1 Automated equations

All equations are derived assuming a Hamiltonian and $\hat{S}$ of the form:

$$\hat{H} = \sum_{a,b} |a\rangle \langle b| \, h_b^a(q) \qquad \hat{S} = \sum_{c,d} |c\rangle \langle d| \, s_d^c(q) \qquad \hat{\Omega} = \hat{i} + \hat{i}^\dagger + \hat{i}\hat{j} + \hat{i}\hat{j}^\dagger + \hat{i}^\dagger\hat{j}^\dagger + \cdots \tag{1}$$

when we limit ourselves to at most 2nd order terms (creation operator: $\hat{b}$, annihilation operator: $\hat{b}^\dagger$)

$$\mathbf{h}(q) = \mathbf{h} + \mathbf{h}_i\hat{i} + \mathbf{h}^i\hat{i}^\dagger + \mathbf{h}_j^i\hat{i}^\dagger\hat{j} + \frac{1}{2}\mathbf{h}_{ij}\hat{i}\hat{j} + \frac{1}{2}\mathbf{h}^{ij}\hat{i}^\dagger\hat{j}^\dagger \tag{2}$$

and

$$\mathbf{s}(q) = \mathbf{s}_i\hat{i} + \mathbf{s}^i\hat{i}^\dagger + \mathbf{s}_j^i\hat{i}^\dagger\hat{j} + \frac{1}{2}\mathbf{s}_{ij}\hat{i}\hat{j} + \frac{1}{2}\mathbf{s}^{ij}\hat{i}^\dagger\hat{j}^\dagger \tag{3}$$

Define the following: (not correct but mechanically useful)

$$f = \overbrace{\hat{b}^\dagger\hat{b}} \qquad \bar{f} = \overbrace{\hat{b}\hat{b}^\dagger} \qquad (\overbrace{\hat{b}\hat{b}^\dagger} - \overbrace{\hat{b}^\dagger\hat{b}}) = (\bar{f} - f) = 1 \qquad \bar{f} = 1, f = 0 \tag{4}$$

and

$$\mathbf{t}_i \equiv f\mathbf{s}_i \qquad \mathbf{t}^i \equiv \bar{f}\mathbf{s}^i \qquad \mathbf{t}_j^i \equiv \bar{f}f\mathbf{s}_j^i \qquad \mathbf{t}^{ij} \equiv \bar{f}^2\mathbf{s}^{ij} \qquad \mathbf{t}_{ij} \equiv f^2\mathbf{s}_{ij} \qquad \text{and so forth ....} \tag{5}$$

The amplitude equation is

$$LHS = RHS \tag{6}$$

$$\langle a|\hat{\Omega}_\lambda \frac{\mathrm{d}}{\mathrm{d}\tau}e^{\hat{S}} + \hat{\Omega}_\lambda e^{\hat{S}}\varepsilon|b\rangle = \langle a|\hat{\Omega}_\lambda \hat{H}e^{\hat{S}}|b\rangle. \tag{7}$$

$$\hat{\Omega} = 1$$

$$LHS = i\left(\varepsilon\right)$$

$$
\begin{aligned}
RHS = (&\mathbf{h}_0 + \mathbf{h}_i \mathbf{t}^i + \frac{1}{2!}\mathbf{h}_{ij}\mathbf{t}^{ij} + \mathbf{h}^i \mathbf{t}_i + \mathbf{h}^i_j \mathbf{t}^j_i + \frac{1}{2!}\mathbf{h}^{ij}\mathbf{t}_{ij} + \frac{1}{2!2!}\mathbf{h}_{ij}\mathbf{t}^i\mathbf{t}^j \\
&+ \frac{1}{3!}\mathbf{h}_{ijk}\mathbf{t}^i\mathbf{t}^{jk} + \frac{1}{4!2!}\mathbf{h}_{ijkl}\mathbf{t}^{ij}\mathbf{t}^{kl} + \mathbf{h}^i_j \mathbf{t}_i \mathbf{t}^j + \frac{1}{2!}\mathbf{h}^i_{jk}\mathbf{t}_i\mathbf{t}^{jk} + \frac{1}{2!}\mathbf{h}^i_{jk}\mathbf{t}^j\mathbf{t}^k_i + \frac{1}{3!}\mathbf{h}^i_{jkl}\mathbf{t}^j_i\mathbf{t}^{kl} + \frac{1}{2!2!}\mathbf{h}^{ij}\mathbf{t}_i\mathbf{t}_j \\
&+ \frac{1}{2!}\mathbf{h}^{ij}_k\mathbf{t}_i\mathbf{t}^k_j + \frac{1}{2!}\mathbf{h}^{ij}_k\mathbf{t}_{ij}\mathbf{t}^k + \frac{1}{2!2!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}^k_i\mathbf{t}^l_j + \frac{1}{2!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_{ij}\mathbf{t}^{kl} + \frac{1}{3!}\mathbf{h}^{ijk}\mathbf{t}_i\mathbf{t}_{jk} + \frac{1}{3!}\mathbf{h}^{ijk}_l\mathbf{t}_{ij}\mathbf{t}^l_k + \frac{1}{4!2!}\mathbf{h}^{ijkl}\mathbf{t}_{ij}\mathbf{t}_{kl} \\
&+ \frac{1}{3!3!}\mathbf{h}_{ijk}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^k + \frac{2!}{4!3!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^{kl} + \frac{1}{3!}\mathbf{h}^i_{jk}\mathbf{t}_i\mathbf{t}^j\mathbf{t}^k + \frac{2!}{3!3!}\mathbf{h}^i_{jkl}\mathbf{t}^j\mathbf{t}^k\mathbf{t}^l_i + \frac{1}{3!}\mathbf{h}^i_{jkl}\mathbf{t}_i\mathbf{t}^j\mathbf{t}^{kl} + \frac{1}{3!}\mathbf{h}^{ij}_k\mathbf{t}_i\mathbf{t}_j\mathbf{t}^k + \frac{1}{3!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_{ij}\mathbf{t}^k\mathbf{t}^l \\
&+ \frac{1}{3!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_i\mathbf{t}_j\mathbf{t}^{kl} + \frac{1}{2!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_i\mathbf{t}^k\mathbf{t}^l_j + \frac{1}{3!3!}\mathbf{h}^{ijk}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_k + \frac{2!}{3!3!}\mathbf{h}^{ijk}_l\mathbf{t}_i\mathbf{t}_j\mathbf{t}^l_k + \frac{1}{3!}\mathbf{h}^{ijk}_l\mathbf{t}_i\mathbf{t}_{jk}\mathbf{t}^l + \frac{2!}{4!3!}\mathbf{h}^{ijkl}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_{kl} + \frac{1}{4!4!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^k\mathbf{t}^l) + () + ()
\end{aligned}
\tag{8}
$$

1.1 Linear Equations

$$\hat{\Omega} = \hat{i}$$

$$LHS = i\left(\frac{\mathrm{d}\mathbf{t}^i}{\mathrm{d}\tau} + \mathbf{t}^i\varepsilon\right)$$

$$
\begin{aligned}
RHS = (&\bar{f}\mathbf{h}^z + \mathbf{h}_i\mathbf{t}^{iz} + \mathbf{h}^i\mathbf{t}^z_i + \bar{f}\mathbf{h}^z_i\mathbf{t}^i + \bar{f}\frac{1}{2!}\mathbf{h}^z_{ij}\mathbf{t}^{ij} + \bar{f}\frac{1}{2!}\mathbf{h}^{iz}\mathbf{t}_i + \bar{f}\frac{1}{2!}\mathbf{h}^{iz}_j\mathbf{t}^j_i \\
&+ \bar{f}\frac{1}{3!}\mathbf{h}^{ijz}\mathbf{t}_{ij} + \frac{1}{2!}\mathbf{h}_{ij}\mathbf{t}^i\mathbf{t}^{jz} + \frac{1}{3!}\mathbf{h}_{ijk}\mathbf{t}^{iz}\mathbf{t}^{jk} + \mathbf{h}^i_j\mathbf{t}^z_i\mathbf{t}^j + \mathbf{h}^i_j\mathbf{t}_i\mathbf{t}^{jz} + \frac{1}{2!}\mathbf{h}^i_{jk}\mathbf{t}^{jz}\mathbf{t}^k_i + \bar{f}\frac{1}{2!2!}\mathbf{h}^z_{ij}\mathbf{t}^i\mathbf{t}^j \\
&+ \frac{1}{2!}\mathbf{h}^i_{jk}\mathbf{t}^z_i\mathbf{t}^{jk} + \bar{f}\frac{1}{3!}\mathbf{h}^z_{ijk}\mathbf{t}^i\mathbf{t}^{jk} + \frac{1}{2!}\mathbf{h}^{ij}\mathbf{t}_i\mathbf{t}^z_j + \bar{f}\frac{1}{2!}\mathbf{h}^{iz}_j\mathbf{t}_i\mathbf{t}^j + \frac{1}{2!}\mathbf{h}^{ij}_k\mathbf{t}^z_i\mathbf{t}^k_j + \frac{1}{2!}\mathbf{h}^{ij}_k\mathbf{t}_{ij}\mathbf{t}^{kz} + \bar{f}\frac{1}{2!2!}\mathbf{h}^{iz}_{jk}\mathbf{t}^j\mathbf{t}^k_i \\
&+ \bar{f}\frac{1}{2!2!}\mathbf{h}^{iz}_{jk}\mathbf{t}_i\mathbf{t}^{jk} + \frac{1}{3!}\mathbf{h}^{ijk}\mathbf{t}^z_i\mathbf{t}_{jk} + \bar{f}\frac{1}{3!2!}\mathbf{h}^{ijz}\mathbf{t}_i\mathbf{t}_j + \bar{f}\frac{1}{3!}\mathbf{h}^{ijz}_k\mathbf{t}_{ij}\mathbf{t}^k + \bar{f}\frac{1}{3!}\mathbf{h}^{ijz}_k\mathbf{t}_i\mathbf{t}^k_j + \bar{f}\frac{1}{4!}\mathbf{h}^{ijkz}\mathbf{t}_i\mathbf{t}_{jk} + \frac{2!}{3!3!}\mathbf{h}_{ijk}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^{kz} \\
&+ \frac{1}{4!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}^{jz}\mathbf{t}^{kl} + \frac{1}{2!}\mathbf{h}^i_{jk}\mathbf{t}_i\mathbf{t}^j\mathbf{t}^{kz} + \frac{1}{3!}\mathbf{h}^i_{jk}\mathbf{t}^z_i\mathbf{t}^j\mathbf{t}^k + \frac{1}{3!}\mathbf{h}^i_{jkl}\mathbf{t}_i\mathbf{t}^{jz}\mathbf{t}^{kl} + \frac{1}{3!}\mathbf{h}^i_{jkl}\mathbf{t}^j\mathbf{t}^{kz}\mathbf{t}^l_i + \bar{f}\frac{1}{3!3!}\mathbf{h}^z_{ijk}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^k + \frac{1}{3!}\mathbf{h}^i_{jkl}\mathbf{t}^z_i\mathbf{t}^j\mathbf{t}^{kl} \\
&+ \frac{1}{3!}\mathbf{h}^{ij}_k\mathbf{t}_i\mathbf{t}_j\mathbf{t}^{kz} + \frac{1}{2!}\mathbf{h}^{ij}_k\mathbf{t}_i\mathbf{t}^z_j\mathbf{t}^k + \frac{1}{2!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}^z_i\mathbf{t}^k\mathbf{t}^l_j + \frac{1}{2!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_{ij}\mathbf{t}^k\mathbf{t}^{lz} + \frac{1}{2!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_i\mathbf{t}^{kz}\mathbf{t}^l_j + \frac{1}{2!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_i\mathbf{t}^z_j\mathbf{t}^{kl} + \bar{f}\frac{1}{3!2!}\mathbf{h}^{iz}_{jk}\mathbf{t}_i\mathbf{t}^j\mathbf{t}^k \\
&+ \frac{2!}{3!3!}\mathbf{h}^{ijk}\mathbf{t}_i\mathbf{t}_j\mathbf{t}^z_k + \frac{1}{3!}\mathbf{h}^{ijk}_l\mathbf{t}_i\mathbf{t}^z_j\mathbf{t}^l_k + \bar{f}\frac{2!}{3!3!}\mathbf{h}^{ijz}_k\mathbf{t}_i\mathbf{t}_j\mathbf{t}^k + \frac{1}{3!}\mathbf{h}^{ijk}_l\mathbf{t}^z_i\mathbf{t}_{jk}\mathbf{t}^l + \frac{1}{3!}\mathbf{h}^{ijk}_l\mathbf{t}_i\mathbf{t}_{jk}\mathbf{t}^{lz} + \bar{f}\frac{1}{4!3!}\mathbf{h}^{ijkz}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_k + \frac{1}{4!}\mathbf{h}^{ijkl}\mathbf{t}_i\mathbf{t}^z_j\mathbf{t}_{kl} \\
&+ \frac{2!}{4!4!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^k\mathbf{t}^{lz} + \frac{1}{4!}\mathbf{h}^i_{jkl}\mathbf{t}_i\mathbf{t}^j\mathbf{t}^k\mathbf{t}^{lz} + \frac{2!}{3!4!}\mathbf{h}^i_{jkl}\mathbf{t}^z_i\mathbf{t}^j\mathbf{t}^k\mathbf{t}^l + \frac{3!}{2!2!4!}\mathbf{h}^{ij}_{kl}\mathbf{t}_i\mathbf{t}^z_j\mathbf{t}^k\mathbf{t}^l + \frac{3!}{2!2!4!}\mathbf{h}^{ij}_{kl}\mathbf{t}_i\mathbf{t}_j\mathbf{t}^k\mathbf{t}^{lz} + \frac{2!}{3!4!}\mathbf{h}^{ijk}_l\mathbf{t}_i\mathbf{t}_j\mathbf{t}_k\mathbf{t}^{lz} + \frac{1}{4!}\mathbf{h}^{ijk}_l\mathbf{t}_i\mathbf{t}_j\mathbf{t}^z_k\mathbf{t}^l)
\end{aligned}
\tag{9}
$$

$$\hat{\Omega} = \hat{i}^{\dagger}$$

$$LHS = i\left(\frac{\mathrm{d}\mathbf{t}_i}{\mathrm{d}\tau} + \mathbf{t}_i\varepsilon\right)$$

$$
\begin{aligned}
RHS = (&f\mathbf{h}_z + \mathbf{h}_i\mathbf{t}_z^i + f\frac{1}{2!}\mathbf{h}_{iz}\mathbf{t}^i + f\frac{1}{3!}\mathbf{h}_{ijz}\mathbf{t}^{ij} + \mathbf{h}^i\mathbf{t}_{iz} + f\mathbf{h}_z^i\mathbf{t}_i + f\frac{1}{2!}\mathbf{h}_{jz}^i\mathbf{t}_i^j \\
&+ f\frac{1}{2!}\mathbf{h}_z^{ij}\mathbf{t}_{ij} + \frac{1}{2!}\mathbf{h}_{ij}\mathbf{t}^i\mathbf{t}_z^j + \frac{1}{3!}\mathbf{h}_{ijk}\mathbf{t}_z^i\mathbf{t}^{jk} + f\frac{1}{3!2!}\mathbf{h}_{ijz}\mathbf{t}^i\mathbf{t}^j + f\frac{1}{4!}\mathbf{h}_{ijkz}\mathbf{t}^i\mathbf{t}^{jk} + \mathbf{h}_j^i\mathbf{t}_{iz}\mathbf{t}^j + \mathbf{h}_j^i\mathbf{t}_i\mathbf{t}_z^j \\
&+ \frac{1}{2!}\mathbf{h}_{jk}^i\mathbf{t}_{iz}\mathbf{t}^{jk} + f\frac{1}{2!}\mathbf{h}_{jz}^i\mathbf{t}_i\mathbf{t}^j + \frac{1}{2!}\mathbf{h}_{jk}^i\mathbf{t}_z^j\mathbf{t}_i^k + f\frac{1}{3!}\mathbf{h}_{jkz}^i\mathbf{t}_i\mathbf{t}^{jk} + f\frac{1}{3!}\mathbf{h}_{jkz}^i\mathbf{t}^j\mathbf{t}_i^k + \frac{1}{2!}\mathbf{h}^{ij}\mathbf{t}_i\mathbf{t}_{jz} + \frac{1}{2!}\mathbf{h}_k^{ij}\mathbf{t}_{iz}\mathbf{t}_j^k \\
&+ \frac{1}{2!}\mathbf{h}_k^{ij}\mathbf{t}_{ij}\mathbf{t}_z^k + f\frac{1}{2!2!}\mathbf{h}_z^{ij}\mathbf{t}_i\mathbf{t}_j + f\frac{1}{2!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_i\mathbf{t}_j^k + f\frac{1}{2!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_{ij}\mathbf{t}^k + \frac{1}{3!}\mathbf{h}^{ijk}\mathbf{t}_{iz}\mathbf{t}_{jk} + f\frac{1}{3!}\mathbf{h}_z^{ijk}\mathbf{t}_i\mathbf{t}_{jk} + \frac{2!}{3!3!}\mathbf{h}_{ijk}\mathbf{t}^i\mathbf{t}^j\mathbf{t}_z^k \\
&+ \frac{1}{4!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}_z^j\mathbf{t}^{kl} + f\frac{1}{4!3!}\mathbf{h}_{ijkz}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^k + \frac{1}{2!}\mathbf{h}_{jk}^i\mathbf{t}_i\mathbf{t}^j\mathbf{t}_z^k + \frac{1}{3!}\mathbf{h}_{jk}^i\mathbf{t}_{iz}\mathbf{t}^j\mathbf{t}^k + \frac{1}{3!}\mathbf{h}_{jkl}^i\mathbf{t}_i\mathbf{t}_z^j\mathbf{t}^{kl} + f\frac{2!}{3!3!}\mathbf{h}_{jkz}^i\mathbf{t}_i\mathbf{t}^j\mathbf{t}^k + \frac{1}{3!}\mathbf{h}_{jkl}^i\mathbf{t}^j\mathbf{t}_z^k\mathbf{t}_i^l \\
&+ \frac{1}{3!}\mathbf{h}_{jkl}^i\mathbf{t}_{iz}\mathbf{t}^j\mathbf{t}^{kl} + \frac{1}{3!}\mathbf{h}_k^{ij}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_z^k + \frac{1}{2!}\mathbf{h}_k^{ij}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}^k + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}^{kl} + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{iz}\mathbf{t}^k\mathbf{t}_j^l + f\frac{1}{3!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_i\mathbf{t}_j\mathbf{t}^k + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{ij}\mathbf{t}^k\mathbf{t}_z^l \\
&+ \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_i\mathbf{t}_z^k\mathbf{t}_j^l + \frac{2!}{3!3!}\mathbf{h}^{ijk}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_{kz} + f\frac{1}{3!3!}\mathbf{h}_z^{ijk}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_k + \frac{1}{3!}\mathbf{h}_l^{ijk}\mathbf{t}_{iz}\mathbf{t}_{jk}\mathbf{t}^l + \frac{1}{3!}\mathbf{h}_l^{ijk}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}_k^l + \frac{1}{3!}\mathbf{h}_l^{ijk}\mathbf{t}_i\mathbf{t}_{jk}\mathbf{t}_z^l + \frac{1}{4!}\mathbf{h}^{ijkl}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}_{kl} \\
&+ \frac{2!}{4!4!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^k\mathbf{t}_z^l + \frac{2!}{3!4!}\mathbf{h}_{jkl}^i\mathbf{t}_{iz}\mathbf{t}^j\mathbf{t}^k\mathbf{t}^l + \frac{1}{4!}\mathbf{h}_{jkl}^i\mathbf{t}_i\mathbf{t}^j\mathbf{t}^k\mathbf{t}_z^l + \frac{3!}{2!2!4!}\mathbf{h}_{kl}^{ij}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}^k\mathbf{t}^l + \frac{3!}{2!2!4!}\mathbf{h}_{kl}^{ij}\mathbf{t}_i\mathbf{t}_j\mathbf{t}^k\mathbf{t}_z^l + \frac{1}{4!}\mathbf{h}_l^{ijk}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_{kz}\mathbf{t}^l + \frac{2!}{3!4!}\mathbf{h}_l^{ijk}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_k\mathbf{t}_z^l)
\end{aligned}
$$

(10)

1.2 Quadratic Equations

$$\hat{\Omega} = \hat{i}\hat{j}$$

$$LHS = i\left(\frac{d\mathbf{t}^{ij}}{d\tau} + \frac{d\mathbf{t}^i}{d\tau}\mathbf{t}^j + \mathbf{t}^i\frac{d\mathbf{t}^j}{d\tau} + \mathbf{t}^{ij}\varepsilon + \mathbf{t}^i\mathbf{t}^j\varepsilon\right)$$

$$
\begin{aligned}
RHS = & \left(\bar{f}^2\mathbf{h}^{zy} + \bar{f}\mathbf{h}^z_i\mathbf{t}^{iy} + \bar{f}\frac{1}{2!}\mathbf{h}^{iz}\mathbf{t}^y_i + \bar{f}^2\frac{1}{2!}\mathbf{h}^{zy}_i\mathbf{t}^i + \bar{f}^2\frac{1}{2!2!}\mathbf{h}^{zy}_{ij}\mathbf{t}^{ij} + \bar{f}^2\frac{1}{3!}\mathbf{h}^{izy}\mathbf{t}_i + \bar{f}^2\frac{1}{3!}\mathbf{h}^{izy}_j\mathbf{t}^j_i\right. \\
& + \bar{f}^2\frac{1}{4!}\mathbf{h}^{ijzy}\mathbf{t}_{ij} + \frac{1}{2!2!}\mathbf{h}_{ij}\mathbf{t}^{iz}\mathbf{t}^{jy} + \mathbf{h}^i_j\mathbf{t}^z_i\mathbf{t}^{jy} + \bar{f}\frac{1}{2!}\mathbf{h}^z_{ij}\mathbf{t}^i\mathbf{t}^{jy} + \bar{f}\frac{1}{3!}\mathbf{h}^z_{ijk}\mathbf{t}^{iy}\mathbf{t}^{jk} + \frac{1}{2!2!}\mathbf{h}^{ij}\mathbf{t}^z_i\mathbf{t}^y_j + \bar{f}\frac{1}{2!}\mathbf{h}^{iz}_j\mathbf{t}^y_i\mathbf{t}^j \\
& + \bar{f}\frac{1}{2!}\mathbf{h}^{iz}_j\mathbf{t}_i\mathbf{t}^{jy} + \bar{f}^2\frac{1}{2!2!2!}\mathbf{h}^{zy}_{ij}\mathbf{t}^i\mathbf{t}^j + \bar{f}\frac{1}{2!2!}\mathbf{h}^{iz}_{jk}\mathbf{t}^{jy}\mathbf{t}^k_i + \bar{f}\frac{1}{2!2!}\mathbf{h}^{iz}_{jk}\mathbf{t}^y_i\mathbf{t}^{jk} + \bar{f}\frac{1}{3!}\mathbf{h}^{ijz}\mathbf{t}_i\mathbf{t}^y_j + \bar{f}\frac{1}{3!}\mathbf{h}^{ijz}_k\mathbf{t}^z_i\mathbf{t}^y_j\mathbf{t}^k + \bar{f}^2\frac{1}{3!}\mathbf{h}^{izy}_j\mathbf{t}_i\mathbf{t}^j \\
& + \bar{f}\frac{1}{3!}\mathbf{h}^{ijz}_k\mathbf{t}_{ij}\mathbf{t}^{ky} + \bar{f}\frac{1}{4!}\mathbf{h}^{ijkz}\mathbf{t}^y_i\mathbf{t}_{jk} + \bar{f}^2\frac{1}{4!2!}\mathbf{h}^{ijzy}\mathbf{t}_i\mathbf{t}_j + \frac{2!}{3!3!}\mathbf{h}_{ijk}\mathbf{t}^i\mathbf{t}^{jz}\mathbf{t}^{ky} + \frac{2!}{4!3!}\mathbf{h}_{ijkl}\mathbf{t}^{iz}\mathbf{t}^{jy}\mathbf{t}^{kl} + \frac{1}{3!}\mathbf{h}^i_{jk}\mathbf{t}_i\mathbf{t}^{jz}\mathbf{t}^{ky} + \frac{1}{2!}\mathbf{h}^i_{jk}\mathbf{t}^z_i\mathbf{t}^j\mathbf{t}^{ky} \\
& + \frac{2!}{3!3!}\mathbf{h}^i_{jkl}\mathbf{t}^{jz}\mathbf{t}^{ky}\mathbf{t}^l_i + \frac{1}{3!}\mathbf{h}^i_{jkl}\mathbf{t}^z_i\mathbf{t}^{jy}\mathbf{t}^{kl} + \bar{f}\frac{2!}{3!3!}\mathbf{h}^z_{ijk}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^{ky} + \frac{1}{3!}\mathbf{h}^{ij}_k\mathbf{t}^z_i\mathbf{t}^y_j\mathbf{t}^k + \frac{1}{2!}\mathbf{h}^{ij}_k\mathbf{t}_i\mathbf{t}^z_j\mathbf{t}^{ky} + \frac{1}{3!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}^z_i\mathbf{t}^y_j\mathbf{t}^{kl} + \bar{f}\frac{1}{2!2!}\mathbf{h}^{iz}_{jk}\mathbf{t}_i\mathbf{t}^j\mathbf{t}^{ky} \\
& + \frac{1}{3!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_{ij}\mathbf{t}^{kz}\mathbf{t}^{ly} + \bar{f}\frac{1}{3!2!}\mathbf{h}^{iz}_{jk}\mathbf{t}^y_i\mathbf{t}^j\mathbf{t}^k + \frac{1}{2!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}^z_i\mathbf{t}^{ky}\mathbf{t}^l_j + \frac{2!}{3!3!}\mathbf{h}^{ijk}\mathbf{t}_i\mathbf{t}^z_j\mathbf{t}^y_k + \bar{f}\frac{2!}{3!3!}\mathbf{h}^{ijz}_k\mathbf{t}_i\mathbf{t}_j\mathbf{t}^{ky} + \frac{1}{3!}\mathbf{h}^{ijk}_l\mathbf{t}^z_i\mathbf{t}_{jk}\mathbf{t}^{ly} + \bar{f}\frac{1}{3!}\mathbf{h}^{ijz}_k\mathbf{t}_i\mathbf{t}^y_j\mathbf{t}^k \\
& \left.+ \frac{2!}{3!3!}\mathbf{h}^{ijk}_l\mathbf{t}^z_i\mathbf{t}^y_j\mathbf{t}^l_k + \frac{2!}{4!3!}\mathbf{h}^{ijkl}\mathbf{t}^z_i\mathbf{t}^y_j\mathbf{t}_{kl} + \bar{f}\frac{2!}{4!3!}\mathbf{h}^{ijkz}\mathbf{t}_i\mathbf{t}_j\mathbf{t}^y_k + \frac{2!}{4!4!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^{kz}\mathbf{t}^{ly} + \frac{1}{4!}\mathbf{h}^i_{jkl}\mathbf{t}^z_i\mathbf{t}^j\mathbf{t}^k\mathbf{t}^{ly} + \frac{1}{4!}\mathbf{h}^i_{jkl}\mathbf{t}_i\mathbf{t}^j\mathbf{t}^{kz}\mathbf{t}^{ly} + \frac{1}{4!2!}\mathbf{h}^{ij}_{kl}\mathbf{t}_i\mathbf{t}_j\mathbf{t}^{kz}\mathbf{t}^{ly}\right)
\end{aligned}
\tag{11}
$$

$$\hat{\Omega} = \hat{i}^\dagger \hat{j}$$

$$LHS = i\left(\frac{\mathrm{d}\mathbf{t}_j^i}{\mathrm{d}\tau} + \frac{\mathrm{d}\mathbf{t}^i}{\mathrm{d}\tau}\mathbf{t}_j + \mathbf{t}^i\frac{\mathrm{d}\mathbf{t}_j}{\mathrm{d}\tau} + \mathbf{t}_j^i\varepsilon + \mathbf{t}^i\mathbf{t}_j\varepsilon\right)$$

$$
\begin{aligned}
RHS = (&f\bar{f}\mathbf{h}_y^z + f\frac{1}{2!}\mathbf{h}_{iz}\mathbf{t}^{iy} + \bar{f}\mathbf{h}_i^z\mathbf{t}_y^i + f\mathbf{h}_z^i\mathbf{t}_i^y + f\bar{f}\frac{1}{2!}\mathbf{h}_{iy}^z\mathbf{t}^i + f\bar{f}\frac{1}{3!}\mathbf{h}_{ijy}^z\mathbf{t}^{ij} + \bar{f}\frac{1}{2!}\mathbf{h}^{iz}\mathbf{t}_{iy} \\
&+ f\bar{f}\frac{1}{2!}\mathbf{h}_y^{iz}\mathbf{t}_i + f\bar{f}\frac{1}{2!2!}\mathbf{h}_{jy}^{iz}\mathbf{t}_i^j + f\bar{f}\frac{1}{3!}\mathbf{h}_y^{ijz}\mathbf{t}_{ij} + \frac{1}{2!}\mathbf{h}_{ij}\mathbf{t}_z^i\mathbf{t}^{jy} + f\frac{1}{3!}\mathbf{h}_{ijz}\mathbf{t}^i\mathbf{t}^{jy} + f\frac{1}{4!}\mathbf{h}_{ijkz}\mathbf{t}^{iy}\mathbf{t}^{jk} + \mathbf{h}_i^t\mathbf{t}_{iz}\mathbf{t}^{jy} \\
&+ \mathbf{h}_j^i\mathbf{t}_i^z\mathbf{t}_y^j + f\frac{1}{2!}\mathbf{h}_{jz}^i\mathbf{t}_i^y\mathbf{t}^j + \bar{f}\frac{1}{2!}\mathbf{h}_{ij}^z\mathbf{t}^i\mathbf{t}_y^j + f\frac{1}{2!}\mathbf{h}_{jz}^i\mathbf{t}_i\mathbf{t}^{jy} + f\bar{f}\frac{1}{3!2!}\mathbf{h}_{ijy}^z\mathbf{t}^i\mathbf{t}^j + f\frac{1}{3!}\mathbf{h}_{jkz}^i\mathbf{t}^{jy}\mathbf{t}_i^k + f\frac{1}{3!}\mathbf{h}_{jkz}^i\mathbf{t}_i^y\mathbf{t}^{jk} \\
&+ \bar{f}\frac{1}{3!}\mathbf{h}_{ijk}^z\mathbf{t}_y^i\mathbf{t}^{jk} + \frac{1}{2!}\mathbf{h}^{ij}\mathbf{t}_{iz}\mathbf{t}_y^j + \bar{f}\frac{1}{2!}\mathbf{h}_j^{iz}\mathbf{t}_{iy}\mathbf{t}^j + \bar{f}\frac{1}{2!}\mathbf{h}_j^{iz}\mathbf{t}_i\mathbf{t}_y^j + f\frac{1}{2!}\mathbf{h}_z^{ij}\mathbf{t}_i\mathbf{t}_y^j + f\frac{1}{2!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_i^y\mathbf{t}_j^k + f\bar{f}\frac{1}{2!2!}\mathbf{h}_{jy}^{iz}\mathbf{t}_i\mathbf{t}^j \\
&+ \bar{f}\frac{1}{2!2!}\mathbf{h}_{jk}^{iz}\mathbf{t}_{iy}\mathbf{t}^{jk} + \bar{f}\frac{1}{2!2!}\mathbf{h}_{jk}^{iz}\mathbf{t}_y^j\mathbf{t}_i^k + f\frac{1}{2!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_{ij}\mathbf{t}^{ky} + \bar{f}\frac{1}{3!}\mathbf{h}^{ijz}\mathbf{t}_i\mathbf{t}_{jy} + \bar{f}\frac{1}{3!}\mathbf{h}_k^{ij}\mathbf{t}_{iy}\mathbf{t}_j^k + f\frac{1}{3!}\mathbf{h}_z^{ijk}\mathbf{t}_i^y\mathbf{t}_{jk} + \bar{f}\frac{1}{3!}\mathbf{h}_k^{ijz}\mathbf{t}_{ij}\mathbf{t}_y^k \\
&+ f\bar{f}\frac{1}{3!2!}\mathbf{h}_y^{ijz}\mathbf{t}_i\mathbf{t}_j + \bar{f}\frac{1}{4!}\mathbf{h}_y^{ijkz}\mathbf{t}_{iy}\mathbf{t}_{jk} + \frac{1}{3!}\mathbf{h}_{ijk}\mathbf{t}^i\mathbf{t}_z^j\mathbf{t}^{ky} + f\frac{2!}{4!3!}\mathbf{h}_{ijkz}\mathbf{t}^i\mathbf{t}^j\mathbf{t}^{ky} + \frac{1}{4!}\mathbf{h}_{ijkl}\mathbf{t}_z^i\mathbf{t}^{jy}\mathbf{t}^{kl} + \frac{1}{2!}\mathbf{h}_{jk}^i\mathbf{t}_i^z\mathbf{t}^j\mathbf{t}_y^k + \frac{1}{2!}\mathbf{h}_{jk}^i\mathbf{t}_{iz}\mathbf{t}^j\mathbf{t}^{ky} \\
&+ \frac{1}{2!}\mathbf{h}_{jk}^i\mathbf{t}_i\mathbf{t}_z^j\mathbf{t}^{ky} + f\frac{1}{3!}\mathbf{h}_{jkz}^i\mathbf{t}_i\mathbf{t}^j\mathbf{t}^{ky} + \frac{1}{3!}\mathbf{h}_{jkl}^i\mathbf{t}_{iz}\mathbf{t}^{jy}\mathbf{t}^{kl} + \frac{1}{3!}\mathbf{h}_{jkl}^i\mathbf{t}_z^z\mathbf{t}_y^j\mathbf{t}^{kl} + f\frac{2!}{3!3!}\mathbf{h}_{jkz}^i\mathbf{t}_i^y\mathbf{t}^j\mathbf{t}^k + \frac{1}{3!}\mathbf{h}_{jkl}^i\mathbf{t}_z^j\mathbf{t}^{ky}\mathbf{t}_i^l + \bar{f}\frac{2!}{3!3!}\mathbf{h}_{ijk}^z\mathbf{t}^i\mathbf{t}^j\mathbf{t}_y^k \\
&+ \frac{1}{2!}\mathbf{h}_k^{ij}\mathbf{t}_i\mathbf{t}_j^z\mathbf{t}_y^k + \frac{1}{2!}\mathbf{h}_k^{ij}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}^{ky} + \frac{1}{2!}\mathbf{h}_k^{ij}\mathbf{t}_{iz}\mathbf{t}_j^y\mathbf{t}^k + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{iz}\mathbf{t}_j^y\mathbf{t}^{kl} + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{ij}\mathbf{t}_z^k\mathbf{t}^{ly} + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_z^i\mathbf{t}_y^k\mathbf{t}_j^l + \bar{f}\frac{1}{2!2!}\mathbf{h}_{jk}^{iz}\mathbf{t}_i\mathbf{t}^j\mathbf{t}_y^k \\
&+ \bar{f}\frac{1}{3!2!}\mathbf{h}_{jk}^{iz}\mathbf{t}_{iy}\mathbf{t}^j\mathbf{t}^k + f\frac{1}{2!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_i\mathbf{t}_y^j\mathbf{t}^k + f\frac{1}{3!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_i\mathbf{t}_j\mathbf{t}^{ky} + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{iz}\mathbf{t}^{ky}\mathbf{t}_j^l + \frac{1}{3!}\mathbf{h}^{ijk}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}_k^y + \bar{f}\frac{2!}{3!3!}\mathbf{h}_k^{ijz}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_y^k + \bar{f}\frac{1}{3!}\mathbf{h}_k^{ijz}\mathbf{t}_i\mathbf{t}_{jy}\mathbf{t}^k \\
&+ \frac{1}{3!}\mathbf{h}_l^{ijk}\mathbf{t}_i^z\mathbf{t}_{jk}\mathbf{t}_y^l + \frac{1}{3!}\mathbf{h}_l^{ijk}\mathbf{t}_{iz}\mathbf{t}_{jk}\mathbf{t}^{ly} + f\frac{2!}{3!3!}\mathbf{h}_z^{ijk}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_k^y + \frac{1}{3!}\mathbf{h}_l^{ijk}\mathbf{t}_{iz}\mathbf{t}_j^y\mathbf{t}_k^l + \frac{1}{4!}\mathbf{h}^{ijkl}\mathbf{t}_{iz}\mathbf{t}_j^y\mathbf{t}_{kl} + \bar{f}\frac{2!}{4!3!}\mathbf{h}^{ijkz}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_{ky} + \frac{3!}{4!4!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}^j\mathbf{t}_z^k\mathbf{t}^{ly} \\
&+ \frac{1}{4!}\mathbf{h}_{jkl}^i\mathbf{t}_{iz}\mathbf{t}^j\mathbf{t}^k\mathbf{t}^{ly} + \frac{1}{3!}\mathbf{h}_{jkl}^i\mathbf{t}_i\mathbf{t}^j\mathbf{t}_z^k\mathbf{t}^{ly} + \frac{1}{4!}\mathbf{h}_{jkl}^i\mathbf{t}_z^i\mathbf{t}^j\mathbf{t}^k\mathbf{t}_y^l + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}^k\mathbf{t}^{ly} + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_i\mathbf{t}_j^z\mathbf{t}_y^k\mathbf{t}^l + \frac{3!}{2!2!4!}\mathbf{h}_{kl}^{ij}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_z^k\mathbf{t}^{ly} + \frac{3!}{2!2!4!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{iz}\mathbf{t}_j^y\mathbf{t}^k\mathbf{t}^l)
\end{aligned}
$$

$$\text{(12)}$$

$$\hat{\Omega} = \hat{i}^\dagger \hat{j}^\dagger$$

$$LHS = i\left(\frac{\mathrm{d}\mathbf{t}_{ij}}{\mathrm{d}\tau} + \frac{\mathrm{d}\mathbf{t}_i}{\mathrm{d}\tau}\mathbf{t}_j + \mathbf{t}_i\frac{\mathrm{d}\mathbf{t}_j}{\mathrm{d}\tau} + \mathbf{t}_{ij}\varepsilon + \mathbf{t}_i\mathbf{t}_j\varepsilon\right)$$

$$
\begin{aligned}
RHS = (&f^2\mathbf{h}_{zy} + f\frac{1}{2!}\mathbf{h}_{iz}\mathbf{t}_y^i + f^2\frac{1}{3!}\mathbf{h}_{izy}\mathbf{t}^i + f^2\frac{1}{4!}\mathbf{h}_{ijzy}\mathbf{t}^{ij} + f\mathbf{h}_z^i\mathbf{t}_{iy} + f^2\frac{1}{2!}\mathbf{h}_{zy}^i\mathbf{t}_i + f^2\frac{1}{3!}\mathbf{h}_{jzy}^i\mathbf{t}_i^j \\
&+ f^2\frac{1}{2!2!}\mathbf{h}_{zy}^{ij}\mathbf{t}_{ij} + \frac{1}{2!2!}\mathbf{h}_{ij}\mathbf{t}_z^i\mathbf{t}_y^j + f\frac{1}{3!}\mathbf{h}_{ijz}\mathbf{t}^i\mathbf{t}_y^j + f\frac{1}{4!}\mathbf{h}_{ijkz}\mathbf{t}_y^i\mathbf{t}^{jk} + f^2\frac{1}{4!2!}\mathbf{h}_{ijzy}\mathbf{t}^i\mathbf{t}^j + \mathbf{h}_j^i\mathbf{t}_{iz}\mathbf{t}_y^j + f\frac{1}{2!}\mathbf{h}_{jz}^i\mathbf{t}_{iy}\mathbf{t}^j \\
&+ f\frac{1}{2!}\mathbf{h}_{jz}^i\mathbf{t}_i\mathbf{t}_y^j + f^2\frac{1}{3!}\mathbf{h}_{jzy}^i\mathbf{t}_i\mathbf{t}^j + f\frac{1}{3!}\mathbf{h}_{jkz}^i\mathbf{t}_{iy}\mathbf{t}^{jk} + f\frac{1}{3!}\mathbf{h}_{jkz}^i\mathbf{t}_y^j\mathbf{t}_i^k + \frac{1}{2!2!}\mathbf{h}^{ij}\mathbf{t}_{iz}\mathbf{t}_{jy} + f\frac{1}{2!}\mathbf{h}_z^{ij}\mathbf{t}_i\mathbf{t}_{jy} + f\frac{1}{2!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_{iy}\mathbf{t}_j^k \\
&+ f\frac{1}{2!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_{ij}\mathbf{t}_y^k + f^2\frac{1}{2!2!2!}\mathbf{h}_{zy}^{ij}\mathbf{t}_i\mathbf{t}_j + f\frac{1}{3!}\mathbf{h}_z^{ijk}\mathbf{t}_{iy}\mathbf{t}_{jk} + \frac{2!}{3!3!}\mathbf{h}_{ijk}\mathbf{t}^i\mathbf{t}_z^j\mathbf{t}_y^k + \frac{2!}{4!3!}\mathbf{h}_{ijkl}\mathbf{t}_z^i\mathbf{t}_y^j\mathbf{t}^{kl} + f\frac{2!}{4!3!}\mathbf{h}_{ijkz}\mathbf{t}^i\mathbf{t}^j\mathbf{t}_y^k + \frac{1}{2!}\mathbf{h}_{jk}^i\mathbf{t}_{iz}\mathbf{t}^j\mathbf{t}_y^k \\
&+ \frac{1}{3!}\mathbf{h}_{jk}^i\mathbf{t}_i\mathbf{t}_z^j\mathbf{t}_y^k + f\frac{1}{3!}\mathbf{h}_{jkz}^i\mathbf{t}_i\mathbf{t}^j\mathbf{t}_y^k + \frac{1}{3!}\mathbf{h}_{jkl}^i\mathbf{t}_{iz}\mathbf{t}_y^j\mathbf{t}^{kl} + \frac{2!}{3!3!}\mathbf{h}_{jkl}^i\mathbf{t}_z^j\mathbf{t}_y^k\mathbf{t}_i^l + f\frac{2!}{3!3!}\mathbf{h}_{jkz}^i\mathbf{t}_{iy}\mathbf{t}^j\mathbf{t}^k + \frac{1}{2!}\mathbf{h}_k^{ij}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}_y^k + \frac{1}{3!}\mathbf{h}_k^{ij}\mathbf{t}_{iz}\mathbf{t}_{jy}\mathbf{t}^k \\
&+ f\frac{1}{3!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_y^k + f\frac{1}{2!2!}\mathbf{h}_{kz}^{ij}\mathbf{t}_i\mathbf{t}_{jy}\mathbf{t}^k + \frac{1}{3!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{iz}\mathbf{t}_{jy}\mathbf{t}^{kl} + \frac{1}{2!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{iz}\mathbf{t}_y^k\mathbf{t}_j^l + \frac{1}{3!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{ij}\mathbf{t}_z^k\mathbf{t}_y^l + \frac{2!}{3!3!}\mathbf{h}^{ijk}\mathbf{t}_i\mathbf{t}_{jz}\mathbf{t}_{ky} + \frac{1}{3!}\mathbf{h}_l^{ijk}\mathbf{t}_{iz}\mathbf{t}_{jk}\mathbf{t}_y^l \\
&+ \frac{2!}{3!3!}\mathbf{h}_l^{ijk}\mathbf{t}_{iz}\mathbf{t}_{jy}\mathbf{t}_k^l + f\frac{2!}{3!3!}\mathbf{h}_z^{ijk}\mathbf{t}_i\mathbf{t}_j\mathbf{t}_{ky} + \frac{2!}{4!3!}\mathbf{h}^{ijkl}\mathbf{t}_{iz}\mathbf{t}_{jy}\mathbf{t}_{kl} + \frac{2!}{4!4!}\mathbf{h}_{ijkl}\mathbf{t}^i\mathbf{t}^j\mathbf{t}_z^k\mathbf{t}_y^l + \frac{1}{4!}\mathbf{h}_{jkl}^i\mathbf{t}_{iz}\mathbf{t}^j\mathbf{t}^k\mathbf{t}_y^l + \frac{1}{4!}\mathbf{h}_{jkl}^i\mathbf{t}_i\mathbf{t}^j\mathbf{t}_z^k\mathbf{t}_y^l + \frac{1}{4!2!}\mathbf{h}_{kl}^{ij}\mathbf{t}_{iz}\mathbf{t}_{jy}\mathbf{t}^k\mathbf{t}^l)
\end{aligned}
$$

$$\text{(13)}$$