Seattle Data Science Journal Club: Wager & Athey

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Seattle Data Science Journal Club

What, why, where, when

Building a platform where data scientists can discuss the latest and greatest in the field and network:

- A seminar series like in graduate school:
  - Discuss important papers with top Seattle data scientists
  - Remain current on latest ideas
  - Occurs every other month
- A speaker series:
  - Hear key data science thinkers
  - Occurs every other month $+ 1$

Discussion of purpose

This is a meetup to discuss the latest data science ideas:

- What topics interest you?
- Are there speakers you want to hear?
- What cadence?
- Do you want to help lead/organize this group?
- Let us know...

Wager & Athey (2017)

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Wager & Athey (2017): *Estimation and Inference of Heterogeneous Treatment Effects Using Random Forest*

- Develop non-parametric *causal forest* to estimate causal effects:
  - Heterogeneous treatment effects
  - Extends on random forest
- First tools to perform valid statistical inference:
  - Asymptotically Gaussian
  - Pointwise consistent
  - Works for any random forest algorithm

Wager & Athey (2017)

Part of a new strand of literature uniting Econometrics and ML to estimate causal effects.

- ML:
  - Great for prediction
  - Great for large datasets
  - Poor for inference
- Econometrics/Applied Statistics/etc.:
  - Great for causality (e.g., Rubin)
  - Great for estimation and inference of causal effects
  - Poor for model selection and many features

Research agenda

Social science problems often consist of prediction + causal inference:

- Use ML for prediction, model selection, and robustness
- Extend to handle inference & estimation of causal effects

Policy evaluation

Data scientists often need to estimate the impact of a policy:

- Is feature X better than feature Y?
- Did our advertising work?

We can apply this literature to many problems we face, such as A/B testing

Real-world example: Ascarza (2016) *Retention futility: Targeting high risk customers might be ineffective*:

- Uses a similar method to measure heterogeneous response to churn intervention
- Computes optimal policy, which is counter to conventional wisdom

Literature

Some classics:

- Breiman (2004). *Random forests*
- Imbens & Rubin (2015). *Causal Inference*

Some recent papers:

- Athey & Imbens (2016). *Recursive partitioning for heterogeneous causal effects*
- Athey, Tibshirani, and Wager (2016). *Generalized random forests*
- Wager, Hastie, and Efron (2014). *Confidence intervals for random forests*

Motivations

Paper tackles several problems:

- Gelman's "Garden of forking paths" – well-intentioned, ex-post data-driven hypothesis testing
  - Should pre-specify analysis plan
  - But, cannot anticipate all forms of heterogeneity ex-ante
- Optimal policy: must estimate treatment effect heterogeneity

Insights of paper

Construct confidence intervals for estimates from modified random forest algorithm using several insights:

- Estimate treatment effects using RF to determine "nearby" observations
  - I.e., with correct splitting, each leaf should be (close to) a random experiment with nigh identical units
- Cross-validation for inference (*honest trees*)
- *Given a tree built on the training set, can use any valid method to estimate τ on test set*
- Prediction at individual and not leaf/group level (e.g., Athey & Imbens (2016))

Applied to decision trees (Athey & Imbens) and random forests (this paper).

Structure of paper

Organization of the paper:

1. Prove consistency & asymptotic normality for a variant of RF
2. Prove infinitesimal jackknife consistent for aVar
3. Extend results to estimation of heterogeneous treatment effects in potential outcomes framework
4. Compare causal forest vs. k-NN using simulations

Notation: potential outcomes notation

Paper uses potential outcomes notation:

- Outcome is $Y_i(W_i)$ for individual i with treatment status W_i
- Treatment is $W_i \in \{0, 1\}$
- Want to measure causal effect, $\tau(x)$ at x

$$\tau(x) = \mathbb{E}[Y_i(1) - Y_i(0) | X_i = x]$$

but we cannot observe both $Y_i(1)$ and $Y_i(0)$...

Estimation

Prediction at x:

$$\hat{\mu}(x) = \frac{1}{|\{i : X_i \in L(x)\}|} \cdot \sum_{\{i : X_i \in L(x)\}} Y_i$$

Let

$$\hat{\mu}(x|w) = \frac{1}{|\{i : W_i = w, X_i \in L(x)\}|} \cdot \sum_{\{i : W_i = w, X_i \in L(x)\}} Y_i$$

Estimation of treatment effect at x for a causal tree:

$$\hat{\tau}(x) = \hat{\mu}(x|w = 1) - \hat{\mu}(x|w = 0)$$

For a RF with B trees, $\hat{\tau}(x) = \frac{1}{B} \cdot \sum_{b=1}^{B} \hat{\tau}_b(x)$

Estimation of variance

Estimation of variance uses:

$$\widehat{V}_{IJ}(x) = \frac{n-1}{n}\left(\frac{n}{n-s}\right)^2 \sum_{i=1}^{n} \mathrm{Cov}_*[\hat{\tau}_b^*(x), N_{ib}^*],$$

where:

- $\mathrm{Cov}_*[\cdot, \cdot]$ is over all trees $b = 1, ..., B$
- N_{ib}^* indicates whether observation i is in tree b

Assumptions

Unconfoundeness:

- $\{Y_i(0), Y_i(1)\} | W_i \perp\!\!\!\perp X_i$
- As if a neighborhood is a randomized experiment

Overlap (probabilistic):

- $\epsilon < \mathbb{P}[W = 1 | X = x] < 1 - \epsilon$
- For large n, a neighborhood contains both treatments

Honest trees

Asymptotic normality and consistency require:

- Subsample size s scales appropriately
- *Honest* trees:
  - *Double-sample* trees use two samples:
    - $\mathcal{I}$: used to estimate effects within each leaf
    - $\mathcal{J}$: used to determine splits
  - *Propensity* trees:
    - Ignore Y_i when computing splits
    - Train classification tree for W_i
    - Estimate leaf-level responses
    - In tradition of propensity matching

Procedure 1: double-sample trees

Plan: split the sample and use one half to build tree and other to estimate:

1. Draw random subsample of size s without replacement
2. Split it into $\mathcal{I}$ and $\mathcal{J}$
3. Grow tree via recursive partitioning:
   - Use any data in $\mathcal{J}$ and only X or W in $\mathcal{I}$
   - Do not use Y in $\mathcal{I}$
4. Estimate $\hat{\tau}(x)$ using only $\mathcal{I}$

Note: must sample without replacement

Procedure 2: propensity trees

Use W_i to determine splits and Y_i to estimate τ:

1. Draw random subsample $\mathcal{I}$ of size s without replacement
2. Train classification tree using $\mathcal{I}$ using W_i as label and X_i as features
   - Must have $\geq k$ observations in each leaf for each treatment
   - Can optimize using Gini criterion, entropy, etc.
3. Estimate $\tau(x)$ on $L(x)$

Definitions

Key definitions:

- **honest** tree:
  - Double-sample tree: does not use Y_i in $\mathcal{I}$ to choose splits
  - Propensity tree: does not use Y_i to choose splits
- **random-split** tree:
  - Marginalize over auxiliary randomness, $\xi \sim \Xi$ in RF
  - At every split, $\pi / d < \mathbb{P}[\texttt{split along } j\texttt{-th feature}], \forall \pi \in (0,1]$
  - Note: ξ contains randomness for splitting features

Definitions

Key definitions:

- α-**regular** $\forall \alpha > 0$ if:
  - standard case:
    - At least α of training observations on each side of split
    - Terminal nodes have at between k and $2k - 1$ observations
  - double-sample: $\mathcal{I}$ satisfies above condition
- **symmetric**:
  - Output of predictor independent of order of training set

Results

Results are theoretical with some confirmation via simulation:

- Theorems on asymptotic normality of mean and treatment effects
- Simulation experiments

Theorems

Then they prove some theorems, given regularity conditions:

- Theorem 1: $\dfrac{\hat{\mu}_n - \mu(x)}{\sigma_n(x)} \overset{a}{\sim} N(0,1)$
- Lemma 2: probability limit on $\mathtt{diam}_j(L(x))$
- Theorem 3: $|\mathbb{E}[\hat{\mu}_n(x)] - \mu(x)| - \mathcal{O}\left(f(s,\alpha)\right)$
- Theorem 11: causal forest has:
  - Predictions $\hat{\tau}(x)$ are consistent and asymptotically Gaussian and centered
  - Variance that is consistently estimated

Simulation experiments

Compare causal forest to k-NN:

- CF provides:
  - Superior matching
  - Stable MSE which is $\ll$ MSE of k-NN
- CF coverage deteriorates for more than ≈ 10 features
- In some simulations, bias dominates variance for RF and causes uncentered CI

$\Rightarrow$ need to improve control for bias, perhaps via better splitting rule

Questions

Some questions:

- Simulations use large enough n?
- How much data needed to succeed?
- How much overlap needed to measure τ well?
- Performance vs. classical methods (propensity score, matching)?
- Why not split on entropy?
- How to identify lack of balance in leaves?
- Performance on real data?

Future meetings

Thanks for attending!

We will meet every month:

- `month %% 2 == 1` $\Rightarrow$ discuss a paper
- `month %% 2 == 0` $\Rightarrow$ listen to a speaker
- Next month's speaker: Jake Vanderplas