2c and 2d present how the mass of firms in Stage 1 and timing of innovation in the home industry 1 change while production subsidy rate s increases.

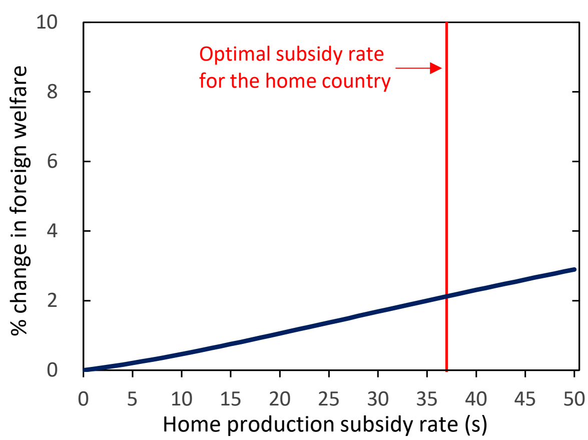
misallocation “within” a targeted industry as argued in Kim et al. (2021), the following policy implication becomes more apparent: policy makers need to pay great attention to how industrial policy can boost growth of targeted industry from a dynamic standpoint as well as how the policy affects economic efficiency in a static setting. In addition, where the targeted industry is in its lifecycle is an important criterion for determining whether the policy will produce growth-boosting effect or not.



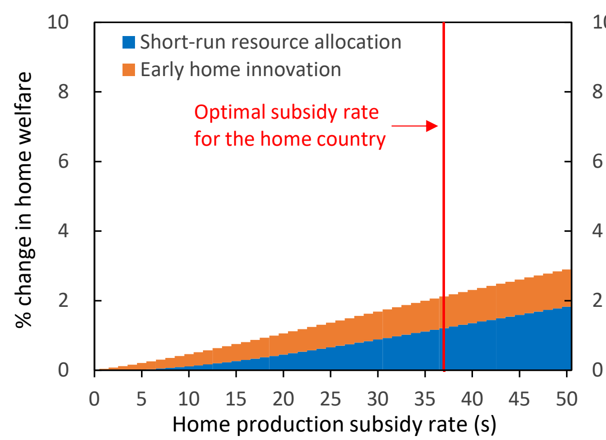
(a) Home welfare



(b) Decomposition of home welfare change



(c) Foreign welfare



(d) Decomposition of foreign welfare change

Figure 3: Overall welfare change by home production subsidy in Catch-up

Notes: Figure 3a and 3c show how the overall home and foreign welfare change while home production subsidy rate s increases in Case 1. Figure 3b and 3d present the decomposition of the home and foreign welfare change into 'short-run resource allocation effect' and 'early home innovation effect'. The red line in the figures represents home country's optimal subsidy rate.

Sensitivity Analysis I also conduct sensitivity analyses to study under what conditions industrial policy can increase home welfare in Case 1, especially focusing on the role of the micro and macro elasticity of substitution (γ and η respectively in the model).

Given a micro elasticity of substitution (γ), a higher macro elasticity of substitution (η) magnifies the positive effects on home welfare in two ways. First, with a higher η, a production subsidy causes the mass of firms in the targeted industry to increase more because of higher substitution between the home and foreign products, which leads a refinement to occur earlier in the industry. Second, under a higher η, the relative home wage increases more in both Stage 1 and 2 as seen in the comparative statics results (96) and (107), and thus the home country benefits more from improvement in the terms of trade. It is worth mentioning that this result contrasts with the implication in Melitz (2005) that the welfare benefit from a production subsidy to an infant industry *decreases* with the level of product differentiation between home and foreign goods. The difference is mainly caused by the above-mentioned effects from change in relative wage or terms of trade do not occur in the small open economy setting of Melitz (2005). This implies that considering changes in relative wage or terms of trade in a large-country open economy model instead of a small open economy model provides quite different welfare implications.

On the other hand, given a macro elasticity of substitution (η), a higher micro elasticity of substitution (γ) dampens home welfare gains from a domestic subsidy in two ways. First, it makes the benefit from correcting monopolistic market power in Stage 1 smaller since the inefficiency caused by monopolistic market power is vanishing with higher γ.[12] Second, a production subsidy is not capable of significantly increasing the mass of firms in the targeted industry subject to high γ. This is because the profit margin of firms in an industry with γ is already low, and thus, a production subsidy does not substantially encourage the entrance of new firms.

Figure 4 shows well the relationships explained above. Based on the results in Feenstra et al. (2018) that the range of estimates of macro elasticity of substitution falls mostly between 1 and 4, Figure 4 indicates that a production subsidy increases home welfare under most plausible parameterizations in the case where the targeted industry's productivity rises 50% after innovation. Even though a production subsidy is likely to be beneficial to the home country in this case, it is worth noting that there is a chance, which is not negligible, that the policy decreases home welfare. If productivity increases only 25% after the refinement, the probability that a production subsidy worsens home welfare increases, as seen in Figure 4. Policy makers have to be cautious of providing a production subsidy—especially when the substitutability between home and foreign goods is low and

[12] If the production subsidy rate exceeds $\frac{1}{\gamma-1}$, the subsidy is already causing distortion even in a partial equilibrium sense. In this case, higher γ brings about more distortion in Stage 1.

thus domestic firms have difficulties expanding their market share in the export market even with policy support.

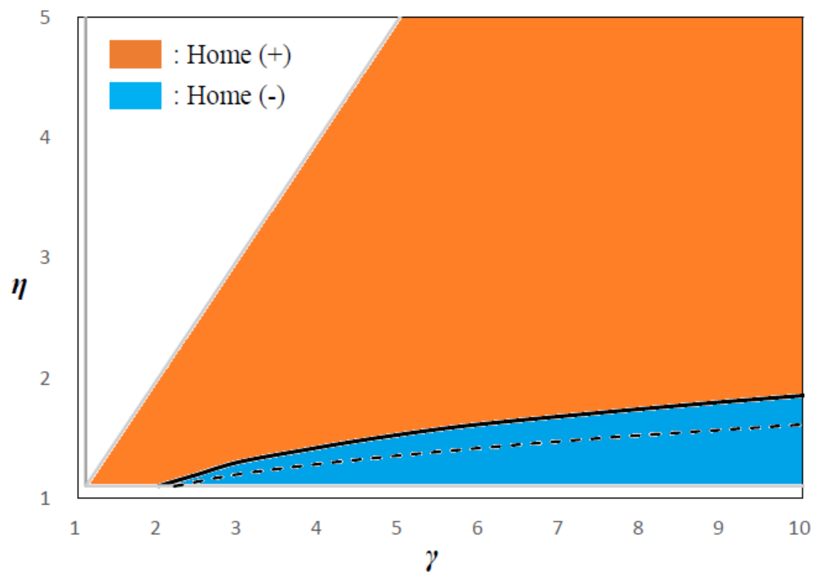


Figure 4: Sensitive analysis in Catch-up
(Effect of $s = 0.1$ on the home welfare, $k = 1.5$ vs $k = 1.25$)

Notes: Figure 4 shows whether a 10 percent production subsidy rate ($s = 0.1$) increases home welfare under $k = 1.5$ and $k = 1.25$ respectively while micro and macro elasticity of substitution (γ and η respectively) change. In the figure, the dotted line represents the boundary line in the case of $k = 1.5$.

5.2 Frontier Technology Races Revisited

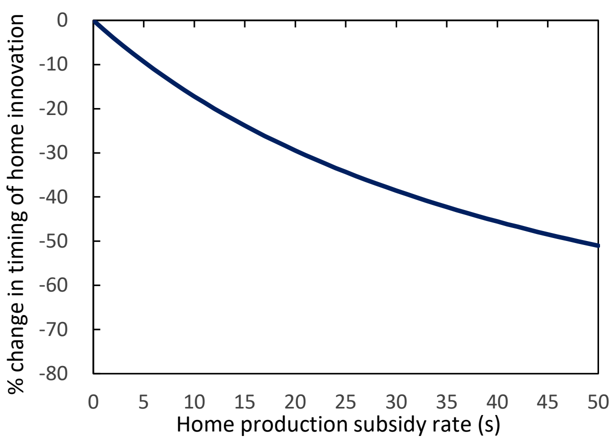
For the benchmark case for frontier technology races (Case 2), I use the same values of parameters as the benchmark for catch-up (Case 1): $\gamma = 4$, $\eta = 2.5$, $\sigma = 1.36$, $\psi = 0.75$, $\tau = 0.25$ and $\rho = 0.042$. It is again assumed that $v = v^* = 1$ and $L = L^* = 10$.

The initial productivity of each industry in the home and foreign country is set at $a_{1,t} = a^*_{1,t} = 6.67$ and $a_{2,t} = a^*_{2,t} = 10$. As in Case 1, I assume that productivity increases by 50 percent after the refinement in the home and foreign industry 1 ($a_{1,t} = 10$ if $t \geq t_r$, $a^*_{1,t} = 10$ if $t \geq t^*_r$). Based on the assumption of the symmetric initial condition, the knowledge stock accumulation function is the same in the home and foreign industry 1:

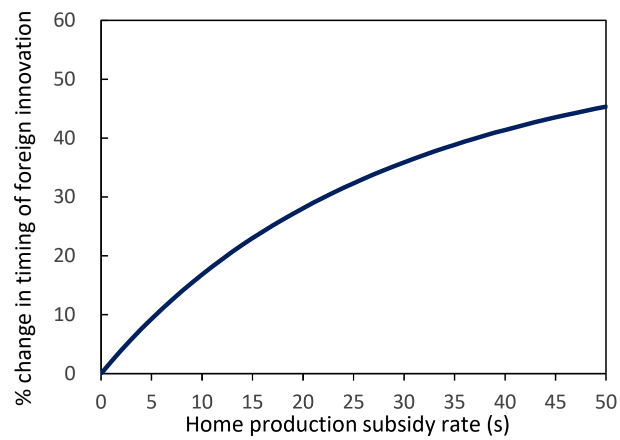
$$Q_{1,t} = \sqrt{\int_0^t n_{1,j} dj}, \; Q^*_{1,t} = \sqrt{\int_0^t n^*_{1,j} dj} \tag{54}$$

The threshold for the level of knowledge stock required for realization of a refinement are $\bar{Q}_1 = \bar{Q}^*_1 = 2.1$ which makes the refinement occur around $t = 10$ without any home or foreign policy intervention.

As shown in Figure 5a and 5b, a higher production subsidy rate causes a refinement to occur earlier in the home industry but later in the foreign industry 1. The delayed foreign innovation negatively affects the home and foreign welfare.



(a) Timing of innovation in home industry 1

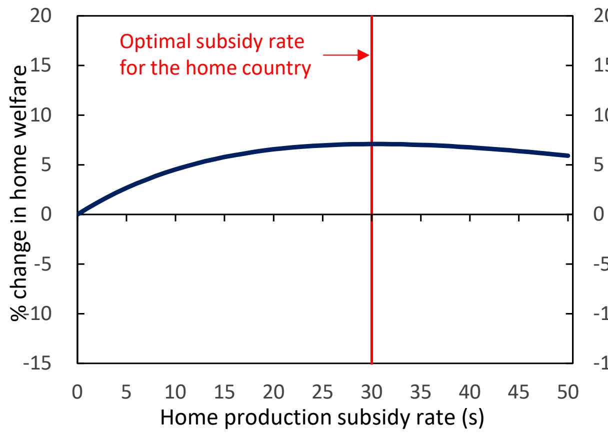


(b) Timing of innovation in foreign industry 1

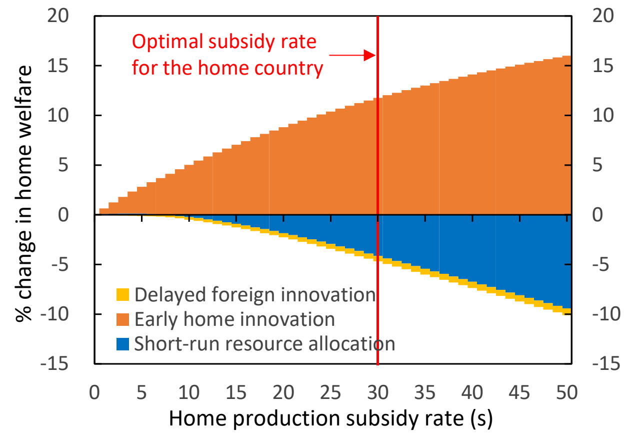
Figure 5: Timing of home and foreign innovation if home only subsidizes a frontier industry

Notes: Figure 5a and 5b show how the timing of refinement in the home and foreign industry 1 change while s increases.

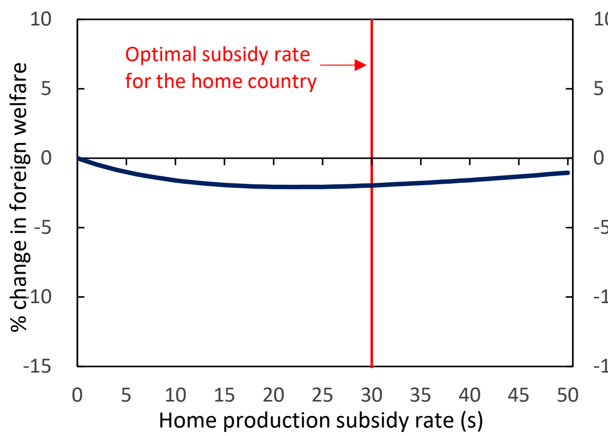
Figure 6 shows the home and foreign welfare change in Case 2 while production subsidy rate s increases. In the benchmark case, the home country's optimal subsidy rate is 0.30 and home welfare increases by 7.1% under the optimal subsidy rate which is not much different from Case 1. This implies that the negative spillover from delayed foreign innovation does not affect home welfare a lot. However, under the home country's optimal production subsidy rate, foreign welfare decreases by 2.0%. It is also worth noting that if $0 < s < 0.68$, the subsidy decreases foreign welfare. This implies that home industrial policy is highly likely to adversely affect foreign welfare in the case of frontier technology races.



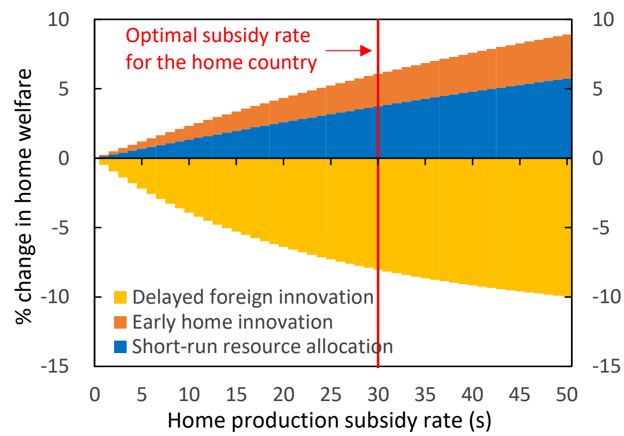
(a) Home welfare



(b) Decomposition of home welfare change



(c) Foreign welfare



(d) Decomposition of foreign welfare change

Figure 6: Overall welfare change from unilateral home production subsidy in Frontier Technology Races

Notes: Figure 6a and 6c show how the overall home and foreign welfare change while the home production subsidy rate s increases in Case 2. Figure 6b and 6d present the decomposition of the home and foreign welfare change into 'short-run resource allocation effect', 'early home innovation effect', and 'delayed foreign innovation effect'. The red line in the figures represents home country's optimal subsidy rate.

Policy Reaction The above result naturally provides an implication that if a country supports a young industry with high-growth potential in which home and foreign firms are competing to take initiative, the foreign country has to conduct countervailing policy in order for its industry not to drop out of the race. Considering the importance of accumulation of data for recent frontier technologies such as Artificial Intelligence and machine learning, such policy reaction becomes even more necessary.

In this context, I analyze the equilibrium in a game situation where both the home and foreign government set a production subsidy rate to foster domestic industry 1. In Figure 7, I present the home and foreign country's reaction curves showing each country's optimal production subsidy rate given any subsidy rate chosen by the other country. The Nash equilibrium in this situation is established at the point where the two curves intersect. The equilibrium shows well the policy competition between two countries in which each country responds to the other country's production subsidy rate by more aggressively setting its own subsidy rate. It is interesting that both home and foreign welfare increase in the Nash equilibrium compared with the benchmark case where the home country sets its optimal production subsidy rate but the foreign country does not conduct any policy. This is because the short-run benefit from the counterpart country's aggressive production subsidy outweighs the negative spillover effects on innovation for both countries.

I also show how the equilibrium changes if the two countries cooperatively conduct industrial policy. Surprisingly, they set a higher production subsidy rate in the cooperative equilibrium than in the Nash equilibrium. The reason for this is that each country internalizes positive spillovers to the counterpart country when deciding its production subsidy rate. As seen in Table 3, the welfare outcome in the cooperative equilibrium is a Pareto improvement compared with the Nash equilibrium.

The above results imply that a policy competition to promote specific economic sectors can be beneficial worldwide if the sector is young in both countries and has high growth-potential. Interestingly, this implication is opposite to a common criticism of industrial policy that it is not efficient for every country to foster the same industry.

Sensitivity Analysis As in Case 1, where the home country subsidizes an industry to catch up with frontier technology, I conduct a sensitivity analysis for Case 2, wherein the home country subsidizes efforts to expedite the discovery of new technology. The micro and macro elasticities of substitution play a pivotal role in determining the welfare effects of industrial policy on the foreign country. A higher η reduces foreign welfare, while a higher γ enhances it. Given a specific γ, a

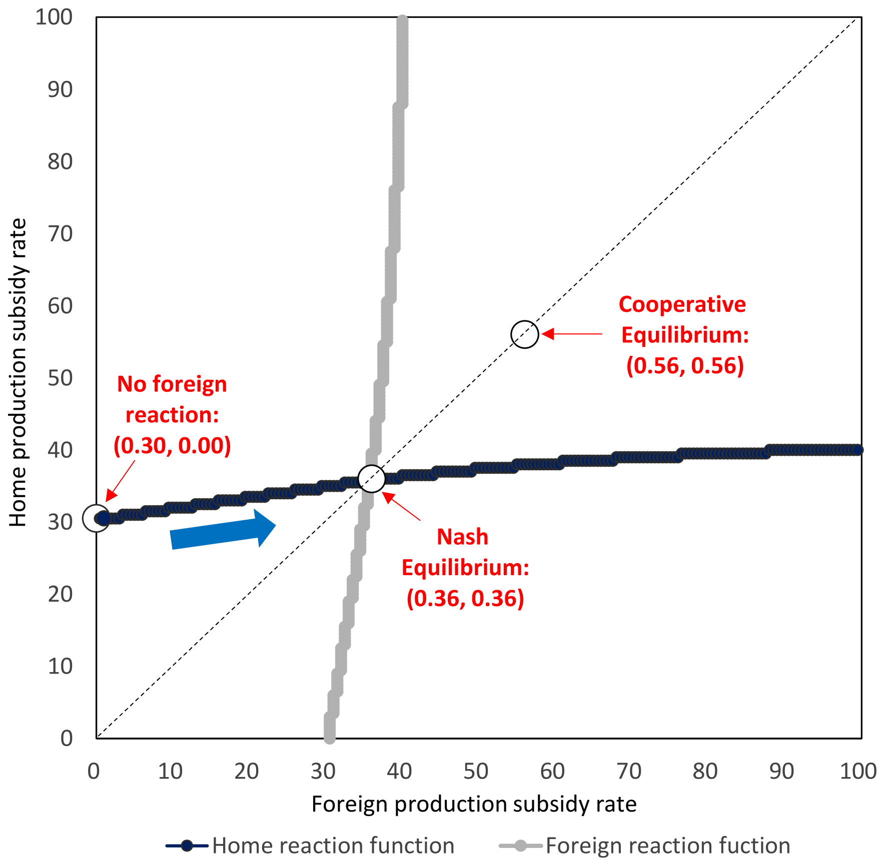


Figure 7: Equilibrium under policy competition

Notes: Figure 7 presents the home and foreign country's reaction function which show each country's optimal production subsidy rate given any subsidy rate chosen by the other country.

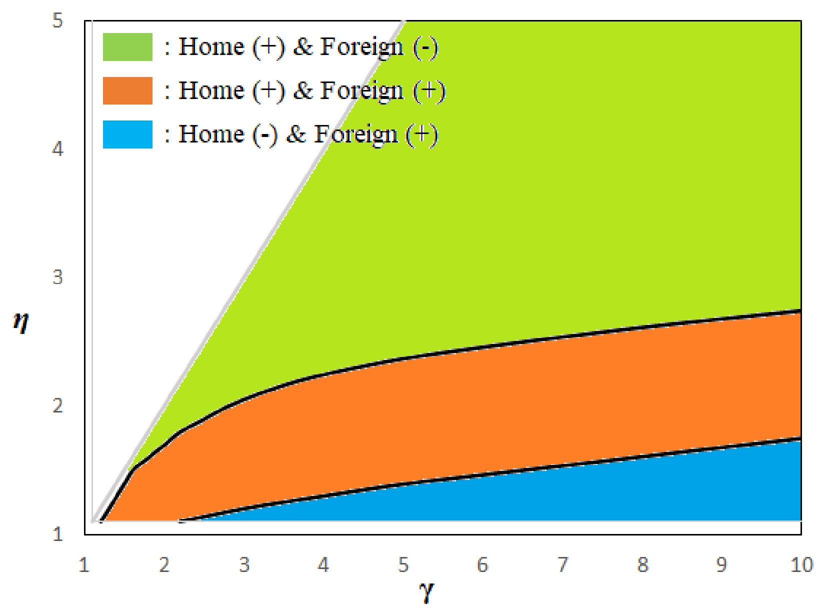
Table 3: Home and foreign welfare in each equilibrium in Figure 7

	Benchmark (No foreign reaction)		Nash		Cooperative	
	Home	Foreign	Home	Foreign	Home	Foreign
Short-run resource reallocation	-4.2%	+3.7%	+0.8%	+0.8%	-1.8%	-1.8%
Timing of home innovation	+11.8%	+2.3%	+9.3%	+9.3%	+12.8%	+12.8%
Timing of foreign innovation	-0.5%	-8.0%				
Overall	+7.1%	-2.0%	+10.1%	+10.1%	+11.0%	+11.0%

Notes: This table presents the home and foreign welfare change in each equilibrium compared with the initial equilibrium.

high η facilitates substitution between home and foreign firms, accelerating innovation at home but impeding it abroad. Moreover, a higher η leads to a larger increase in the home wage compared to the foreign wage, resulting in more favorable terms of trade for the home country and a correspondingly less favorable situation for the foreign country. On the other hand, given a specific η, higher γ implies a more positive effect on reducing the price for the foreign country due to both the home subsidy and increased home productivity. However, it also indicates a greater subsidy expenditure for the home country.

Figure 8 illustrates how the relative sizes of these elasticities influence which country benefits from the home country's subsidy. Considering estimates of the micro and macro elasticity of substitution in Feenstra et al. (2018), where the former mostly lies between 1 and 10 and the latter mostly lies between 1 and 4, the sensitivity analysis shows that the home production subsidy worsens foreign welfare under wide range of parameters.



Effect of $s = 0.1$ on the home and foreign welfare, $k = 1.5$

Figure 8: Sensitive analysis in Frontier Technology Races

Notes: Figure 8 shows whether 10 percent of production subsidy rate ($s = 0.1$) increases the home and foreign welfare while micro and macro elasticity of substitution (γ and η respectively) change.

6 Empirical Examination of Catch-Up

I revisit the Heavy and Chemical Drive (HCI drive), conducted by the Korean government during the period 1973-1979, through the lens of this model. I examine the policy in a simple event study framework to check whether outcomes are consistent with the central mechanism in my model.

HCI Drive The Korean government launched its HCI drive in 1973, which aimed to support 6 industries: steel, non-ferrous metal, petrochemical, machinery, shipbuilding, and electronics. Supports included tax cuts, foreign credit allocation and providing new industrial complexes for those industries.[13] The table in Appendix C provides a detailed description of the targeted industries. The HCI drive unexpectedly ended after the assassination of President Park in October 1979. Among various policy supports, a tax cut is closely related to this paper. Baek and Kim (2023) estimates the wage subsidy rate if tax supports were implemented in the form of wage subsidies, based on the effective marginal corporate tax rate provided by Yoo (1991). Using the method proposed by Baek and Kim (2023), production is estimated to be about 11 percent.[14]

There are several reasons why the HCI drive is a good example to use for the empirical examination to check whether the policy outcomes are consistent with the central mechanism in the catch up case in the model. First, heavy and chemical industries were literally infant industries even compared with other industries in Korea at the beginning of the drive. During the 1960s, Korean economy had grown, led by labor-intensive light industries such as textiles, wearing apparel, and leather products. With the comparative advantage in light industries at that time (Lane (2022)), the HCI drive was criticized by many economists and businesses even when President Park was alive. In addition, since firms in Korea did not have any experience manufacturing HCI products, some even argued the government ought to foster light industries further based on the advantage of low labor costs. Second, different policy implications can be drawn depending on the time horizon. The HCI drive is often considered to be a successful industrial policy in the long-run aspect (Kim and Leipziger (1997),

[13] See Kim et al. (2021), Choi and Levchenko (2021) and Lane (2022) for more details.

[14] In this model, profit is proportional to output, and thus corporate tax can be transformed into a production subsidy.

Choi and Levchenko (2021), and Lane (2022)). Table 4 presents how HCI's value added, labor, and capital stock share in total manufacturing change during 1970-1990. As shown in the table, those shares significantly increased. However, as observed by Kim et al. (2021), the policy also causes inefficiency such as resource misallocation within the targeted industries, which was a reason why the policy was withdrawn right after President Park's assassination. In this context, it is plausible that there might be conflicting effects from the policy in the short-run and the long-run. Lastly, the timing of the policy can be used to identify its impact (Choi and Levchenko (2021), and Lane (2022)): i) the policy was partly initiated by external political event that President Nixon announced the end of direct U.S. military support for Asia-Pacific allies in 1969 and ii) it unexpectedly ended right after the assassination of President Park.

Table 4: HCI's Value added, Labor, Capital Stock Share in Total Manufacturing

Year	Value Added	Labor Input	Capital Stock
1970	39.7%	28.3%	44.1%
1990	57.3%	43.5%	60.1%

Source: Pyo et al. (1993)

6.1 Data

I compare outcomes in the targeted industries with those in the non-targeted industries to see how resource allocation, output and productivity changed during and after the HCI drive. For this comparison between the targeted and the non-targeted industries in Korea, I use 1) annual data for the number of plants and workers, and valued added for 28 Korean manufacturing industries between 1967 and 1986 from Statistics Korea's annual Mining and Manufacturing Survey and 2) annual data on total factor productivity for 28 Korean manufacturing industries between 1970 and 1986 from Pyo et al. (1993).

I also analyze how the relative wage between South Korea and other countries and how South Korea's real exchange rate changed during and after the HCI drive. For the comparison between Korea and other countries, I use annual wage data between 1970 and 1986 from OECD and annual real exchange rate data between 1967 and 1986 from Bank of International Settlements (BIS).

6.2 Empirical Specifications

As in Kim et al. (2021) and Lane (2022), I use a difference-in-difference estimation to explore the effect of the HCI drive on output, input, and productivity in the targeted industries compared with the non-targeted industries. The following equation is the difference-in-difference specification I use:

$$logY_{it} = \alpha + \sum_{j=\{1967-1971\}\cup\{1973-1990\}} \beta_j [D_i \times Year_t^j] + \delta_i + \delta_t + \epsilon_{it} \tag{55}$$

where Y_{it} is outcome variable for industry i in year t, and D_i is dummy variable which is equal to one if the industry were treated and otherwise zero. 9 industries are subject to the targeted industries out of 28 industries (See Appendix C for details). $Year_t^j$ is a year dummy variable. δ_i and δ_t are industry and time fixed effects respectively. Since I don't include year 1972 in the set of year dummy variables, β_j means the differential evolution of targeted and non-targeted industries relative to 1972. Standard errors are clustered by industry.

I also use another difference-in-difference estimation to see how the HCI drive affected wage and real exchange rate of Korea relative to other countries.

$$logY_{ct} = \alpha + \sum_{j=\{1967-1971\}\cup\{1973-1990\}} \beta_h [D_c \times Year_t^j] + \gamma_c + \delta_t + \epsilon_{ct} \tag{56}$$

where Y_{ct} is outcome variable for country c in year t, and D_c is dummy variable which is equal to one if the country is Korea and otherwise zero. $Year_t^j$ is a year dummy variable. δ_c and δ_t are country and time fixed effects respectively. Similarly, β_h means the differential evolution of Korea and other countries relative to 1972. Standard errors are clustered by country.

Findings Figure 9 plots β_j for four outcome variables from the equation (55), which are labor input, number of plants, value added, and TFP, and β_h for two outcome variables from the equation (56), which are wage and real exchange rate, with 95% confidence interval.

The results from the difference-in-difference estimations are consistent with the model's central mechanisms. First, the pattern of productivity growth in the targeted industries matches the productivity evolution described in Section 3.1.1. Total factor productivity of the targeted heavy and chemical industries increased significantly relative to that of non-treated manufacturing industries with persistence after 9 years from the implementation of the HCI drive—later than the end of the policy. In addition, after 9 years, the total factor productivity kept increasing even though labor input and the number of plants didn't increase much. These imply that, as industry lifecycle theory anticipates, 1) even though the HCI were young and had high growth-potential, some amount of time was required for a realization of productivity improvement and 2) the productivity growth in the targeted industries was maintained at such higher rate for a while after it outpaced that in other matured industries. From there results, it can be reasonably inferred that the targeted industries moved from the early stage to the high-productivity stage around 1982.

Second, the estimation results on labor input, number of plants and value added also correspond to the model's predictions on resource allocation. Figures 9a, 9b and 9c show that labor input, number of plants and value added of the targeted HCI increased significantly relative to the non-targeted manufacturing industries only a few years after the policy implementation, which is consistent with the comparative statics results in (92). After the end of the HCI drive, labor input and the number of plants in the targeted industries were nearly constant compared with the non-targeted industries but value-added kept increasing because of the productivity improvement.

Third, the movement of Korea's wage and real exchange rate compared with other countries is consistent with the model's mechanisms. Figures 9e and 9f show that 1) Korea's real exchange rate depreciated and 2) Korea's wage increased relative to other countries right after the policy implementation and after 1982. Since the period of the HCI drive and after 1982 correspond to Stage 1 and 2 in the model, respectively, the results match the comparative statics results in (96), (99), (107), and (110). The positive spillovers of the home country's industrial policy for the foreign country primarily materialize through a depreciation of the home country's real exchange rate in the model, as observed in Figure 9c. As the model predicts, foreign countries were likely to benefit from lower prices of Korean products, which was mainly caused by the supports from the HCI drive in the short-run and by Korea's productivity improvement in the long-run.

7 Extention: Production Subsidy vs R&D Subsidy

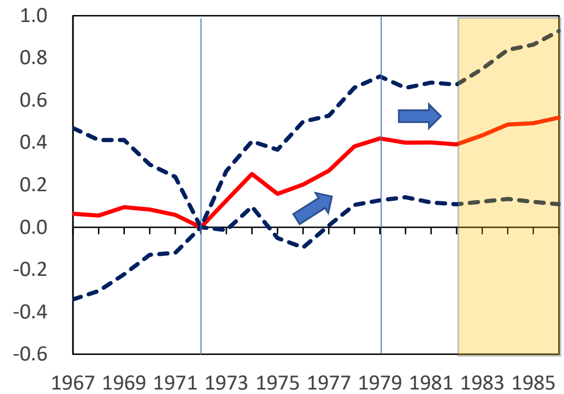
In this section, I study how a home R&D subsidy affects innovation and welfare both at home and abroad, compared to a home production subsidy. For this policy comparison, I focus solely on Case 2, where both the home and foreign industry 1 are in their nascent stages and have high growth potential—the frontier technology races scenario—but only the home country provides the subsidy.

Since the fixed cost in equation (2) is assumed to be devoted to R&D cost in the early stage of the indutry lifecycle, an R&D subsidy works by decreasing the fixed cost in the model. Thus, the zero profit condition for a firm in the home industry 1 changes to equation (57).

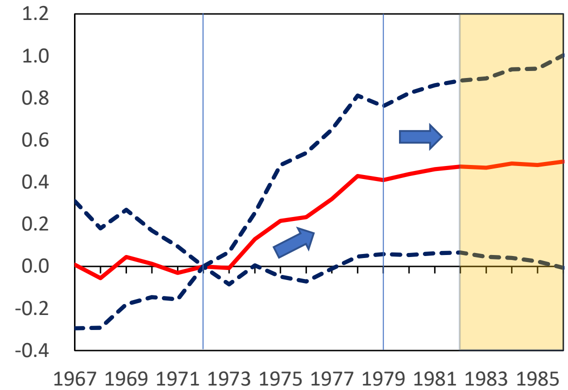
$$\begin{aligned}
\Pi_{1,t}(h) &= \frac{p_{1,t}(h)y_{1,t}(h)}{\gamma} \\
&= \frac{p_{1,t}(h)}{\gamma}\left[c_{1,t}(h)L + (1+\tau)c^*_{1,t}(h)L^*\right] \\
&= \frac{1}{\gamma}\left(\frac{p_{i,t}(h)}{P_{hi,t}}\right)^{1-\gamma}\left[\left(\frac{P_{hi,t}}{P_{i,t}}\right)^{1-\eta}\left(\frac{P_{i,t}}{P_t}\right)^{1-\sigma}P_t^{1-\psi}L + \phi\left(\frac{P_{hi,t}}{\epsilon_t P^*_{i,t}}\right)^{1-\eta}\left(\frac{P^*_{i,t}}{P^*_t}\right)^{1-\sigma}P_t^{*1-\psi}L^*\right] \\
&= (1-s_d)\frac{1}{v}
\end{aligned} \tag{57}$$

where s_d is the R&D subsidy rate. In contrast to a production subsidy, an R&D subsidy does not affect the price of the home varieties in industry 1 and thus the pricing equation (22) remains unchanged.

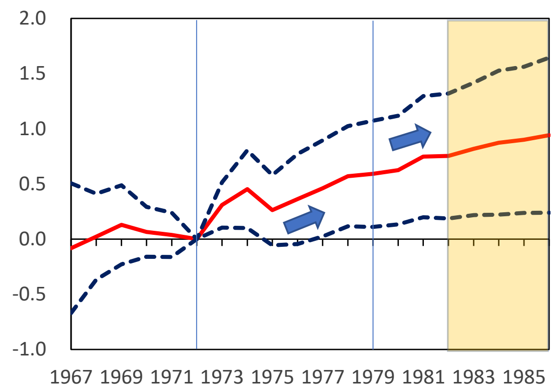
I again assume that the home government provides an R&D subsidy to firms in industry 1 until a refinement occurs in the industry. Under this assumption, a R&D subsidy has different effects only in Stage 1 (short-run) compared with a production subsidy. In more detail, it differently affects the home and foreign welfare by changing i) the instantaneous utility in Stage 1 (U_1 and U_1^*) and ii) the timing of refinement in the two countries (t_r and t_r^*) in a different way. Thus, I focus on how the



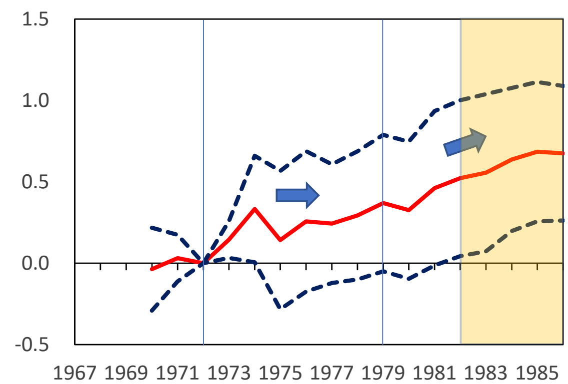
(a) Labor



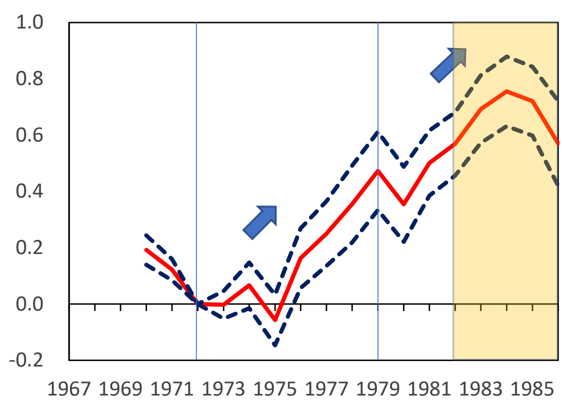
(b) Number of plants



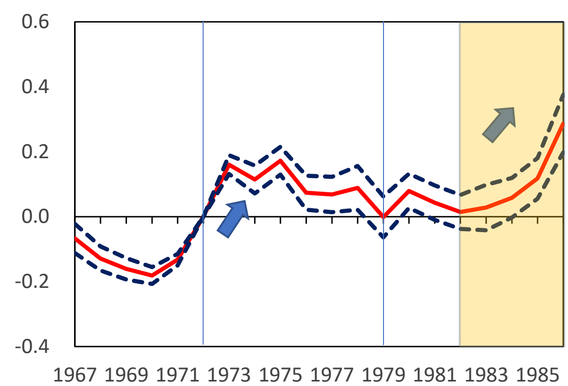
(c) Value added



(d) Total Factor Productivity



(e) Wage



(f) Real exchange rate

Figure 9: Impact of HCI drive, [1973-1979]

Notes: Figure 9a, 9b, 9c and 9d plot the estimated coefficients β_j along with a 95 percent confidence interval from equation (55). Figure 9e and 9f plot β_h along with a 95 percent confidence interval from equation (56). The vertical lines indicate the start and end year of the HCI drive. The shaded area represents the period after the evolution of targeted industries' productivity becomes significantly different relative to non-targeted industries.

short-run reallocation effect is different under a R&D subsidy. The comparative statics results for R&D subsidy are reported in Table 9 in Appendix D.

The results in Table 9 show that a R&D subsidy has qualitatively similar effects with a production subsidy. First, it unambiguously increases the mass of firms in the home industry 1 and decreases that in the foreign industry 1, which causes innovation to occur earlier in the home industry 1 and to be delayed in the foreign industry 1. Second, R&D subsidy can either increase or decrease the home instantaneous utility in Stage 1 while it unambiguously increases the foreign instantaneous utility in Stage 1.

Even though the direction of the aforementioned two effects are the same for both policies, the magnitude of those effects is different depending on which policy is implemented. For a fair comparison, it is necessary to compare effects of two policies given the same amount of required tax. The following equations present the change in tax by introducing a production and a R&D subsidy respectively in the neighborhood of the initial equilibrium.

$$\frac{dT}{ds} = \frac{\gamma}{v} n_{1,t}, \;\; \frac{dT}{ds_d} = \frac{1}{v} n_{1,t} \tag{58}$$

From the above equations, to the first order near the initial equilibrium, the relation between a production and a R&D subsidy rate which cause the same amount of tax is derived as follows.

$$\gamma ds = ds_d \tag{59}$$

Intuitively, this relationship is satisfied because the government subsidizes total revenue, which is equivalent to $\frac{\gamma}{v}$ in the model, with a production subsidy while it only subsidizes the fixed cost ($\frac{1}{v}$), which is equivalent to the operating profit, with a R&D subsidy. Equation (59) implies police-makers can set a R&D subsidy rate at γ times higher than a production subsidy rate with the same amount of a consequent tax.

Bringing the previous results from equation (59), Table 7 and Table 9 together allows me to compare the effects of two policies analytically. The following propositions show how the two policies differently affect the home and foreign economies.

Proposition 6 *To the first order in the neighborhood of the initial equilibrium, if a production and a R&D subsidy impose the same amount of tax, i) the home utility in Stage 1 under the production subsidy is greater than that under the R&D subsidy, and ii) the R&D subsidy increases the mass of firms in the home industry 1 more than the production subsidy, and consequently iii) the R&D subsidy causes a refinement to occur more earlier in the home industry 1 than the production subsidy.*

$$dU_1^p - dU_1^d \mid_{T_p=T_d} = \frac{1}{2} P_t^{1-\psi} (\gamma - 1) ds > 0 \tag{60}$$

$$d \ln n_{1,t}^p - d \ln n_{1,t}^d \mid_{T_p=T_d} = -(\gamma - 1) ds < 0 \tag{61}$$

$$dt_r^p - dt_r^d \mid_{T_p=T_d} = t_r(0)(\gamma - 1) ds > 0 \tag{62}$$

Proof. See Appendix E ■

Proposition 7 *To the first order in the neighborhood of the initial equilibrium, if a production and a R&D subsidy impose the same amount of tax, i) the policy effect on the foreign utility in Stage 1 and the mass of firms in the foreign industry 1 is the same under both policies, and ii) the R&D subsidy causes the refinement to be more delayed in the foreign industry 1 than the production subsidy.*

$$dU^{*p}_1 - dU^{*d}_1 \mid_{T_p=T_d} = 0 \tag{63}$$

$$d \ln n^{*p}_{1,t} - d \ln n^{*d}_{1,t} \mid_{T_p=T_d} = 0 \tag{64}$$

$$dt^{*p}_r - dt^{*d}_r \mid_{T_p=T_d} = -\left(\frac{n_1^*(1)}{n_1^*(2)} - 1 \right) t_r(0)(\gamma - 1) ds < 0 \tag{65}$$

Proof. See Appendix E ■

where a variable with superscript p (or d) means that corresponding to a production subsidy (or R&D subsidy).

For the home country, Proposition 6 shows a R&D subsidy is more effective for increasing the mass of firms in the targeted industry and consequently hastening innovation more than a production subsidy. This mainly arises from two reasons. First, as mentioned above, the a R&D subsidy rate is set at γ times higher than a production subsidy with the same amount of a consequent tax. Second, decrease in price by firms in the targeted industry in response to a production subsidy prevents new entry to the industry in some degree.

On the other hand, a R&D subsidy causes more distortion in Stage 1 than a production subsidy with the same amount of subsidy because it changes more the resource allocation in Stage 1. In short, a R&D subsidy hastens the long-run innovation more at the cost of greater distortion in the short-run. The following equation summarizes the difference in welfare effect for the home country between two policies.

$$\begin{aligned} d\ln W^p - d\ln W^d \mid_{T_p=T_d} = (\gamma - 1) \Bigg[& \underbrace{\frac{1}{2}\frac{1}{\rho}(1-\psi)\left(1-e^{-\rho t_r(0)}\right)}_{\substack{\text{Short-run resource reallocation} \\ :\ \text{Production} \succ \text{R\&D}}} \underbrace{-e^{-\rho t_r(0)}(\ln U_2 - \ln U_1)t_r(0)}_{\substack{\text{Early home innovation} \\ :\ \text{Production} \prec \text{R\&D}}} \\ & \underbrace{+e^{-\rho t_r(0)}(\ln U_3 - \ln U_2)\left(\frac{n_1^*(1)}{n_1^*(2)} - 1\right) t_r(0)}_{\substack{\text{Delayed foreign innovation} \\ :\ \text{Production} \succ \text{R\&D}}} \Bigg] ds \end{aligned} \tag{66}$$

Since each policy has its advantages and disadvantages compared with the other, which policy is better for the home welfare depends on the values of parameters. If the targeted industry's growth potential is very high, the positive welfare effect from the early home innovation will dominate the other effects for the home country. Thus, the higher the growth-potential of the targeted industry is, the better R&D subsidy is for the home welfare.

Regarding the foreign country, Proposition 7 shows, to the first order, there is no difference in the effects on the foreign utility and the mass of firms in the foreign industry 1 between two polices in Stage 1. However, a R&D subsidy causes more faster transition from Stage 1 to Stage 2, which makes innovation be more delayed in the foreign industry 1. The following equation shows how differently the two policies affect the foreign welfare.

$$d\ln W^{*p} - d\ln W^{*d}\,|_{T_p=T_d} = e^{-\rho t_r(0)} t_r(0)(\gamma-1)\left[\underbrace{-(\ln U_2^* - \ln U_1^*)}_{\substack{\text{Early home innovation}\\ \text{Production} \prec \text{R\&D}}} + \underbrace{(\ln U_3^* - \ln U_2^*)\left(\frac{n_1^*(1)}{n_1^*(2)}-1\right)}_{\substack{\text{Delayed foreign innovation}\\ :\ \text{Production} \succ \text{R\&D}}}\right] ds \tag{67}$$

From equation (67), an implication can be drawn that a production subsidy is likely to be better for the foreign welfare than a R&D subsidy because i) the direct innovation effect ($\ln U_3^* - \ln U_2^*$) is much greater than the spillover effect ($\ln U_2^* - \ln U_1^*$), and ii) the mass of firms in the foreign industry 1 drops a lot in transition from Stage 1 ($n_1^*(1)$) to Stage 2 ($n_1^*(2)$) due to the productivity jump in the home industry 1. Overall, contrary to the home welfare, the higher the growth potential of the targeted industry is, the better a production subsidy is for the foreign welfare. Table 5 summarizes the welfare effect comparison between the two policies.

Table 5: Summary of welfare effect comparison between production and R&D subsidy

<table>
<tr><th rowspan="2">Effect</th><th colspan="2">Home</th><th colspan="2">Foreign</th></tr>
<tr><th>Production</th><th>R&D</th><th>Production</th><th>R&D</th></tr>
<tr><td>Short-run resource reallocation effect</td><td colspan="2">$\succ$</td><td colspan="2">$=$</td></tr>
<tr><td>Gain from speeding up home innovation</td><td colspan="2">$\prec$</td><td colspan="2">$\prec$</td></tr>
<tr><td>Loss from delaying foreign innovation</td><td colspan="2">$\succ$</td><td colspan="2">$\succ$</td></tr>
</table>

7.1 Model Simulation

The analysis in the previous section implies that the growth potential of the targeted industry plays an important role in determining the relative performance of two policies. In this context, I carry out numerical exercises for two scenarios. Three conditions are different for each scenario: productivity growth by innovation, required time for innovation without policy, and discount rate. Table 6 presents the difference in those conditions in detail. I use the same values for all the other parameters as those in the benchmark case in Section 5.2.

In the first scenario, relative to the second scenario, the growth potential of the targeted industry

Table 6: Conditions in two scenarios

Condition	Scenario 1	Scenario 2
Productivity growth by innovation ($\frac{a_{1,t}(2)}{a_{1,t}(1)}$)	1.5	1.05
Required time for innovation without policy ($t_r(0)$)	10	20
Discount rate (ρ)	0.042	0.111

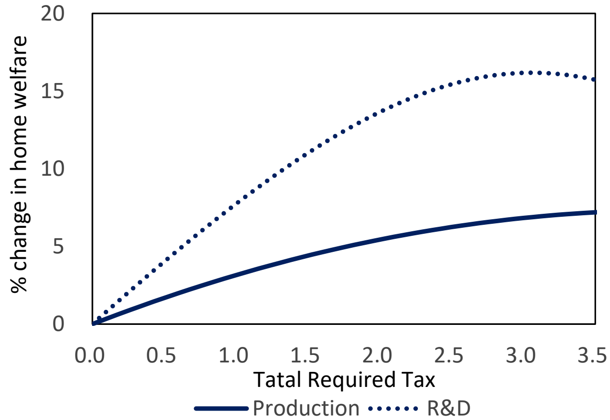
is higher and less time is required for innovation without any policy support, and discount rate is smaller.

Figure 10 shows how each policy affects the home and foreign welfare under each scenario. For an appropriate comparison, the welfare is calculated by controlling the total amount of subsidy the government has to provide until emergence of innovation to be the same under each policy. The results show a conflict of of interest between the home and foreign country regarding choice of policy instrument. If the targeted industry has very high growth-potential (Scenario 1), a R&D subsidy performs better for the home welfare than a production subsidy by hastening innovation more, which worsens the foreign welfare more. In contrast, the growth-potential of the targeted industry is not that high (Scenario 2), a production subsidy is superior to a R&D subsidy for the home country while a R&D subsidy performs slightly better for the foreign country.

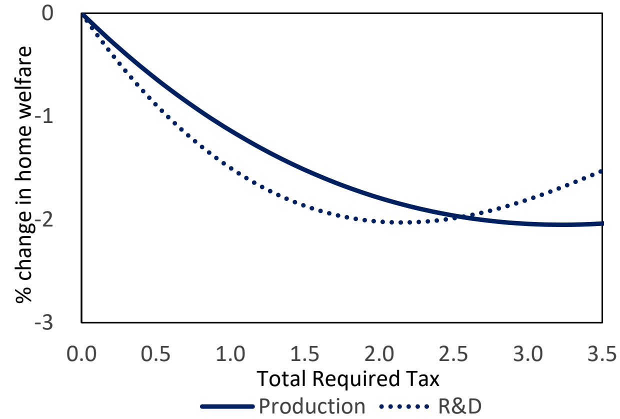
8 Conclusion

In this paper, I study how the timing of industrial policy affects its welfare effects on the home and foreign country with a consideration of industry dynamics based on industry lifecycle theory. To answer this question, I propose an open economy macroeconomic model incorporating industry lifecycle theory. In this model, the home industrial policy affects the home and foreign welfare by hastening or delaying the timing of innovation in the targeted industry.

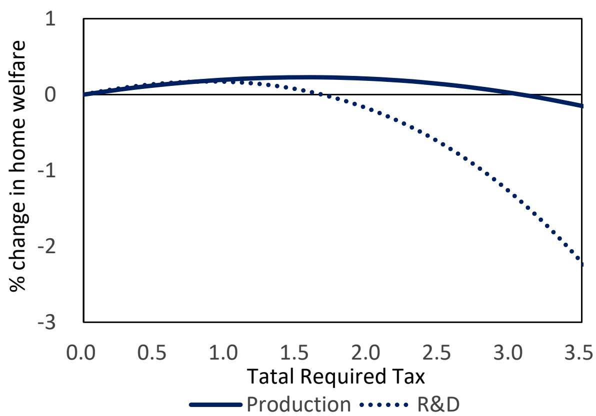
I derive several policy implications from the model. First, if industrial policy supports an young and high growth-potential industry, it hastens innovation in the industry and thus has a positive long-run growth effect. Even with the positive growth effect, overall welfare effect can be positive or negative since the policy can cause short-run welfare loss before realization of the innovation. This



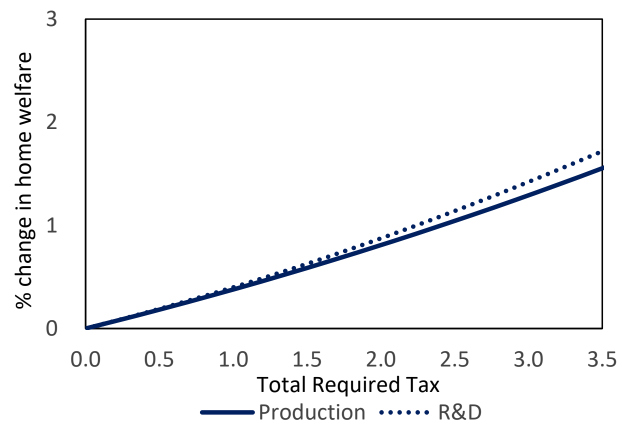
(a) Home Welfare in Scenario 1



(b) Foreign Welfare in Scenario 1



(c) Home Welfare in Scenario 2



(d) Foreign Welfare in Scenario 2

Figure 10: Comparison of Welfare Effect: Production vs R&D Subsidy

Notes: Figure 10a and 10b (Figure 10c and 10d) show the change in the home and foreign welfare in Scenario 1 (Scenario 2) while the overall tax amount which the home government has to spend in Stage 1 changes under production and R&D subsidy respectively.

suggests the importance of welfare analysis of industrial policy in a dynamic view.

Second, home industrial policy can increase or decrease the foreign welfare, depending on whether the targeted industry is already mature in the foreign country. If the industry is already mature abroad, the home policy unambiguously increases the foreign welfare. This result is consistent with the positive international spillovers from home productivity increases suggested by Corsetti et al. (2007) with the standard Dixit-Stiglitz consumption index. On the other hand, if the foreign industry is also young and has high growth-potential, the home policy can worsen the foreign welfare by delaying innovation in the foreign industry. This is a novel channel through which home industrial policy causes "beggar-thy-neighbor" consequences.

Third, home industrial policy can trigger a policy competition between the home and foreign country. In the case where home industrial policy have a beggar-thy-neighbor effect, the model predicts that the foreign country responds by implementing more aggressive policy to offset the negative spillovers from the home policy and take a leadership in the high growth-potential industry. This give an explanation why advanced countries are actively seeking to support high-tech industries in response to recent Chinese industrial policy, while they did not for China's catching-up policy of manufacturing industries such as steel and shipbuilding. The model suggests that if the home and foreign country cooperatively support the high growth-potential industry in such game situation, the welfare outcome is a Pareto improvement compared with the Nash equilibrium.

I also compare welfare effects of a production and R&D subsidy on home and foreign country. The results show that a R&D subsidy is more effective for hastening innovation in the targeted home industry than a production subsidy at the cost of more short-run distortion. Thus, the higher the growth potential of the targeted industry is, the better a R&D subsidy performs than a production subsidy by more accelerating R&D activities and competition in the industry. This mechanism works in the opposite way for the foreign country. Since the home R&D subsidy delays more innovation in the foreign industry which also has high growth-potential, the foreign country benefits much later from the rapid productivity growth after innovation.

The implications derived in this paper can be useful for policy decision making since it provides a criteria for deciding "a right timing" to policy makers. For example, based on Vernon (1966) or Klepper (1996), if policy makers observe that firms start to set up their production facilities in other countries with lower wage rate, or the market share of a few large firms increases, or firms focus on process R&D in the industry that they are considering to support, it indicates the industry is maturing and thus it might be not good time to support the industry in aspect of long-run growth. However, I have to admit that the measurements of industry lifecyle suggested by the existing literature might not perfectly work for establishing industrial policy. For future research, providing a measurement method of industry lifecycle in the context of policy can complement this paper.

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A Comparative Statics for Change in s

An equilibrium with a production subsidy at each period is given by $\{n_{1,t}, n_{2,t}, n^*_{1,t}, n^*_{2,t}, \epsilon_t\}$ that satisfies the following five equations: the zero profit condition for industry 1 in the home country from (36), the zero profit condition for industry 2 in the home country from (28), two zero profit conditions for each industry in the foreign country from (29) and balance of payment equilibrium from (30).

As in Corsetti et al. (2007), I set a symmetric initial condition where $v = v^* = L = L^* = a_{1,t} = a_{2,t} = a^*_{1,t} = a^*_{2,t} = 1$ and $s = 0$. With this initial condition, there is a symmetric equilibrium such that $\epsilon_t = 1$, $n_{1,t} = n_{2,t} = n^*_{1,t} = n^*_{2,t}$, $l_t = l^*_t = P_t^{1-\psi} = P_t^{*1-\psi} = \gamma(n_{1,t} + n_{2,t}) = \gamma(n^*_{1,t} + n^*_{2,t})$. I take a first-order approximation of this model in the neighborhood of this initial symmetric equilibrium and analyze the local effects of industrial policy.

For computational convenience, I extend the system of equilibrium into 24 equations with 24 endogenous variables. These variables include the mass of firms in each industry, $\{n_{1,t}, n_{2,t}, n^*_{1,t}, n^*_{2,t}\}$, the exchange rate ($\epsilon_t$), the composite bundle of home varieties and the composite bundle of foreign varieties for the home representative consumer in each industry, $\{C_{h1,t}, C_{h2,t}, C_{f1,t}, C_{f2,t}\}$, the composite bundle of home varieties and the composite bundle of foreign varieties for the foreign representative consumer in each industry, $\{C^*_{h1,t}, C^*_{h2,t}, C^*_{f1,t}, C^*_{f2,t}\}$, the price index of the composite bundle of home varieties for the home representative consumer in each industry, $\{P_{h1,t}, P_{h2,t}\}$, the price index of the composite bundle of foreign varieties for the foreign representative consumer in each industry, $\{P^*_{f1,t}, P^*_{f2,t}\}$, the composite prices index in industry, $\{P_{1,t}, P_{2,t}, P^*_{1,t}, P^*_{2,t}\}$, utility based CPI, $\{P_t, P^*_t\}$, and the real exchange rate, RER_t. I take a first-order approximation of the system with respect to s and obtain the following equations.

$$\text{(36): } \gamma(1+\phi) + \gamma(1+\phi)\frac{d\ln P_{h1,t}}{ds} + \frac{d\ln C_{h1,t}}{ds} + \phi\frac{d\ln C^*_{h1,t}}{ds} = 0 \tag{68}$$

$$\text{(28): } \gamma(1+\phi)\frac{d\ln P_{h2,t}}{ds} + \frac{d\ln C_{h2,t}}{ds} + \phi\frac{d\ln C^*_{h2,t}}{ds} = 0 \tag{69}$$

$$\text{(29): } \gamma(1+\phi)\frac{d\ln P^*_{f1,t}}{ds} + \frac{d\ln C^*_{f1,t}}{ds} + \phi\frac{d\ln C_{f1,t}}{ds} = 0 \tag{70}$$

$$\text{(29): } \gamma(1+\phi)\frac{d\ln P^*_{f2,t}}{ds} + \frac{d\ln C^*_{f2,t}}{ds} + \phi\frac{d\ln C_{f2,t}}{ds} = 0 \tag{71}$$

$$\frac{d\ln P_{h1,t}}{ds} + \frac{d\ln C^*_{h1,t}}{ds} + \frac{d\ln P_{h2,t}}{ds} + \frac{d\ln C^*_{h2,t}}{ds} = 2\frac{d\ln \epsilon_t}{ds} + \frac{d\ln P^*_{f1,t}}{ds} + \frac{d\ln C_{f1,t}}{ds} + \frac{d\ln P^*_{f2,t}}{ds} + \frac{d\ln C_{f2,t}}{ds} \tag{72}$$

$$\text{(12), (13), (15): } \frac{d\ln C_{h1,t}}{ds} = -\eta\frac{d\ln P_{h1,t}}{ds} + (\eta-\sigma)\frac{d\ln P_{1,t}}{ds} + (\sigma-\psi)\frac{d\ln P_t}{ds} \tag{73}$$

$$\text{(12), (13), (15): } \frac{d\ln C_{h2,t}}{ds} = -\eta\frac{d\ln P_{h2,t}}{ds} + (\eta-\sigma)\frac{d\ln P_{2,t}}{ds} + (\sigma-\psi)\frac{d\ln P_t}{ds} \tag{74}$$

$$(12),\ (13),\ (15):\ \frac{d\ln C_{f1,t}}{ds} = -\eta\frac{d\ln\epsilon_t}{ds} - \eta\frac{d\ln P^*_{f1,t}}{ds} + (\eta-\sigma)\frac{d\ln P_{1,t}}{ds} + (\sigma-\psi)\frac{d\ln P_t}{ds} \tag{75}$$

$$(12),\ (13),\ (15):\ \frac{d\ln C_{f2,t}}{ds} = -\eta\frac{d\ln\epsilon_t}{ds} - \eta\frac{d\ln P^*_{h2,t}}{ds} + (\eta-\sigma)\frac{d\ln P_{2,t}}{ds} + (\sigma-\psi)\frac{d\ln P_t}{ds} \tag{76}$$

$$(12),\ (13),\ (15):\ \frac{d\ln C^*_{h1,t}}{ds} = \eta\frac{d\ln\epsilon_t}{ds} - \eta\frac{d\ln P_{h1,t}}{ds} + (\eta-\sigma)\frac{d\ln P^*_{1,t}}{ds} + (\sigma-\psi)\frac{d\ln P^*_t}{ds} \tag{77}$$

$$(12),\ (13),\ (15):\ \frac{d\ln C^*_{h2,t}}{ds} = \eta\frac{d\ln\epsilon_t}{ds} - \eta\frac{d\ln P_{h2,t}}{ds} + (\eta-\sigma)\frac{d\ln P_{2,t}}{ds} + (\sigma-\psi)\frac{d\ln P_t}{ds} \tag{78}$$

$$(12),\ (13),\ (15):\ \frac{d\ln C^*_{f1,t}}{ds} = -\eta\frac{d\ln P^*_{f1,t}}{ds} + (\eta-\sigma)\frac{d\ln P^*_{1,t}}{ds} + (\sigma-\psi)\frac{d\ln P^*_t}{ds} \tag{79}$$

$$(12),\ (13),\ (15):\ \frac{d\ln C^*_{f2,t}}{ds} = -\eta\frac{d\ln P^*_{h2,t}}{ds} + (\eta-\sigma)\frac{d\ln P^*_{2,t}}{ds} + (\sigma-\psi)\frac{d\ln P^*_t}{ds} \tag{80}$$

$$(18),\ (35):\ \frac{d\ln P_{h1,t}}{ds} = \frac{1}{1-\gamma}\frac{d\ln n_{1,t}}{ds} - 1 \tag{81}$$

$$(18):\ \frac{d\ln P_{h2,t}}{ds} = \frac{1}{1-\gamma}\frac{d\ln n_{2,t}}{ds} \tag{82}$$

$$(18):\ \frac{d\ln P^*_{f1,t}}{ds} = \frac{1}{1-\gamma}\frac{d\ln n^*_{1,t}}{ds} \tag{83}$$

$$(18):\ \frac{d\ln P^*_{f2,t}}{ds} = \frac{1}{1-\gamma}\frac{d\ln n^*_{2,t}}{ds} \tag{84}$$

$$(17):\ (1+\phi)\frac{d\ln P_{1,t}}{ds} = \frac{d\ln P_{h1,t}}{ds} + \phi\left(\frac{d\ln\epsilon_t}{ds} + \frac{d\ln P^*_{f1,t}}{ds}\right) \tag{85}$$

$$(17):\ (1+\phi)\frac{d\ln P_{2,t}}{ds} = \frac{d\ln P_{h2,t}}{ds} + \phi\left(\frac{d\ln\epsilon_t}{ds} + \frac{d\ln P^*_{f2,t}}{ds}\right) \tag{86}$$

$$(17):\ (1+\phi)\frac{d\ln P^*_{1,t}}{ds} = \frac{d\ln P^*_{f1,t}}{ds} + \phi\left(-\frac{d\ln\epsilon_t}{ds} + \frac{d\ln P_{h1,t}}{ds}\right) \tag{87}$$

$$(17):\ (1+\phi)\frac{d\ln P^*_{2,t}}{ds} = \frac{d\ln P^*_{f2,t}}{ds} + \phi\left(-\frac{d\ln\epsilon_t}{ds} + \frac{d\ln P_{h2,t}}{ds}\right) \tag{88}$$

$$2\frac{d\ln P_t}{ds} = \frac{d\ln P_{1,t}}{ds} + \frac{d\ln P_{2,t}}{ds} \tag{89}$$

$$2\frac{d\ln P^*_t}{ds} = \frac{d\ln P^*_{1,t}}{ds} + \frac{d\ln P^*_{2,t}}{ds} \tag{90}$$

$$\frac{dRER_t}{ds} = \frac{d\ln\epsilon_t}{ds} + \frac{d\ln P^*_t}{ds} - \frac{d\ln P_{1,t}}{ds} \tag{91}$$

I obtain the results presented in Table 7 by solving the aforementioned system of 24 equations. Additionally, I follow the same procedure to obtain the results shown in Table 8 and Table 9.

Table 7: Comparative Statics: Production subsidy

$$\frac{d\ln n_{1,t}}{ds} = \frac{1}{2}\left[\frac{(\gamma-1)\psi}{2(\gamma-\psi)} + \frac{(\gamma-1)\psi}{2(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(\sigma+\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] > 0 \tag{92}$$

$$\frac{d\ln n_{2,t}}{ds} = \frac{1}{2}\left[\frac{(\gamma-1)\psi}{2(\gamma-\psi)} + \frac{(\gamma-1)\psi}{2(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right) - \frac{\gamma-1}{\gamma-\sigma}\left(\sigma+\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] \tag{93}$$

$$\frac{d\ln n^*_{1,t}}{ds} = \frac{1}{2}\left[\frac{(\gamma-1)\psi}{2(\gamma-\psi)} - \frac{(\gamma-1)\psi}{2(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right) - \frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] < 0 \tag{94}$$

$$\frac{d\ln n^*_{2,t}}{ds} = \frac{1}{2}\left[\frac{(\gamma-1)\psi}{2(\gamma-\psi)} - \frac{(\gamma-1)\psi}{2(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] \tag{95}$$

$$\frac{d\ln \epsilon_t}{ds} = -\frac{1}{2}\frac{[(\eta-\psi)(1-\phi)+(\eta-1)(1+\phi)]\,\gamma}{\triangle} < 0 \tag{96}$$

$$\frac{d\ln P_t}{ds} = \frac{1}{2}\frac{1}{1-\psi}\left(\frac{d\ln n_{1,t}}{ds}+\frac{d\ln n_{2,t}}{ds}-1\right) = -\frac{1}{2}\left[\frac{\gamma}{\gamma-\psi}\left(1-\frac{\psi(\gamma-1)\phi}{\triangle}\right)\right] < 0 \tag{97}$$

$$\frac{d\ln P^*_t}{ds} = \frac{1}{2}\frac{1}{1-\psi}\left(\frac{d\ln n^*_{1,t}}{ds}+\frac{d\ln n^*_{2,t}}{ds}\right) = -\frac{1}{2}\frac{\gamma}{\gamma-\psi}\left[\frac{\psi(\gamma-1)\phi}{\triangle}\right] \leq 0 \tag{98}$$

$$\frac{dRER_t}{ds} = \frac{1}{2}\frac{\psi(1-\phi)\gamma}{\triangle} > 0 \tag{99}$$

$$\frac{dU_1}{ds} = -P_t^{1-\psi}\left(\frac{d\ln P_t}{ds}+\frac{1}{2}\right) = \frac{1}{2}P_t^{1-\psi}\left[\frac{\gamma}{\gamma-\psi}\left(1-\frac{\psi(\gamma-1)\phi}{\triangle}\right)-1\right] \tag{100}$$

$$\frac{dU^*_1}{ds} = -P_t^{*1-\psi}\frac{d\ln P^*_t}{ds} \geq 0 \tag{101}$$

$$\triangle \equiv (2\eta-1)[\gamma-\psi(1-\phi)] + 2\psi\phi(\gamma-\eta) - \gamma\phi > 0 \tag{102}$$

B Comparative Statics for Change in $a_{1,t}$

As in Section 4.1.1, I add an intuitive explanation on how resource allocation is affected by an increase in $a_{1,t}$. When productivity increases in the home industry 1, as in the short-run (Stage 1), there are three effects: an income effect, a relative wage effect and a substitution effect. Again, Equation (43) helps to understand an income effect. 1 percent increase in productivity of firms in the home industry 1 leads to change the world demand by $\frac{1}{4}(\psi - 1)$. However, since the home government does not give subsidy anymore, there is no direct change in the right hand side. Thus, with the assumption $\psi < 1$, the excess supply exists by $\frac{1}{4}(1 - \psi)$. To clear the excess supply, the mass of firms in all industries in both the home and foreign country needs to uniformly decrease by $\frac{(\gamma-1)(1-\psi)}{4(\gamma-\psi)}$.

The relative wage effect causes the mass of firms in the home industries ($n_{1,t}$ and $n_{2,t}$) to decrease and that in the foreign industries ($n^*_{1,t}$ and $n^*_{2,t}$) to increase. After innovation, the increase in productivity in the home industry 1 leads home firms to demand more labor and thus the relative wage of the home labor increases. This is represented by a decrease in ϵ_t in (107). With $\psi < 1$, when real wage increases, the mass of firms decreases.

The substitution effect is qualitatively the same as that in Stage 1. The increase in productivity in the home industry 1 makes the industry 1 home varieties relatively cheaper than the industry 1 foreign varieties. Thus, the home and foreign consumers substitutes the industry 1 foreign varieties with the industry 1 home varieties, which increases the mass of firm in the home industry 1 and decreases the mass of firms in the foreign industry 1. In contrast, the decrease in ϵ_t leads the home and foreign consumers to substitute the industry 2 home varieties with the industry 2 foreign varieties. Because of this substitution, the mass of firms in the home industry 2 decreases and that in the foreign industry 2 increases.

By taking the three effects into account together, the mass of firms in the home industry 2, $n_{2,t}$, and that in the foreign industry 1, $n^*_{1,t}$, unambiguously decreases. Changes in $n_{1,t}$ and $n^*_{2,t}$ are ambiguous since the direction of the substitution effect is opposite from the other two effects for $n_{1,t}$ and the direction of the income effect is opposite from the other two effects for $n^*_{2,t}$.

Table 8: Comparative Statics: change in $a_{1,t}$

$$\frac{d\ln n_{1,t}}{d\ln a_{1,t}} = \frac{1}{2}\left[-\frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} - \frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)}\left(1-\frac{2\psi(\gamma-1)\phi}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(\sigma-1+\frac{2\phi(\gamma-1)(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] \tag{103}$$

$$\frac{d\ln n_{2,t}}{d\ln a_{1,t}} = \frac{1}{2}\left[-\frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} - \frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)}\left(1-\frac{2\psi(\gamma-1)\phi}{\triangle}\right) - \frac{\gamma-1}{\gamma-\sigma}\left(\sigma-1+\frac{2\phi(\gamma-1)(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] < 0 \tag{104}$$

$$\frac{d\ln n^*_{1,t}}{d\ln a_{1,t}} = \frac{1}{2}\left[-\frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} + \frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)}\left(1-\frac{2\psi\phi}{\triangle}\right) - \frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi(\gamma-1)(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] < 0 \tag{105}$$

$$\frac{d\ln n^*_{2,t}}{d\ln a_{1,t}} = \frac{1}{2}\left[-\frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)} + \frac{(\gamma-1)(1-\psi)}{2(\gamma-\psi)}\left(1-\frac{2\psi(\gamma-1)\phi}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi(\gamma-1)(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] \tag{106}$$

$$\frac{d\ln \epsilon_t}{d\ln a_{1,t}} = -\frac{1}{2}\frac{[(\gamma-\psi)(1-\phi)+(\gamma-1)(1+\phi)]\,(\gamma-1)}{\triangle} < \frac{1}{2} \tag{107}$$

$$\frac{d\ln P_t}{d\ln a_{1,t}} = \frac{1}{2}\frac{1}{1-\psi}\left(\frac{d\ln n_{1,t}}{ds}+\frac{d\ln n_{2,t}}{ds}\right) = -\frac{1}{2}\frac{\gamma-1}{\gamma-\psi}\left[1-\frac{\psi(\gamma-1)\phi}{\triangle}\right] < 0 \tag{108}$$

$$\frac{d\ln P^*_t}{d\ln a_{1,t}} = \frac{1}{2}\frac{1}{1-\psi}\left(\frac{d\ln n^*_{1,t}}{d\ln a_{1,t}}+\frac{d\ln n^*_{2,t}}{d\ln a_{1,t}}\right) = -\frac{1}{2}\frac{\gamma-1}{\gamma-\psi}\left[\frac{\psi(\gamma-1)\phi}{\triangle}\right] \leq 0 \tag{109}$$

$$\frac{dRER_t}{d\ln a_{1,t}} = \frac{1}{2}\frac{\psi(1-\phi)(\gamma-1)}{\triangle} > 0 \tag{110}$$

$$\frac{dU_1}{d\ln a_{1,t}} = -P_t^{1-\psi}\frac{d\ln P_t}{d\ln a_{1,t}} > 0 \tag{111}$$

$$\frac{dU^*_1}{d\ln a_{1,t}} = -P_t^{*1-\psi}\frac{d\ln P^*_t}{d\ln a_{1,t}} \geq 0 \tag{112}$$

$$\triangle \equiv (2\eta-1)[\gamma-\psi(1-\phi)] + 2\psi\phi(\gamma-\eta) - \gamma\phi > 0 \tag{113}$$

C List of Treated and Untreated Industries during HCI Drive

Industry Name	HCI
Manufacture of food products	N
Manufacture of beverages	N
Manufacture of tobacco products	N
Manufacture of textiles, except apparel	N
Manufacture of wearing apparel	N
Manufacture of leather and fur articles	N
Manufacture of footwear	N
Manufacture of wood and of products of wood and cork; except furniture	N
Manufacture of furniture	N
Manufacture of pulp, paper and paper products	N
Printing and service activities related to printing	N
Manufacture of chemical products	Y
Manufacture of other chemical products	N
Manufacture of refined petroleum products	Y
Manufacture of coke and briquettes	Y
Manufacture of Rubber	N
Manufacture of Plastic	N
Manufacture of ceramic products	N
Manufacture of glass and glass products	N
Other non-metallic mineral products	N
Manufacture of basic iron and steel	Y
Manufacture of basic precious and other non-ferrous metals	Y
Manufacture of fabricated metal products, except machinery and equipment	Y
Manufacture of machinery and equipment n.e.c.	Y
Manufacture of electronic and electrical equipment	Y
Manufacture of transport equipment	Y
Manufacture of medical, precision and optical instruments	N
Other manufacturing	N

D Comparative Statistics: R&D subsidy

Table 9: Comparative Statistics: R&D subsidy

$$\frac{d\ln n_{1,t}}{ds_d} = \frac{1}{2}\left[\frac{\gamma-1}{2(\gamma-\psi)} + \frac{\gamma-1}{2(\gamma-\psi)}\left(1+\frac{2\psi\phi(1-\psi)}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(1+\frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] > 0 \tag{114}$$

$$\frac{d\ln n_{2,t}}{ds_d} = \frac{1}{2}\left[\frac{\gamma-1}{2(\gamma-\psi)} + \frac{\gamma-1}{2(\gamma-\psi)}\left(1+\frac{2\psi\phi(1-\psi)}{\triangle}\right) - \frac{\gamma-1}{\gamma-\sigma}\left(1+\frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] \tag{115}$$

$$\frac{d\ln n^*_{1,t}}{ds_d} = \frac{1}{2}\left[\frac{\gamma-1}{2(\gamma-\psi)} - \frac{\gamma-1}{2(\gamma-\psi)}\left(1+\frac{2\psi\phi(1-\psi)}{\triangle}\right) - \frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] < 0 \tag{116}$$

$$\frac{d\ln n^*_{2,t}}{ds_d} = \frac{1}{2}\left[\frac{\gamma-1}{2(\gamma-\psi)} - \frac{\gamma-1}{2(\gamma-\psi)}\left(1+\frac{2\psi\phi(1-\psi)}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right] \tag{117}$$

$$\frac{d\ln \epsilon_t}{ds_d} = -\frac{1}{2}\frac{(\eta-\psi)(1-\phi)+(\eta-1)(1+\phi)}{\triangle} < 0 \tag{118}$$

$$\frac{d\ln P_t}{ds_d} = \frac{1}{2}\frac{1}{1-\psi}\left(\frac{d\ln n_{1,t}}{ds_d} + \frac{d\ln n_{2,t}}{ds_d} - 1\right) = -\frac{1}{2}\left[\frac{1}{\gamma-\psi}\left(1-\frac{\psi(\gamma-1)\phi}{\triangle}\right)\right] < 0 \tag{119}$$

$$\frac{d\ln P^*_t}{ds_d} = \frac{1}{2}\frac{1}{1-\psi}\left(\frac{d\ln n^*_{1,t}}{ds_d} + \frac{d\ln n^*_{2,t}}{ds_d}\right) = -\frac{1}{2}\frac{1}{\gamma-\psi}\left[\frac{\psi(\gamma-1)\phi}{\triangle}\right] \leq 0 \tag{120}$$

$$\frac{dU_1}{ds_d} = -P_t^{1-\psi}\left(\frac{d\ln P_t}{ds_d} + \frac{1}{2}\right) = \frac{1}{2}P_t^{1-\psi}\left[\frac{1}{\gamma-\psi}\left(1-\frac{\psi(\gamma-1)\phi}{\triangle}\right) - 1\right] \tag{121}$$

$$\frac{dU^*_1}{ds_d} = -P_t^{*1-\psi}\frac{d\ln P^*_t}{ds_d} \geq 0 \tag{122}$$

$$\triangle \equiv (2\eta-1)[\gamma-\psi(1-\phi)] + 2\psi\phi(\gamma-\eta) - \gamma\phi > 0 \tag{123}$$

E Proof of Proposition 6 and 7

I derive the difference in policy effects on the variables of interest by utilizing the previous results from equation (59), Table 7 and Table 9 together. For a suitable comparison, I calculate the difference under the condition that the total amount of subsidy, which the home government must provide until the occurrence of the refinement, is the same for both policies.

$$
\begin{aligned}
& dU_1^p - dU_1^d \\
&= \frac{1}{2}P_t^{1-\psi}\left[\frac{\gamma}{\gamma-\psi}\left(1-\frac{\psi(\gamma-1)\phi}{\triangle}\right)-1\right]ds - \frac{1}{2}P_t^{1-\psi}\left[\frac{1}{\gamma-\psi}\left(1-\frac{\psi(\gamma-1)\phi}{\triangle}\right)-1\right]ds_d \\
&= \frac{1}{2}P_t^{1-\psi}\left[\frac{\gamma}{\gamma-\psi}\left(1-\frac{\psi(\gamma-1)\phi}{\triangle}\right)-1\right]ds - \frac{1}{2}P_t^{1-\psi}\left[\frac{1}{\gamma-\psi}\left(1-\frac{\psi(\gamma-1)\phi}{\triangle}\right)-1\right]\gamma ds \\
&= \frac{1}{2}P_t^{1-\psi}(\gamma-1)ds > 0 \qquad (124)
\end{aligned}
$$

$$
\begin{aligned}
& d\ln n_{1,t}^p - d\ln n_{1,t}^d \\
&= \frac{1}{2}\left[\frac{(\gamma-1)\psi}{2(\gamma-\psi)} + \frac{(\gamma-1)\psi}{2(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(\sigma+\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right]ds \\
&- \frac{1}{2}\left[\frac{\gamma-1}{2(\gamma-\psi)} + \frac{\gamma-1}{2(\gamma-\psi)}\left(1+\frac{2\psi\phi(1-\psi)}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(1+\frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right]ds_d \\
&= \frac{1}{2}\left[\frac{(\gamma-1)\psi}{2(\gamma-\psi)} + \frac{(\gamma-1)\psi}{2(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(\sigma+\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right]ds \\
&- \frac{1}{2}\left[\frac{\gamma-1}{2(\gamma-\psi)} + \frac{\gamma-1}{2(\gamma-\psi)}\left(1+\frac{2\psi\phi(1-\psi)}{\triangle}\right) + \frac{\gamma-1}{\gamma-\sigma}\left(1+\frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right]\gamma ds \\
&= -(\gamma-1)ds < 0 \qquad (125)
\end{aligned}
$$

$$
dt_r^p - dt_r^d = -t_r(0)\left(d\ln n_{1,t}^p - d\ln n_{1,t}^d\right) = t_r(0)(\gamma-1)ds > 0 \qquad (126)
$$

$$
\begin{aligned}
& dU_1^{*p} - dU_1^{*d} \\
&= \frac{1}{2}P_t^{*1-\psi}\left[\frac{\gamma}{\gamma-\psi}\left(\frac{\psi(\gamma-1)\phi}{\triangle}\right)\right]ds - \frac{1}{2}P_t^{*1-\psi}\left[\frac{1}{\gamma-\psi}\left(\frac{\psi(\gamma-1)\phi}{\triangle}\right)\right]ds_d \\
&= \frac{1}{2}P_t^{*1-\psi}\left[\frac{\gamma}{\gamma-\psi}\left(\frac{\psi(\gamma-1)\phi}{\triangle}\right)\right]ds - \frac{1}{2}P_t^{*1-\psi}\left[\frac{1}{\gamma-\psi}\left(\frac{\psi(\gamma-1)\phi}{\triangle}\right)\right]\gamma ds = 0 \qquad (127)
\end{aligned}
$$

$$
\begin{aligned}
& d\ln {n^*}_{1,t}^{p} - d\ln {n^*}_{1,t}^{d} \\
&= \frac{1}{2}\left[\frac{(\gamma-1)\psi}{2(\gamma-\psi)} - \frac{(\gamma-1)\psi}{2(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right) - \frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right]ds
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{2}\left[\frac{\gamma-1}{2(\gamma-\psi)}-\frac{\gamma-1}{2(\gamma-\psi)}\left(1+\frac{2\psi\phi(1-\psi)}{\triangle}\right)-\frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right]ds_d\\
&=\frac{1}{2}\left[\frac{(\gamma-1)\psi}{2(\gamma-\psi)}-\frac{(\gamma-1)\psi}{2(\gamma-\psi)}\left(1+\frac{2\gamma\phi(1-\psi)}{\triangle}\right)-\frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi\gamma(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right]ds\\
&-\frac{1}{2}\left[\frac{\gamma-1}{2(\gamma-\psi)}-\frac{\gamma-1}{2(\gamma-\psi)}\left(1+\frac{2\psi\phi(1-\psi)}{\triangle}\right)-\frac{\gamma-1}{\gamma-\sigma}\left(\frac{2\phi(\eta-\sigma)}{(\gamma-\sigma)(\phi-1)^2+4\phi(\gamma-\eta)}\right)\right]\gamma ds=0
\end{aligned}
\tag{128}
$$

$$
\begin{aligned}
&dt_r^{*p}-dt_r^{*d}\\
&=-\left(\frac{n^*_{1,t}(1)}{n^*_{1,t}(2)}-1\right)(dt_r^p-dt_r^d)-t_r(0)\frac{n^*_{1,t}(1)}{n^*_{1,t}(2)}\left(d\ln n^{*p}{}_{1,t}-d\ln n^{*d}{}_{1,t}\right)\\
&=-\left(\frac{n^*_{1,t}(1)}{n^*_{1,t}(2)}-1\right)t_r(0)(\gamma-1)ds<0
\end{aligned}
\tag{129}
$$