CS545–Introduction to Robotics

Homework Assignment 3 (Due April 12)

In the following problems, you should use MATLAB to compute numerical results and visualize the data, and Simulink for simulations. A handout about getting started with MATLAB is in

http://www-clmc.usc.edu/~cs545/homework.php

This web page also contains all the files needed below. IMPORTANT: In your solutions of the homework, also provide intermediate steps how you derived the solution to a problem.

1. (100 Points) This problem is to familiarize you with data filtering, by comparing a 2nd Order Butterworth Filter with a Kalman Filter. The data in the file noisy.data was generated from the following difference equations:

$$\begin{aligned} x^{n+1} &= 0.5x^n + 3.5u^n + \varepsilon_x \\ y^n &= x^n + \varepsilon_y \end{aligned} \tag{1}$$

where $\varepsilon_x \sim N\left(0,\sigma^2{}_x\right)$, $\varepsilon_y \sim N\left(0,\sigma^2{}_y\right)$ are Gaussian noise variables. The file noisy.data contains 1000 observations of $\left(y^n,x^n,u^n\right)$ from 1000 time steps of the system sampled at $\Delta t = 0.01$ seconds.

a) A second order Butterworth filter has the following mathematical form:

$$x_f{}^n = b_1 y^n + b_2 y^{n-1} + b_3 y^{n-2} - a_2 x_f{}^{n-1} - a_3 x_f{}^{n-2}$$

where the subscript "f" denotes the filtered variable. The "butter" function in Matlab allows you to generate the filter coefficients for a discrete Butterworth filter (try "help butter" in Matlab). The function takes two arguments, the filter order and the cutoff frequency, expressed as a fraction of half of the sampling frequency, also called the Nyquist frequency. The filter order will be "2" for a second order filter as described above, and our sampling frequency is 100Hz. Calculate the filter coefficients b_1,b_2,b_3,a_2,a_3 for a low pass filter with cutoff frequency 5Hz. Provide a print-out of the coefficients.

b) Use the Matlab "filter" function and apply your filter to the y^n data from noisy.data. Plot the filtered, i.e., x_f^n and the true, i.e., x^n data on top of each other, provide the print-out, and comment on the quality of the filter. Estimate the delay of the filtered data from your plots.

c) Assume $\sigma_x^2 = 0.01$ and $\sigma_y^2 = 1.0$. Implement a Kalman filter from the equations provided in class for the system in Matlab, using your knowledge of $\left(y^n,u^n\right)$ (but not x^n -- this is only used for comparisons.) Provide a print-out of your Matlab program, and a plot of the filtered data, i.e., $\hat{x}^n$, and the true data x^n. Plot the gain K and the posterior covariance matrix P as a function of time (iterations). How did you initialize P? Estimate the delay of the filtered data from your plots. Comment on the quality of the filter in comparison to the Butterworth filter.