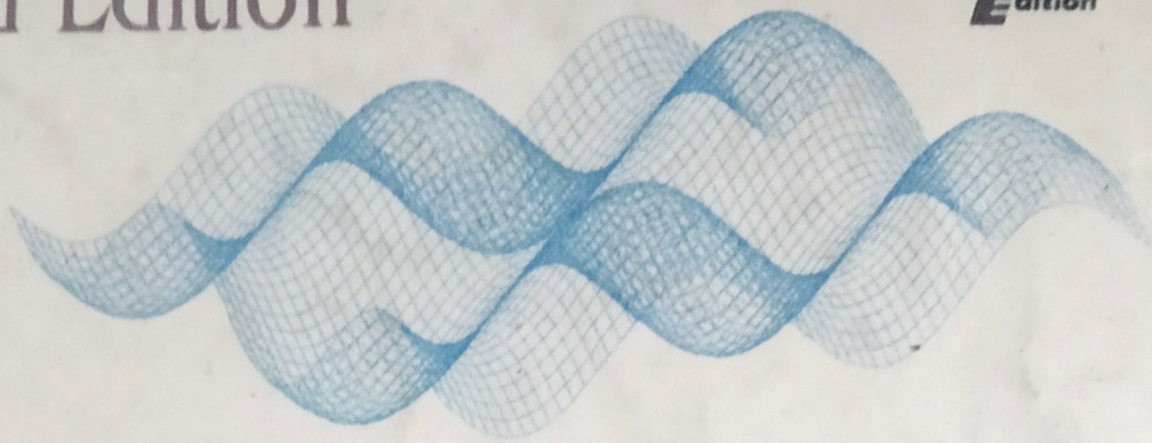
Third Edition

Eastern Economy Edition



# Signals and Systems

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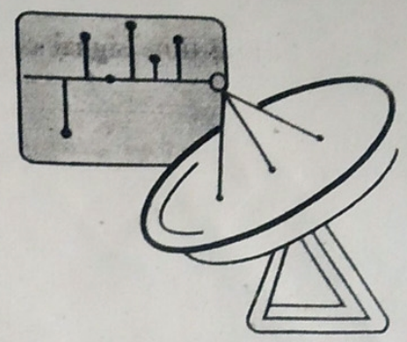

# Signals

## 1.1 INTRODUCTION

Anything that carries information can be called a signal. Signals constitute an important part of our daily life. A signal is defined as a single-valued function of one or more independent variables which contain some information A signal may also be defined as any physical quantity that varies with time, space or any other independent variable. A signal may be represented in time domain or frequency domain. Human speech is a familiar example of a signal. Electric current and voltage are also examples of signals. A signal can be a function of one or more independent variables. A signal may be a function of time, temperature, position, pressure, distance, etc. If a signal depends on only one independent variable, it is called a *one-dimensional signal*, and if a signal depends on two independent variables, it is called a *two-dimensional signal*. In this book we discuss only about one-dimensional signals. In this chapter we discuss about various basic signals available, various operations on signals and classification of signals.

## 1.2 REPRESENTATION OF DISCRETE-TIME SIGNALS

In general signals may be continuous-time signals or discrete-time signals. Continuous-time signals are defined for all instants of time, whereas discrete-time signals are defined only at discrete instants of time. Continuous-time signals are represented by $x(t)$ and discrete-time signals are represented by $x(n)$ where $t$ and $n$ are independent variables in time domain. Continuous-time signals are represented by a function or a graph.

There are four ways of representing discrete-time signals. They are:

1. Graphical representation
2. Functional representation
3. Tabular representation
4. Sequence representation

### 1.2.1 Graphical Representation

Consider a signal $x(n)$ with values

$$x(-2) = -3,\ \ x(-1) = 2,\ \ x(0) = 0,\ \ x(1) = 3,\ \ x(2) = 1 \ \text{ and } \ x(3) = 2$$

This discrete-time signal can be represented graphically as shown in Figure 1.1.

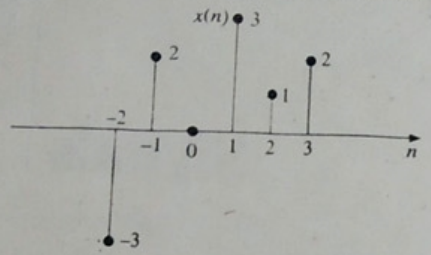


**Figure 1.1** Graphical representation of discrete-time signal.

### 1.2.2 Functional Representation

In this, the amplitude of the signal is written against the values of $n$. The signal given in 1.2.1 can be represented using functional representation as given below.

$$x(n) = \begin{cases} -3 & \text{for } n = -2 \\ 2 & \text{for } n = -1 \\ 0 & \text{for } n = 0 \\ 3 & \text{for } n = 1 \\ 1 & \text{for } n = 2 \\ 2 & \text{for } n = 3 \end{cases}$$

Another example is

$$x(n) = \begin{cases} 2^n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

### 1.2.3 Tabular Representation

In this, the sampling instant $n$ and the magnitude of the signal at the sampling instant are represented in tabular form. The signal given in 1.2.1 can be represented in tabular form as shown below.

| $n$ | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| $x(n)$ | -3 | 2 | 0 | 3 | 1 | 2 |



### 1.2.4 Sequence Representation

A finite duration sequence given in 1.2.1 can be represented as:

$$x(n) = \left\{ \begin{matrix} -3, 2, 0, 3, 1, 2 \\ \uparrow \end{matrix} \right\}$$

Another example is

$$x(n) = \left\{ \begin{matrix} \ldots 2, 3, 0, 1, -2, \ldots \\ \uparrow \end{matrix} \right\}$$

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The arrow mark ↑ denotes the $n = 0$ term. When no arrow is indicated, the first term corresponds to $n = 0$

So a finite duration sequence, that satisfies the condition $x(n) = 0$ for $n < 0$ can be represented as $x(n) = \{3, 5, 2, 1, 4, 7\}$.

***Sum and product of discrete-time sequences***

The sum of two discrete-time sequences is obtained by adding the corresponding elements of sequences

$$\{C_n\} = \{a_n\} + \{b_n\} \ \rightarrow \ C_n = a_n + b_n$$

The product of two discrete-time sequences is obtained by multiplying the corresponding elements of the sequences.

$$\{C_n\} = \{a_n\}\{b_n\} \ \rightarrow \ C_n = a_n b_n$$

The multiplication of a sequence by a constant $k$ is obtained by multiplying each element of the sequence by that constant.

$$\{C_n\} = k\{a_n\} \ \rightarrow \ C_n = k a_n$$

## 1.3 ELEMENTARY SIGNALS

There are several elementary signals which play vital role in the study of signals and systems. These elementary signals serve as basic building blocks for the construction of more complex signals. Infact, these elementary signals may be used to model a large number of physical signals which occur in nature. These elementary signals are also called standard signals.

The standard signals are:

1. Unit step function
2. Unit ramp function
3. Unit parabolic function
4. Unit impulse function
5. Sinusoidal function
6. Real exponential function
7. Complex exponential function, etc.

### 1.3.1 Unit Step Function

The step function is an important signal used for analysis of many systems. The step function is that type of elementary function which exists only for positive time and is zero for negative time. It is equivalent to applying a signal whose amplitude suddenly changes and remains constant forever after application.

If a step function has unity magnitude, then it is called unit step function. The usefulness of the unit-step function lies in the fact that if we want a signal to start at $t = 0$, so that it may have a value of zero for $t < 0$, we only need to multiply the given signal with unit step function $u(t)$. A unit step function is useful as a test signal because the response of the system for a unit step reveals a great deal about how quickly the system responds to a sudden change in the input signal.

The continuous-time unit step function $u(t)$ is defined as:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

From the above equation for $u(t)$, we can observe that when the argument $t$ in $u(t)$ is less than zero, then the unit step function is zero, and when the argument $t$ in $u(t)$ is greater than or equal to zero, then the unit step function is unity.

The shifted unit step function $u(t - a)$ is defined as:

$$u(t-a) = \begin{cases} 1 & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

It is zero if the argument $(t - a) < 0$ and equal to 1 if the argument $(t - a) \geq 0$.
The graphical representations of $u(t)$ and $u(t - a)$ are shown in Figure 1.2[(a) and (b)].

If a is positive then delay otherwise advance.

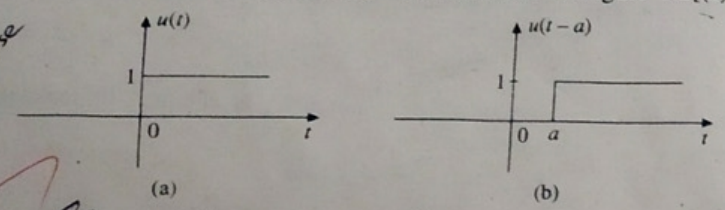


Figure 1.2 (a) Unit step function, (b) Delayed unit step function.

The discrete-time unit step sequence $u(n)$ is defined as:

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

The shifted version of the discrete-time unit step sequence $u(n - k)$ is defined as

$$u(n-k) = \begin{cases} 1 & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

The graphical representations of $u(n)$ and $u(n - k)$ are shown in Figure 1.3[(a) and (b)].

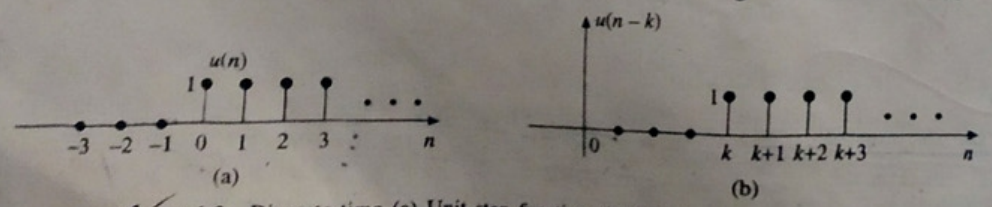


Figure 1.3 Discrete-time (a) Unit step function, (b) Shifted unit step function.



### 1.3.2 Unit Ramp Function

The continuous-time unit ramp function $r(t)$ is that function which starts at $t = 0$ and increases linearly with time and is defined as:

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

or

$$r(t) = t\,u(t)$$

The unit ramp function has unit slope. It is a signal whose amplitude varies linearly. It can be obtained by integrating the unit step function. That means, a unit step signal can be obtained by differentiating the unit ramp signal.

i.e.

$$r(t) = \int u(t)\,dt = \int dt = t \quad \text{for } t \geq 0$$

$$u(t) = \frac{d}{dt} r(t)$$

The delayed unit ramp signal $r(t - a)$ is given by

$$r(t-a) = \begin{cases} t-a & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

or

$$r(t-a) = (t-a)\,u(t-a)$$

The graphical representations of $r(t)$ and $r(t - a)$ are shown in Figure 1.4[(a) and (b)].

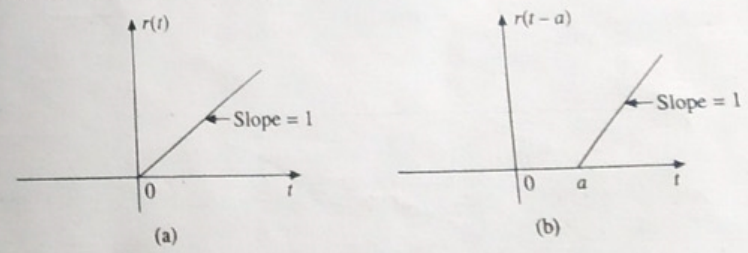


Figure 1.4 (a) Unit ramp signal, (b) Delayed unit ramp signal.

The discrete-time unit ramp sequence $r(n)$ is defined as

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

or

$$r(n) = n\,u(n)$$

The shifted version of the discrete-time unit-ramp sequence $r(n - k)$ is defined as

$$r(n-k) = \begin{cases} n-k & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

or

$$r(n-k) = (n-k)\,u(n-k)$$

The graphical representations of $r(n)$ and $r(n-2)$ are shown in Figure 1.5[(a) and (b)].

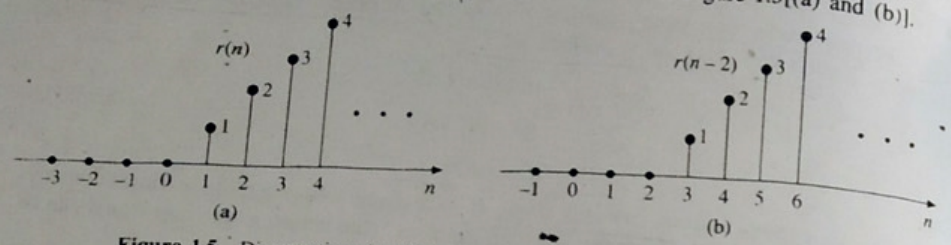


**Figure 1.5** Discrete-time (a) Unit-ramp sequence, (b) Shifted-ramp sequence.

### 1.3.3 Unit Parabolic Function

The continuous-time unit parabolic function $p(t)$, also called unit acceleration signal starts at $t = 0$, and is defined as:

$$p(t) = \begin{cases} \dfrac{t^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

or

$$p(t) = \frac{t^2}{2} u(t)$$

The shifted version of the unit parabolic sequence $p(t-a)$ is given by

$$p(t-a) = \begin{cases} \dfrac{(t-a)^2}{2} & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases}$$

or

$$p(t-a) = \frac{(t-a)^2}{2} u(t-a)$$

The graphical representations of $p(t)$ and $p(t-a)$ are shown in Figure 1.6[(a) and (b)].

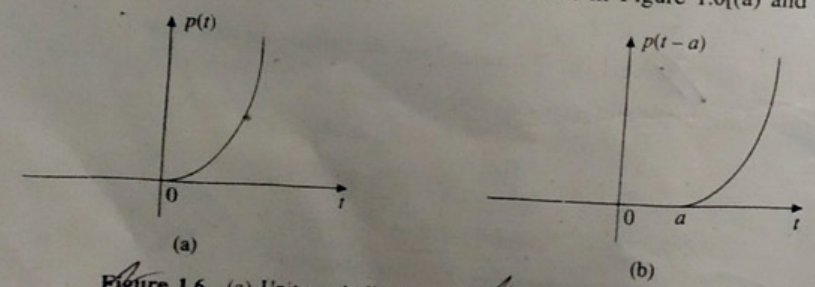


**Figure 1.6** (a) Unit parabolic signal, (b) Delayed parabolic signal.

The unit parabolic function can be obtained by integrating the unit ramp function or double integrating the unit step function.

$$p(t) = \iint u(t)\,dt = \int r(t)\,dt = \int t\,dt = \frac{t^2}{2} \quad \text{for } t \geq 0$$



The ramp function is derivative of parabolic function and step function is double derivative of parabolic function

$$r(t) = \frac{d}{dt} p(t); \quad u(t) = \frac{d^2}{dt^2} p(t)$$

The discrete-time unit parabolic sequence $p(n)$ is defined as:

$$p(n) = \begin{cases} \dfrac{n^2}{2} & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

or

$$p(n) = \frac{n^2}{2} u(n)$$

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The shifted version of the discrete-time unit parabolic sequence $p(n-k)$ is defined as:

$$p(n-k) = \begin{cases} \dfrac{(n-k)^2}{2} & \text{for } n \geq k \\ 0 & \text{for } n < k \end{cases}$$

or

$$p(n-k) = \frac{(n-k)^2}{2} u(n-k)$$

The graphical representations of $p(n)$ and $p(n-3)$ are shown in Figure 1.7[(a) and (b)].

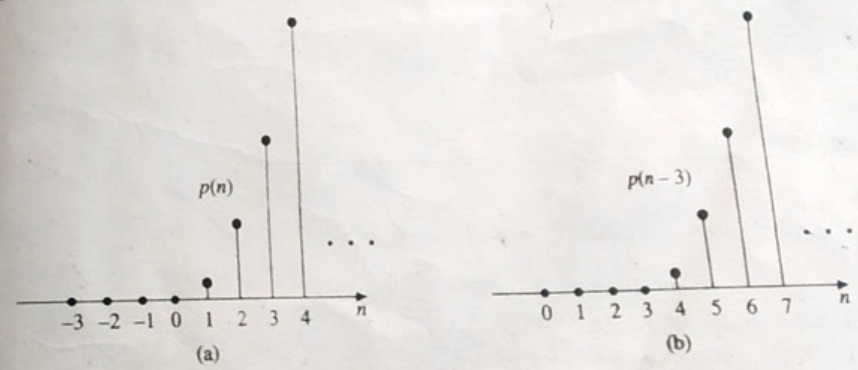


**Figure 1.7** Discrete-time (a) Parabolic sequence, (b) Shifted parabolic sequence.

### 1.3.4 Unit Impulse Function

The unit impulse function is the most widely used elementary function used in the analysis of signals and systems. The continuous-time unit impulse function $\delta(t)$, also called Dirac delta function, plays an important role in signal analysis. It is defined as:

$$\int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

and
$$\delta(t) = 0, \quad \text{for } t \neq 0$$

i.e. as
$$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$$

That is, the impulse function has zero amplitude everywhere except at $t = 0$. At $t = 0$, the amplitude is infinity so that the area under the curve is unity. $\delta(t)$ can be represented as a limiting case of a rectangular pulse function.

As shown in Figure 1.8(a),

$$x(t) = \frac{1}{\Delta}[u(t) - u(t - \Delta)]$$

$$\delta(t) = \underset{\Delta \to 0}{\text{Lt}}\, x(t) = \underset{\Delta \to 0}{\text{Lt}}\, \frac{1}{\Delta}[u(t) - u(t - \Delta)]$$

A delayed unit impulse function $\delta(t - a)$ is defined as:

$$\delta(t - a) = \begin{cases} 1 & \text{for } t = a \\ 0 & \text{for } t \neq a \end{cases}$$

The graphical representations of $\delta(t)$ and $\delta(t - a)$ are shown in Figure 1.8[(b) and (c)].

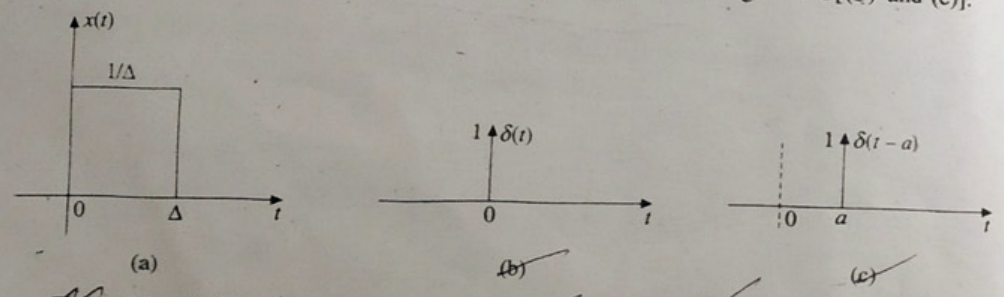


**Figure 1.8** (a) $\delta(t)$ as limiting case of a pulse, (b) Unit impulse, (c) Delayed unit impulse.

If unit impulse function is assumed in the form of a pulse, then the following points may be observed about a unit impulse function.

(i) The width of the pulse is zero. This means the pulse exists only at $t = 0$.
(ii) The height of the pulse goes to infinity.
(iii) The area under the pulse curve is always unity.
(iv) The height of arrow indicates the total area under the impulse.

The integral of unit impulse function is a unit step function and the derivate of unit step function is a unit impulse function.

$$u(t) = \int_{-\infty}^{\infty} \delta(t)\, dt$$

and
$$\delta(t) = \frac{d}{dt} u(t)$$



***Properties of continuous-time unit impulse function***

1. It is an even function of time $t$, i.e. $\delta(t) = \delta(-t)$
2. $\int_{-\infty}^{\infty} x(t)\, \delta(t)\, dt = x(0)$; $\int_{-\infty}^{\infty} x(t)\, \delta(t - t_0)\, dt = x(t_0)$
3. $\delta(at) = \frac{1}{|a|}\delta(t)$
4. $x(t)\, \delta(t - t_0) = x(t_0)\, \delta(t - t_0) = x(t_0)$; $x(t)\, \delta(t) = x(0)\, \delta(t) = x(0)$
5. $x(t) = \int_{-\infty}^{\infty} x(\tau)\, \delta(t - \tau)\, d\tau$

The discrete-time unit impulse function $\delta(n)$, also called unit sample sequence, is defined as:

$$\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$$

The shifted unit impulse function $\delta(n - k)$ is defined as:

$$\delta(n - k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$$

The graphical representations of $\delta(n)$ and $\delta(n - 3)$ are shown in Figure 1.9[(a) and (b)].

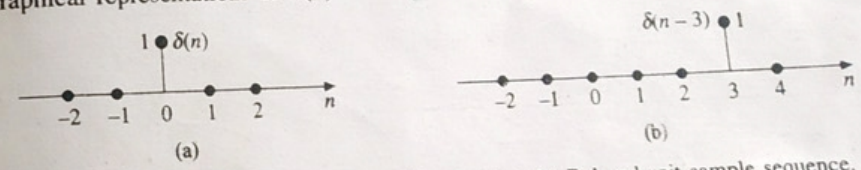


**Figure 1.9** Discrete-time (a) Unit sample sequence, (b) Delayed unit sample sequence.

***Properties of discrete-time unit sample sequence***

1. $\delta(n) = u(n) - u(n - 1)$
2. $\delta(n - k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$
3. $x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n - k)$
4. $\sum_{n=-\infty}^{\infty} x(n)\, \delta(n - n_0) = x(n_0)$

## 1.3.5 Sinusoidal Signal

A continuous-time sinusoidal signal in its most general form is given by

$$x(t) = A \sin(\omega t + \phi)$$

where

$A$ = Amplitude

$\omega$ = Angular frequency in radians

$\phi$ = Phase angle in radians

Figure 1.10 shows the waveform of a sinusoidal signal. A sinusoidal signal is an example of a periodic signal. The time period of a continuous-time sinusoidal signal is given by

$$T = \frac{2\pi}{\omega}$$

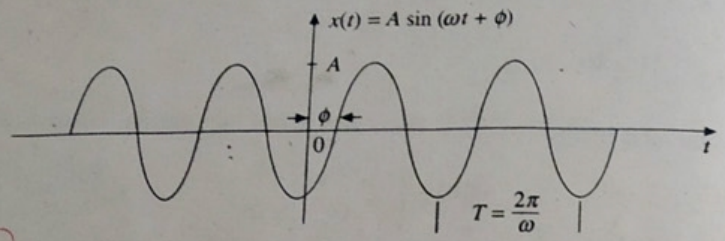


**Figure 1.10** Sinusoidal waveform.

The discrete-time sinusoidal sequence is given by

$$x(n) = A \sin(\omega n + \phi)$$

where $A$ is the amplitude, $\omega$ is angular frequency, $\phi$ is phase angle in radians and $n$ is an integer.

The period of the discrete-time sinusoidal sequence is:

$$N = \frac{2\pi}{\omega} m$$

where $N$ and $m$ are integers.

*All continuous-time sinusoidal signals are periodic but discrete-time sinusoidal sequences may or may not be periodic depending on the value of $\omega$.*

For a discrete-time signal to be periodic, the angular frequency $\omega$ must be a rational multiple of $2\pi$.

The graphical representation of a discrete-time sinusoidal signal is shown in Figure 1.11.

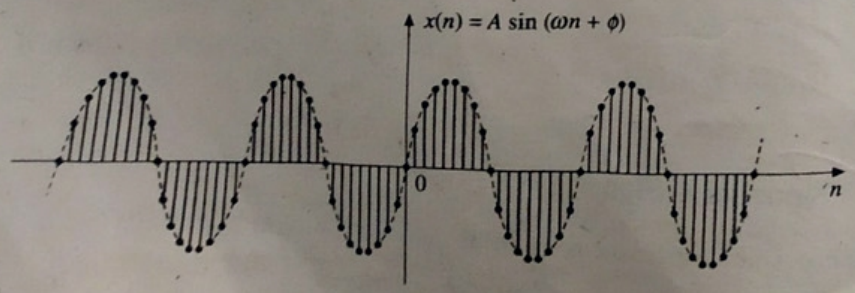


**Figure 1.11** Discrete-time sinusoidal signal.



### 1.3.6 Real Exponential Signal

A continuous-time real exponential signal has the general form as:

$$x(t) = Ae^{\alpha t}$$

where both $A$ and $\alpha$ are real.

The parameter $A$ is the amplitude of the exponential measured at $t = 0$. The parameter $\alpha$ can be either positive or negative. Depending on the value of $\alpha$, we get different exponentials.

1. If $\alpha = 0$, the signal $x(t)$ is of constant amplitude for all times.
2. If $\alpha$ is positive, i.e. $\alpha > 0$, the signal $x(t)$ is a growing exponential signal.
3. If $\alpha$ is negative, i.e. $\alpha < 0$, the signal $x(t)$ is a decaying exponential signal.

These three waveforms are shown in Figure 1.12[(a), (b) and (c)].

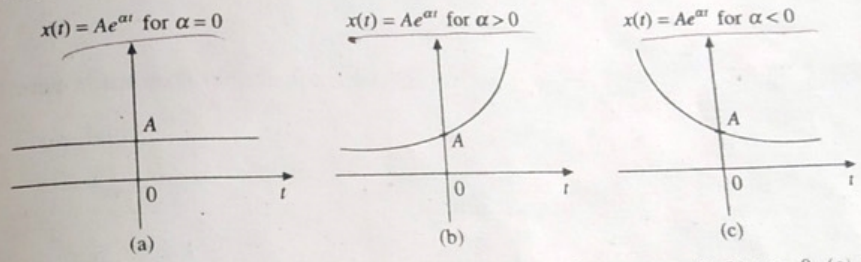


**Figure 1.12** Continuous-time real exponential signals $x(t) = Ae^{\alpha t}$ for (a) $\alpha = 0$, (b) $\alpha > 0$, (c) $\alpha < 0$.

The discrete-time real exponential sequence $a^n$ is defined as:

$$x(n) = a^n \quad \text{for all } n$$

Figure 1.13 illustrates different types of discrete-time exponential signals.

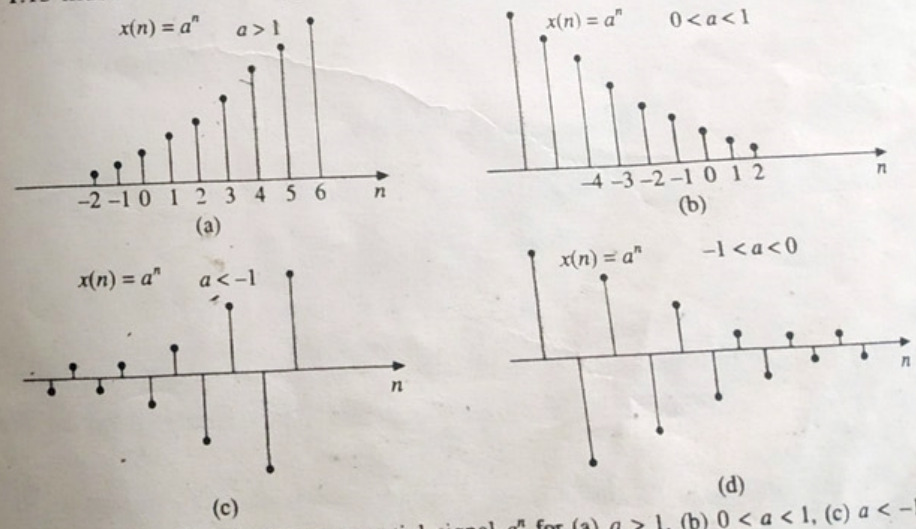


**Figure 1.13** Discrete-time exponential signal $a^n$ for (a) $a > 1$, (b) $0 < a < 1$, (c) $a < -1$, (d) $-1 < a < 0$.

When $a > 1$, the sequence grows exponentially as shown in Figure 1.13(a).
When $0 < a < 1$, the sequence decays exponentially as shown in Figure 1.13(b).
When $a < 0$, the sequence takes alternating signs as shown in Figure 1.13[(c) and (d)].

### 1.3.7 Complex Exponential Signal

The complex exponential signal has a general form as

$$x(t) = Ae^{st}$$

where $A$ is the amplitude and $s$ is a complex variable defined as

$$s = \sigma + j\omega$$

Therefore,

$$x(t) = Ae^{st} = Ae^{(\sigma + j\omega)t} = Ae^{\sigma t}e^{j\omega t}$$

$$= Ae^{\sigma t}[\cos \omega t + j \sin \omega t]$$

Depending on the values of $\sigma$ and $\omega$, we get different waveforms as shown in Figure 1.14.

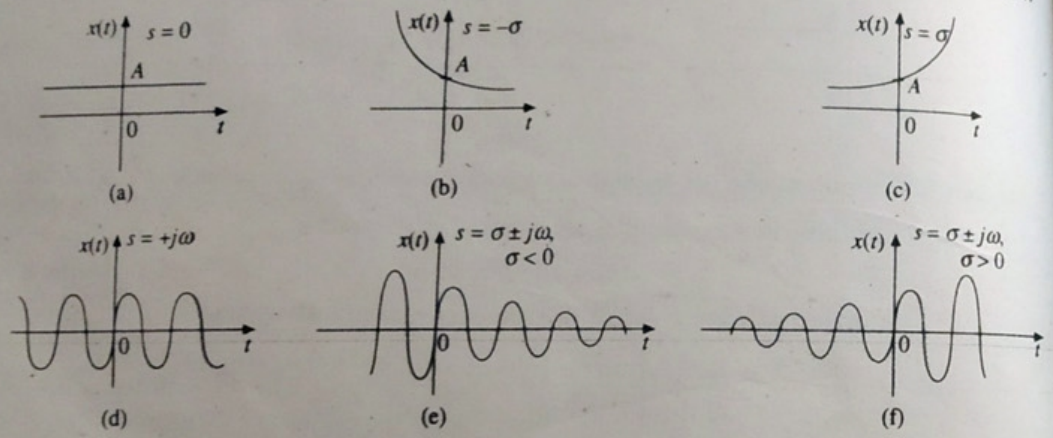


**Figure 1.14** Complex exponential signals.

The discrete-time complex exponential sequence is defined as

$$x(n) = a^n e^{j(\omega_0 n + \phi)}$$

$$= a^n \cos(\omega_0 n + \phi) + ja^n \sin(\omega_0 n + \phi)$$

For $|a| = 1$, the real and imaginary parts of complex exponential sequence are sinusoidal.

For $|a| > 1$, the amplitude of the sinusoidal sequence exponentially grows as shown in Figure 1.15(a).

For $|a| < 1$, the amplitude of the sinusoidal sequence exponentially decays as shown in Figure 1.15(b)



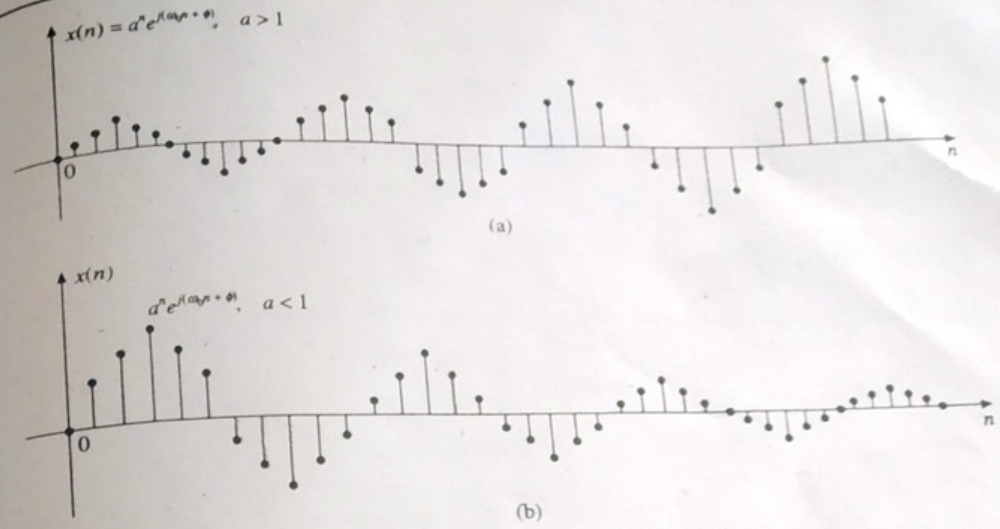


**Figure 1.15** Complex exponential sequence $x(n) = a^n e^{j(\omega_0 n + \phi)}$ for (a) $a > 1$, (b) $a < 1$.

### 1.3.8 Rectangular Pulse Function

The unit rectangular pulse function $\Pi(t/\tau)$ shown in Figure 1.16 is defined as

$$\Pi\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \text{for } |t| \le \dfrac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

It is an even function of $t$.

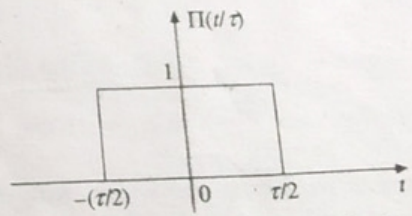


**Figure 1.16** Rectangular pulse function.

### 1.3.9 Triangular Pulse Function

The unit triangular pulse function $\Delta(t/\tau)$ shown in Figure 1.17 is defined as:

$$\Delta\left(\frac{t}{\tau}\right) = \begin{cases} 1 - (2|t|/\tau) & \text{for } |t| < (\tau/2) \\ 0 & \text{for } |t| > (\tau/2) \end{cases}$$

It is an even function of $t$.

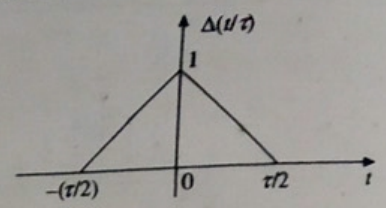


**Figure 1.17** Triangular pulse function.

### 1.3.10 Signum Function

The unit signum function sgn $(t)$ shown in Figure 1.18 is defined as:

$$\text{sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$$

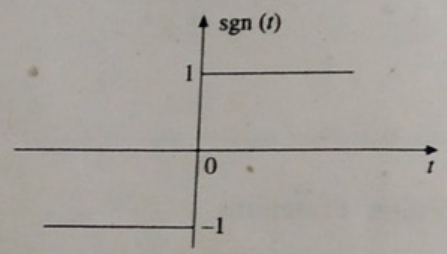


**Figure 1.18** Signum function.

The signum function can be expressed in terms of unit step function as:

$$\text{sgn}(t) = -1 + 2u(t)$$

### 1.3.11 Sinc Function

The sinc function sinc $(t)$ shown in Figure 1.19 is defined as:

$$\text{sinc}(t) = \frac{\sin t}{t} \quad \text{for } -\infty < t < \infty$$

The sinc function oscillates with period $2\pi$ and decays with increasing $t$. Its value is zero at $n\pi$, $n = \pm 1, \pm 2, \ldots$. It is an even function of $t$.

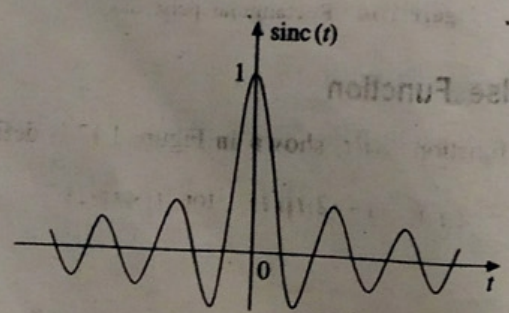


**Figure 1.19** Sinc function.



### 1.3.12 Gaussian Function

The Gaussian function $g_a(t)$ shown in Figure 1.20 is defined as:

$$g_a(t) = e^{-at^2} \quad \text{for } -\infty < t < \infty$$

This function is extremely useful in probability theory.

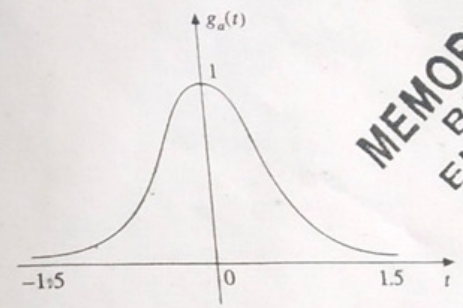


**Figure 1.20** Gaussian function.

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**EXAMPLE 1.1** Evaluate the following integrals:

(a) $\int_{-\infty}^{\infty} e^{-at^2} \delta(t-5)\, dt$

(b) $\int_{0}^{\infty} t^2 \delta(t-6)\, dt$

(c) $\int_{0}^{3} \delta(t) \sin 5\pi t\, dt$

(d) $\int_{-\infty}^{\infty} \delta(t+2) e^{-2t}\, dt$

(e) $\int_{-\infty}^{\infty} (t-2)^3 \delta(t-2)\, dt$

(f) $\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt$

(g) $\int_{-\infty}^{\infty} [\delta(t) \cos 2t + \delta(t-2) \sin 2t]\, dt$

***Solution:***

(a) Given $\int_{-\infty}^{\infty} e^{-at^2} \delta(t-5)\, dt$

We know that $\delta(t-5) = \begin{cases} 1 & \text{for } t = 5 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \quad \int_{-\infty}^{\infty} e^{-at^2} \delta(t-5)\, dt = \left[ e^{-at^2} \right]_{t=5} = e^{-25a}$$

(b) Given $$\int_0^{\infty} t^2 \delta(t-6)\, dt$$

We know that $$\delta(t-6) = \begin{cases} 1 & \text{for } t = 6 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \int_0^{\infty} t^2 \delta(t-6)\, dt = [t^2]_{t=6} = 36$$

(c) Given $$\int_0^{3} \delta(t) \sin 5\pi t\, dt$$

We know that $$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \int_0^{3} \delta(t) \sin 5\pi t\, dt = [\sin 5\pi t]_{t=0} = 0$$

(d) Given $$\int_{-\infty}^{\infty} \delta(t+2) e^{-2t}\, dt$$

We know that $$\delta(t+2) = \begin{cases} 1 & \text{for } t = -2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \int_{-\infty}^{\infty} \delta(t+2) e^{-2t}\, dt = [e^{-2t}]_{t=-2} = e^4$$

(e) Given $$\int_{-\infty}^{\infty} (t-2)^3 \delta(t-2)\, dt$$

We know that $$\delta(t-2) = \begin{cases} 1 & \text{for } t = 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \int_{-\infty}^{\infty} (t-2)^3 \delta(t-2)\, dt = [(t-2)^3]_{t=2} = 0$$

(f) Given $$\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt$$

We know that $$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{elsewhere} \end{cases}$$



$$\therefore \quad \int_{-\infty}^{\infty} \delta(t) e^{-j\omega t}\, dt = [e^{-j\omega t}]_{t=0} = 1$$

(g) Given $$\int_{-\infty}^{\infty} [\delta(t) \cos 2t + \delta(t-2) \sin 2t]\, dt$$

We know that $\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{elsewhere} \end{cases}$ and $\delta(t-2) = \begin{cases} 1 & \text{for } t = 2 \\ 0 & \text{elsewhere} \end{cases}$

$$\therefore \quad \int_{-\infty}^{\infty} [\delta(t) \cos 2t + \delta(t-2) \sin 2t]\, dt = [\cos 2t]_{t=0} + [\sin 2t]_{t=2} = 1 + \sin 4t$$

**EXAMPLE 1.2** Find the following summations

(a) $\sum_{n=-\infty}^{\infty} e^{3n} \delta(n-3)$

(b) $\sum_{n=-\infty}^{\infty} \delta(n-2) \cos 3n$

(c) $\sum_{n=-\infty}^{\infty} n^2 \delta(n+4)$

(d) $\sum_{n=-\infty}^{\infty} \delta(n-2) e^{n^2}$

(e) $\sum_{n=0}^{\infty} \delta(n+1)\, 4^n$

***Solution:***

(a) Given $$\sum_{n=-\infty}^{\infty} e^{3n} \delta(n-3)$$

We know that $$\delta(n-3) = \begin{cases} 1 & \text{for } n = 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \sum_{n=-\infty}^{\infty} e^{3n} \delta(n-3) = [e^{3n}]_{n=3} = e^9$$

(b) Given $$\sum_{n=-\infty}^{\infty} \delta(n-2) \cos 3n$$

We know that $$\delta(n-2) = \begin{cases} 1 & \text{for } n = 2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \sum_{n=-\infty}^{\infty} \delta(n-2) \cos 3n = [\cos 3n]_{n=2} = \cos 6$$

(c) Given $$\sum_{n=-\infty}^{\infty} n^2 \delta(n+4)$$

We know that $$\delta(n+4) = \begin{cases} 1 & \text{for } n=-4 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \sum_{n=-\infty}^{\infty} n^2 \delta(n+4) = [n^2]_{n=-4} = 16$$

(d) Given $$\sum_{n=-\infty}^{\infty} \delta(n-2)\, e^{n^2}$$

We know that $$\delta(n-2) = \begin{cases} 1 & \text{for } n=2 \\ 0 & \text{elsewhere} \end{cases}$$

$$\therefore \quad \sum_{n=-\infty}^{\infty} \delta(n-2)\, e^{n^2} = [e^{n^2}]_{n=2} = e^{2^2} = e^4$$

(e) Given $$\sum_{n=0}^{\infty} \delta(n+1)\, 4^n$$

We know that $$\delta(n+1) = \begin{cases} 1 & \text{for } n=-1 \\ 0 & \text{for } n \neq -1 \end{cases}$$

$$\therefore \quad \sum_{n=0}^{\infty} \delta(n+1)\, 4^n = 0$$

**EXAMPLE 1.3** Prove the properties of impulse function.

***Solution:*** First property $$\int_{-\infty}^{\infty} x(t)\, \delta(t)\, dt = x(0)$$

*Proof:* Let $x(t)$ be continuous at $t = 0$; the value of $x(t)$ at $t = 0$ is $x(0)$. The impulse $\delta(t)$ exists only at $t = 0$. For all other time $\delta(t) = 0$. Therefore, the integration of $x(t)\, \delta(t)$ from $t = -\infty$ to $\infty$ has a value only at $t = 0$.

$$\therefore \quad \int_{-\infty}^{\infty} x(t)\, \delta(t)\, dt = \int_{-\infty}^{\infty} x(0)\, \delta(t)\, dt = x(0) \int_{-\infty}^{\infty} \delta(t)\, dt = x(0)$$

Second property $$x(t)\, \delta(t-t_0) = x(t_0)\, \delta(t-t_0)$$

*Proof:* Let the signal $x(t)$ be continuous at $t = t_0$. The value of $x(t)$ at $t = t_0$ is $x(t_0)$. $\delta(t - t_0)$ is an impulse function that exists only at $t = t_0$. Therefore,

$$x(t)\, \delta(t-t_0) = x(t_0)\, \delta(t-t_0)$$

Third property $$\int_{-\infty}^{\infty} x(t)\, \delta(t-t_0)\, dt = x(t_0)$$



*Proof:* Let $x(t)$ be continuous at $t = t_0$ and let its value at $t = t_0$ be $x(t_0)$. We know that $\delta(t - t_0) = 1$ only for $t = t_0$. For all other time it is equal to zero. Therefore, the integration of the product term $x(t)\, \delta(t - t_0)$ from $-\infty$ to $\infty$ has a value only at $t = t_0$.

$$\therefore \quad \int_{-\infty}^{\infty} x(t)\, \delta(t-t_0)\, dt = \int_{-\infty}^{\infty} x(t_0)\, \delta(t-t_0)\, dt = x(t_0) \int_{-\infty}^{\infty} \delta(t-t_0)\, dt = x(t_0)$$

Fourth property $$\int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau = x(t)$$

*Proof:* Let $x(\tau)$ be continuous at $\tau = t$. Let the value of $x(\tau)$ at $\tau = t$ be $x(t)$. We know that $\delta(t - \tau) = 1$ only at $\tau = t$. For all other $\tau$ it is zero. So the integration of the product $x(\tau)\, \delta(t - \tau)$ has a value only at $\tau = t$.

$$\therefore \quad \int_{-\infty}^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau = x(\tau)\big|_{\tau=t} = x(t)$$

This is the formula for convolution of $x(t)$ with $\delta(t)$. This property says that the convolution of any signal with an impulse results in the original signal itself.

Fifth property $$\delta(at) = \frac{1}{|a|}\, \delta(t) \qquad \text{[Scaling property]}$$

*Proof:* Let $x(t)$ be some function. Consider the integral

$$\int_{-\infty}^{\infty} x(t)\, \delta(at)\, dt \quad \text{for } a > 0$$

Let $$at = \tau$$

$$\therefore \quad t = \frac{\tau}{a} \quad \text{and} \quad dt = \frac{d\tau}{a}$$

If $a > 0$ $$\int_{-\infty}^{\infty} x(t)\, \delta(at)\, dt = \frac{1}{a} \int_{-\infty}^{\infty} x\left(\frac{\tau}{a}\right) \delta(\tau)\, d\tau$$

$$= \left[\frac{1}{a}\, x\left(\frac{\tau}{a}\right)\right]_{\tau=0}$$

$$= \frac{1}{a}\, x(0)$$

Similarly, for $a < 0$ $$\int_{-\infty}^{\infty} x(t)\, \delta(at)\, dt = \frac{1}{-a}\, x(0)$$

$$\therefore \quad \int_{-\infty}^{\infty} x(t)\, \delta(at)\, dt = \frac{1}{|a|}\, x(0)$$

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Now, consider $$\frac{1}{|a|}x(0)$$

We know that $$x(0) = \int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt$$

$$\therefore \quad \frac{1}{|a|}x(0) = \frac{1}{|a|}\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt$$

$$= \int_{-\infty}^{\infty} x(t)\,\frac{1}{|a|}\,\delta(t)\,dt$$

which indicates that $$\delta(at) = \frac{1}{|a|}\,\delta(t)$$

Sixth property $$\delta(t) = \delta(-t)$$

i.e. impulse function is an even function.

*Proof:* Consider the scaling property,

$$\delta(at) = \frac{1}{|a|}\,\delta(t)$$

Let $$a = -1$$

$$\therefore \quad \delta(-t) = \frac{1}{|-1|}\,\delta(t) = \delta(t)$$

## 1.4 BASIC OPERATIONS ON SIGNALS

When we process a signal, this signal may undergo several manipulations involving the independent variable or the amplitude of the signal. The basic operations on signals are as follows:

1. Time shifting
2. Time reversal
3. Time scaling
4. Amplitude scaling
5. Signal addition
6. Signal multiplication

The first three operations correspond to transformation in independent variable $t$ or $n$ of a signal. The last three operations correspond to transformation on amplitude of a signal.

### 1.4.1 Time Shifting

Mathematically, the time shifting of a continuous-time signal $x(t)$ can be represented by

$$y(t) = x(t - T)$$

The time shifting of a signal may result in time delay or time advance. In the above equation if $T$ is positive the shift is to the right and then the shifting delays the signal, and if $T$ is negative the shift is to the left and then the shifting advances the signal. An arbitrary



signal $x(t)$, its delayed version and advanced version are shown in Figure 1.21[(a), (b) and (c)]. Shifting a signal in time means that a signal may be either advanced in the time axis or delayed in the time axis.

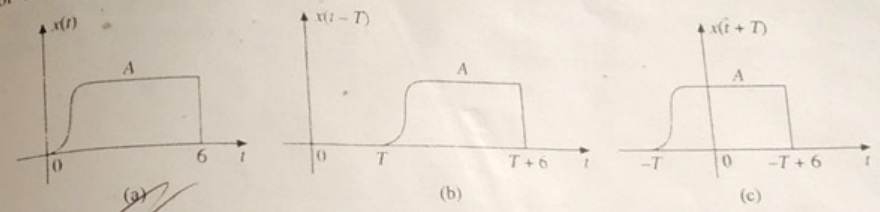


Figure 1.21 (a) Signal, (b) Its delayed version, (c) Its time advanced version.

Similarly, the time shifting operation of a discrete-time signal $x(n)$ can be represented by

$$y(n) = x(n - k)$$

This shows that the signal $y(n)$ can be obtained by time shifting the signal $x(n)$ by $k$ units. If $k$ is positive, it is delay and the shift is to the right, and if $k$ is negative, it is advance and the shift is to the left.

An arbitrary signal $x(n)$ is shown in Figure 1.22(a). $x(n - 3)$ which is obtained by shifting $x(n)$ to the right by 3 units [i.e. delay $x(n)$ by 3 units] is shown in Figure 1.22(b). $x(n + 2)$ which is obtained by shifting $x(n)$ to the left by 2 units (i.e. advancing $x(n)$ by 2 units) is shown in Figure 1.22(c).

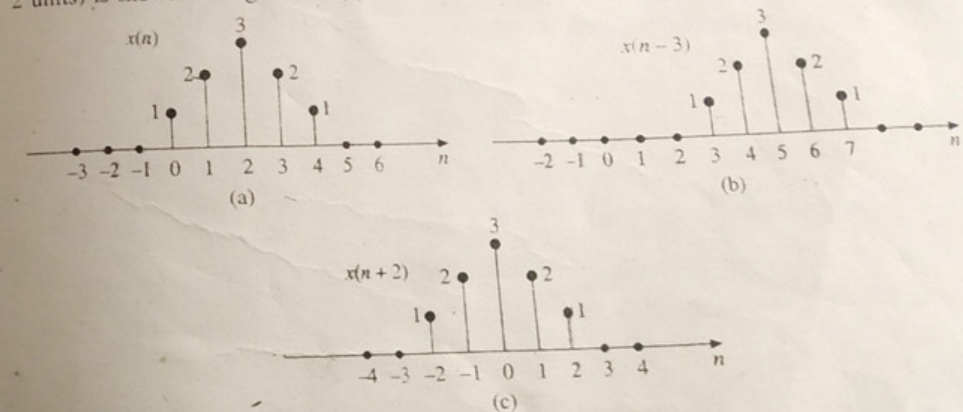


Figure 1.22 (a) Sequence $x(n)$, (b) $x(n - 3)$, (c) $x(n + 2)$.

### 1.4.2 Time Reversal

The time reversal, also called time folding of a signal $x(t)$ can be obtained by folding the signal about $t = 0$. This operation is very useful in convolution. It is denoted by $x(-t)$. It is obtained by replacing the independent variable $t$ by $(-t)$. Folding is also called as the

reflection of the signal about the time origin $t = 0$. Figure 1.23(a) shows an arbitrary signal $x(t)$, and Figure 1.23(b) shows its reflection $x(-t)$.

The signal $x(-t + 3)$ obtained by shifting the reversed signal $x(-t)$ to the right by 3 units (delay by 3 units) is shown in Figure 1.23(c). The signal $x(-t - 3)$ obtained by shifting the reversed signal $x(-t)$ to the left by 3 units (advance by 3 units) is shown in Figure 1.23(d).

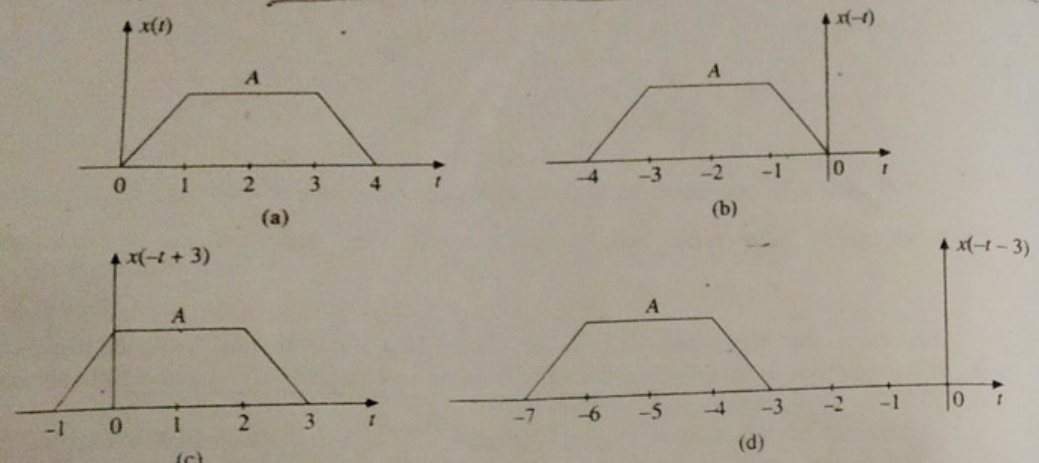


**Figure 1.23** (a) An arbitrary signal $x(t)$, (b) Time reversed signal $x(-t)$, (c) Time reversed and delayed signal $x(-t + 3)$, (d) Time reversed and advanced signal $x(-t - 3)$.

Other examples for time reversal operation are shown in Figure 1.24.

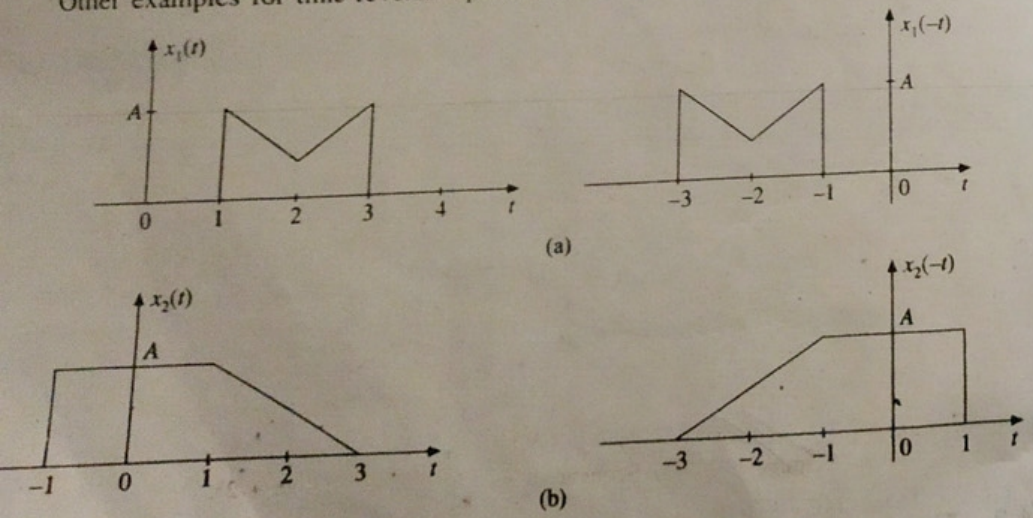


**Figure 1.24** Time reversal operations.

The time reversal of a discrete-time signal $x(n)$ can be obtained by folding the sequence $n = 0$. Figure 1.25(a) shows an arbitrary discrete-time signal $x(n)$ and its time reversed $x(-n)$ is shown in Figure 1.25(b). Figure 1.25[(c) and (d)] shows the delayed and ed versions of reversed signal $x(-n)$.



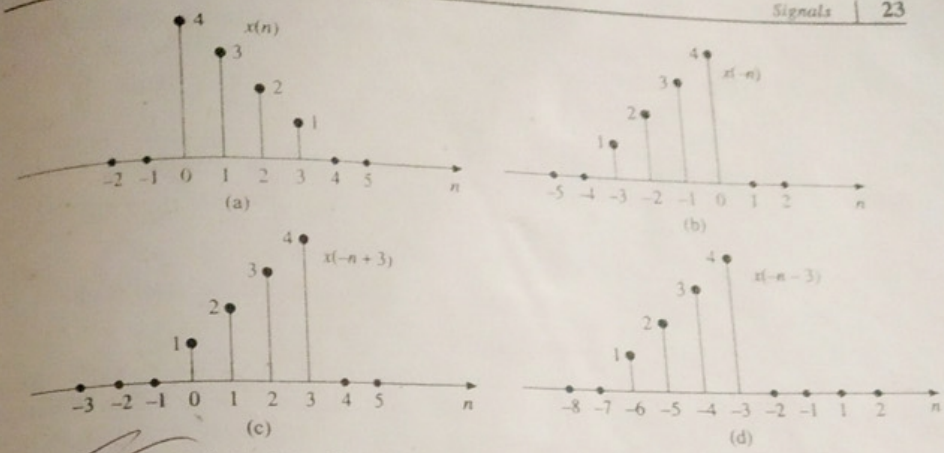


**Figure 1.25** (a) Original signal $x(n)$, (b) Time reversed signal $x(-n)$, (c) Time reversed and delayed signal $x(-n + 3)$, (d) Time reversed and advanced signal $x(-n - 3)$.

The signal $x(-n + 3)$ is obtained by delaying (shifting to the right) the time reversed signal $x(-n)$ by 3 units of time. The signal $x(-n - 3)$ is obtained by advancing (shifting to the left) the time reversed signal $x(-n)$ by 3 units of time.

Figure 1.26 shows other examples for time reversal of signals.

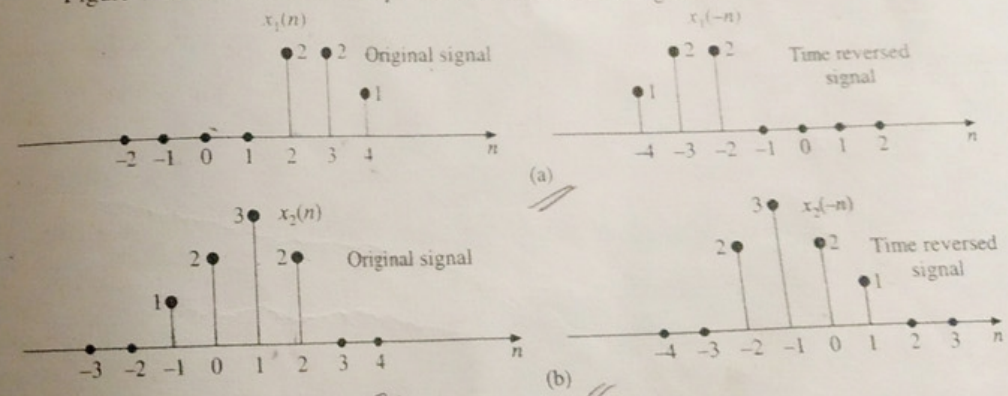


**Figure 1.26** Time reversal operations.

**EXAMPLE 1.4** Sketch the following signals

(a) $u(-t + 2)$ (b) $-2u(t + 2)$
(c) $-4r(t)$ (d) $2r(t - 2)$
(e) $r(-t + 3)$ (f) $\Pi(t - 2)$

***Solution:***

(a) Given $x(t) = u(-t + 2)$

The signal $u(-t + 2)$ can be obtained by first drawing the unit step signal $u(t)$ as shown in Figure 1.27(a), then time reversing the signal $u(t)$ about $t = 0$ to obtain

$u(-t)$ as shown in Figure 1.27(b), and then shifting the reversed signal to the right by 2 units of time to obtain $u(-t+2)$ as shown in Figure 1.27(c).

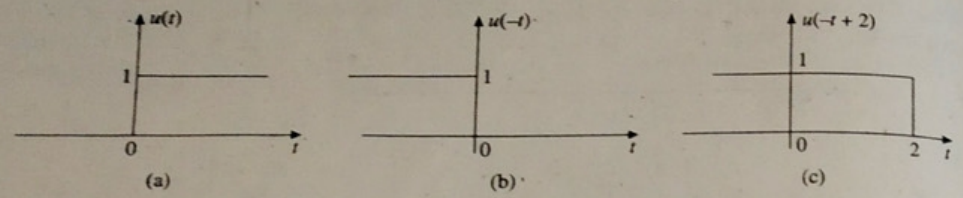


**Figure 1.27** (a) Unit step signal, (b) Folded unit step signal, (c) Delayed folded signal.

(b) Given $\qquad x(t) = -2u(t+2)$

The signal $-2u(t+2)$ can be obtained by first drawing the unit step signal $u(t)$ as shown in Figure 1.28(a), then shifting the signal $u(t)$ to the left by 2 units of time to obtain $u(t+2)$ as shown in Figure 1.28(b), and then multiplying that signal $u(t+2)$ by $-2$ to obtain $-2u(t+2)$ as shown in Figure 1.28(c).

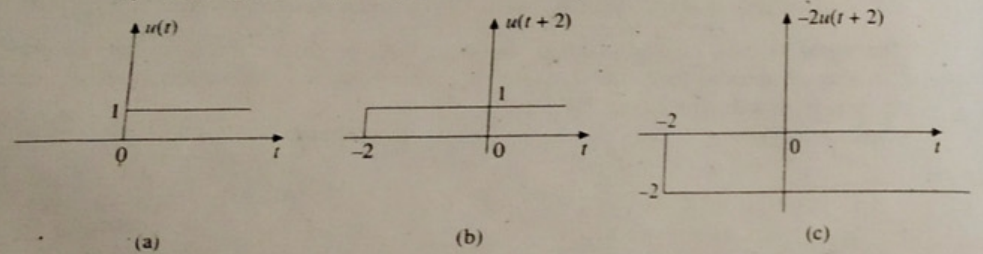


**Figure 1.28** (a) Unit step signal $u(t)$, (b) Shifted signal $u(t+2)$, (c) Scaled signal $-2u(t+2)$.

(c) Given $\qquad x(t) = -4r(t)$

The signal $x(t)$ is a ramp signal with a slope of $-4$ as shown in Figure 1.29.

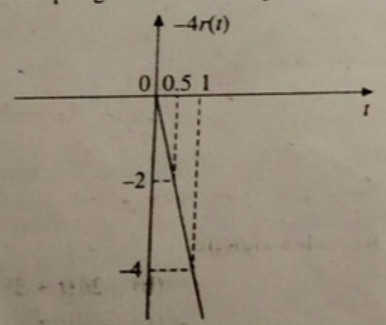


**Figure 1.29** Ramp signal $x(t) = -4r(t)$.

(d) Given $\qquad x(t) = 2r(t-2)$

The signal $2r(t-2)$ can be obtained by first drawing the ramp signal $2r(t)$ with slope of 2 as shown in Figure 1.30(a), and then shifting it to the right by 2 units to obtain $2r(t-2)$ as shown in Figure 1.30(b).



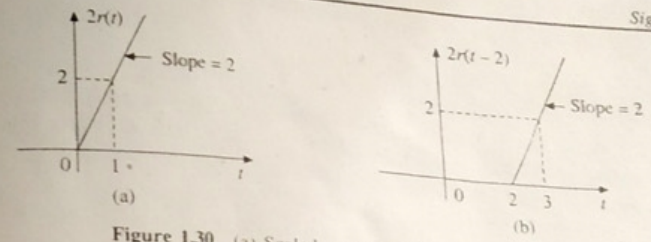


**Figure 1.30** (a) Scaled ramp, (b) Delayed scaled ramp.

(e) Given $\qquad x(t) = r(-t+3)$

The signal $r(-t+3)$ can be obtained by first drawing the unit ramp signal $r(t)$ as shown in Figure 1.31(a), folding the signal $r(t)$ about $t = 0$ to obtain $r(-t)$ as shown in Figure 1.31(b) and then shifting it to the right (delaying) by 3 units of time to obtain $r(-t+3)$ as shown in Figure 1.31(c).

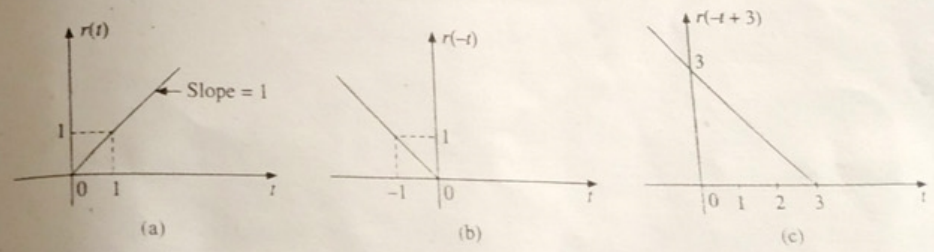


**Figure 1.31** (a) Ramp signal, (b) Folded ramp, (c) Delayed folded ramp.

(f) Given $\qquad x(t) = \Pi(t-2)$

The signal $\Pi(t-2)$ can be obtained by first drawing $\Pi(t)$ as shown in Figure 1.32(a) and then shifting it to the right by 2 units to obtain $\Pi(t-2)$ as shown in Figure 1.32(b).

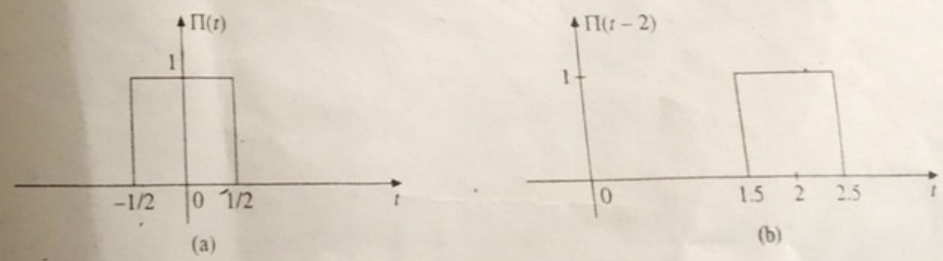


**Figure 1.32** (a) Signal $\Pi(t)$, (b) Signal $\Pi(t-2)$.

**EXAMPLE 1.5** Sketch the following signals

(a) $u(n+2)\,u(-n+3)$ $\qquad$ (b) $x(n) = u(n+4) - u(n-2)$

***Solution:***

(a) Given $\qquad x(n) = u(n+2)\,u(-n+3)$

The signal $u(n + 2)\,u(-n + 3)$ can be obtained by first drawing the signal $u(n + 2)$ as shown in Figure 1.33(a), then drawing $u(-n + 3)$ as shown in Figure 1.33(b) and then multiplying these sequences element by element to obtain $u(n + 2)\,u(-n + 3)$ as shown in Figure 1.33(c).

$x(n) = 0$ for $n < -2$ and $n > 3$; $x(n) = 1$ for $-2 < n < 3$

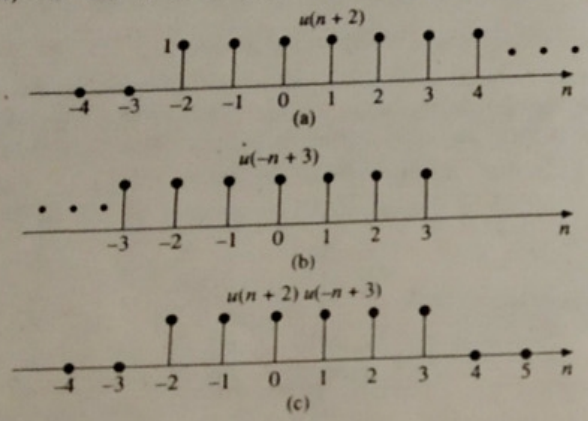


**Figure 1.33** Plots of (a) $u(n + 2)$, (b) $u(-n + 3)$, (c) $u(n + 2)\,u(-n + 3)$.

(b) Given $\qquad x(n) = u(n + 4) - u(n - 2)$

The signal $u(n + 4) - u(n - 2)$ can be obtained by first plotting $u(n + 4)$ as shown in Figure 1.34(a), then plotting $u(n - 2)$ as shown in Figure 1.34(b) and then subtracting each element of $u(n - 2)$ from the corresponding element of $u(n + 4)$ to obtain the result shown in Figure 1.34(c).

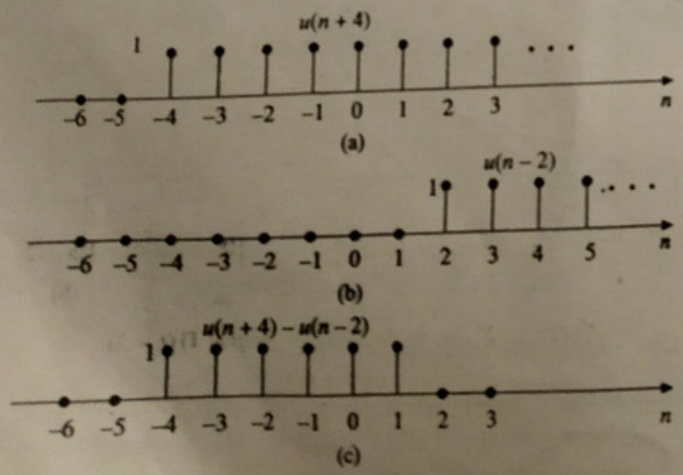


**Figure 1.34** Plots of (a) $u(n + 4)$, (b) $u(n - 2)$, (c) $u(n + 4) - u(n - 2)$.



## 1.4.3 Amplitude Scaling

The amplitude scaling of a continuous-time signal $x(t)$ can be represented by

$$y(t) = Ax(t)$$

where $A$ is a constant.

The amplitude of $y(t)$ at any instant is equal to $A$ times the amplitude of $x(t)$ at that instant, but the shape of $y(t)$ is same as the shape of $x(t)$. If $A > 1$, it is amplification and if $A < 1$, it is attenuation.

Here the amplitude is rescaled. Hence the name amplitude scaling. Figure 1.35(a) shows an arbitrary signal $x(t)$ and Figure 1.35(b) shows $y(t) = 2x(t)$.

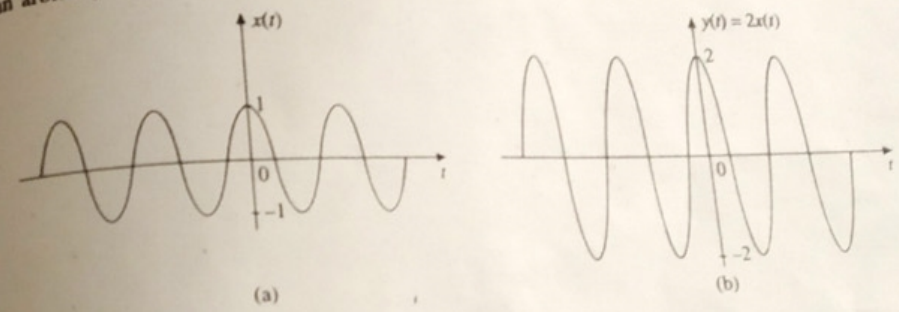


**Figure 1.35** Plots of (a) $x(t) = \cos \omega t$, (b) $y(t) = 2x(t)$.

Similarly, the amplitude scaling of a discrete-time signal can be represented by

$$y(n) = ax(n)$$

where $a$ is a constant.

Figure 1.36(a) shows a signal $x(n)$ and Figure 1.36(b) shows a scaled signal $y(n) = 2x(n)$.

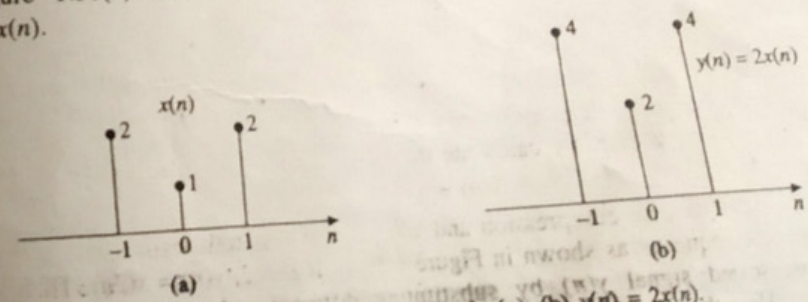


**Figure 1.36** Plots of (a) Signal $x(n)$, (b) $y(n) = 2x(n)$.

## 1.4.4 Time Scaling

Time scaling may be time expansion or time compression. The time scaling of a signal $x(t)$ can be accomplished by replacing $t$ by $at$ in it. Mathematically, it can be expressed as:

$$y(t) = x(at)$$

If $a > 1$, it results in time compression by a factor $a$ and if $a < 1$, it results in time expansion by a factor $a$ because with that transformation a point at '$at$' in signal $x(t)$ becomes a point at '$t$' in $y(t)$.

Consider a signal shown in Figure 1.37(a). For a transformation $y(t) = x(2t)$, the time compressed signal is as shown in Figure 1.37(b) and for a transformation $y(t) = x(t/2)$ the time expanded signal is as shown in Figure 1.37(c).

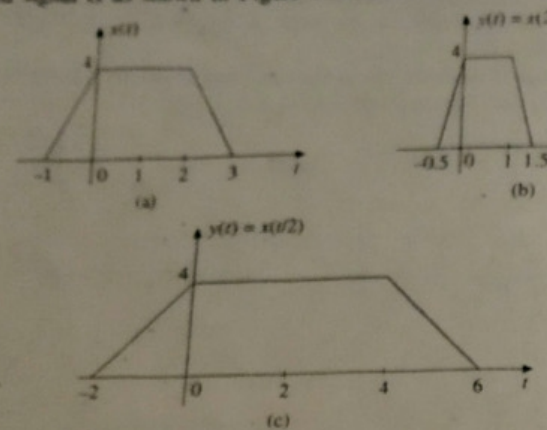


**Figure 1.37** (a) Original signal. (b) Compressed signal. (c) Expanded signal.

Observe that in Figure 1.37(a), $x(t)$ is increasing linearly from 0 to 4 in the interval $t = -1$ to $t = 0$ and remaining constant at 4 in the interval $t = 0$ to $t = 2$ and decreasing linearly from 4 to 0 in the interval $t = 2$ to $t = 3$.

In Figure 1.37(b), the time scaled (compressed) signal $x(2t)$ increases linearly from 0 to 4 in the interval $t = -(1/2)$ to $t = (0/2)$, remains constant at 4 from $t = (0/2)$ to $t = (2/2)$ and then decreases linearly from 4 to 0 in the interval $t = (2/2)$ to $t = (3/2)$.

In Figure 1.37(c), the time scaled (expanded) signal $x(t/2)$ increases linearly from 0 to 4, in the interval $t = -1 \times 2$ to $t = 0 \times 2$, remains constant at 4 in the interval $t = 0 \times 2$ to $t = 2 \times 2$ and then decreases linearly from 4 to 0 in the interval $t = 2 \times 2$ to $t = 2 \times 3$.

In the discrete-time case, we can write the time scaling as follows.

$$y(n) = x(an)$$

again when $a > 1$, it is time compression and when $a < 1$, it is time expansion.

Let $x(n)$ be a sequence as shown in Figure 1.38(a). If $a = 2$, $y(n) = x(2n)$. Then we can plot the time scaled signal $y(n)$ by substituting different values for $n$ as shown in Figure 1.38(b). Here the signal is compressed by 2.

$$y(0) = x(0) = 1$$
$$y(-1) = x(-2) = 3$$
$$y(-2) = x(-4) = 0$$
$$y(1) = x(2) = 3$$
$$y(2) = x(4) = 0$$

and so on.



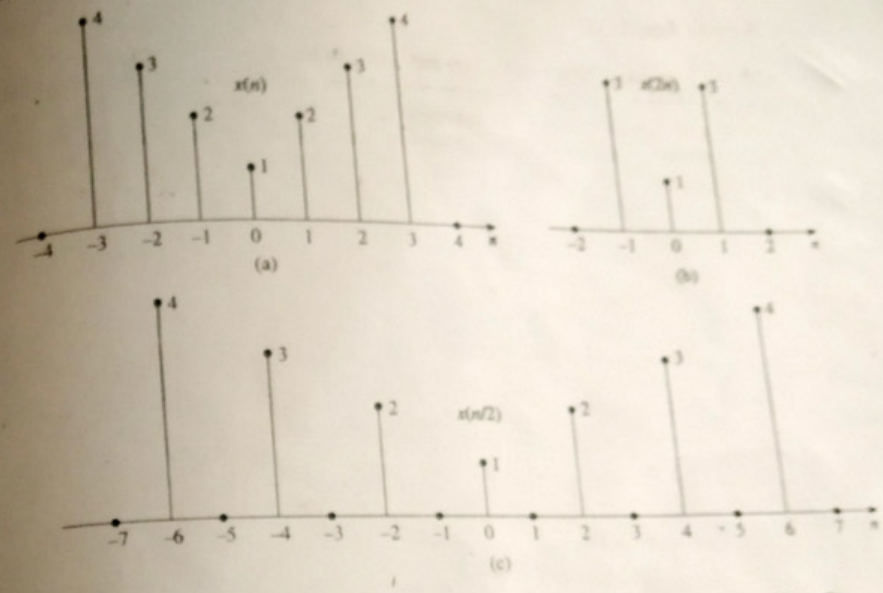


**Figure 1.38** Discrete-time scaling (a) Plot of $x(n)$, (b) Plot of $x(2n)$, (c) Plot of $x(n/2)$.

So to plot $x(2n)$ we have to skip odd numbered samples in $x(n)$.

If $a = (1/2)$, $y(n) = x(n/2)$, then

$$y(0) = x(0) = 1$$
$$y(2) = x(1) = 2$$
$$y(4) = x(2) = 3$$
$$y(6) = x(3) = 4$$
$$y(8) = x(4) = 0$$
$$y(-2) = x(-1) = 2$$
$$y(-4) = x(-2) = 3$$
$$y(-6) = x(-3) = 4$$
$$y(-8) = x(-4) = 0$$

We can plot $y(n) = x(n/2)$ as shown in Figure 1.38(c). Here the signal is expanded by 2. All odd components in $x(n/2)$ are zero because $x(n)$ does not have any value in between the sampling instants.

Time scaling is very useful when data is to be fed at some rate and is to be taken out at a different rate.

### 1.4.5 Signal Addition

The sum of two continuous-time signals $x_1(t)$ and $x_2(t)$ can be obtained by adding their values at every instant of time. Similarly, the subtraction of one continuous-time signal $x_2(t)$ from another signal $x_1(t)$ can be obtained by subtracting the value of $x_2(t)$ from that of $x_1(t)$ at every instant. Consider two signals $x_1(t)$ and $x_2(t)$ shown in Figure 1.39[(a) and (b)].

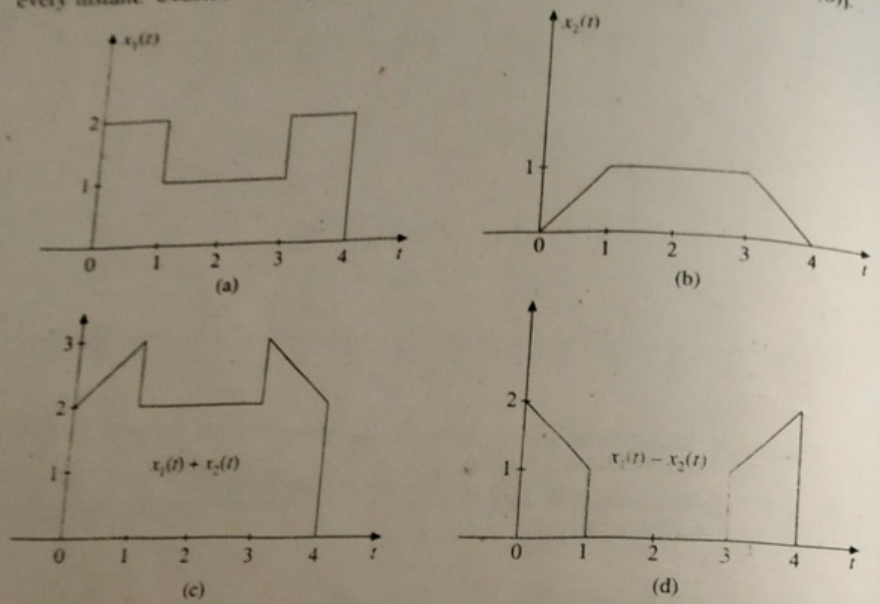


**Figure 1.39** Addition and subtraction of continuous-time signals.

The addition of those two signals $x_1(t)$ and $x_2(t)$ can be obtained by considering different time intervals as follows:

For $0 \le t \le 1$ $x_1(t) = 2$ and $x_2(t)$ is rising linearly from 0 to 1.
Hence $x_1(t) + x_2(t)$ will rise linearly from $2 + 0 = 2$ to $2 + 1 = 3$.

For $1 \le t \le 3$ $x_1(t) = 1$ and $x_2(t) = 1$
Hence $x_1(t) + x_2(t)$ will be equal to $1 + 1 = 2$.

For $3 \le t \le 4$ $x_1(t) = 2$ and $x_2(t)$ falls linearly from 1 to 0.
Hence $x_1(t) + x_2(t)$ will fall linearly from $2 + 1 = 3$ to $2 + 0 = 2$.

The sum $x_1(t) + x_2(t)$ is as shown in Figure 1.39(c).

The subtraction of signal $x_2(t)$ from $x_1(t)$ can be performed by considering different time intervals as follows:

For $0 \le t \le 1$ $x_1(t) = 2$ and $x_2(t)$ rises linearly from 0 to 1.
Hence $x_1(t) - x_2(t)$ falls linearly from $2 - 0 = 2$ to $2 - 1 = 1$.



For $1 \le t \le 3$ $x_1(t) = 1$ and $x_2(t) = 1$
Hence $x_1(t) - x_2(t) = 1 - 1 = 0$

For $3 \le t \le 4$ $x_1(t) = 2$ and $x_2(t)$ is falling linearly from 1 to 0.
Hence $x_1(t) - x_2(t)$ rises linearly, from $2 - 1 = 1$ to $2 - 0 = 2$.

The difference $x_1(t) - x_2(t)$ is as shown in Figure 1.39(d).

In discrete-time domain, the sum of two signals $x_1(n)$ and $x_2(n)$ can be obtained by adding the corresponding sample values and the subtraction of $x_2(n)$ from $x_1(n)$ can be obtained by subtracting each sample of $x_2(n)$ from the corresponding sample of $x_1(n)$ as illustrated below.

If $x_1(n) = \{1, 2, 3, 1, 5\}$

and $x_2(n) = \{2, 3, 4, 1, -2\}$

Then $x_1(n) + x_2(n) = \{1 + 2, 2 + 3, 3 + 4, 1 + 1, 5 - 2\} = \{3, 5, 7, 2, 3\}$

and $x_1(n) - x_2(n) = \{1 - 2, 2 - 3, 3 - 4, 1 - 1, 5 + 2\} = \{-1, -1, -1, 0, 7\}$

### 1.4.6 Signal Multiplication

The multiplication of two continuous-time signals can be performed by multiplying their values at every instant. Two continuous-time signals $x_1(t)$ and $x_2(t)$ shown in Figure 1.40[(a) and (b)] are multiplied as shown below to obtain $x_1(t)\,x_2(t)$ shown in Figure 1.40(c).

For $0 \le t \le 1$ $x_1(t) = 2$ and $x_2(t) = 1$
Hence $x_1(t)\,x_2(t) = 2 \times 1 = 2$

For $1 \le t \le 2$ $x_1(t) = 1$ and $x_2(t) = 1 + (t - 1)$
Hence $x_1(t)\,x_2(t) = (1)[1 + (t - 1)] = 1 + (t - 1)$

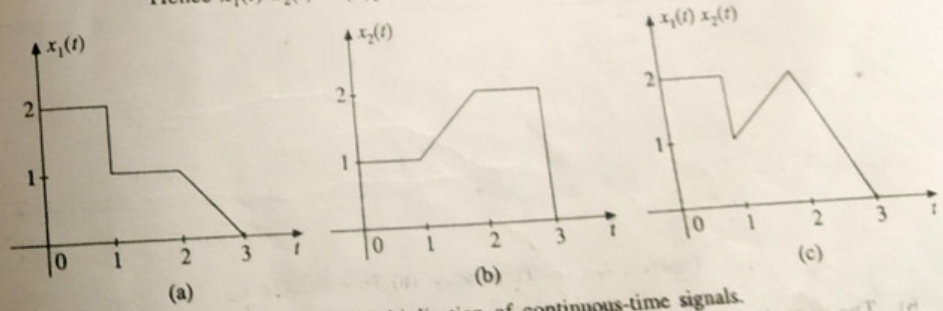


**Figure 1.40** Multiplication of continuous-time signals.

For $2 \le t \le 3$ $x_1(t) = 1 - (t - 2)$ and $x_2(t) = 2$
Hence $x_1(t)\,x_2(t) = [1 - (t - 2)]\,2 = 2 - 2(t - 2)$

Multiplication of two discrete-time sequences can be performed by multiplying their values at the sampling instants.

If $\quad x_1(n) = \{1, -3, 2, 4, 1.5\}$ and $x_2(n) = \{2, -1, 3, 1.5, 2\}$

Then $\quad x_1(n)\, x_2(n) = \{1 \times 2, -3 \times -1, 2 \times 3, 4 \times 1.5, 1.5 \times 2\}$

$$= \{2, 3, 6, 6, 3\}$$

**EXAMPLE 1.6** For the signal $x(t)$ shown in Figure 1.41, find the signals.

(a) $x(t-3)$ and $x(t+3)$

(b) $x(2t+2)$ and $x\left(\frac{1}{2}t - 2\right)$

(c) $x\left(\frac{5}{3}t\right)$ and $x\left(\frac{3}{5}t\right)$

(d) $x(-t+2)$ and $x(-t-2)$

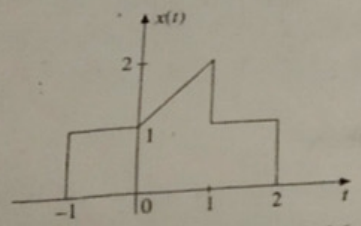


**Figure 1.41** Signal for Example 1.6.

***Solution:***

(a) The signal $x(t-3)$ can be obtained by shifting $x(t)$ to the right (delay) by 3 units as shown in Figure 1.42(a).

The signal $x(t+3)$ can be obtained by shifting $x(t)$ to the left (advance) by 3 units as shown in Figure 1.42(b).

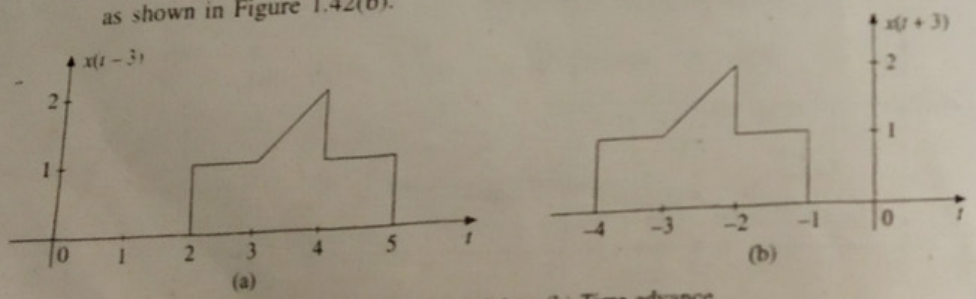


**Figure 1.42** (a) Time delay, (b) Time advance.

(b) The signal $x(2t+2)$ can be obtained by first advancing $x(t)$ by two units to get $x(t+2)$ as shown in Figure 1.43(a) and then time scaling (compression) $x(t+2)$ by a factor of 2 as shown in Figure 1.43(b).

The signal $x\left(\frac{1}{2}t - 2\right)$ can be obtained by first time delaying $x(t)$ by 2 units to obtain $x(t-2)$ as shown in Figure 1.43(c) and then time scaling (expanding) $x(t-2)$ by a factor of (1/2) as shown in Figure 1.43(d).



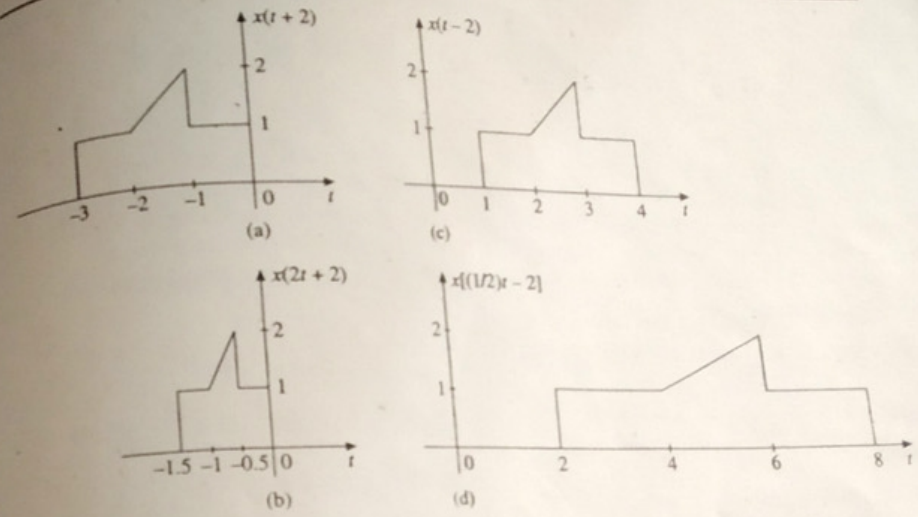


**Figure 1.43** (a) Time advance, (b) Time compression, (c) Time delay, (d) Time expansion.

(c) The signal $x[(5/3)t]$ can be obtained by time scaling $x(t)$ by a factor of (5/3) i.e. by compressing the signal $x(t)$ by (3/5) times as shown in Figure 1.44(a).

The signal $x[(3/5)t]$ can be obtained by time scaling $x(t)$ by a factor of (3/5), i.e. by expanding $x(t)$ by (5/3) times as shown in Figure 1.44(b). The zero point remains as it is because $0 \times a = 0$ itself.

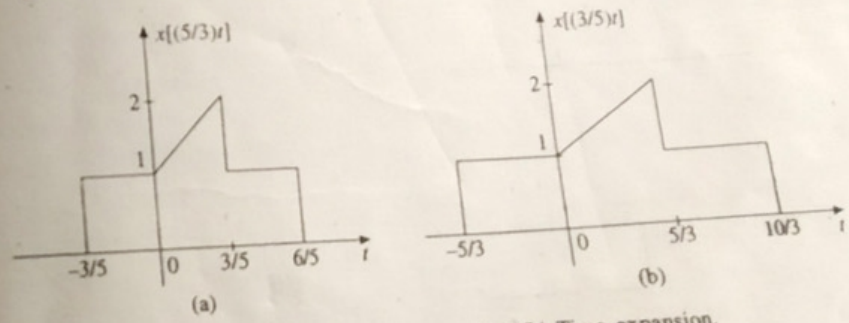


**Figure 1.44** (a) Time compression, (b) Time expansion.

(d) The signal $x(-t+2)$ can be obtained by folding the signal $x(t)$ about $t = 0$ to obtain the time reversed signal $x(-t)$ as shown in Figure 1.45(a) and then shifting $x(-t)$ to the right by 2 units as shown in Figure 1.45(b).

The signal $x(-t-2)$ can be obtained by shifting $x(-t)$ to the left by two units as shown in Figure 1.45(c).

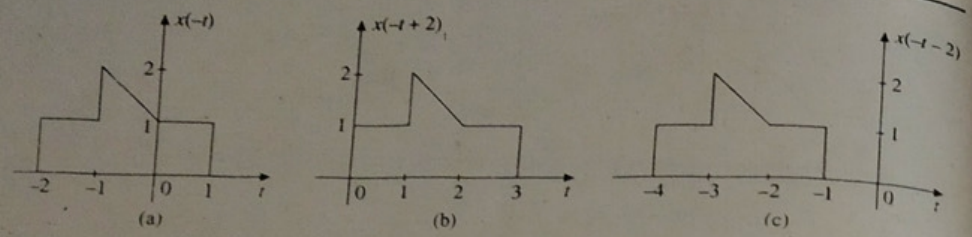


**Figure 1.45** (a) Time reversal, (b) Time delay, and (c) Time advance operations on $x(t)$.

**EXAMPLE 1.7** Sketch the following signals

(a) $2u(t+2) - 2u(t-3)$ (b) $u(t+4)\,u(-t+4)$

(c) $\Pi\left(\frac{t-2}{2}\right) + \Pi(2t-3.5)$. (d) $r(t) - r(t-1) - r(t-3) + r(t-4)$

(e) $r(-t)\,u(t+2)$ (f) $r(-0.25t+1)$

***Solution:***

(a) Given $2u(t+2) - 2u(t-3)$

The signal $u(t)$ is shown in Figure 1.46(a). The signal $2u(t+2)$ is obtained by shifting $u(t)$ to the left by 2 units and multiplying by 2 as shown in Figure 1.46(b).

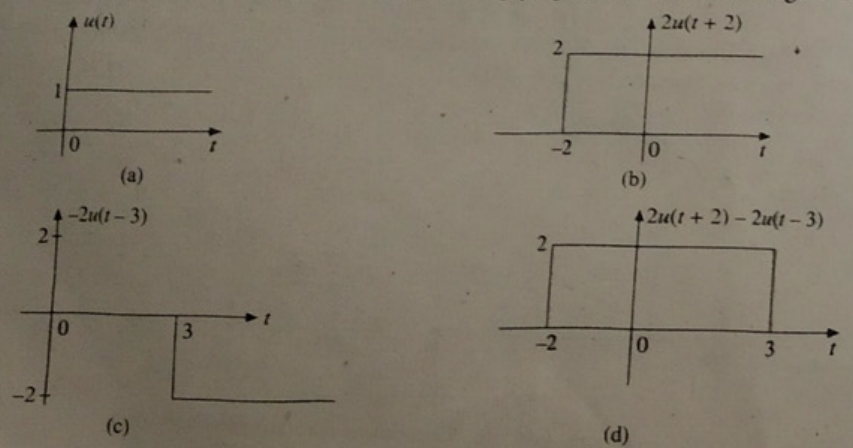


**Figure 1.46** Waveforms for Example 1.7(a).

The signal $-2u(t-3)$ is obtained by shifting $u(t)$ to the right by 3 units and multiplying by $-2$ as shown in Figure 1.46(c). The signal $2u(t+2) - 2u(t-3)$ obtained by adding signals $2u(t+2)$ and $-2u(t-3)$ is shown in Figure 1.46(d).

$$2u(t+2) = 2 \quad \text{for } t \geq -2$$

and

$$-2u(t-3) = -2 \quad \text{for } t \geq 3$$



Therefore, $$2u(t+2) - 2u(t-3) = \begin{cases} 2 & \text{for } -2 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

(b) Given $u(t+4)\,u(-t+4)$

The signal $u(t)$ is shown in Figure 1.47(a). The signal $u(t+4)$ is obtained by shifting $u(t)$ to the left by 4 units as shown in Figure 1.47(b). The signal $u(-t+4)$ is obtained by reversing $u(t)$ to obtain $u(-t)$ and then shifting $u(-t)$ to the right by 4 units as shown in Figure 1.47(c). The signal $u(t+4)\,u(-t+4)$ obtained by multiplying the signals $u(t+4)$ and $u(-t+4)$ is shown in Figure 1.47(d).

$$u(t+4) = \begin{cases} 1 & \text{for } t \geq -4 \\ 0 & \text{elsewhere} \end{cases}$$

and

$$u(-t+4) = \begin{cases} 1 & \text{for } t \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

Therefore, $$u(t+4)\,u(-t+4) = \begin{cases} 1 & \text{for } -4 \leq t \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

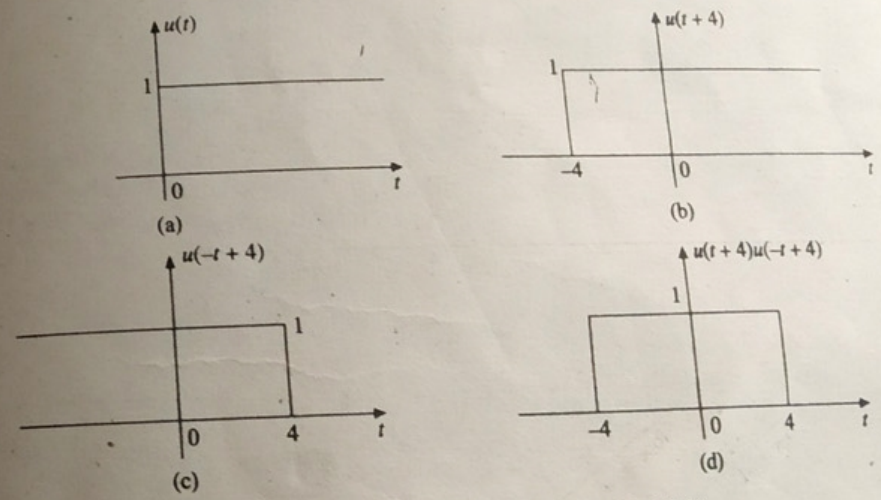


**Figure 1.47** Waveforms for Example 1.7(b).

(c) Given $\Pi\left(\frac{t-2}{2}\right) + \Pi(2t-3.5)$

The signal $\Pi(t)$ is shown in Figure 1.48(a). The signal $\Pi[(t-2)/2] = \Pi(0.5t-1)$ is obtained by first shifting $\Pi(t)$ by 1 unit right and then expanding the time scale by 2 as shown in Figure 1.48(b). The signal $\Pi(2t-3.5)$ is obtained by first shifting $\Pi(t)$ by 3.5 units right and then compressing the time scale by 2 as shown in Figure 1.48(c). The signal $\Pi[(t-2)/2] + \Pi(2t-3.5)$ is obtained by adding the signals $\Pi[(t-2)/2]$ and $\Pi(2t-3.5)$ as shown in Figure 1.48(d).

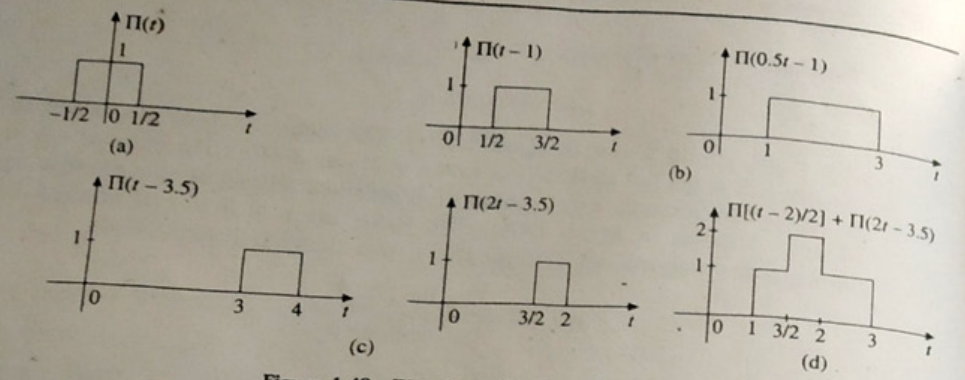


**Figure 1.48** Waveforms for Example 1.7(c).

$$\Pi\left(\frac{t-2}{2}\right)=1 \quad \text{for } 1\le t\le 3$$

$$\Pi(2t-3.5)=1 \quad \text{for } 1.5\le t\le 2$$

$$\therefore \quad \Pi\left(\frac{t-2}{2}\right)+\Pi(2t-3.5)=\begin{cases}1 & \text{for } 1\le t\le 1.5\\ 2 & \text{for } 1.5\le t\le 2\\ 1 & \text{for } 2\le t\le 3\end{cases}$$

(d) Given $\qquad r(t)-r(t-1)-r(t-3)+r(t-4)$

The signal $r(t)$ is drawn as shown in Figure 1.49(a). It is a straight line starting at origin with a slope 1. $-r(t-1)$ is a straight line starting at $t = 1$ with a slope $-1$ as shown in Figure 1.49(a). $-r(t-3)$ is a straight line starting at $t = 3$ with a

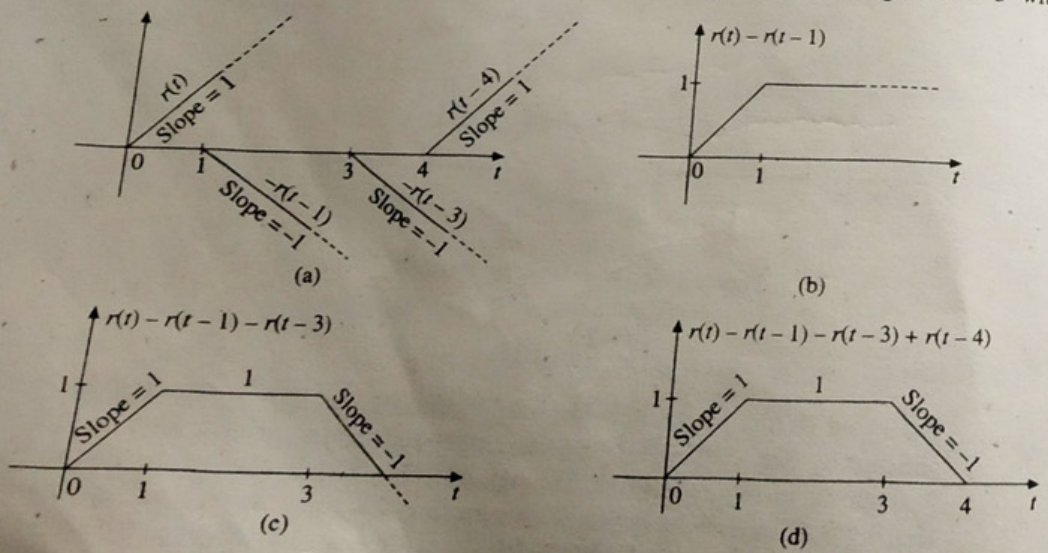


**Figure 1.49** Waveforms for Example 1.7(d).



slope $-1$ as shown in Figure 1.49(a) and $r(t-4)$ is a straight line starting at $t = 4$ with a slope 1 as shown in Figure 1.49(a). All these straight lines extend upto $\infty$.

The sum $r(t) - r(t-1) - r(t-3) + r(t-4)$ is a trapezoidal signal as shown in Figure 1.49(d).

In the interval $0 \le t < 1$, only $r(t)$ is existing and the amplitudes of $-r(t-1)$, $-r(t-3)$ and $r(t-4)$ are zero. Therefore, the resultant output is $r(t)$ alone. In the interval $1 \le t \le 3$, the amplitudes of $-r(t-3)$ and $r(t-4)$ are zero. Therefore, the resultant output is the sum of $r(t)$ and $-r(t-1)$. That is, the slope is changed from 1 to 0. In the interval $3 \le t \le 4$, the amplitude of $r(t-4)$ is zero. Therefore, the resultant output is the sum of $r(t)$, $-r(t-1)$ and $-r(t-3)$. That is, the slope is changed from 0 to $-1$. For $4 \le t \le \infty$, the amplitude of resultant output is the sum of $r(t)$, $-r(t-1)$, $-r(t-3)$ and $r(t-4)$. It is equal to zero.

(e) Given $\qquad r(-t)\,u(t+2)$

$r(-t)$ is time reversal of $r(t)$ as shown in Figure 1.50(a).

$$r(-t)=\begin{cases}-t & \text{for } t<0\\ 0 & \text{for } t>0\end{cases}$$

$$u(t+2)=\begin{cases}1 & \text{for } t>-2\\ 0 & \text{for } t<-2\end{cases}$$

as shown in Figure 1.50(a).

The product

$$r(-t)u(t+2)=\begin{cases}r(-t)=-t & \text{for } -2\le t<0\\ 0 & \text{for } t<-2\\ 0 & \text{for } t>0\end{cases}$$

as shown in Figure 1.50(b).

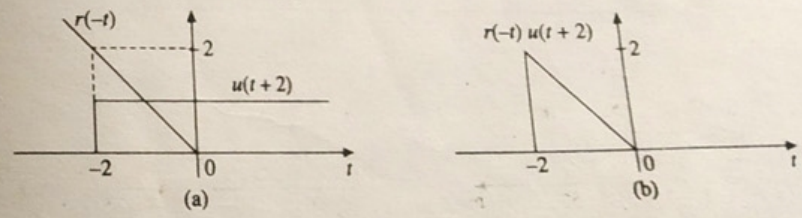


**Figure 1.50** Waveforms for Example 1.7(e).

(f) Given $\qquad r(-0.25t+1)$

$r(-0.25t+1)$ is obtained by time reversing the signal $r(t)$ to obtain $r(-t)$ delaying it (moving to right) by 1 unit to obtain $r(-t+1)$ and then expanding the signal $r(-t+1)$ by 4 times. The entire operation is shown in Figure 1.51.

$$r(-0.25t+1)=\begin{cases}-0.25t+1 & \text{for } -0.25t+1>0\\ 0 & \text{otherwise}\end{cases}$$

i.e. $$r(-0.25t+1)=\begin{cases}-0.25t+1 & \text{for } t<4\\ 0 & \text{otherwise}\end{cases}$$

i.e.
$$r(-0.25t+1) = 0 \quad \text{for } t \geq 4$$
$$= 0.25 \quad \text{for } t = 3$$
$$= 0.75 \quad \text{for } t = 1$$
$$= 1 \quad \text{for } t = 0$$
$$= 1.5 \quad \text{for } t = -2$$

and so on.

This can be plotted as shown in Figure 1.51(c).

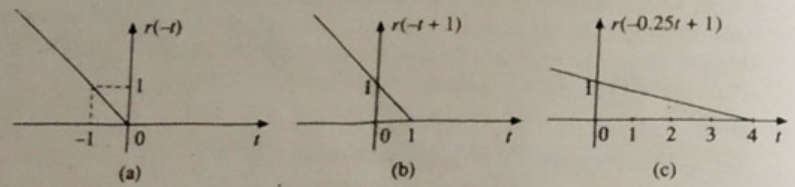


**Figure 1.51** Waveforms for Example 1.7(f).

**EXAMPLE 1.8** Express the following signals as sum of singular functions:

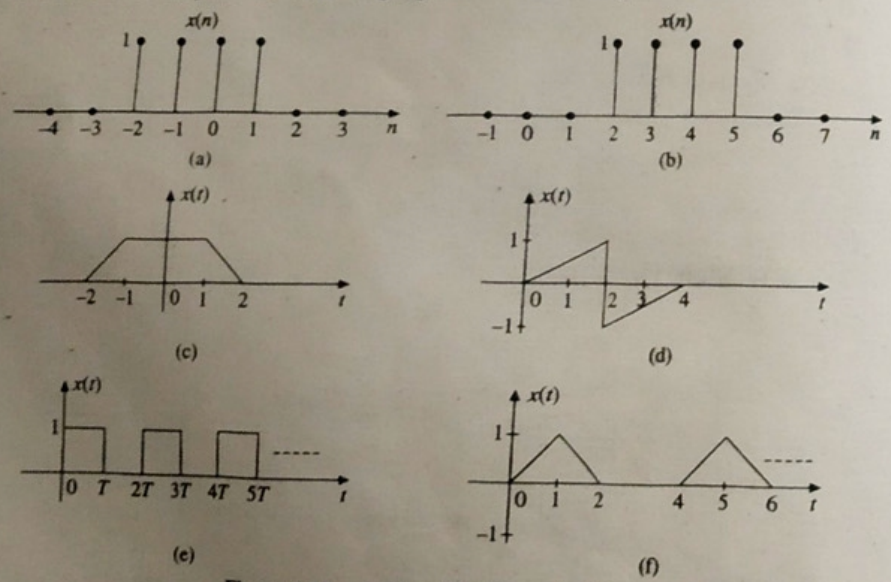


**Figure 1.52** Waveforms for Example 1.8.

***Solution:***

(a) The given signal shown in Figure 1.52(a) is:

$$x(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1)$$

$$x(n) = \begin{cases} 0 & \text{for } n \leq -3 \\ 1 & \text{for } -2 \leq n \leq 1 \\ 0 & \text{for } n \geq 2 \end{cases}$$

$$\therefore \quad x(n) = u(n+2) - u(n-2)$$



(b) The signal shown in Figure 1.52(b) is:

$$x(n) = \delta(n-2) + \delta(n-3) + \delta(n-4) + \delta(n-5)$$

$$x(n) = \begin{cases} 0 & \text{for } n \leq 1 \\ 1 & \text{for } 2 \leq n \leq 5 \\ 0 & \text{for } n \geq 6 \end{cases}$$

$$\therefore \quad x(n) = u(n-2) - u(n-6)$$

(c) The signal shown in Figure 1.52(c) is starting at $t = -2$ with a slope 1 and extends up to $t = -1$. Therefore

From $-2 \leq t \leq -1$ $\quad x(t) = r(t+2)$

At $t = -1$, the slope is changing from 1 to 0. So at $t = 1$, we have to add a ramp with slope $-1$. This 0 slope is maintained up to $t = 1$. Therefore

From $-2 \leq t \leq 1$ $\quad x(t) = r(t+2) - r(t+1)$

At $t = 1$, the slope is changing from 0 to $-1$. So at $t = 1$, we have to add a ramp with a slope $-1$. This $-1$ slope is maintained up to $t = 2$. Therefore

From $-2 \leq t \leq 2$ $\quad x(t) = r(t+2) - r(t+1) - r(t-1)$

At $t = 2$, the slope is changing from $-1$ to 0. So at $t = 2$, we have to add a ramp with a slope 1. This 0 slope is maintained up to $t = \infty$. Therefore

From $-2 \leq t \leq \infty$ $\quad x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$

$\therefore \quad x(t) = r(t+2) - r(t+1) - r(t-1) + r(t-2)$

The analysis is shown in Figure 1.53.

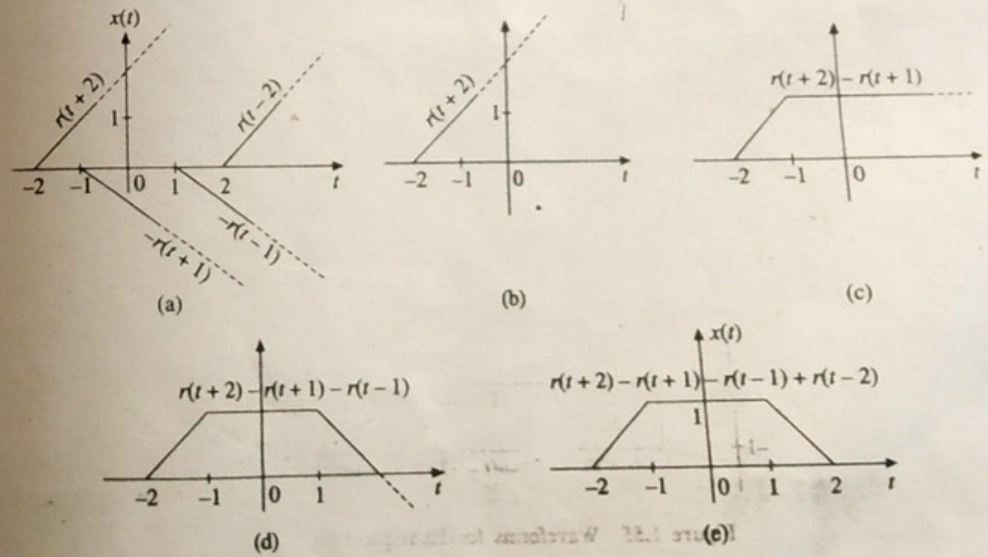


**Figure 1.53** Waveforms for Example 1.8(c).

(d) The signal shown in Figure 1.52(d) is starting from $t = 0$ with a slope 1/2. So it is ramp $(1/2)r(t)$. At $t = 2$, the value of $(1/2)r(t) = 1$. At $t = 2$, the amplitude is suddenly falling to $-1$ and then rising linearly with same slope 1/2. Hence at $t = 2$, we have to add a step of $-2$ amplitude, i.e. $-2u(t-2)$. At $t = 4$, the signal terminates. So at

$t = 4$, we have to add a ramp with a slope $-(1/2)$, i.e. $-(1/2)r(t-4)$. The analysis is shown in Figure 1.54.

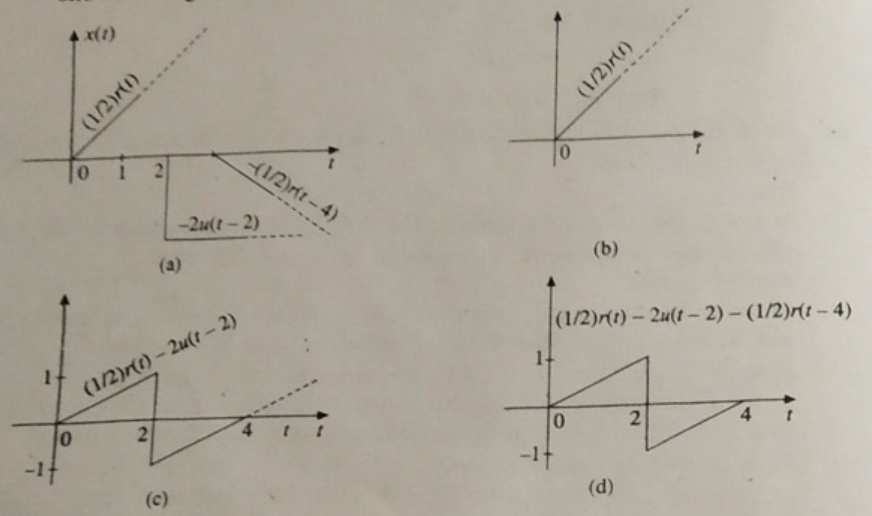


**Figure 1.54** Waveforms for Example 1.8(d).

(e) Each rectangular pulse in the given signal of Figure 1.52(e) can be expressed as the sum of two step functions as shown in Figure 1.55. The pulse train extends up to infinity.

$$\therefore \quad x(t) = u(t) - u(t-T) + u(t-2T) - u(t-3T) + u(t-4T) - u(t-5T) + \cdots$$

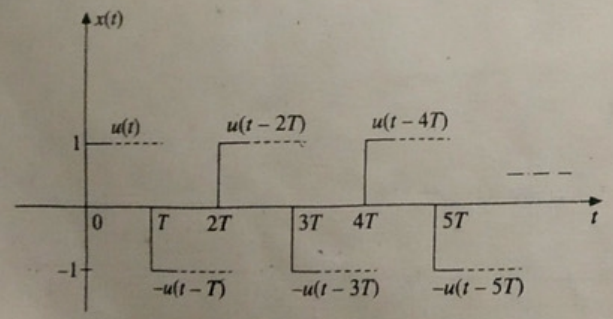


**Figure 1.55** Waveforms for Example 1.8(e).

(f) Each triangular pulse in the given signal of Figure 1.52(f) can be expressed as the sum of 3 ramp functions as shown in Figure 1.56. The pulse train extends upto infinity.

$$\therefore \quad x(t) = r(t) - 2r(t-1) + r(t-2) + r(t-4) - 2r(t-5) + r(t-6) + \cdots$$



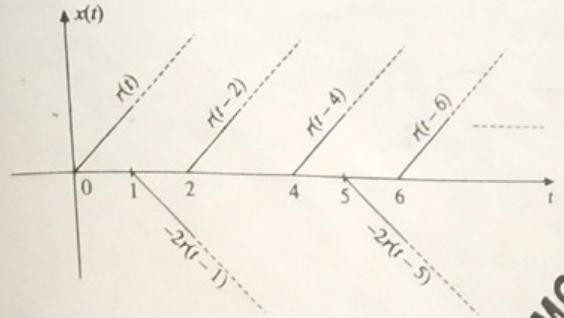


**Figure 1.56** Example 1.8(f).

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## 1.5 CLASSIFICATION OF SIGNALS

Based upon their nature and characteristics in the time domain, the signals may be broadly classified as under

(a) Continuous-time signals
(b) Discrete-time signals

### *Continuous-time signals*

The signals that are defined for every instant of time are known as continuous-time signals. Continuous-time signals are also called analog signals. For continuous-time signals, the independent variable is time. They are denoted by $x(t)$. They are continuous in amplitude as well as in time. Most of the signals available are continuous-time signals. Figure 1.57(a) and (b) shows the graphical representation of continuous-time signals.

### *Discrete-time signals*

The signals that are defined only at discrete instants of time are known as discrete-time signals. The discrete-time signals are continuous in amplitude but discrete in time. For discrete-time signals, the amplitude between two time instants is just not defined. For discrete-time signals, the independent variable is time $n$. Since they are defined only at discrete instants of time, they are denoted by a sequence $x(nT)$ or simply by $x(n)$ where $n$ is an integer.

The discrete-time signals may be inherently discrete or may be discrete versions of the continuous-time signals. Figure 1.57(c) and (d) show the graphical representation of discrete-time signals.

Both continuous-time and discrete-time signals may further be classified as under

1. Deterministic and random signals
2. Periodic and non-periodic signals
3. Energy and power signals
4. Causal and non-causal signals
5. Even and odd signals

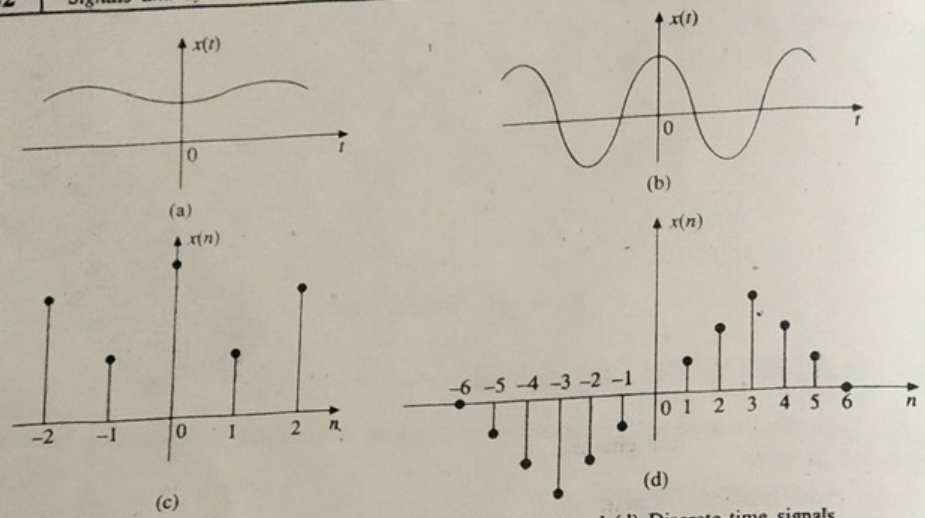


**Figure 1.57** (a) and (b) Continuous-time signals, (c) and (d) Discrete-time signals.

### 1.5.1 Deterministic and Random Signals

A signal exhibiting no uncertainty of its magnitude and phase at any given instant of time is called deterministic signal. A deterministic signal has a regular pattern and can be completely represented by mathematical equation at any time. Its amplitude and phase at any time instant can be predicted in advance.

*Examples:* Sine wave, $x(t) = \cos \omega t$ or $x(n) = \cos \omega n$, Exponential signals, square wave, triangular wave, etc.

A signal characterized by uncertainty about its occurrence is called a random signal. A random signal cannot be represented by any mathematical equation. The pattern of such a signal is quite irregular. Its amplitude and phase at any time instant cannot be predicted in advance.

A typical example of non deterministic signals is thermal noise generated in an electric circuit. Such a noise has a probabilistic nature.

Figure 1.58 shows the graphical representation of deterministic and random signals.

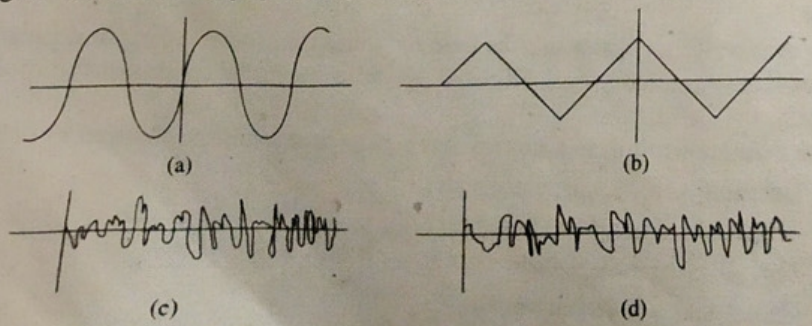


**Figure 1.58** (a) and (b) Deterministic signals, (c) and (d) Random signals.



### 1.5.2 Periodic and Non-periodic Signals

A signal which has a definite pattern and which repeats itself at regular intervals of time is called a periodic signal, and a signal which does not repeat at regular intervals of time is called a non-periodic or aperiodic signal.

Mathematically, a continuous-time signal $x(t)$ is called periodic if and only if

$$x(t + T) = x(t) \text{ for all } t, \text{ i.e. for } -\infty < t < \infty$$

where $t$ denotes time and $T$ is a constant representing the period. The smallest value of $T$ which satisfies the above condition is called the fundamental period or simply period of $x(t)$. A signal is aperiodic if the above condition is not satisfied even for one value of $t$. Sometimes aperiodic signals are said to have a period equal to infinity.

The reciprocal of fundamental period $T$ is called the fundamental frequency $f$ of $x(t)$,

$$f = \frac{1}{T}$$

i.e.

The angular frequency is given by

$$\omega = 2\pi f = \frac{2\pi}{T}$$

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$\therefore$ Fundamental period $T = \dfrac{2\pi}{\omega}$

Similarly, a discrete-time signal $x(n)$ is said to be periodic if it satisfies the condition $x(n) = x(n + N)$ for all integers $n$.

The smallest value of $N$ which satisfies the above condition is known as fundamental period.

If the above condition is not satisfied even for one value of $n$, then the discrete-time signal is aperiodic.

The angular frequency is given by

$$\omega = \frac{2\pi}{N}$$

$\therefore$ Fundamental period $N = \dfrac{2\pi}{\omega}$

1. The sum of two continuous-time periodic signals $x_1(t)$ and $x_2(t)$ with periods $T_1$ and $T_2$ may or may not be periodic depending on the relation between $T_1$ and $T_2$.
2. The sum of two periodic signals is periodic only if the ratio of their respective periods $T_1/T_2$ is a rational number or ratio of two integers.
3. The fundamental period is the least common multiple (LCM) of $T_1$ and $T_2$.
4. If the ratio $T_1/T_2$ is an irrational number, then the signals $x_1(t)$ and $x_2(t)$ do not have a common period and $x(t)$ cannot be periodic.
5. The sum of two discrete-time periodic sequences is always periodic.

Some examples of continuous-time and discrete-time periodic/non-periodic signals are shown in Figure 1.59.

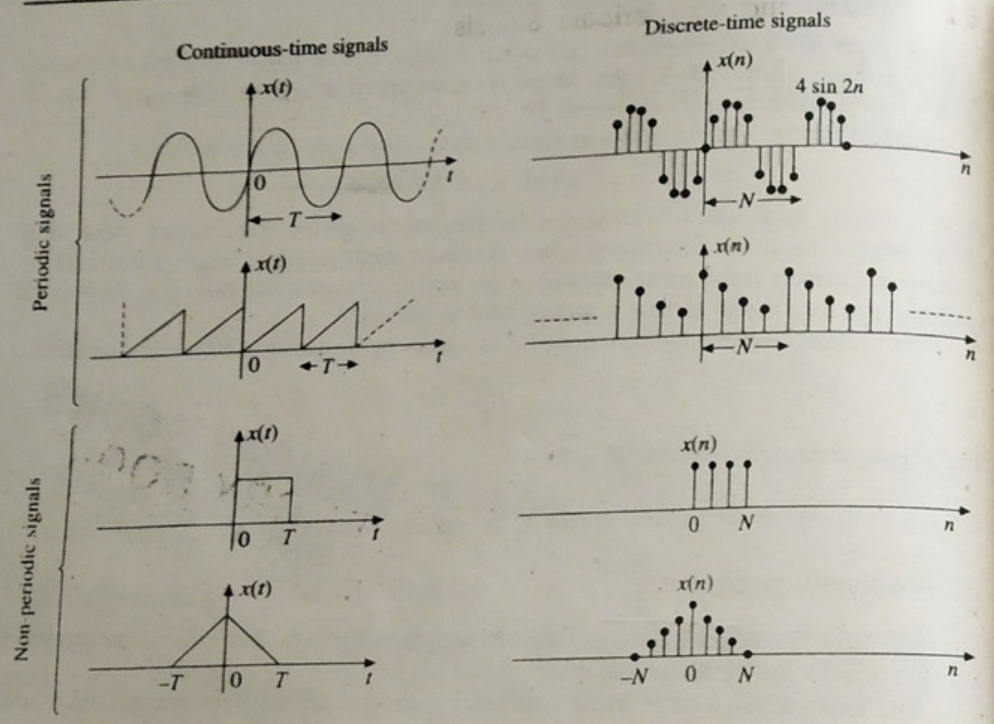


**Figure 1.59** Examples of periodic and non-periodic signals.

**EXAMPLE 1.9** Show that the complex exponential signal $x(t) = e^{j\omega_0 t}$ is periodic with period $2\pi/\omega_0$.

*Solution:* Given $\quad x(t) = e^{j\omega_0 t}$

$x(t)$ will be periodic if $\quad x(t + T) = x(t)$

i.e. if $\quad e^{j\omega_0(t+T)} = e^{j\omega_0 t}$

i.e. if $\quad e^{j\omega_0 t} e^{j\omega_0 T} = e^{j\omega_0 t}$

i.e. if $\quad e^{j\omega_0 T} = 1$

This is valid if $\quad \omega_0 T = n(2\pi)$

where $n$ is an integer.

$$\therefore \quad T = \frac{2\pi n}{\omega_0}$$

Thus, the fundamental period, i.e. the smallest positive $T$ of $x(t)$ is given by

$$T = \frac{2\pi}{\omega_0}$$



**EXAMPLE 1.10** Show that the sinusoidal signal $x(t) = \sin(\omega_0 t + \theta)$ is periodic with period $2\pi/\omega_0$.

*Solution:* Given $\quad x(t) = \sin(\omega_0 t + \theta)$

$x(t)$ will be periodic if $\quad x(t) = x(t + T)$

But $\quad x(t + T) = \sin[\omega_0(t + T) + \theta] = \sin[\omega_0 t + \theta + \omega_0 T]$

For $\sin(\omega_0 t + \theta)$ to be equal to $\sin(\omega_0 t + \theta + \omega_0 T)$, $\omega_0 T$ must be equal to $2n\pi$, where $n$ is a positive integer.

Therefore, $\quad T = \dfrac{2\pi n}{\omega_0}$

This shows that the sinusoidal signal $x(t) = \sin(\omega_0 t + \theta)$ is periodic with fundamental period $T = (2\pi/\omega_0)$.

**EXAMPLE 1.11** Let $x_1(t)$ and $x_2(t)$ be periodic signals with fundamental periods $T_1$ and $T_2$ respectively. Under what conditions is the sum $x(t) = x_1(t) + x_2(t)$ periodic and what is the fundamental period of $x(t)$ if it is periodic?

Or

Show that a composite signal is periodic if the ratio of their fundamental periods is a rational number.

*Solution:* Given signals $x_1(t)$ and $x_2(t)$ are periodic with fundamental periods $T_1$ and $T_2$ respectively. So

$$x_1(t) = x_1(t + T_1) = x_1(t + mT_1), \ m \text{ is a positive integer}$$

$$x_2(t) = x_2(t + T_2) = x_2(t + kT_2), \ k \text{ is a positive integer}$$

Given signal is $\quad x(t) = x_1(t) + x_2(t)$

$$\therefore \quad x(t) = x_1(t + mT_1) + x_2(t + kT_2)$$

For $x(t)$ to be periodic with period $T$, we require

$$x(t) = x(t + T) = x_1(t + T) + x_2(t + T)$$

$$= x_1(t + mT_1) + x_2(t + kT_2)$$

$$\therefore \quad mT_1 = kT_2 = T$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{k}{m} = \text{rational number}$$

Then the fundamental period $T$ of $x(t)$ is the LCM of $T_1$ and $T_2$. Therefore,

$$T = mT_1 = kT_2$$

So we can say that, the sum of two periodic signals is periodic only if the ratio of their respective periods is a rational number. Then the fundamental period is the LCM of the respective fundamental periods.

**Note:** The same condition holds for discrete-time signals also.

All continuous time sinusoidal and complex exponential signals are periodic.

**EXAMPLE 1.12** Examine whether the following signals are periodic or not? If periodic determine the fundamental period.

(a) $\sin 12\pi t$ (b) $e^{j4\pi t}$

(c) $\sin \pi t\, u(t)$ (d) $e^{-|t|}$

(e) $\cos 2t + \sin \sqrt{3}t$ (f) $3 \sin 200\pi t + 4 \cos 100t$

(g) $\sin 10\pi t + \cos 20\pi t$ (h) $\sin (10t + 1) - 2 \cos (5t - 2)$

(i) $je^{j6t}$ (j) $3u(t) + 2 \sin 2t$

(k) $6e^{j[4t+(\pi/3)]} + 8e^{j[3\pi+(\pi/4)]}$ (l) $u(t) - 2u(t - 5)$

(m) $2 + \cos 2\pi t$

***Solution:***

(a) Given $x(t) = \sin 12\pi t$

Since $x(t)$ is a sinusoidal signal it is periodic.

Comparing it with $\sin \omega t$, we have

$$\omega = 12\pi \quad \text{or} \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{12\pi} = \frac{1}{6}$$

So, $x(t)$ is periodic with fundamental period $T = 1/6$ sec.

(b) Given $x(t) = e^{j4\pi t}$

Since $x(t)$ is a complex exponential signal it is periodic.

Comparing it with $e^{j\omega t}$, we have

$$\omega = 4\pi \quad \text{or} \quad T = \frac{2\pi}{\omega} = \frac{2\pi}{4\pi} = \frac{1}{2}$$

So, $x(t)$ is periodic with fundamental period $T = 1/2$ sec.

(c) Given $x(t) = \sin \pi t\, u(t)$

$\sin \pi t$ is periodic with period $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{\pi} = 2$ sec. $u(t)$ exists only between $t = 0$ to $t = \infty$. Hence it is not periodic.

Therefore, $\sin \pi t\, u(t)$ which is the product of a periodic and a non-periodic signal is not periodic.

(d) Given $x(t) = e^{-|t|}$

The plot of $x(t) = e^{-|t|}$ versus $t$ is shown in Figure 1.60. It does not repeat at all. So it is aperiodic.



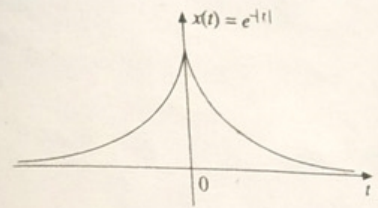


**Figure 1.60** Plot of $x(t) = e^{-|t|}$.

(e) Given $x(t) = \cos 2t + \sin \sqrt{3}t$

Let $x(t) = x_1(t) + x_2(t)$

where $x_1(t) = \cos 2t$ and $x_2(t) = \sin \sqrt{3}t$

Comparing $x_1(t) = \cos 2t$ with $\cos \omega_1 t$, we have

$$\omega_1 = 2, \text{ i.e. } 2\pi f_1 = 2 \quad \text{or} \quad f_1 = \frac{1}{\pi}$$

$\therefore$ Period of $x_1(t)$ is $T_1 = \dfrac{1}{f_1} = \pi$

Comparing $x_2(t) = \sin \sqrt{3}t$ with $\sin \omega_2 t$, we have

$$\omega_2 = \sqrt{3}, \text{ i.e. } 2\pi f_2 = \sqrt{3} \quad \text{or} \quad f_2 = \frac{\sqrt{3}}{2\pi}$$

$\therefore$ Period of $x_2(t)$ is $T_2 = \dfrac{1}{f_2} = \dfrac{2\pi}{\sqrt{3}}$

The ratio of two periods $= \dfrac{T_1}{T_2} = \dfrac{\pi}{2\pi/\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

Since $T_1/T_2$ is not a ratio of two integers (i.e. not a rational number), the given signal $x(t)$ is non-periodic.

(f) Given $x(t) = 3 \sin 200\pi t + 4 \cos 100t$

Let $x(t) = x_1(t) + x_2(t)$

where $x_1(t) = 3 \sin 200\pi t$ and $x_2(t) = 4 \cos 100t$

Comparing $x_1(t) = 3 \sin 200\pi t$ with $A \sin \omega_1 t$, we have

$$\omega_1 = 200\pi, \text{ i.e. } 2\pi f_1 = 200\pi \quad \text{or} \quad f_1 = 100$$

$\therefore$ Period of $x_1(t)$ is $T_1 = \dfrac{1}{f_1} = \dfrac{1}{100}$

Comparing $x_2(t) = 4 \cos 100t$ with $B \cos \omega_2 t$, we have

$$\omega_2 = 100, \text{ i.e. } 2\pi f_2 = 100 \quad \text{or} \quad f_2 = \frac{50}{\pi}$$

$\therefore$ Period of $x_2(t)$ is $\quad T_2 = \dfrac{1}{f_2} = \dfrac{\pi}{50}$

The ratio of two periods $= \dfrac{T_1}{T_2} = \dfrac{1/100}{\pi/50} = \dfrac{1}{2\pi}$

Since $T_1/T_2$ is not a ratio of two integers (i.e. not a rational number), the given signal $x(t)$ is non-periodic.

(g) Given $\quad x(t) = \sin 10\pi t + \cos 20\pi t$

Let $\quad x(t) = x_1(t) + x_2(t)$

where $\quad x_1(t) = \sin 10\pi t$ and $x_2(t) = \cos 20\pi t$

Comparing $x_1(t) = \sin 10\pi t$ with $\sin \omega_1 t$, we have

$$\omega_1 = 10\pi, \text{ i.e. } 2\pi f_1 = 10\pi \quad \text{or} \quad f_1 = 5$$

$\therefore$ Period of $x_1(t)$ is $\quad T_1 = \dfrac{1}{f_1} = \dfrac{1}{5}$

Comparing $x_2(t) = \cos 20\pi t$ with $\cos \omega_2 t$, we have

$$\omega_2 = 20\pi, \text{ i.e. } 2\pi f_2 = 20\pi \quad \text{or} \quad f_2 = 10$$

$\therefore$ Period of $x_2(t)$ is $\quad T_2 = \dfrac{1}{f_2} = \dfrac{1}{10}$

The ratio of two periods $= \dfrac{T_1}{T_2} = \dfrac{1/5}{1/10} = 2$

$\therefore \quad T_1 = 2T_2$

Since $T_1/T_2$ is a rational number (ratio of two integers 2 and 1), the given signal $x(t)$ is periodic.

The fundamental period $\quad T = T_1 = 2T_2 = \dfrac{1}{5}$ sec

(h) Given $\quad x(t) = \sin(10t + 1) - 2\cos(5t - 2)$

Let $\quad x(t) = x_1(t) + x_2(t)$

where $\quad x_1(t) = \sin(10t + 1)$ and $x_2(t) = 2\cos(5t - 2)$

Comparing $x_1(t) = \sin(10t + 1)$ with $\sin(\omega_1 t + \theta_1)$, we have

$$\omega_1 = 10, \text{ i.e. } 2\pi f_1 = 10 \quad \text{or} \quad f_1 = \frac{10}{2\pi} = \frac{5}{\pi}$$



The period of $x_1(t)$ is:

$$T_1 = \frac{1}{f_1} = \frac{\pi}{5} \text{ sec}$$

Comparing $x_2(t) = 2\cos(5t - 2)$ with $A\cos(\omega_2 t + \theta_2)$, we have

$$\omega_2 = 5, \text{ i.e. } 2\pi f_2 = 5 \quad \text{or} \quad f_2 = \frac{5}{2\pi}$$

The period of $x_2(t)$ is:

$$T_2 = \frac{1}{f_2} = \frac{2\pi}{5} \text{ sec}$$

The ratio of two periods $= \dfrac{T_1}{T_2} = \dfrac{\pi/5}{2\pi/5} = \dfrac{1}{2}$

$\therefore \quad 2T_1 = T_2$

Since $T_1/T_2$ is a ratio of two integers (i.e. a rational number), the given signal $x(t)$ is periodic.

$$\text{Fundamental period } T = 2T_1 = T_2 = \frac{2\pi}{5} \text{ sec}$$

(i) Given $\quad x(t) = je^{j6t}$

Since $x(t)$ is a complex exponential signal it is periodic.

Comparing this with the standard form $x(t) = e^{j\omega t}$, we have

$$\omega = 6, \text{ i.e. } 2\pi f = 6 \quad \text{or} \quad f = \frac{3}{\pi}$$

$\therefore$ Time period $T = \dfrac{1}{f} = \dfrac{\pi}{3}$

So, $x(t)$ is periodic with period $T = \pi/3$.

(j) Given $x(t) = 3u(t) + 2\sin 2t$ (Figure 1.61).

Period of $3u(t)$ is zero, i.e. it is aperiodic. Period of $2 \sin 2t$ is $T = (2\pi/2) = \pi$ sec $2 \sin 2t$ is periodic. Therefore, $x(t)$, the sum of an aperiodic signal and a periodic signal, is aperiodic.

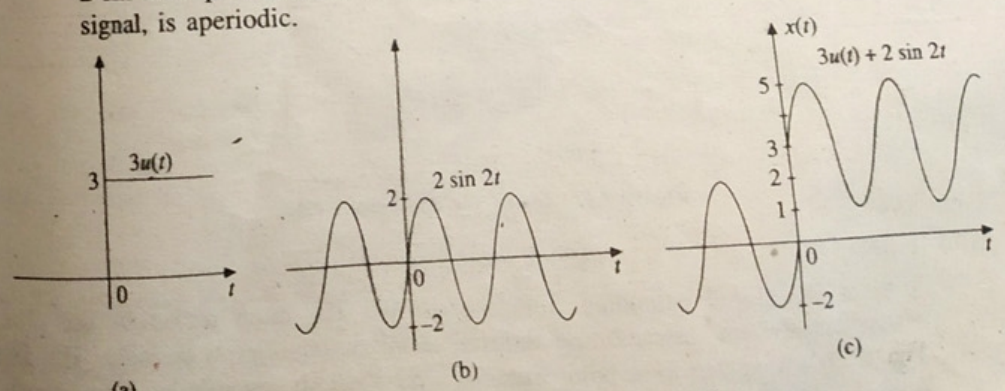


**Figure 1.61** Signal for Example 1.12(j).

(k) Given $$x(t) = 6e^{j[4t+(\pi/3)]} + 8e^{j[3\pi t+(\pi/4)]}$$

Let $$x(t) = x_1(t) + x_2(t)$$

where $x_1(t) = 6e^{j[4t+(\pi/3)]}$ and $x_2(t) = 8e^{j[3\pi t+(\pi/4)]}$

Comparing $x_1(t) = 6e^{j[4t+(\pi/3)]}$ with $Ae^{j(\omega_1 t+\theta_1)}$, we have

$$\omega_1 = 4, \text{ i.e. } 2\pi f_1 = 4 \quad \text{or} \quad f_1 = \frac{2}{\pi}$$

$\therefore$ Time period of $6e^{j[4t+(\pi/3)]}$ is $T_1 = \dfrac{1}{f_1} = \dfrac{\pi}{2}$.

Comparing $x_2(t) = 8e^{j[3\pi t+(\pi/4)]}$ with $Be^{j(\omega_2 t+\theta_2)}$, we have

$$\omega_2 = 3\pi, \text{ i.e. } 2\pi f_2 = 3\pi \quad \text{or} \quad f_2 = \frac{3}{2}$$

$\therefore$ Time period of $8e^{j[3\pi t+(\pi/4)]}$ is $T_2 = \dfrac{1}{f_2} = \dfrac{2}{3}$.

The ratio of two periods $= \dfrac{T_1}{T_2} = \dfrac{\pi/2}{2/3} = \dfrac{3\pi}{4}$.

Since the ratio $T_1/T_2$ is not rational, the given signal $x(t)$ is not periodic.

(l) Given $$x(t) = u(t) - 2u(t-5)$$

Period of $u(t)$ as well as $u(t-5)$ is zero. So both $u(t)$ and $u(t-5)$ are aperiodic. Therefore, $x(t)$, the sum of two aperiodic signals, is aperiodic as shown in Figure 1.62.

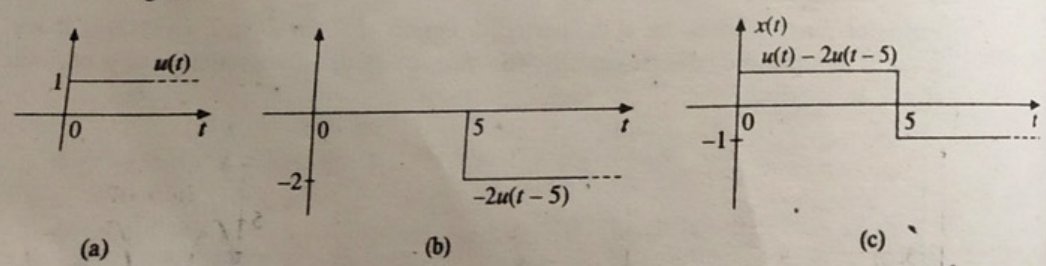


**Figure 1.62** Signal for Example 1.12(l).

(m) Given $$x(t) = 2 + \cos 2\pi t$$

2 is a dc signal extending from $-\infty$ to $\infty$. The time period of $\cos 2\pi t$ is $T = (2\pi/2\pi) = 1$. Since it is a rational number, $\cos 2\pi t$ is periodic. The signal $x(t) = 2 + \cos 2\pi t$ is nothing but $\cos 2\pi t$ shifted upwards by 2 as shown in Figure 1.63. So $x(t)$ is also periodic with a fundamental period $T = 1$ sec.



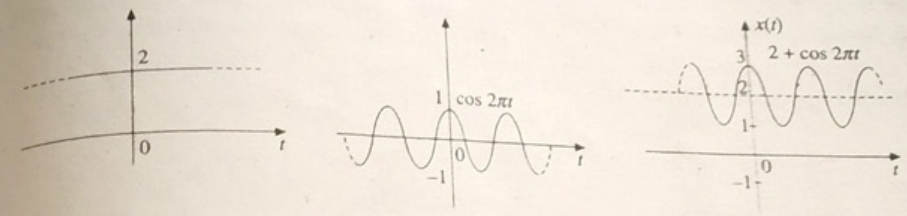


**Figure 1.63** Signal for Example 1.12(m).

**EXAMPLE 1.13** Show that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic only if $\omega_0/2\pi$ is a rational number.

***Solution:*** Given $$x(n) = e^{j\omega_0 n}$$

$x(n)$ will be periodic if $$x(n+N) = x(n)$$

i.e. $$e^{j[\omega_0(n+N)]} = e^{j\omega_0 n}$$

i.e. $$e^{j\omega_0 N} e^{j\omega_0 n} = e^{j\omega_0 n}$$

This is possible only if $$e^{j\omega_0 N} = 1$$

This is true only if $$\omega_0 N = 2\pi k$$

where $k$ is a positive integer.

$$\therefore \quad \frac{\omega_0}{2\pi} = \frac{k}{N} = \text{Rational number}$$

This shows that the complex exponential sequence $x(n) = e^{j\omega_0 n}$ is periodic if $\omega_0/2\pi$ is a rational number.

**EXAMPLE 1.14** Let $x(t)$ be the complex exponential signal, $x(t) = e^{j\omega_0 t}$ with radian frequency $\omega_0$ and fundamental period $T = 2\pi/\omega_0$. Consider the discrete-time sequence $x(n)$ obtained by the uniform sampling of $x(t)$ with sampling interval $T_s$, i.e.

$$x(n) = x(nT_s) = e^{jn\omega_0 T_s}$$

Show that $x(n)$ is periodic if the ratio of the sampling interval $T_s$ to the fundamental period $T$ of $x(t)$, i.e. $T_s/T$ is a rational number.

***Solution:*** Given $$x(t) = e^{j\omega_0 t}$$

$$x(n) = x(nT_s) = e^{jn\omega_0 T_s}$$

$T_s$ is the sampling interval.

Now, fundamental period $T = \dfrac{2\pi}{\omega_0}$

$$\therefore \qquad \omega_0 = \frac{2\pi}{T}$$

If $x(n)$ is periodic with the fundamental period $N$, then

$$x(n+N) = x(n)$$

i.e. $$e^{j(n+N)\omega_0 T_s} = e^{jn\omega_0 T_s}$$

i.e. $$e^{jn\omega_0 T_s} e^{jN\omega_0 T_s} = e^{jn\omega_0 T_s}$$

This is true only if $$e^{jN\omega_0 T_s} = 1$$

i.e. $$N\omega_0 T_s = 2\pi m$$

where $m$ is a positive integer.

i.e. $$N\frac{2\pi}{T}T_s = 2\pi m$$

$$\therefore \qquad \frac{T_s}{T} = \frac{m}{N} = \text{Rational number}$$

This shows that $x(n)$ is periodic if the ratio of the sampling interval to the fundamental period of $x(t)$, $T_s/T$ is a rational number.

**EXAMPLE 1.15** Obtain the condition for discrete-time sinusoidal signal to be periodic.

***Solution:*** In case of continuous-time signals, all sinusoidal signals are periodic. But in discrete-time case, not all sinusoidal sequences are periodic.

Consider a discrete-time signal given by

$$x(n) = A\sin(\omega_0 n + \theta)$$

where $A$ is amplitude, $\omega_0$ is frequency and $\theta$ is phase shift.

A discrete-time signal is periodic if and only if

$$x(n) = x(n+N) \text{ for all } n$$

Now, $$x(n+N) = A\sin[\omega_0(n+N)+\theta)] = A\sin(\omega_0 n + \theta + \omega_0 N)$$

Therefore, $x(n)$ and $x(n+N)$ are equal if $\omega_0 N = 2\pi m$. That is, there must be an integer $m$ such that

$$\omega_0 = \frac{2\pi m}{N} = 2\pi\left[\frac{m}{N}\right]$$

or $$N = 2\pi\left[\frac{m}{\omega_0}\right]$$

From the above equation we find that, for the discrete-time signal to be periodic, the fundamental frequency $\omega_0$ must be a rational multiple of $2\pi$. Otherwise the discrete-time signal is aperiodic. The smallest value of positive integer $N$, for some integer $m$, is the fundamental period.



**EXAMPLE 1.16** Determine whether the following discrete-time signals are periodic or not? If periodic, determine the fundamental period.

(a) $\sin(0.02\pi n)$ (b) $\sin(5\pi n)$

(c) $\cos 4n$ (d) $\sin\frac{2\pi n}{3} + \cos\frac{2\pi n}{5}$

(e) $\cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$ (f) $\cos\left(\frac{\pi}{2} + 0.3n\right)$

(g) $e^{j(\pi/2)n}$ (h) $1 + e^{j2\pi n/3} - e^{j4\pi n/7}$

***Solution:***

(a) Given $$x(n) = \sin(0.02\pi n)$$

Comparing it with $$x(n) = \sin(2\pi f n),$$

we have $$0.02\pi = 2\pi f \quad \text{or} \quad f = \frac{0.02\pi}{2\pi} = 0.01 = \frac{1}{100} = \frac{k}{N}$$

Here $f$ is expressed as a ratio of two integers with $k = 1$ and $N = 100$. So it is rational. Hence the given signal is periodic with fundamental period $N = 100$.

(b) Given $$x(n) = \sin(5\pi n)$$

Comparing it with $$x(n) = \sin(2\pi f n),$$

we have $$2\pi f = 5\pi \quad \text{or} \quad f = \frac{5}{2} = \frac{k}{N}$$

Here $f$ is a ratio of two integers with $k = 5$ and $N = 2$. Hence it is rational. Therefore, the given signal is periodic with fundamental period $N = 2$.

(c) Given $$x(n) = \cos 4n$$

Comparing it with $$x(n) = \cos 2\pi f n,$$

we have $$2\pi f = 4 \quad \text{or} \quad f = \frac{2}{\pi}$$

Since $f = (2/\pi)$ is not a rational number, $x(n)$ is not periodic.

(d) Given $$x(n) = \sin\frac{2\pi n}{3} + \cos\frac{2\pi n}{5}$$

Comparing it with $$x(n) = \sin 2\pi f_1 n + \cos 2\pi f_2 n$$

we have $$2\pi f_1 = \frac{2\pi}{3} \quad \text{or} \quad f_1 = \frac{1}{3} = \frac{k_1}{N_1}$$

$$\therefore \quad N_1 = 3$$

and $$2\pi f_2 = \frac{2\pi}{5} \quad \text{or} \quad f_2 = \frac{1}{5}$$

$$\therefore \quad N_2 = 5$$

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Since $\frac{N_1}{N_2} = \frac{3}{5}$ is a ratio of two integers, the sequence $x(n)$ is periodic. The period of $x(n)$ is the LCM of $N_1$ and $N_2$. Here LCM of $N_1 = 3$ and $N_2 = 5$ is 15. Therefore, the given sequence is periodic with fundamental period $N = 15$.

(e) Given
$$x(n) = \cos\left(\frac{n}{6}\right)\cos\left(\frac{n\pi}{6}\right)$$

Comparing it with $x(n) = \cos(2\pi f_1 n)\cos(2\pi f_2 n)$,

we have
$$2\pi f_1 n = \frac{n}{6} \quad \text{or} \quad f_1 = \frac{1}{12\pi}$$

which is not rational

and
$$2\pi f_2 n = \frac{n\pi}{6} \quad \text{or} \quad f_2 = \frac{1}{12}$$

which is rational

Thus, $\cos(n/6)$ is non-periodic and $\cos(n\pi/6)$ is periodic. $x(n)$ is non-periodic because it is the product of periodic and non-periodic signals.

(f) Given
$$x(n) = \cos\left(\frac{\pi}{2} + 0.3n\right)$$

Comparing it with $x(n) = \cos(2\pi fn + \theta)$,

we have $2\pi fn = 0.3n$ and phase shift $\theta = \frac{\pi}{2}$

$$\therefore \quad f = \frac{0.3}{2\pi} = \frac{3}{20\pi}$$

which is not rational.

Hence, the signal $x(n)$ is non-periodic.

(g) Given
$$x(n) = e^{j(\pi/2)n}$$

Comparing it with $x(n) = e^{j2\pi fn}$,

we have
$$2\pi f = \frac{\pi}{2} \quad \text{or} \quad f = \frac{1}{4} = \frac{k}{N}$$

which is rational.

Hence, the given signal $x(n)$ is periodic with fundamental period $N = 4$.

(h) Given
$$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7}$$

Let
$$x(n) = 1 + e^{j2\pi n/3} - e^{j4\pi n/7} = x_1(n) + x_2(n) + x_3(n)$$

where $x_1(n) = 1$, $x_2(n) = e^{j2\pi n/3}$ and $x_3(n) = e^{j4\pi n/7}$



$x_1(n) = 1$ is a dc signal with an arbitrary period $N_1 = 1$

$$x_2(n) = e^{j2\pi n/3} = e^{j2\pi f_2 n}$$

$$\therefore \quad \frac{2\pi n}{3} = 2\pi f_2 n \quad \text{or} \quad f_2 = \frac{1}{3} = \frac{k_2}{N_2} \quad \text{where } N_2 = 3.$$

Hence $x_2(n)$ is periodic with period $N_2 = 3$.

$$x_3(n) = e^{j4\pi n/7} = e^{j2\pi f_3 n}$$

$$\therefore \quad \frac{4\pi n}{7} = 2\pi f_3 n \quad \text{or} \quad f_3 = \frac{2}{7} = \frac{k_3}{N_3} \quad \text{where } N_3 = \frac{7}{2}$$

Now,
$$\frac{N_1}{N_2} = \frac{1}{3} = \text{Rational number}$$

$$\frac{N_1}{N_3} = \frac{1}{7/2} = \frac{2}{7} = \text{Rational number}$$

The LCM of
$$N_1, N_2, N_3 = \frac{7}{2} \times 3 = \frac{21}{2}$$

The given signal $x(n)$ is periodic with fundamental period $N = 10.5$.

### 1.5.3 Energy and Power Signals

In electrical systems, signals may represent voltage or current. Consider a voltage signal $v(t)$ across a resistance $R$ producing a current $i(t)$ as shown in Figure 1.64.

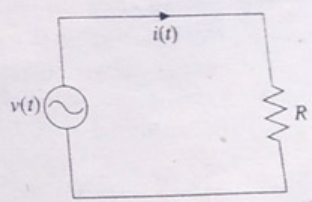


**Figure 1.64** Simple resistive circuit.

The instantaneous power developed in $R$ is given by

$$p(t) = v(t)\, i(t)$$

$$= v(t)\frac{v(t)}{R} = \frac{v^2(t)}{R}$$

$$= i(t)\, Ri(t) = i^2(t)R$$

When $R = 1$ ohm, the power dissipated is called *normalized power*.

So
$$\text{Normalized power } p(t) = v^2(t) \quad \text{or} \quad i^2(t)$$

If $v(t)$ or $i(t)$ is denoted by a signal $x(t)$, then the instantaneous power is equal to the square of the amplitude of the signal.

i.e.
$$p(t) = |x(t)|^2$$

Thus, for current as well as voltage, the equation for normalized power is same.

The total energy or normalized energy of a continuous-time signal $x(t)$ is given by

$$E = \underset{T\to\infty}{\text{Lt}} \int_{-T}^{T} |x(t)|^2\, dt \text{ joules}$$

### *When the signal is given as a waveform*

The even part of the signal can be found by folding the signal about the $y$-axis and adding the folded signal to the original signal and dividing the sum by two. The odd part of the signal can be found by folding the signal about $y$-axis and subtracting the folded signal from the original signal and dividing the difference by two as illustrated in Figure 1.73.

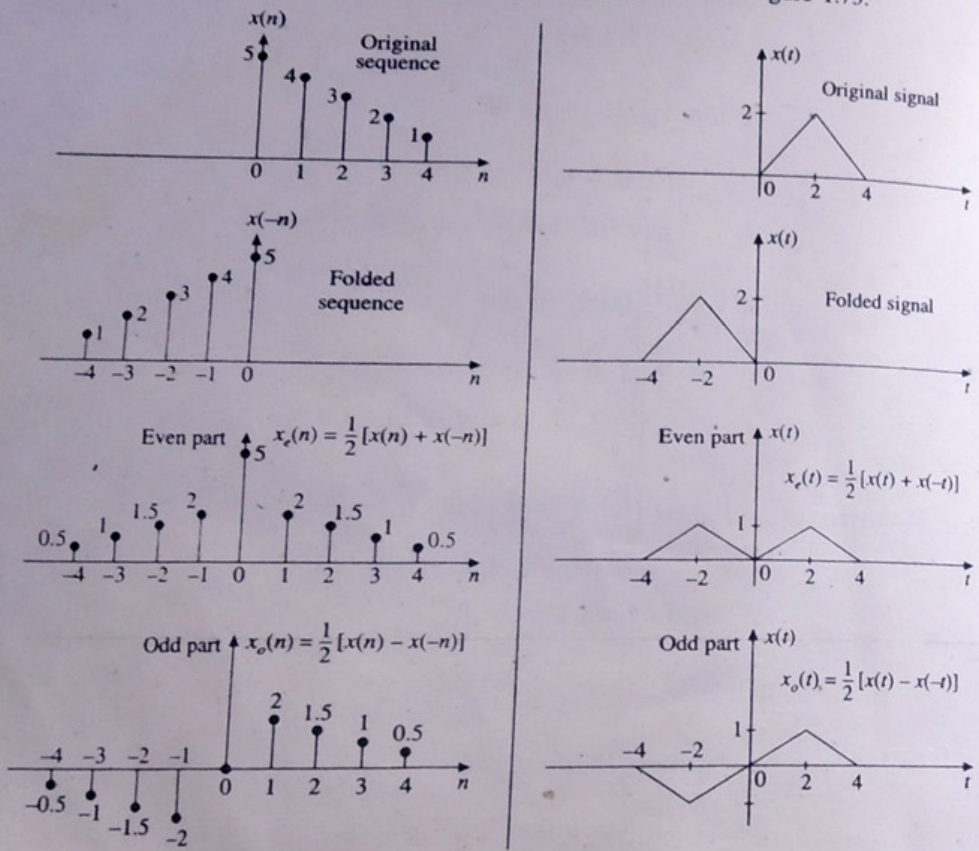


**Figure 1.73** Graphical evaluation of even and odd parts.

## MATLAB PROGRAMS

### Program 1.1

```
% Generation of elementary signals in continuous-time
clc; close all; clear all;
% Unit Impulse signal
t=-1:0.001:1;
```



```
impulse=[zeros(1,1000),ones(1,1),zeros(1,1000)];
subplot(2,2,1);plot(t,impulse);
xlabel('time —>');ylabel('Amplitude —>');
title('Unit Impulse signal');
axis([-1 1 0 1.5]);
% Unit Step signal
t=-5:0.01:5;
step=[zeros(1,500),ones(1,501)];
subplot(2,2,2);plot(t,step);
xlabel('time —>');ylabel('Amplitude —>');
title('Unit Step signal');
axis([-5 5 0 1.5]);
% Unit Ramp signal
t=0:0.01:6;
ramp=t;
subplot(2,2,3);plot(t,ramp);
xlabel('time —>');ylabel('Amplitude —>');
title('Unit Ramp signal');
% Unit Parabolic signal
t=0:0.01:6;
parabola=0.5*(t.^2);
subplot(2,2,4);plot(t,parabola);
xlabel('time —>');ylabel('Amplitude —>');
title('Unit Parabolic signal');
```

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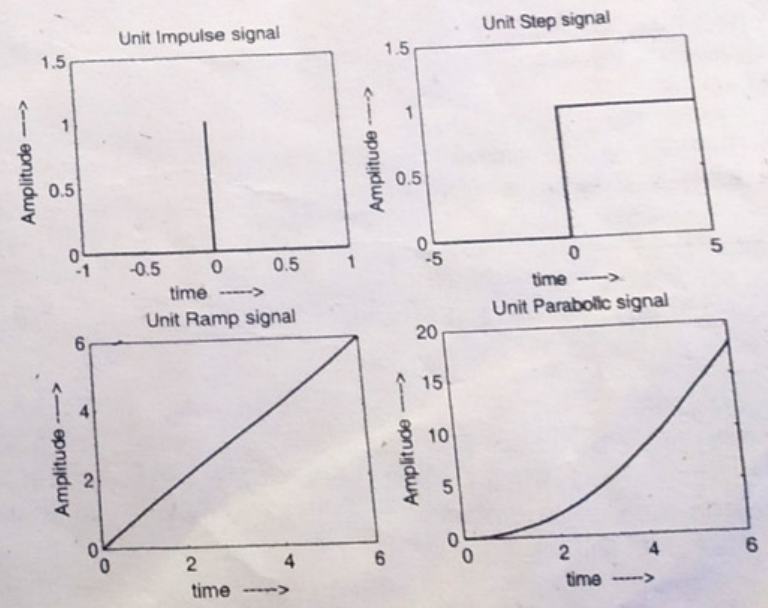
Output:

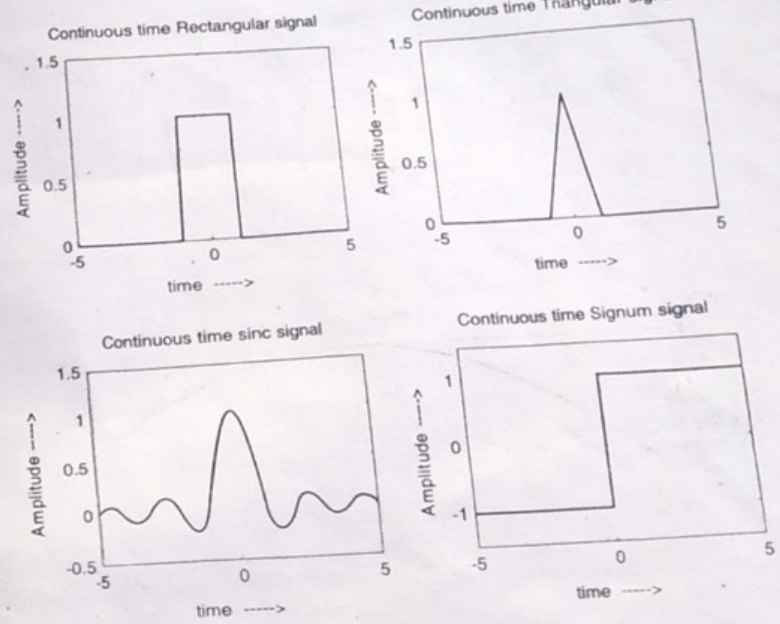


## Program 1.2

```
% Generation of elementary signals in continuous-time
clc; close all; clear all;
% Rectangular signal
t=-5:0.01:5;
width=2;
y=rectpuls(t,width);
% alternate procedure
%y=[zeros(1,400),ones(1,201),zeros(1,400)];
subplot(2,2,1); plot(t,y);
xlabel('time —>');ylabel('Amplitude —>');
title('Continuous time Rectangular signal');
axis([-5 5 0 1.5]);
% Triangular signal
t=-5:0.01:5;
width=2;
y=tripuls(t,width);
% alternate procedure
%y=trimf(t,[-1 0 1]);
subplot(2,2,2); plot(t,y);
xlabel('time —>');ylabel('Amplitude —>');
title('Continuous time Triangular signal');
axis([-5 5 0 1.5]);
% sinc function
t=-5:0.01:5;
y=sin(pi*t)./(pi*t);
% alternate procedure
%y=sinc(t);
subplot(2,2,3); plot(t,y);
xlabel('time —>');ylabel('Amplitude —>');
title('Continuous time sinc signal');
axis([-5 5 -0.5 1.5]);
% Signum signal
t=-10:0.01:10;
y=[-ones(1,1000),zeros(1,1),ones(1,1000)];
% alternate procedure
%y=sign(t);
%y=t./abs(t);
subplot(2,2,4); plot(t,y);
xlabel('time —>');ylabel('Amplitude —>');
title('Continuous time Signum signal');
axis([-5 5 -1.5 1.5]);
```



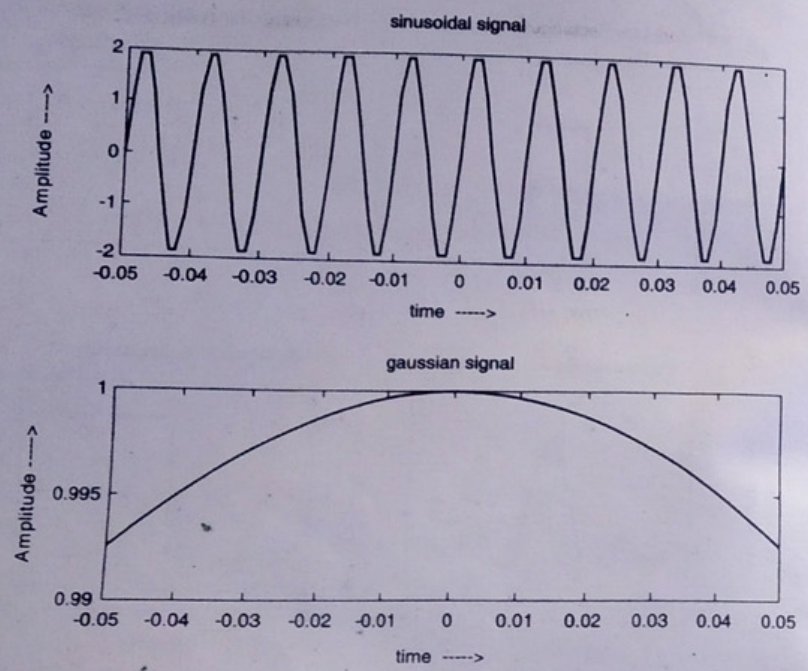
Output:




## Program 1.3

```
% Generation of continuous-time sinusoidal and gaussian signals
clc;close all;clear all;
t=-0.05:0.001:0.05;
f=100;
a=2;
yt=a*sin(2*pi*f*t);
subplot(2,1,1);plot(t,yt);
xlabel('time —>');ylabel('Amplitude —>');
title('sinusoidal signal')
a=3;
yt=exp(-a*t.^2);
subplot(2,1,2);plot(t,yt);
xlabel('time —>');ylabel('Amplitude —>');
title('gaussian signal')
```

Output:

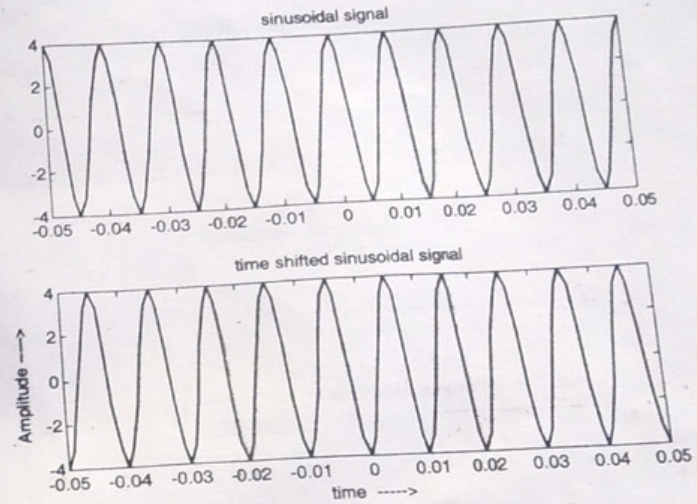



## Program 1.4

```
% Generation of a continuous-time sinusoidal signal of frequency
100Hz in a time duration of 100 msec
% with a time shift of 5 msec.
t=-0.05:0.001:0.05;
f=100;
a=4;
y1=a*cos(2*pi*f*t);
y2=a*cos(2*pi*f*(t-0.005));
subplot(2,1,1);plot(t,y1);
title('sinusoidal signal');
subplot(2,1,2);plot(t,y2);
xlabel('time --->');ylabel('Amplitude --->');
title('time shifted sinusoidal signal');
```



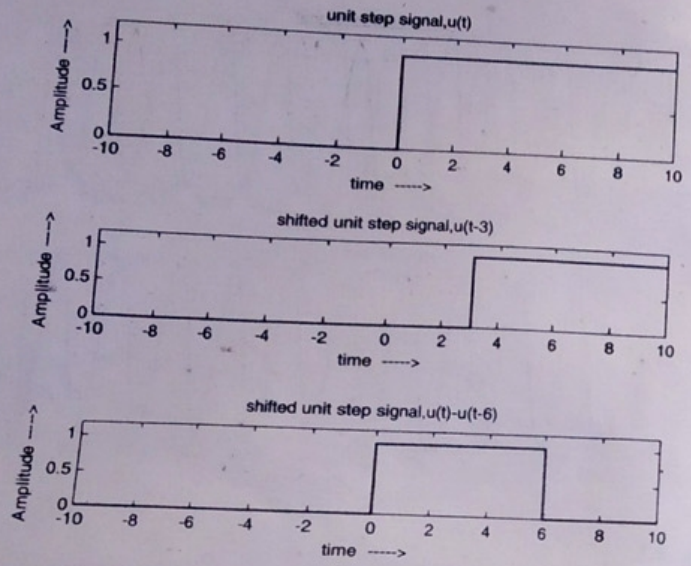
Output:




## Program 1.5

```
% Generation of unit step function using Signum function
% u(t)=0.5sgn(t)+0.5
t=-10:0.01:10;
step=0.5*sign(t)+0.5;
subplot(3,1,1);plot(t,step);
xlabel('time --->');ylabel('Amplitude --->');
title('unit step signal,u(t)');
axis([-10 10 0 1.2]);
% y(t)=u(t-3)
t=-10:0.01:10;
y=0.5*sign(t-3)+0.5;
subplot(3,1,2);plot(t,y);
xlabel('time --->');ylabel('Amplitude --->');
title('shifted unit step signal,u(t-3)');
axis([-10 10 0 1.2]);
% y(t)=u(t)-u(t-6)
t=-10:0.01:10;
y=(0.5*sign(t)+0.5)-(0.5*sign(t-6)+0.5);
subplot(3,1,3);plot(t,y);
xlabel('time --->');ylabel('Amplitude --->');
title('shifted unit step signal,u(t)-u(t-6)');
axis([-10 10 0 1.2]);
```

Output:

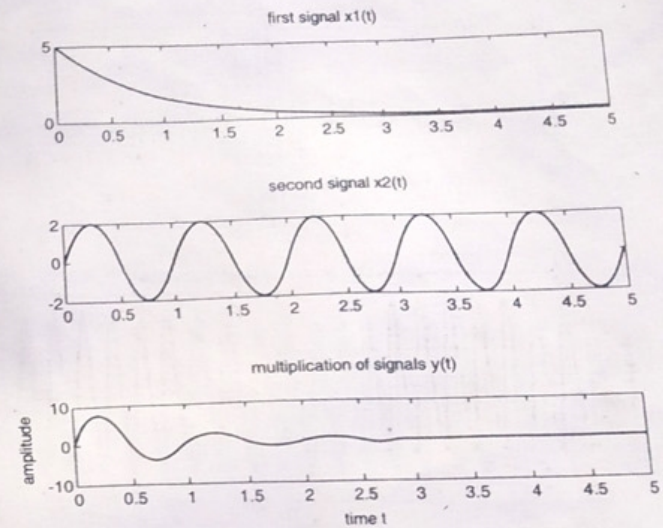



## Program 1.6

```
% Multiplication of two continuous-time signals
% x1(t)=5exp(-at);
t=0:0.001:5;
a=1.2;
x1=5*exp(-a*t);
subplot(3,1,1);plot(t,x1);
title('first signal x1(t)');
% x2(t)=2sin(wt)
f=1;
x2=2*sin(2*pi*f*t);
subplot(3,1,2);plot(t,x2);
title('second signal x2(t)');
% multiplication of x1(t)and x2(t)
y=x1.*x2;
subplot(3,1,3);plot(t,y);
xlabel('time t');ylabel('amplitude');
title('multiplication of signals y(t)');
```



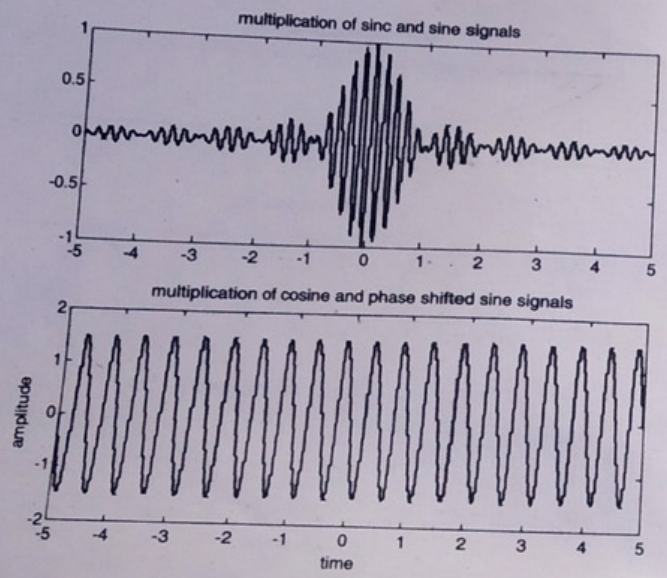
Output:




## Program 1.7

```
% Multiplication of two continuous-time signals
clc;clear all;clear all;
t=-5:0.001:5;
f1=5;
x1=sinc(t).*sin(2*pi*f1*t);
f2=1;
x2=3*cos(2*pi*f2*t).*sin(2*pi*f2*(t+0.5));
subplot(2,1,1);plot(t,x1);
title('multiplication of sinc and sine signals');
subplot(2,1,2);plot(t,x2);
xlabel('time');ylabel('amplitude');
title('multiplication of cosine and phase shifted sine signals');
```

Output:




**Program 1.8**

```
% Shifting and scaling operations using sinc signal
% y1=sinc(t)
t=-10:0.01:10;
y1=sinc(t);
subplot(2,2,1);plot(t,y1);
xlabel('time —>');ylabel('Amplitude —>');
title('sinc signal');
axis([-10 10 -0.5 1.5]);
% y2=sinc(4t)
t=-10:0.01:10;
y2=sinc(4*t);
subplot(2,2,2);plot(t,y2);
xlabel('time —>');ylabel('Amplitude —>');
title('compressed sinc signal');
axis([-10 10 -0.5 1.5]);
% y3=sinc(2t-5)
t=-10:0.01:10;
y3=sinc(2*(t-5));
subplot(2,2,3);plot(t,y3);
xlabel('time —>');ylabel('Amplitude —>');
title('shifted sinc signal');
```



```
axis([-10 10 -0.5 1.5]);
% y4=sinc(0.25t)
t=-10:0.01:10;
y4=sinc(0.5*t);
subplot(2,2,4);plot(t,y4);
xlabel('time —>');ylabel('Amplitude —>');
title('expanded sinc signal');
axis([-10 10 -0.5 1.5]);
```

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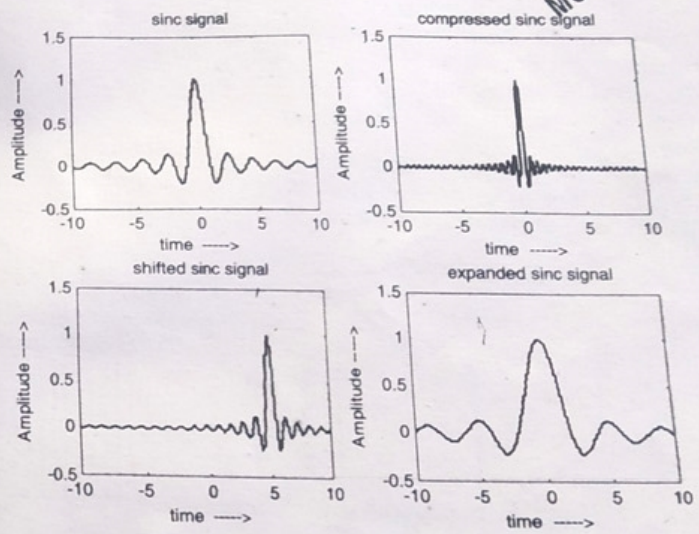
Output:

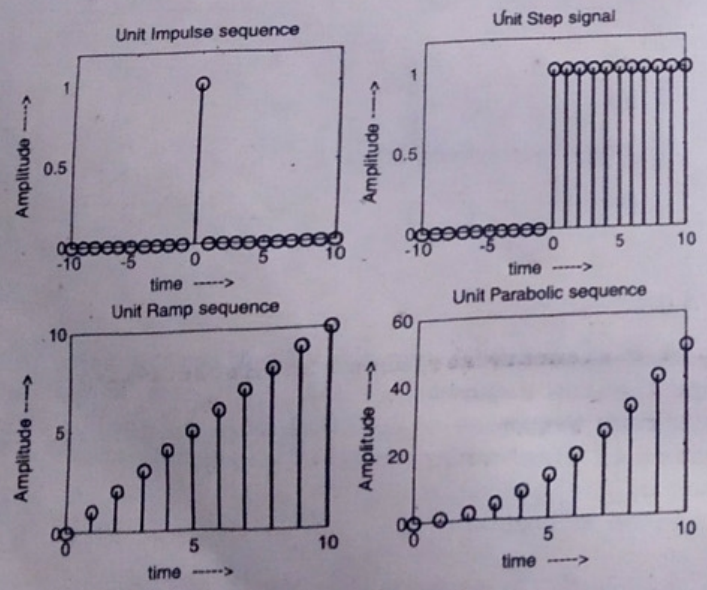



**Program 1.9**

```
Generation of elementary sequences in discrete-time
  (i) Unit Impulse sequence      (ii) Unit step sequence
(iii) Unit ramp sequence         (iv) Unit parabolic sequence
% Generation of Elementary signals in Discrete time
clc; close all; clear all;
% Unit Impulse sequence
n=-10:1:10;
impulse=[zeros(1,10),ones(1,1),zeros(1,10)];
subplot(2,2,1);stem(n,impulse);
xlabel('Discrete time n —>');ylabel('Amplitude —>');
title('Unit Impulse sequence');
axis([-10 10 0 1.2]);
% Unit Step sequence
```

```
n=-10:1:10;
step=[zeros(1,10),ones(1,11)];
subplot(2,2,2);stem(n,step);
xlabel(''Discrete time n —>');ylabel('Amplitude —>');
title('Unit Step sequence');
axis([-10 10 0 1.2]);
% Unit Ramp sequence
n=0:1:10;
ramp=n;
subplot(2,2,3);stem(n,ramp);
xlabel(''Discrete time n —>');ylabel('Amplitude —>');
title('Unit Ramp sequence');
% Unit Parabolic sequence
n=0:1:10;
parabola=0.5*(n.^2);
subplot(2,2,4);stem(n,parabola);
xlabel(''Discrete time n —>');ylabel('Amplitude —>');
title('Unit Parabolic sequence');
```

Output:




## Program 1.10

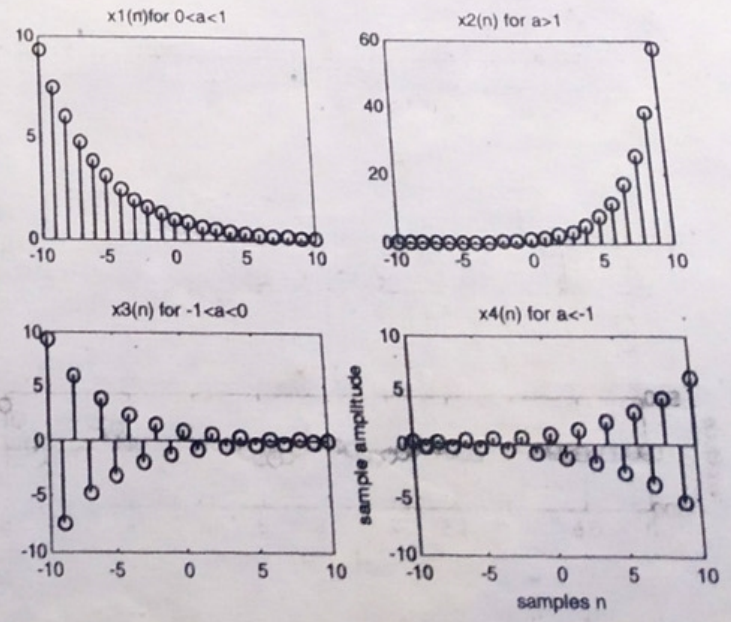
**Generation of discrete-time Exponential sequence for various values of 'a'.**

```
% Program to generate a DT exponential sequence
```



```
clc;close all;clear all;
n=-10:1:10;
% for 0<a<1
a=0.8;
x1=a.^n;
subplot(2,2,1);stem(n,x1);
title('x1(n)for 0<a<1');
% for a>1
a=1.5;
x2=a.^n;
subplot(2,2,2);stem(n,x2);
title('x2(n) for a>1');
% for -1<a<0
a=-0.8;
x3=a.^n;
subplot(2,2,3);stem(n,x3);
title('x3(n) for -1<a<0');
% for a<-1
a=-1.2;
x4=a.^n;
subplot(2,2,4);stem(n,x4);
title('x4(n) for a<-1');
xlabel('samples n');ylabel('sample amplitude')
```

Output:

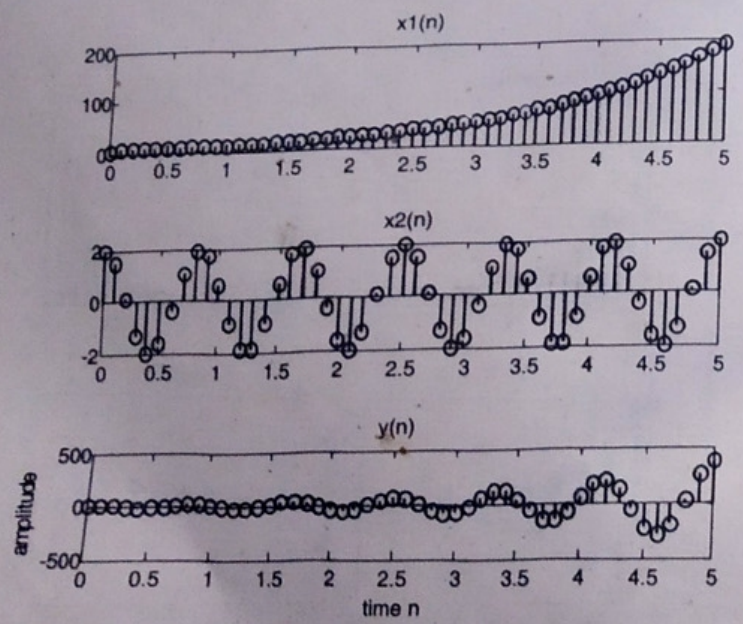


## Program 1.11

**Multiplication of two discrete-time sequences**

```
% mutiplication of DT signals
% x1(n)=6*a^n;
n=0:0.1:5;
a=2;
x1=6*(a.^n);
subplot(3,1,1);stem(n,x1);
title('x1(n)');
% x2(n)=2*cos(wn)
f=1.2;
x2=2*cos(2*pi*f*n);
subplot(3,1,2);stem(n,x2);
title('x2(n)');
% multiplication of two sequences
y=x1.*x2;
subplot(3,1,3);stem(n,y);
xlabel('time n');ylabel('amplitude');
title('y(n)');
```

Output:



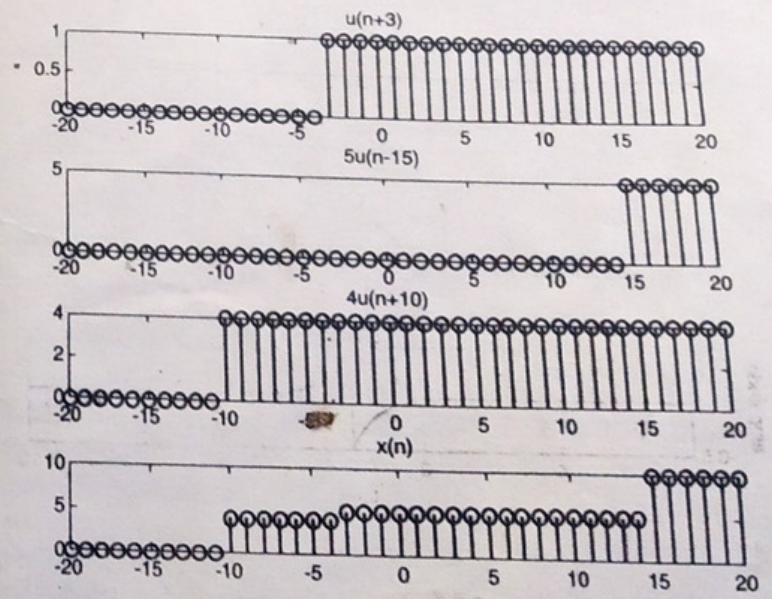



## Program 1.12

**Generation of the discrete-time sequence x(n)=u(n+3)+5u(n-15)+4u(n+10)**

```
% Generate the following step sequences.
n=-20:1:20;
u=[zeros(1,20),ones(1,21)];
u1=[zeros(1,17),ones(1,24)];
u2=[zeros(1,35),ones(1,6)];u2=5*u2;
u3=[zeros(1,10),ones(1,31)];u3=4*u3;
x=u1+u2+u3;
subplot(4,1,1);stem(n,u1);
title('u(n+3)');
subplot(4,1,2);stem(n,u2);
title('5u(n-15)');
subplot(4,1,3);stem(n,u3);
title('4u(n+10)');
subplot(4,1,4);stem(n,x);
title('x(n)');
```

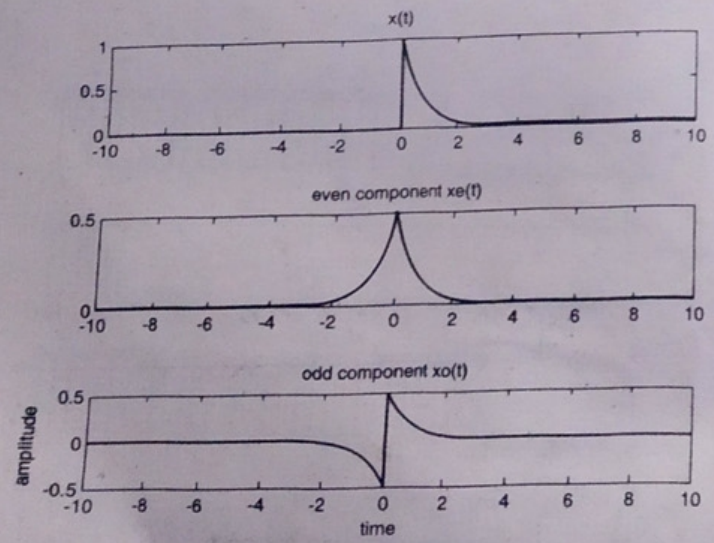
Output:



### Program 1.13

```
% Even and odd components of a continuous-time signal
% x(t)=exp(-2t)u(t)
clc;close all;clear all;
t=-10:0.01:10;
% actual signal x(t)
u=0.5*sign(t)+0.5
x1=exp(-1.5*t).*u;
subplot(3,1,1);plot(t,x1);
title('x(t)');
% time reversed signal x(-t)
x2=fliplr(x1);
% Even component xe(t)
xe=0.5*(x1+x2);
subplot(3,1,2);plot(t,xe);
title('even component xe(t)');
% Odd component xo(t)
xo=0.5*(x1-x2);
subplot(3,1,3);plot(t,xo);
xlabel('time');ylabel('amplitude');
title('odd component xo(t)');
```

Output:



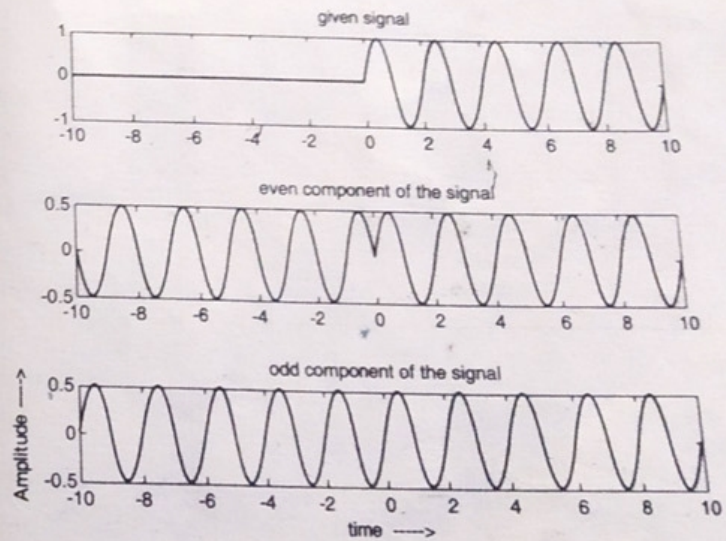

### Program 1.14

```
% Evaluation of the even and odd components of a signal
% x(t)=sin(wt)u(t)
clc;close all;clear all;
```



```
t=-10:0.01:10;
u=0.5*sign(t)+0.5;
w=2*pi*0.5;
x1=sin(w*t).*u;
x2=fliplr(x1);
xe=0.5*(x1+x2);
xo=0.5*(x1-x2);
subplot(3,1,1);plot(t,x1);
title('given signal')
subplot(3,1,2);plot(t,xe);
title('even component of the signal')
subplot(3,1,3);plot(t,xo);
title('odd component of the signal')
xlabel('time --->');ylabel('Amplitude --->');
```

Output:

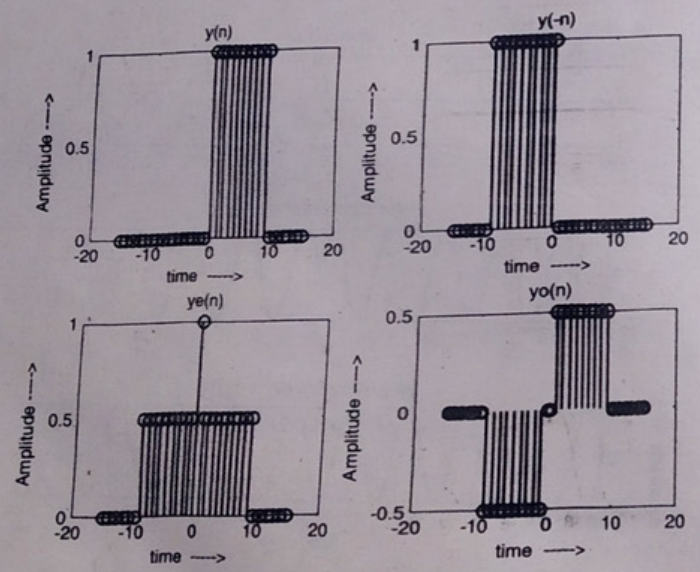



### Program 1.15

```
Evaluation of the even and odd components of a sequence
y(n)=u(n)-u(n-10)
% Even and Odd components of a sequence
n=-15:1:15;
y1=[zeros(1,15),ones(1,10),zeros(1,6)];
y2=fliplr(y1);
ye=0.5*(y1+y2);
```

```
yo=0.5*(y1-y2);
subplot(2,2,1);stem(n,y1);
xlabel('time —>');ylabel('Amplitude —>');
title('y(n)');
subplot(2,2,2);stem(n,y2);
xlabel('time —>');ylabel('Amplitude —>');
title('y(-n)');
subplot(2,2,3);stem(n,ye);
xlabel('time —>');ylabel('Amplitude —>');
title('ye(n)');
subplot(2,2,4);stem(n,yo);
xlabel('time —>');ylabel('Amplitude —>');
title('yo(n)');
```

**Output:**




## SHORT QUESTIONS WITH ANSWERS

1. Define a signal.

   *Ans.* A signal is defined as a single-valued function of one or more independent variables which contain some information.

2. What is one dimensional signal?

   *Ans.* A signal which depends on only one independent variable is called a one dimensional signal.



3. What is signal modelling?

   *Ans.* The representation of a signal by mathematical expression is known as signal modelling.

4. What are the different types of representing discrete-time signals?

   *Ans.* There are four different types of representation of discrete-time signals. They are:

   (a) Graphical representation
   (b) Functional representation
   (c) Tabular representation
   (d) Sequence representation

5. Define unit step function.

   *Ans.* The continuous-time unit step function $u(t)$ is defined as:

$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

The discrete-time unit step sequence $u(n)$ is defined as:

$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

6. Define unit ramp function.

   *Ans.* The continuous-time unit ramp function $r(t)$ is defined as:

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

or

$$r(t) = tu(t)$$

The discrete-time unit ramp sequence $r(n)$ is defined as:

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

or

$$r(n) = nu(n)$$

7. Define unit parabolic function.

   *Ans.* The continuous-time unit parabolic function $p(t)$ is defined as:

$$p(t) = \begin{cases} t^2/2 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

or

$$p(t) = \frac{t^2}{2}u(t)$$

The discrete-time unit parabolic sequence $p(n)$ is defined as:

$$p(n) = \begin{cases} n^2/2 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$