FOREST FIRE MODEL SIMULATION - PROJECT REPORT

A Project report for PH4505 Computational Physics

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1 Background

The forest fire model studied in this report was introduced originally by Bak et al. 1990. The system usually investigated based on the cellular automaton framework. The model can be studied extensively in this framework as the system consisted of finite choice of state and constructed in a grid manner. The study of the forest fire model which originally only focus on the spread of wild fire can be taken in different context such as the spread of disease within a population. In general, there are three states to be considered in the cell of the in the model :

1. Tree is existing in the cell (healthy, not on fire)
2. Burnt tree is in the cell (infected, on fire)
3. Empty cell (tree is died due to the fire or infection)

The rule fixated for the grid are as follow:

- A tree grows with a probability p from an empty sites for every time step.
- A burning tree becomes an empty on the next time step.
- A tree becomes on fire on the next time step if at least one of the nearest neighbors is burning.
- A tree have the probability to catch fire (sometimes noted as lightning strike) or infection with probability f.

The first three rules were introduced from the original study of Bak et al. 1990 and the last rule item was an addition by Drossel and Schwabl 1992 to allow small forest cluster to burn. Bak et al. 1990 claimed that the system approach its critical state for the value of $p \to 0$. It was found via computer simulation that with the original set of rule the system is deterministic and displaying regular spiral-shaped waves of fires.

The model is a member of a system with self-organizing criticality (SOC). System with this particular property exhibits a spatial and/or temporal scale-invariance characteristic of the critical point of a phase transition and the system self tunes the control parameters as the system evolves toward criticality.

The forest fire model is a toy model which initially was purposed for demonstration of the emergence of scaling and fractal energy dissipation by Bak et al. 1990. The "energy" injected uniformly (trees grow uniformly) and the dissipation (trees burn) form a fractal. Later, Drossel and Schwabl 1992 concluded that the number of growing trees and of burned trees is maximum for a critical state i.e. energy dissipation in the system is maximum and a large amount of energy is deposited in the system between two fire ignition and consequently a large number of trees is destroyed by a fire ignition.

2 Objective

In this study, we aim to explore several characteristics for the forest-fire model. One of our particular interest is how the parameters inputted initially for the system affects the configuration of the system. This parameter including the value of the tree growth probability p, and the value of the tree burn probability f. This two parameter often use altogether for the calculation. Hence, it's convinient to introduce the following parameter:

$$\zeta = p/f \tag{1}$$

where $\zeta \in [0, \infty)$.

We also interested to learn the characteristic of the system such as the distribution of number of cluster $N(s)$ and the radius of a cluster $R(s)$ which discussed later. Also, we consider one modification of forest fire system where the tree has some resistance against fire/disease. The study was conducted mostly in computer simulation. With computational method, it enables us to better understand the property of the system. It is rather difficult to solve and describe the system in the analytical form. Yet, some limitation are the numerical correctness of the simulation result due to the finite system studied, in contrary to the theoretical work.

3 Methodology & Analysis

As covered in the above discussion, it is in particular our main of interest to understand the underlying characteristic of a forest fire model. The simulation carried out mostly in Python program. As mentioned earlier, the simulation was done in a cellular automaton grid, mainly is 2 dimensional grid. The shape of the grid decided to be in a square form i.e. width of the grid and the length of the grid considered to be the same dimension.

The state of the respective tree updated according to the rules listed above. There are certain condition which required for the simulation which explicitly were not mentioned by some the author of the original paper as follow:

- Choice of nearest neighbourhood

  In general, there are two types of nearest neighbour considered in the study of cellular automaton, von Neumann neighborhood and Moore neighborhood. Von Neumann neighborhood consider the site on the four cardinal directions (north, south, west and east) of the reference cell while Moore neighborhood consider all the four cardinal directions and their intermediate directions (i.e. diagonal direction). Figure 1 visualize the difference of the nearest neighborhood criteria.

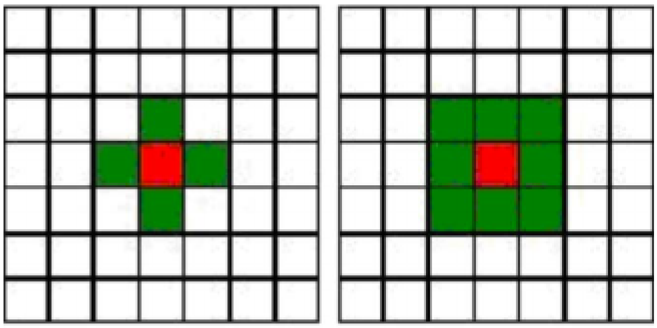


Figure 1: (a) Von Neumann neighborhood and (b) Moore neighborhood. Taken from Espínola et al. 2015

- Choice of boundary.
  - Absorbing Boundary Condition : Cell next to perimeter are inactive (No rows of the northern side and southern side of rows N and rows 1. No columns of the western side and northern side of rows 1 and rows N)
  - Periodic Boundary Condition : Column 1 and column N are neighbors. Row 1 and row N are neighbors. In topological form, it is the surface of a torus.

In this study, we considered only the von Neumann neighborhood for simplicity and using the absorbing boundary condition for realistic real world scenario (where one fire at the boundary very unlikely to ignite another tree at the other end boundary). The shape of the grid decided to be a square grid.

3.1 Spiral Firefront

From the literature, Bak et al. 1990 original rule of the system resulting in deterministic fire front in spiral shape. The work done by Bak et al. 1990 interpreted by starting with random forest configuration and ignite several random tree initially (instead of the tree ignited independently following probability f). An explicit and clear procedure of the original rule hardly found and the work rather provide a vague explanation. The update rules that were imposed to the system in the work done by Chen et al. 1990 are different.

One of our simulation attempt shows the described behaviour. The simulation was done based on the following site with parameter sweep across the probability of tree grown and the probability of tree burnt. The simulation was based on the Drossel and Schwabl 1992 update rules.

The system size shape are rectangle with width of 367 cell and length of 768 cell. Periodic boundary condition imposed in the system and Moore neighborhood were used. For large ζ, essentially the probability of fire f will approach zero and reduce to the original rule of the forest fire by Bak et al. 1990. The observation of spiral fire front is in the following video where green is the tree, white is the burnt tree and black is empty sites.

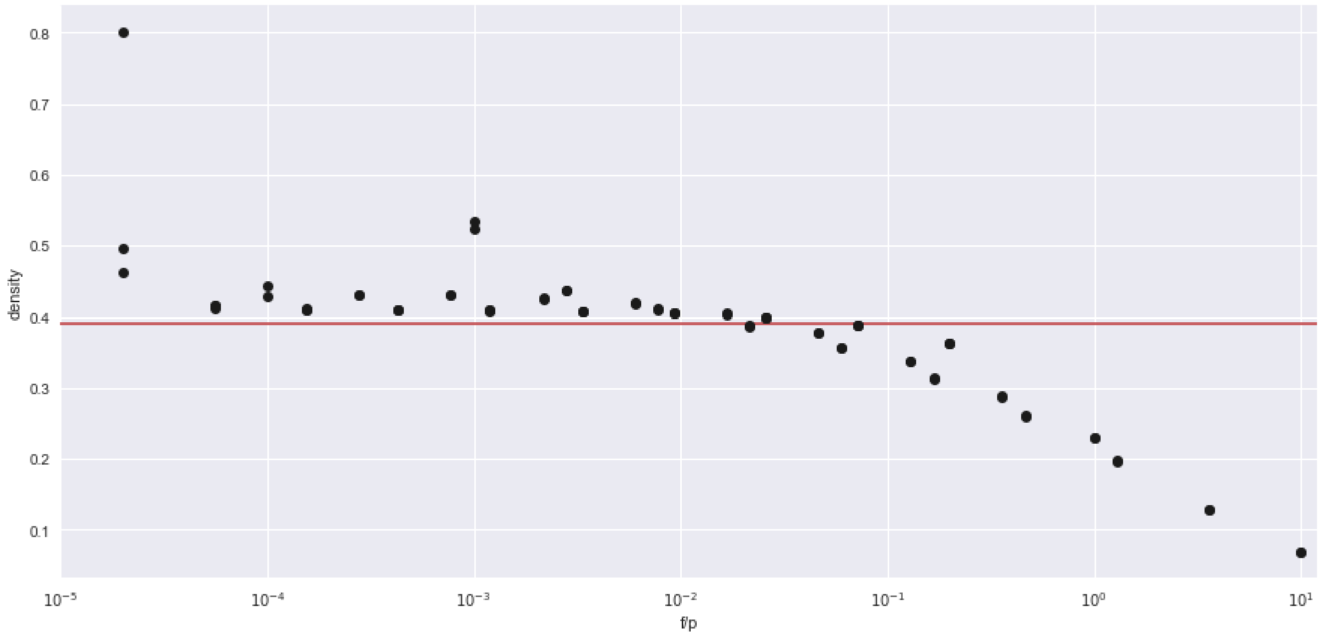


Figure 2: Tree Density as a function of ζ^{-1} (or f/p). Simulation time = 2000. Lattice size = 50×50. Red line is density equal 0.39

3.2 Density of Tree

One subject of interest in the model in particular is the tree density ρ_{tree}. In fact, out of the three possible states in the system, the following relation can be established:

$$\rho_{\text{tree}} + \rho_{\text{empty}} + \rho_{\text{fire}} = 1 \tag{2}$$

which is the mathematical equation that describes the system. In this instance, the value of $\rho_{\text{fire}} \ll \{\rho_{\text{tree}}, \rho_{\text{empty}}\}$. Therefore, the density of empty sites can be approximated as $\rho_{\text{empty}} \approx 1 - \rho_{\text{tree}}$

Drossel and Schwabl 1992 proved in the original study that the density of tree ρ_{tree} eventually will grow until a certain upper limit with the decreasing value of ζ^{-1} due extremum principle. This upper limit was found to be 0.39. Verification of this value was done with the simulation of forest fire model. Our simulation for this verification was completed within 2000 iteration on the grid of size 50 by 50. Different values of f and p were used to obtain different value of ζ. The plot of density with respect to ζ shown in Figure 2.

From our simulation, it observed that the density of tree ρ_{tree} for large ζ^{-1} is small and gradually increasing which agrees with the work done by Drossel and Schwabl 1992. The density for small ζ^{-1} tend to fluctuate around the value of 0.39. However, some anomaly observed is that for $\zeta^{-1} \to 0$, the density tends to jump which contradict the original claim.

This was due to the finite nature of the grid. For an infinite size grid, the upper limit obeyed. Yet, the grid size used which is small have higher tendency to be fully occupied by tree across each iteration when ζ^{-1} is small (i.e. f is small). The time evolution of the density showed in Figure 3. From Figure 3a, Figure 3c and 3e, it can be seen that the system mostly occupied by tree and the value is stable across time for small system size and small ζ^{-1}. This is the finite size effect for small value of ζ^{-1}. The effect is an analogue of the real phenomena - the

"Yellowstone effect" - where Yellowstone Park had a strictly policy of suppressing fires (small fire probability f) until 1972, a large number of dead trees accumulated which resulted in huge fires in 1988. This full occupation and stability property diminish as the system size increased even when the value of ζ^{-1} is small. On the other hand, Figure 3b, Figure 3d and 3f showed that system with large size will have less fluctuation of density across time given large ζ^{-1}.

One interesting insight from this time evolution dynamics are the number of tree burn in one instance with regard to the probability of fire f. For fixed p, large ζ^{-1} implies large f and $\zeta^{-1} \to 0$ implies $f \to 0$. The insight for a finite size system are as follow:

- Small probability of fire, f, (large ζ^{-1}) will result in larger drop of the number of tree (i.e. significant decrease of tree density within small period)
- Large probability of fire, f, ($\zeta^{-1} \to 0$) will result in marginal decrease of the number of the tree (i.e. density of tree is stable)

Figure 3 illustrate the insight drawn above. Despite the counter-intuitive interpretation drawn from the observation, this is true. When the probability of f is small (suppression of fire ignition), this allow all the tree in the system to be interconnected creating a giant cluster. This giant cluster is prone to any fire damage once a particular member of the cluster catch fire. Mathematically, the average number of tree burnt for one fire source expressed in Equation (3) by Drossel and Schwabl 1992:

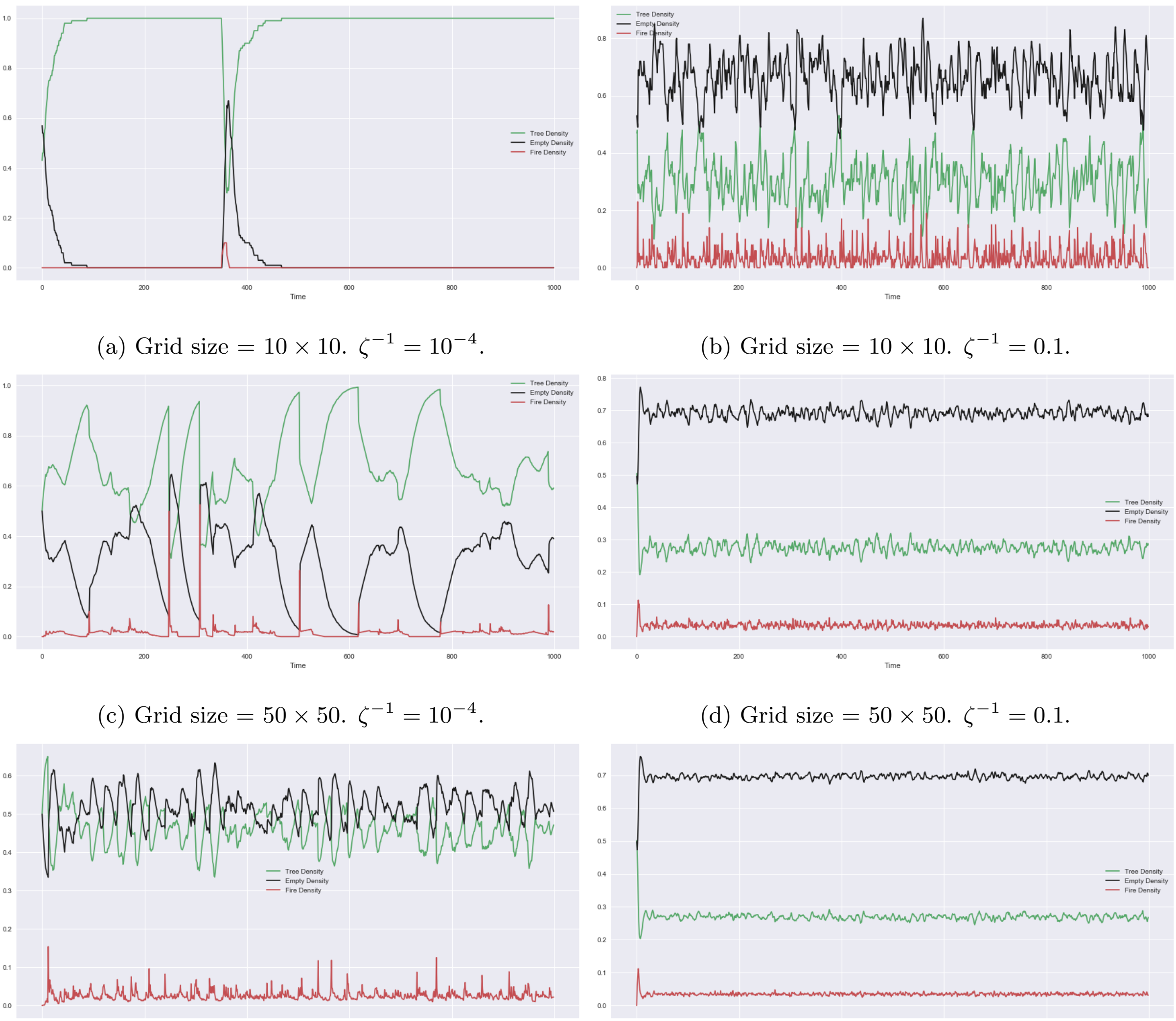
$$\begin{aligned} \langle \tilde{s} \rangle_{\text{fire}} &= \frac{(1-\rho_{\text{tree}})pL^d}{\rho_{\text{tree}} f L^d} \\ \langle \tilde{s} \rangle_{\text{fire}} &= \frac{p}{f}\frac{1-\rho_{\text{tree}}}{\rho_{\text{tree}}} = \zeta\frac{1-\rho_{\text{tree}}}{\rho_{\text{tree}}} \end{aligned} \tag{3}$$

where L^d are the volume for d-dimension square grid with side length L ($d = 2$ is area for a square).

The Equation (3) comes from the following:

- Denominator, $\rho_{\text{tree}} f L^d$: The average number of ignited tree (or 'zero patient' of disease).
- Numerator, $(1-\rho_{\text{tree}})pL^d$: The average number of trees growing in the system.

The formulation shows that Equation (3) follows power law $\langle \tilde{s} \rangle_{\text{fire}} \propto \zeta^{-1}$



(a) Grid size = 10×10. $\zeta^{-1} = 10^{-4}$.

(b) Grid size = 10×10. $\zeta^{-1} = 0.1$.

(c) Grid size = 50×50. $\zeta^{-1} = 10^{-4}$.

(d) Grid size = 50×50. $\zeta^{-1} = 0.1$.

(e) Grid size = 100×100. $\zeta^{-1} = 10^{-4}$.

(f) Grid size = 100×100. $\zeta^{-1} = 0.1$.

Figure 3: Time evolution of tree density, empty site density and fire density for different system size and different ζ^{-1}. Green line is the tree density, red line is the density of tree on fire and black are the density of empty site. $p = 0.05$. Iteration time = 1000

3.3 Cluster Radius and Number of Cluster

Cluster defined as the coherent set of cells. Coherent cells are cells that able to reach each other cell with same state via nearest neighbor relations. Typically, the choice of nearest neighbor for a cluster are von Neumann neighborhood.

There are two main properties that characterize a cluster, the cluster radius $R(s)$ and the number of cluster $N(s)$:

- Cluster Radius, $R(s)$: Square root of the mean quadratic distance of the cluster member from their center of mass i.e.

$$R(s) = \sqrt{R_x(s)^2 + R_y(s)^2}$$
$$R(s) = \sqrt{\left(\frac{1}{N}\sum_{x_i \in \mathbf{C}(s)} (x_i - x_{com})^2\right)^2 + \left(\frac{1}{N}\sum_{y_i \in \mathbf{C}(s)} (y_i - y_{com})^2\right)^2} \tag{4}$$

where (x_i, y_i) are the coordinate that belong to the cluster with size s, $\mathbf{C}(s)$.

The choice of radius definition may vary for different author. In this study, we use the above definition that was used by Drossel and Schwabl 1992. Other common definition of a cluster radius are the maximum distance from one point at the perimeter of the cluster to another point at the perimeter.

- Number of Cluster, $N(s)$: Number of cluster consisted of s element.

These two characteristic are the main measure for the critical exponent of the system. It defined in the following manner

$$\xi = R(s) \propto \zeta^{\nu} \tag{5}$$
$$R(s) \propto s^{1/\mu}\mathcal{C}(s/s_{\max}) \tag{6}$$
$$N(s) \propto s^{-\tau} \times \begin{cases} \mathcal{C}(s/s_{\max}) & \tau > 2 \\ \mathcal{C}(s/s_{\max}) \ln^{-1}(s_{\max}) & \tau = 2 \end{cases} \tag{7}$$
$$s_{\max} \propto \zeta^{\lambda} \tag{8}$$

where ξ are the correlation length and $\mathcal{C}(x) = 1$ if $x \ll 1$ and 0 otherwise.

The critical exponent have the following relation:

$$\lambda = \nu\mu, \qquad d = \mu(\tau - 1), \qquad \mu = 1/\nu \tag{9}$$

The theoretical value of critical exponents was found to be

$$\lambda = 1, \quad \tau = 2, \quad \mu = d, \quad \nu = 1/d \tag{10}$$

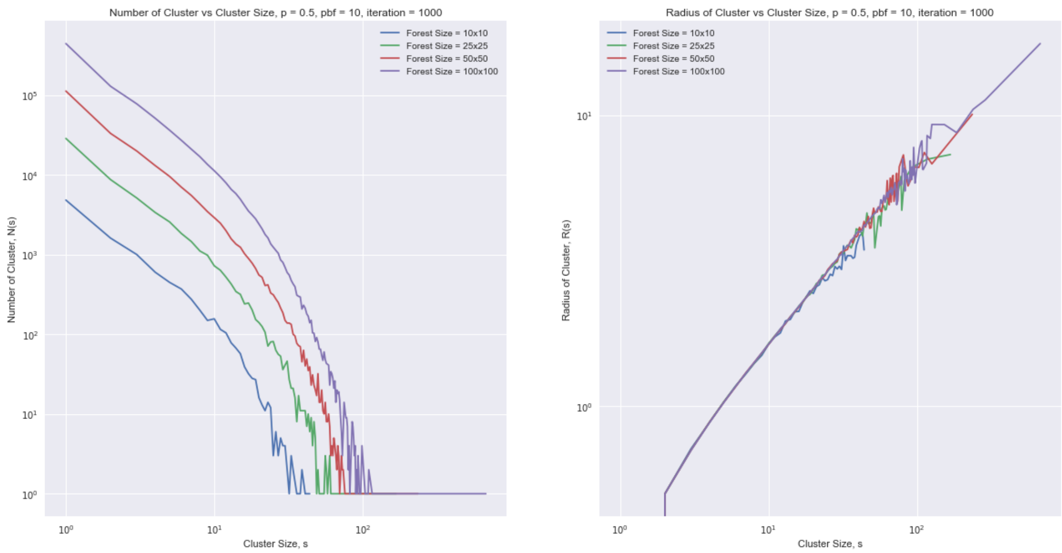
by Drossel and Schwabl 1992 where d are the number of the grid dimension. In this study, the number of dimension are two, $d = 2$. Given this relation of critical exponent, it is sufficient to calculate μ and ν to infer the other critical exponent.

The investigation for radius and number of a cluster as a function of the size was done by tuning the parameter. Due the nature of the system which consist of scale-free behaviour, any parameter change unlikely to shows a drastic difference between each setup.

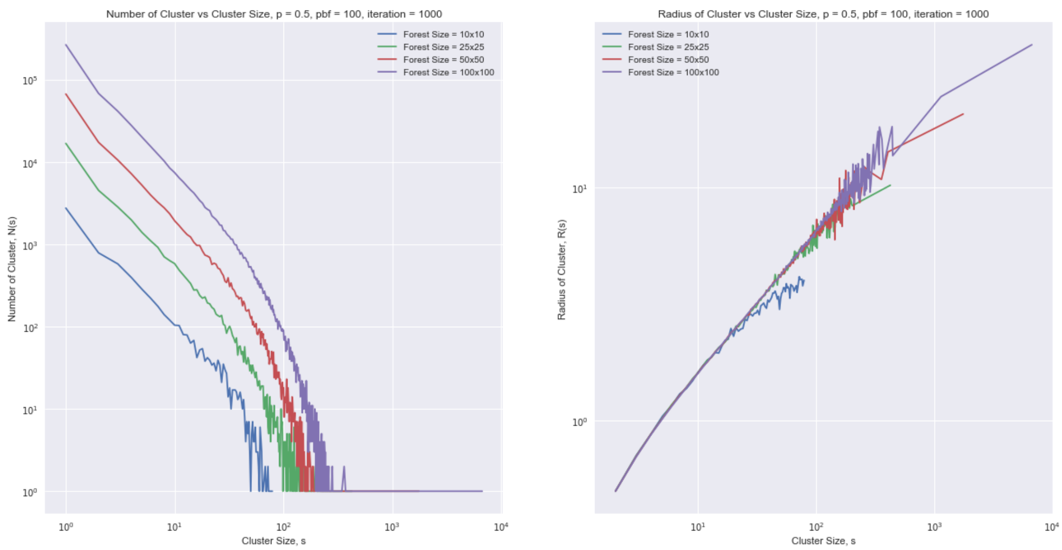
3.3.1 Fix p, fix ζ, different system size

First, we simulated the system for fixed p, ζ and varying the system size. The plot of the number of the cluster and the radius of cluster with respect to the cluster size shown in Figure 4. The number of cluster with size s increases for larger system size given the same value of p, ζ and iteration time. Also, the maximum size of the cluster increased as the system size enlarged. This increases expected as there are more possible site to form a cluster due to larger system size. Yet, the number of cluster size for large size tend to fluctuate for smaller ζ. This observed from the right tail of the number of cluster plot in Figure 4a and 4b.

On the other hand, the radius of the cluster as a function of cluster size is consistent for any system size. For small cluster size, there is less variability of the radius of the cluster. This is expected as there is less configuration possibility for extremely small cluster size. In addition, it is very unlikely for the formation of a cluster with size in the magnitude of the system size i.e. $s \propto \mathcal{O}(L^2)$.



(a) $\zeta = 10$



(b) $\zeta = 100$

Figure 4: Number of cluster $N(s)$ and radius of cluster $R(s)$ for different system size. Grid size $= 10 \times 10, 25 \times 25, 50 \times 50, 100 \times 100$. $p = 0.5$. Iteration $= 1000$

3.3.2 Fix p, fix system size, different ζ

Second, we simulated the system for fixed p and system size for various ζ value. The trend of cluster radius still consistent with our previous result as the choice of configuration given the size does not change i.e. independent of ζ and system size. In contrary, the number of cluster depends on the ζ in addition of the system size.

For the same iteration time, the number of cluster with small size are within the same magnitude for different. As the size of the cluster increase, the number of cluster drop and the drop is more rapid for larger ζ (small f). This is consistent with the earlier discussion in the previous section where small f in finite size system will result in higher connectivity within sites and a higher chance of the large cluster formation.

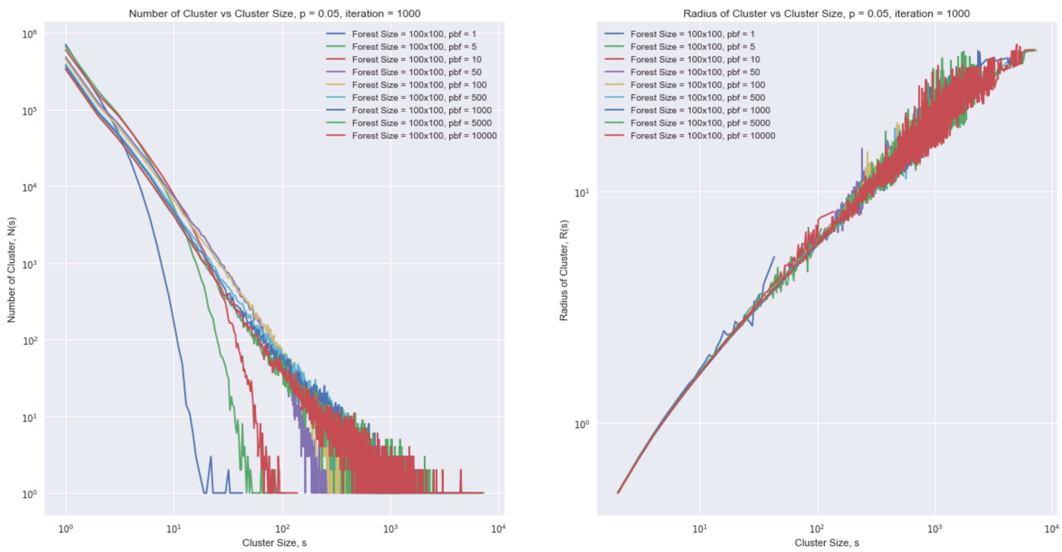


Figure 5: Number of cluster $N(s)$ and radius of cluster $R(s)$ for different ζ. $\zeta = 1, 5, 10, 50, 100, 500, 1000, 5000, 10000$. Grid size $= 100 \times 100$. $p = 0.5$. Iteration $= 1000$

The simulation for varying ζ and system size were done for completeness and the distribution plot of the number of cluster and the cluster radius appended in the Appendix B.2. Additional figure for Section 3.3.1 attached in Appendix B.2

3.4 Numerical Calculation of Critical Exponent

Equation (5), Equation (6), Equation (7), Equation (8) and Equation (9) provide the formulation to evaluate the critical exponent for a system. From the simulation data, the value of critical exponent can be inferred by fitting the suitable function and appropriate critical exponent relationship. The simulation data used for this calculation from the system with size 500-by-500 square cell, the probability of tree growth $p = 0.05$, $\rho = 0.43727 \pm 0.00729$, $\zeta = 10000$, von Neumann nearest neighborhood and absorbing boundary condition. The calculated critical exponent are as follow

$$\mu = 1.83475 \pm 0.01942 \tag{11}$$

$$\tau = 1.95596 \pm 0.00432 \tag{12}$$

$$\nu = 0.48300 \pm 0.00164 \tag{13}$$

$$\lambda = 0.97715 \pm 0.01046 \tag{14}$$

The results of the critical exponent numerical calculation agree with the value from (10) with $d = 2$.

3.5 Modified Forest Fire Model

3.5.1 Tree with Immunity Rate

One modification of the forest fire model can be done by adding another probability/rate for the immunity of the tree, i. The value of i ranged from 0 to 1 i.e. $r \in [0, 1]$. This probability/rate control the spread of fire/disease for tree surrounded by nearest neighborhood tree that on fire. This adds the possibility for a tree not to be burnt/infected despite their surrounding due some natural factor such as tree with high moisture (catch dew) or mutated tree which has resistance to disease.

For $i = 0$, the modified system reduced to the regular forest fire model. For $i = 1$, the modified system will result in a burnt tree which flame is not spreading across their neighborhood; this case produces a stable giant cluster of trees. Figure 6 and Figure 9 shows the system number of cluster distribution $N(s)$ along with the cluster radius $R(s)$ and tree density with average number of cluster for different immunity probability/rate respectively. The total number of cluster decreases as the immunity rate increases (within same iteration period). It expected due the frequent formation of large cluster which require most of the sites in the system. This result in a payoff in the form of less small cluster. The behaviour described in the following manner:

$$i \to 1 \Rightarrow N(s) \sim \begin{cases} \text{high}, & \text{for large } s \\ \text{low}, & \text{for small } s \end{cases} \tag{15}$$

$$i \to 0 \Rightarrow N(s) \sim \begin{cases} \text{low}, & \text{for large } s \\ \text{high}, & \text{for small } s \end{cases} \tag{16}$$

From Figure 9, it observed that there is an immunity probability/rate where the density stop increasing and reach the maximum before $i = 1$. This can be seen as the critical "temperature" with the value of $i \in [0.5, 0.6]$. Phase transition occurred from high i (highly connected system) to low i (low to medium connected system, depend on p and ζ).

This modification of forest-fire model is possible to be extended for different scenario. Three potential consideration are evolving value of i across time, unique i for each individual tree and combination of both. Other modification includes the addition of another tree type which have different properties (different p, i and f) and wind direction that may influence the direction of the fire. These case may be considered for improvement and future work to investigate whether the dynamics of the system remain the same or change.

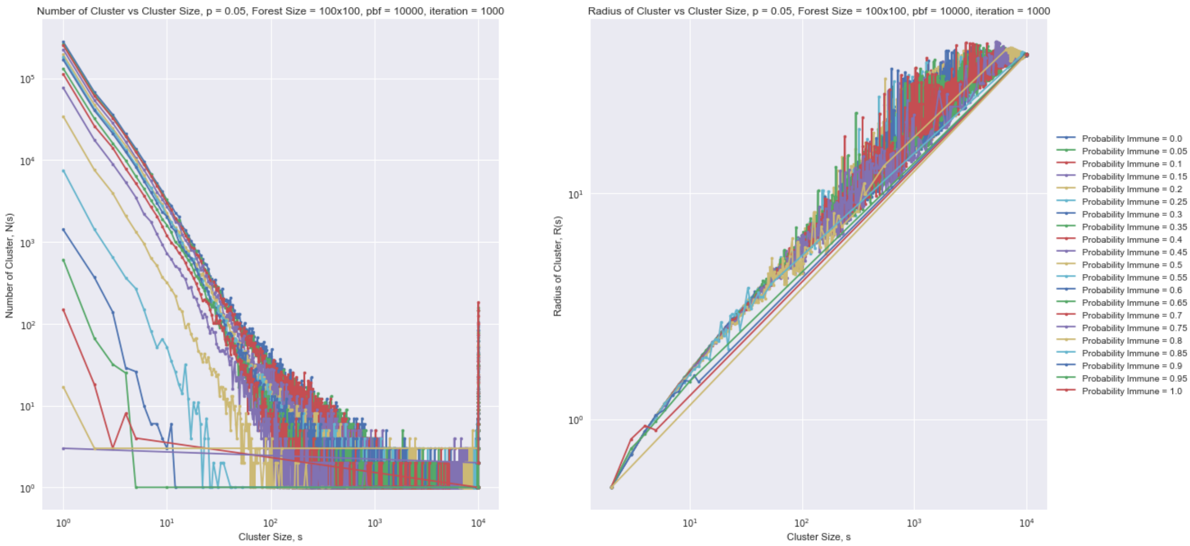


Figure 6: Number of cluster $N(s)$ and radius of cluster $R(s)$ for different i. $\{i \in \mathbb{R} | 0 \leq i \leq 1, i \mod 0.1 = 0\}$. Grid size $= 100 \times 100$. $p = 0.05$. Iteration $= 1000$

4 Conclusion

In this project, we interested to explore the properties and characteristic of a forest-fire model proposed initially by Bak et al. 1990 and continued in the work done by Drossel and Schwabl 1992. From previous study, it was concluded that the original set of rule produce a deterministic characteristic rather than approaching its critical state for small parameter p. Drossel and Schwabl 1992 work includes one additional parameter which is the probability for a tree to be ignited, f, which allow small forest clusters to burn. There is only one relevant parameter of the system which is $\zeta^{-1} = f/p$.

From our simulation, we encountered the phenomena where the fire fronts are in spiral shape which consistent with the counterargument by Drossel and Schwabl 1992 for Bak et al. 1990 work. One characteristic properties of the system are the density of the tree. Drossel and Schwabl 1992 showed that the value of the tree density to be $\rho = 0.39$ as an upper limit of the system. Our simulation agrees with the given value except that there are anomaly for $\zeta^{-1} \to 0$ where the value tend to deviate away from the upper limit. This is the result of the finite-size effect of the system size. The number of large fires increases, since the forest clusters spread nearly the whole lattice, large clusters are dominant and large cluster are prone to fire due to highly connected sites.

The cluster radius and number of cluster with size s were studied based on the simulation by clustering the forest sites. It was found that the cluster radius of different system size and parameter ζ to be in the same trend and have less variability for small cluster size s due to the limited configuration with s sites. Meanwhile, the number of the cluster with size s is highly dependent with the parameter ζ and system size. Large ζ and large system size increase the largest possible size of the cluster and the occurrence of cluster respectively.

Calculation of critical exponents was done following the definition provided from the work done by Drossel and Schwabl 1992. The numerically calculated critical exponent are $\mu = 1.83475 \pm 0.01942$, $\tau = 1.95596 \pm 0.00432$, $\nu = 0.48300 \pm 0.00164$, and $\lambda = 0.97715 \pm 0.01046$. These values agree with the theoretical value derived by Drossel and Schwabl 1992 shown in (10).

At last, we modified the system by imposing a resistance/immunity probability i where the model with $i = 0$ reduce to the original definition and the model with $i = 1$ implies the ignited/infected tree is not able affecting their neighbor. The new parameter i was found to be not affecting the cluster radius trend. Yet, the value i affects the average number of cluster and the average cluster size of the system. This observed from Figure 6 and Figure 9. The change of the characteristic value suggest a critical value of $i = 0.55 - 0.60$.

Further investigation of the system includes the study of the modified forest-fire system with more parameter that better reflect real life scenario. Nevertheless, the standard assumptions of the model are able to provide a guidance on managing forest fire in real life, especially regarding the "Yellowstone effect". The study can be improved by optimizing the algorithm for clustering and calculation to able perform calculation for larger system size within appropriate time.

References

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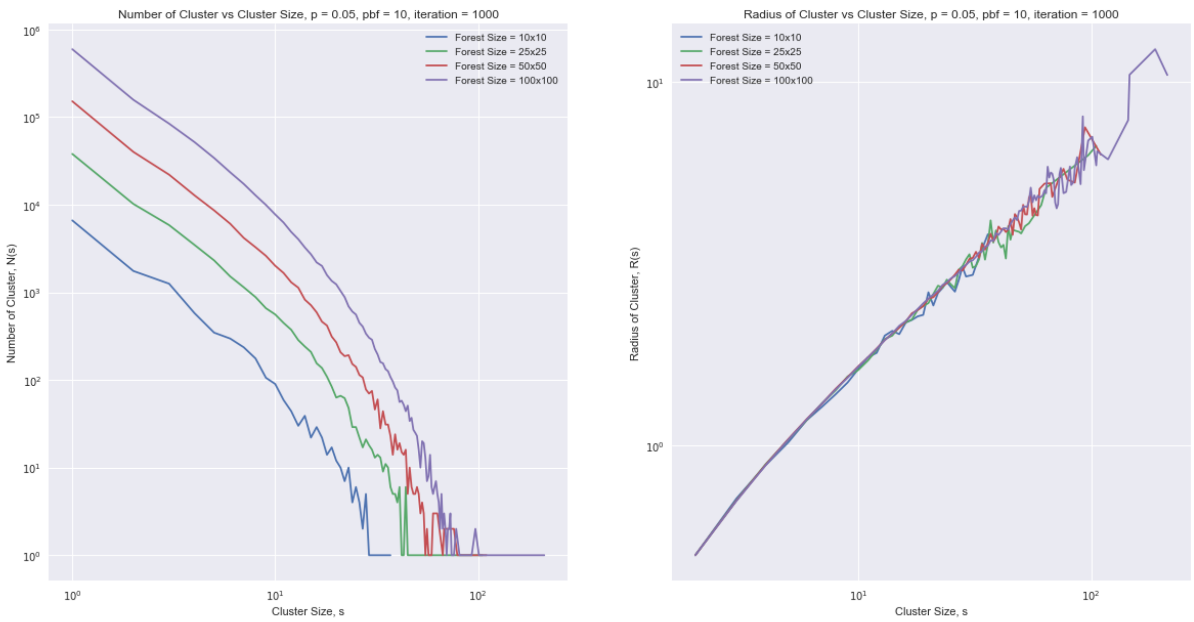
A Simulation Result and Relevant Code

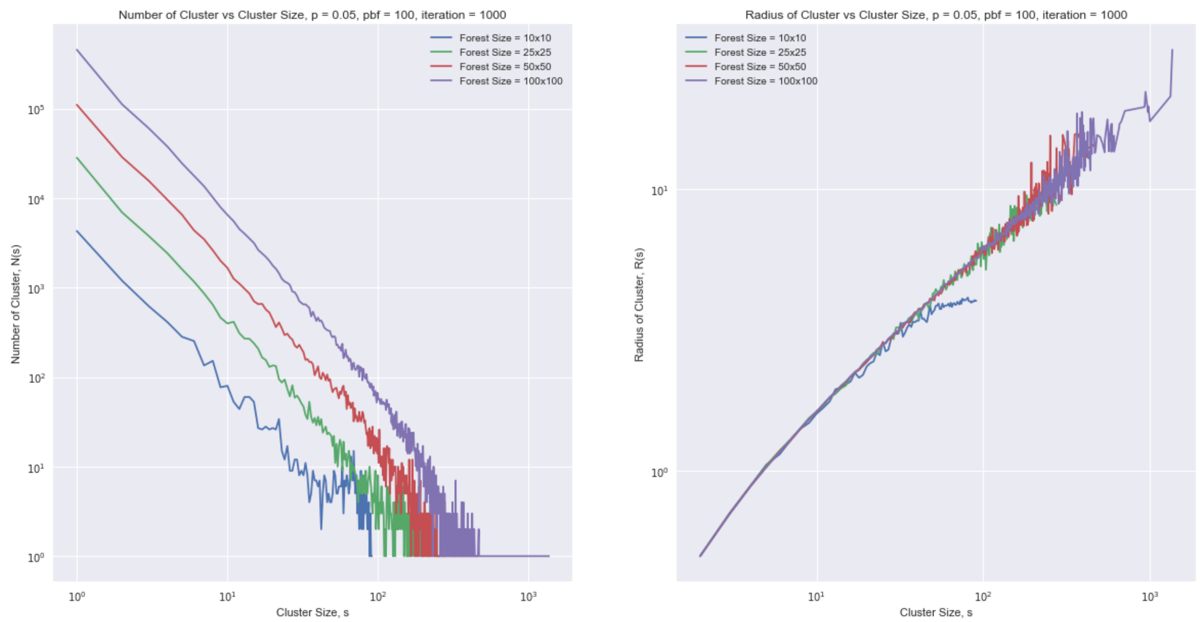
The result of the simulation and relevant code used for the simulation can be accessed from here (click here)

B Cluster Radius and Number of Cluster Figure

B.1 Fix $p = 0.05$, fix ζ, different system size



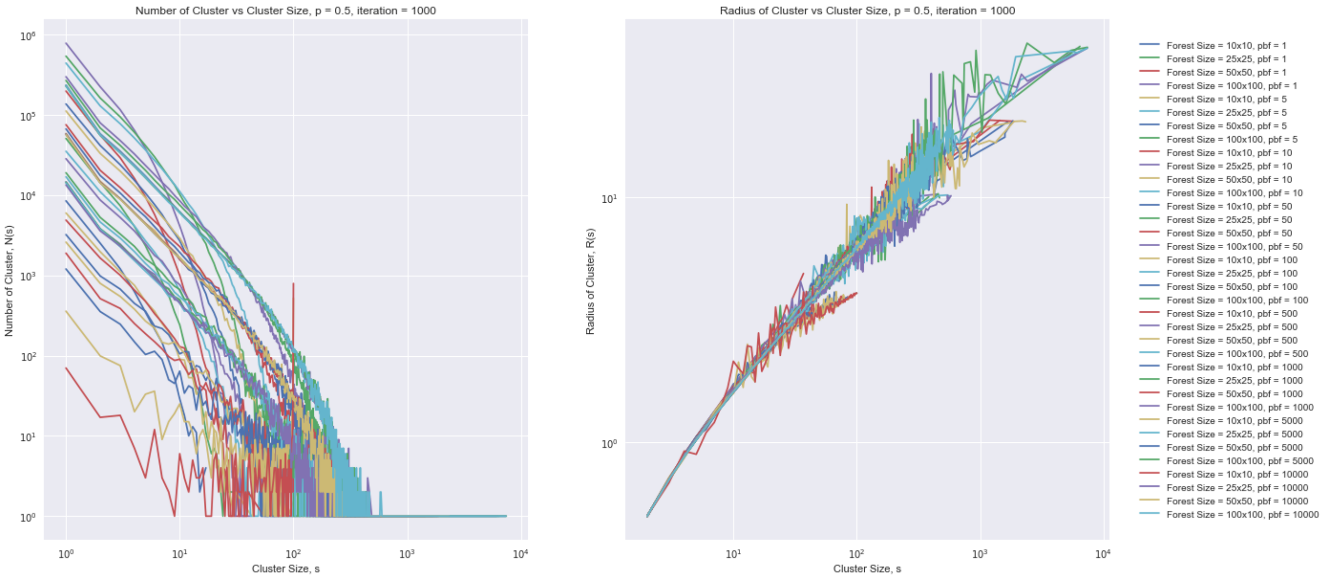
(a) $\zeta = 10$



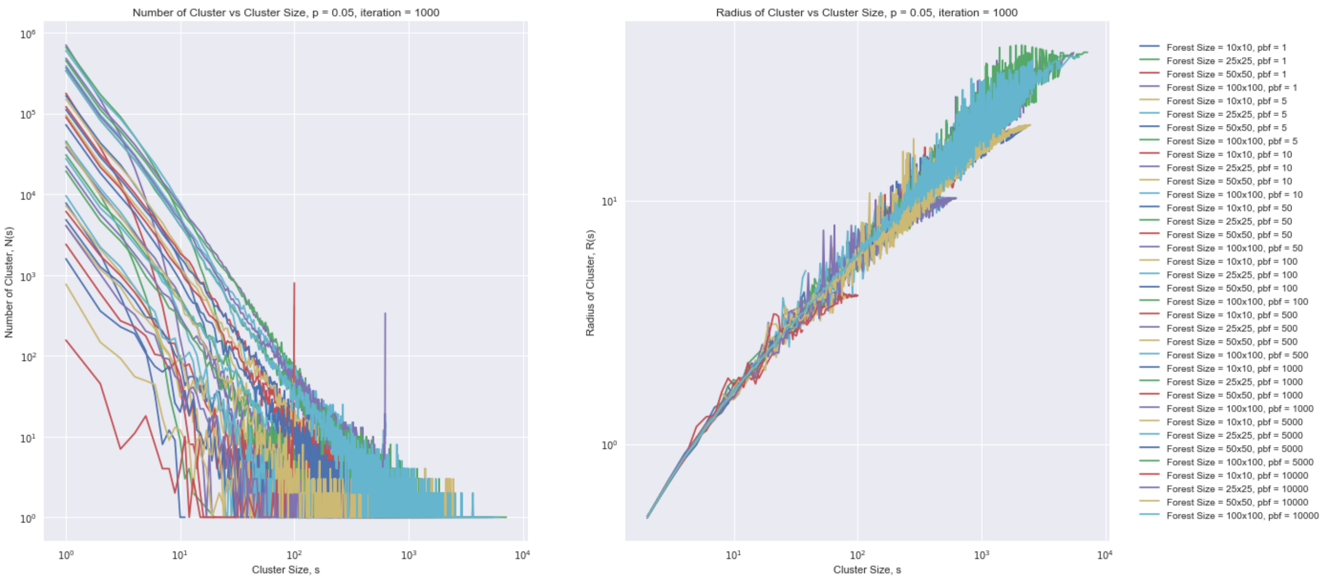
(b) $\zeta = 100$

Figure 7: Number of cluster $N(s)$ and radius of cluster $R(s)$ for different system size. Grid size $= 10 \times 10, 25 \times 25, 50 \times 50, 100 \times 100$. $p = 0.05$. Iteration $= 1000$

B.2 Fix p, different system size and ζ



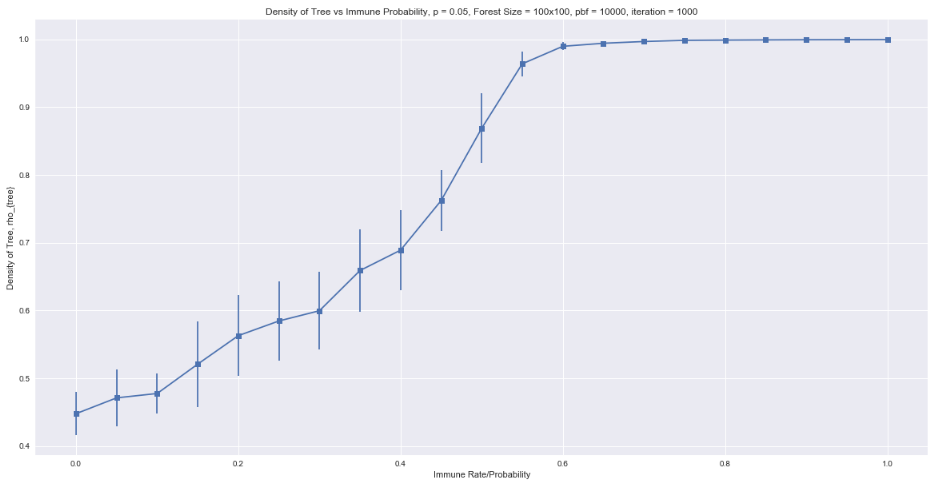
(a) $p = 0.5$



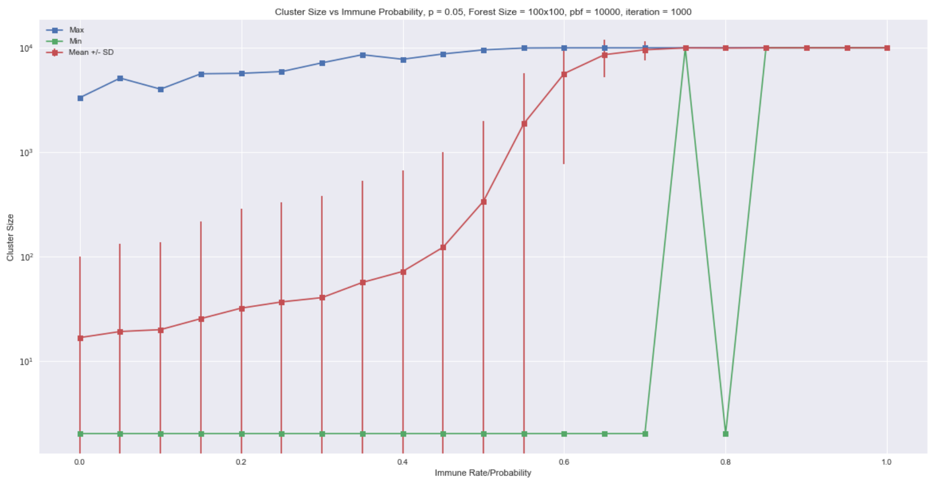
(b) $p = 0.05$

Figure 8: Number of cluster $N(s)$ and radius of cluster $R(s)$ for different system size and ζ. Grid size $= 10 \times 10, 25 \times 25, 50 \times 50, 100 \times 100$. $\zeta = 1, 5, 10, 50, 100, 500, 1000, 5000, 10000$. Iteration $= 1000$

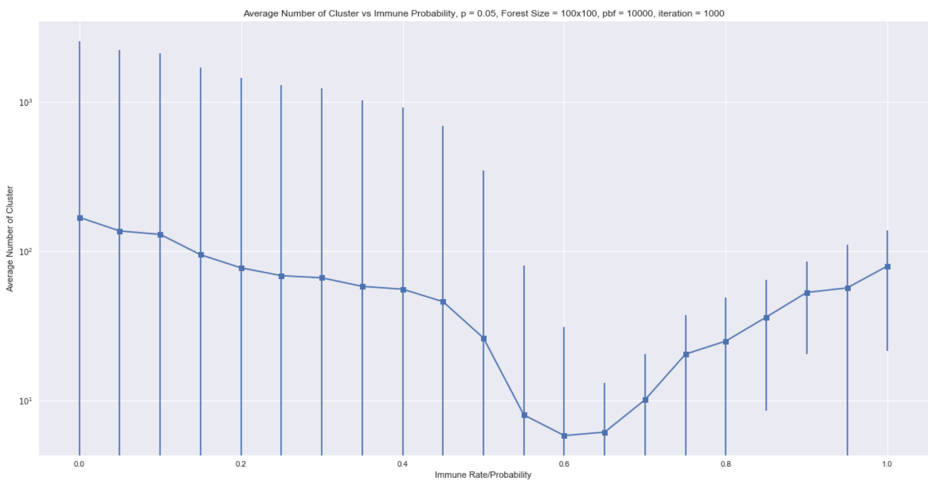
B.3 Modified Forest Fire Model - Immune Probability



(a) Tree Density vs Immunity Rate

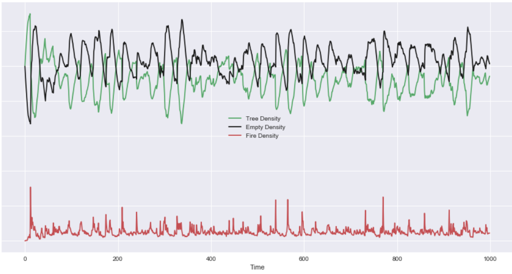


(b) Cluster Size vs Immunity Rate



(c) Average Number of Cluster vs Immunity Rate

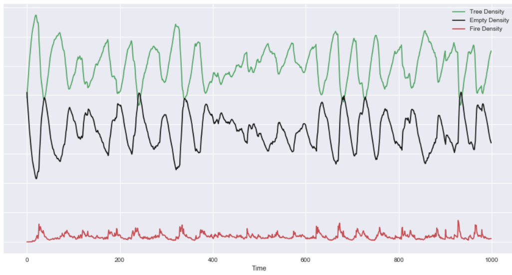
Figure 9: Tree density ρ_{tree}, average cluster size and average number of cluster $\langle N(s) \rangle$ for different immunity probability/rate. Grid size $= 100 \times 100$. $\zeta = 10000$, $p = 0.05$. Iteration $= 1000$



(a) $i = 0$

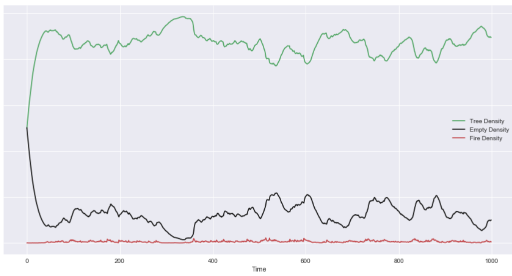
(b) $i = 0.1$

(c) $i = 0.2$

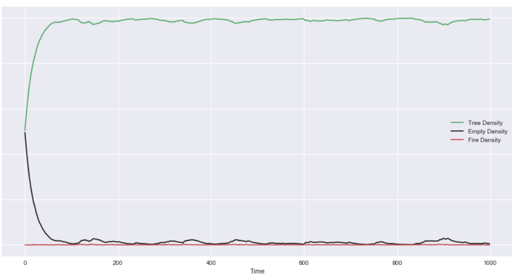


(d) $i = 0.3$

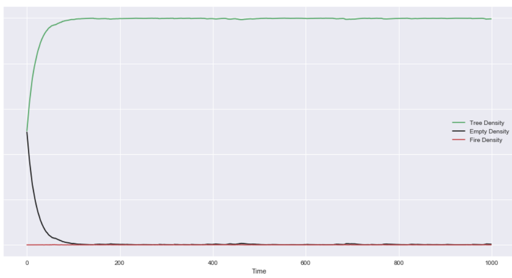
(e) $i = 0.4$



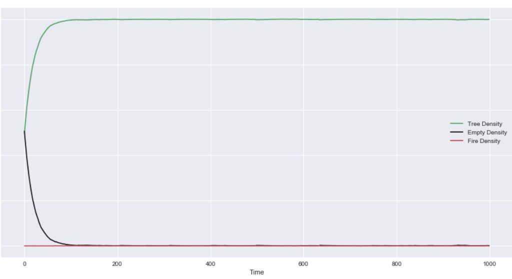
(f) $i = 0.5$



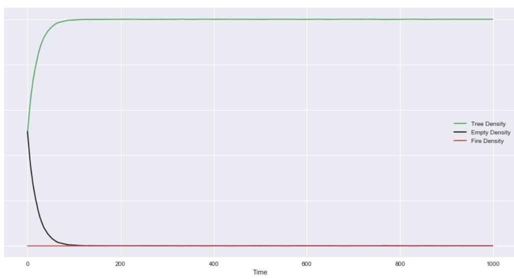
(g) $i = 0.6$



(h) $i = 0.7$



(i) $i = 0.8$



(j) $i = 0.9$

Figure 10: Tree density ρ_{tree} time evolution for different immunity probability/rate. Grid size = 100×100. $\zeta = 10000$, $p = 0.05$. Iteration = 1000