Using Losistic Regression to Predict Boston Neighborhood Crime Levels

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The purpose of this analysis is to build a logistic regression model that will predict whether a particular neighborhood in Boston is above or below the median crime level for the city.

Our dataset includes information on 466 Boston neighborhoods. Each neighborhood has 13 potential predictor variables, and 1 response variable. The response variable is "target", which is "1" if the neighborhood is above the city's median crime level, and 0 if not.

1 Data Exploration

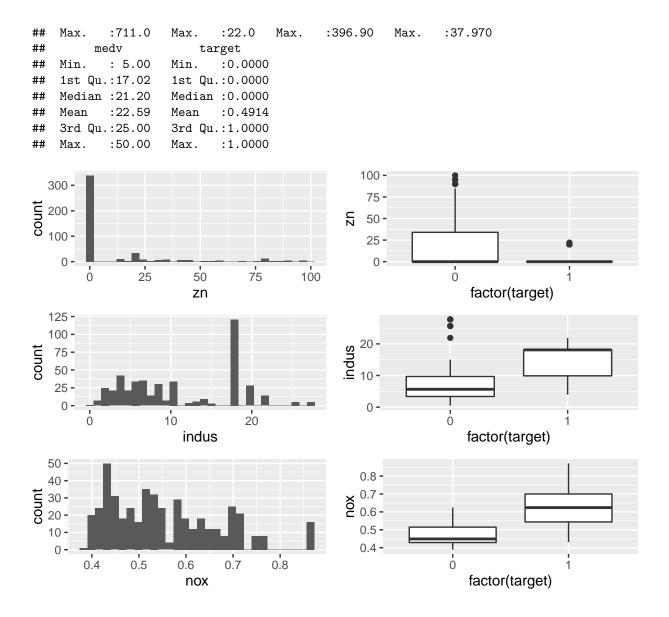
The first thing we checked for was if there was missing data in any of the variables. There was no missing data.

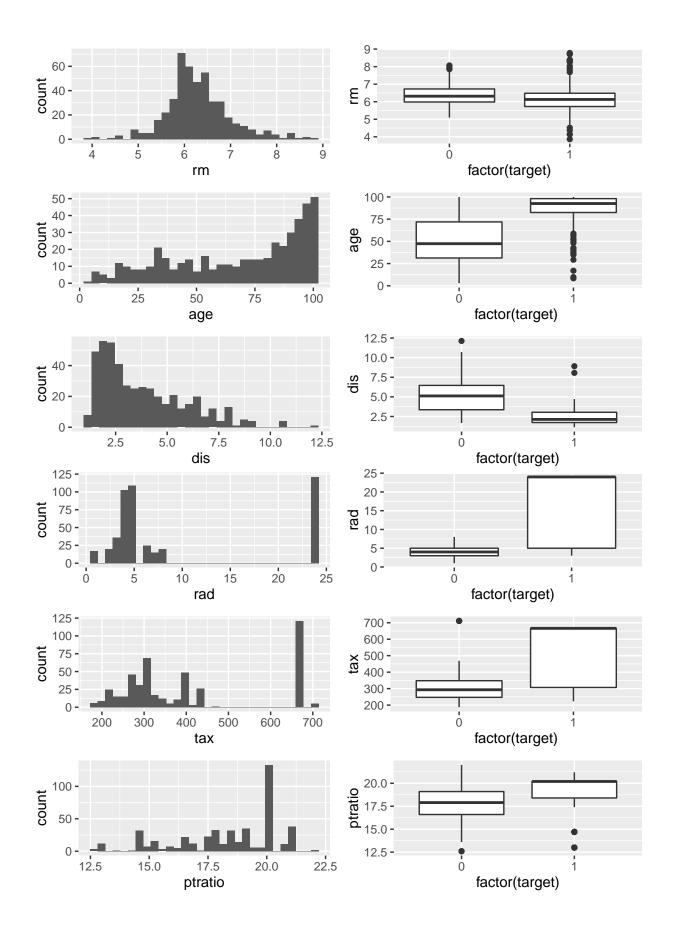
Of the 13 predictor variables, 12 were numeric and 1 was caterogical. The categorical data would need to be converted in order to be appropriately used in a generalized linear model.

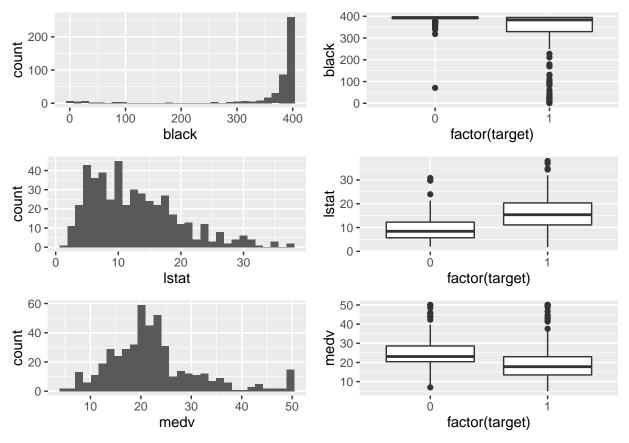
We also examined the distribution of the response variable, target. 229 neighborhoods were marked with a 1, thus above the median crime level, and 237 were not. The fact that the two numbers were not within 1 of each other implies that some neighborhood data was not included. However, the split was roughly even, allowing us to proceed.

Figure 1.1

```
##
           zn
                           indus
                                               chas
                                                                  nox
##
    Min.
            :
               0.00
                      Min.
                              : 0.460
                                         Min.
                                                 :0.00000
                                                             Min.
                                                                     :0.3890
##
    1st Qu.:
               0.00
                       1st Qu.: 5.145
                                         1st Qu.:0.00000
                                                             1st Qu.:0.4480
    Median :
               0.00
                      Median: 9.690
                                         Median: 0.00000
                                                             Median: 0.5380
            : 11.58
##
    Mean
                      Mean
                              :11.105
                                         Mean
                                                 :0.07082
                                                             Mean
                                                                     :0.5543
##
    3rd Qu.: 16.25
                       3rd Qu.:18.100
                                         3rd Qu.:0.00000
                                                             3rd Qu.:0.6240
##
    Max.
            :100.00
                      Max.
                              :27.740
                                         Max.
                                                 :1.00000
                                                             Max.
                                                                     :0.8710
##
                                             dis
                                                                rad
          rm
                           age
                     Min.
##
    Min.
            :3.863
                             : 2.90
                                        Min.
                                                : 1.130
                                                          Min.
                                                                  : 1.00
    1st Qu.:5.887
                                                           1st Qu.: 4.00
##
                     1st Qu.: 43.88
                                        1st Qu.: 2.101
##
    Median :6.210
                     Median: 77.15
                                        Median : 3.191
                                                          Median: 5.00
##
    Mean
            :6.291
                     Mean
                             : 68.37
                                        Mean
                                                : 3.796
                                                          Mean
                                                                  : 9.53
##
    3rd Qu.:6.630
                     3rd Qu.: 94.10
                                        3rd Qu.: 5.215
                                                           3rd Qu.:24.00
            :8.780
                             :100.00
                                                :12.127
##
                                                                  :24.00
    Max.
                     Max.
                                        Max.
                                                          Max.
                         ptratio
##
         tax
                                          black
                                                             lstat
##
    Min.
            :187.0
                             :12.6
                                             : 0.32
                                                                : 1.730
                     \mathtt{Min}.
                                      Min.
                                                        Min.
    1st Qu.:281.0
                     1st Qu.:16.9
                                      1st Qu.:375.61
                                                        1st Qu.: 7.043
##
##
    Median :334.5
                     Median:18.9
                                      Median :391.34
                                                        Median :11.350
    Mean
            :409.5
                     Mean
                             :18.4
                                      Mean
                                             :357.12
                                                        Mean
                                                                :12.631
    3rd Qu.:666.0
                     3rd Qu.:20.2
                                      3rd Qu.:396.24
                                                        3rd Qu.:16.930
```







Using histograms of variables and boxplots of variables grouped by predictor (Figure 1.1), we could see some outliers that may need to be dealt with. Also, correlations with the response variable (target) showed that variables such as zn, indus, dis and rad may have more potential as predictor variables while chas and rm may not be as useful.

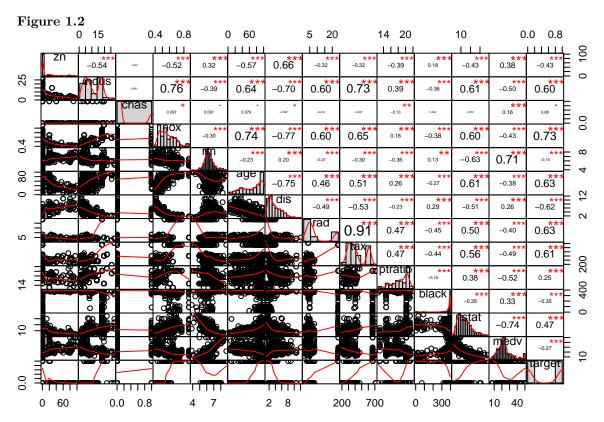


Figure 1.2 shows that every variable other than adjancency to the Charles river has correlation significance level of .001 with our response variable, target. This graph also shows that multicollinearity is something that will have to be dealt with in this model. Principal component analysis could be useful to mitigate this.

The fact that the predictor variables show a high degree of correlation with each other makes intuitive sense. Most variables relate to a notion of "desirability", which would hypothetically have significant impacts on other variables.

For instance, one might suppose that a high degree of industrial real estate in a neighborhood would have a negative effect on real estate values. One might than hypothesize that a neighborhood with lower median real estate values would be more highly susceptible to higher than usual crime.

At least in the provided dataset, there are strong correlations that bear out both of those hypotheses. Indus, which describes the proportion of non-retail businesses in a neighborhood, is negatively correlated by a factor of -.49617 with Medv, the median value of owner occupied homes; Medv is negative correlated (-.27) with higher than normal crime rates.

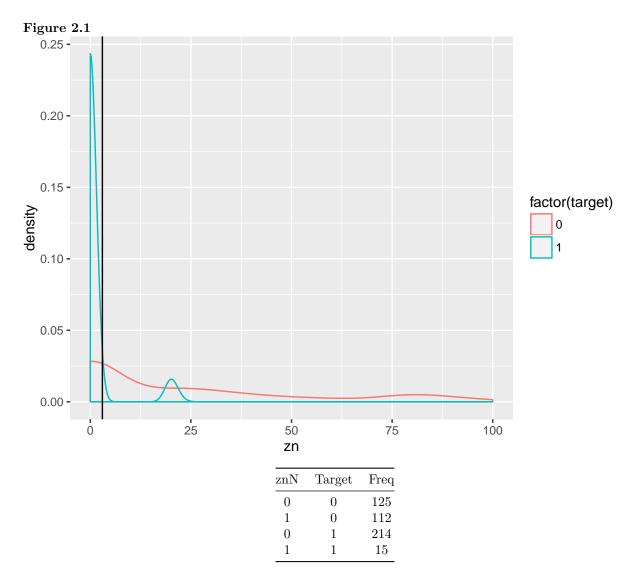
2 Data Preparation

We used two separate approaches to preparing the data. One approach dealt with each variable separately, while the second approach normalized all variables.

2.1 Individual Variable Preparation Approach

This approach looked at each variable independently of the others.

2.1.1 zn

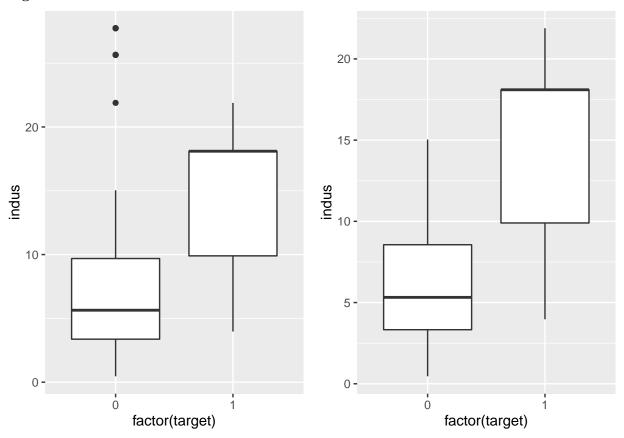


Using Figure 2.1, we decided to transform the numeric z variable to a derived categorical variable called znN. For this new znN variable, 1 means more than 3% of residential land zoned for large lots (over 25000 square feet), and θ means less than or equal to 3% of residential land zoned for large lots (over 25000 square feet).

2.1.2 indus

Each observation in the dataset is for a different Boston area neighborhood. This means the data are somewhat arbitrarily binned by geography - the area that is considered a neighborhood is influenced by historical factors. For instance, we see high-value outliers in the indus variable because historic land-use and zoning laws mean that non-retail business use is concentrated in specific industrial areas. In order to remove these potential leverage points, we removed neighborhoods with indus values above 20. However, we do not consider this to be invalid data.

Figure 2.3



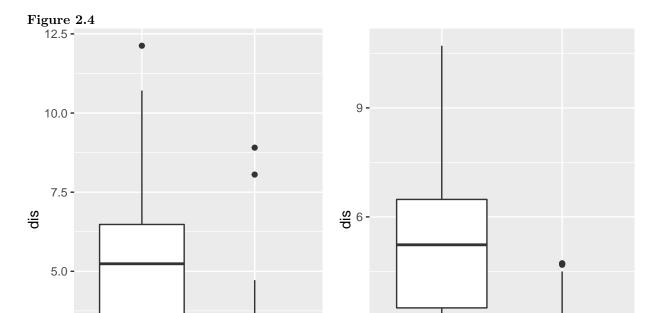
For indus, we removed observations where indus was greater than 20 and target was 0.

2.1.3 dis

2.5 -

0

factor(target)



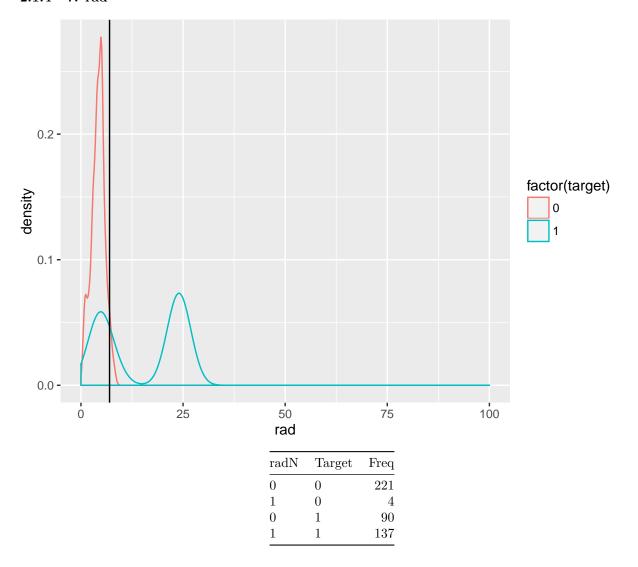
We removed observations where dis was greater than 11 and target was 0, and observations where dis was greater than 7.5 and target was 1.

3 -

0

factor(target)

2.1.4 7. rad



Here we applied the same method as zn. We set up a new variable called radN, where 1 means index of accessibility to radial highways was greater than 7 and 0 means index of accessibility to radial highways was less than or equal to 7.

2.1.5 Summary

Under our first approach, we removed 14 rows and added 2 new variables: znN and radN.

2.2 Normalization Approach

Since the variables were on a variety of scales, we created a dataset of centered (predictor variable mean subtracted from each observation) and scaled (each observation divided by the predictor variable's standard deviation) data. The two datasets are distinguished as 'non-normalized' and 'normalized' below, with the latter having been centered and scaled using R's 'scale' function.

3 Build Models

Since the evaluation data set did not include the response variable, we were not able to use it to cross validate our models. Instead, we split the training data on a 70-30 training/testing split to evaluate our models.

3.1 Model 1: Non-normalized Baseline

This model used the non-normalized original variables. It utilized the probit for the link function. The model began with all the original values, and via backwards selection removed the zn, chas, rm, dis, and black variables.

```
##
## Call:
  glm(formula = target ~ indus + nox + age + rad + tax + ptratio +
       lstat + medv, family = binomial(link = "probit"), data = training)
##
##
## Deviance Residuals:
##
      Min
               1Q Median
                                3Q
                                       Max
## -2.343 -0.026
                    0.000
                            0.000
                                     2.882
##
## Coefficients:
                 Estimate Std. Error z value Pr(>|z|)
##
## (Intercept) -24.171357
                            4.325079
                                      -5.589 2.29e-08 ***
## indus
                 0.134991
                            0.060490
                                        2.232 0.025639 *
                                        5.239 1.61e-07 ***
## nox
                28.874385
                            5.510981
                                        2.101 0.035654 *
## age
                 0.015291
                            0.007279
                 0.491912
                            0.136730
                                        3.598 0.000321 ***
## rad
## tax
                -0.017047
                            0.004355
                                       -3.914 9.07e-05 ***
                                        3.310 0.000933 ***
## ptratio
                 0.344176
                            0.103985
## 1stat
                 0.099590
                            0.035125
                                        2.835 0.004579 **
## medv
                 0.079078
                            0.029423
                                        2.688 0.007196 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 438.75
                              on 316
                                      degrees of freedom
## Residual deviance: 102.98
                              on 308
                                      degrees of freedom
## AIC: 120.98
##
## Number of Fisher Scoring iterations: 11
```

From the output, the model is as follows:

the log odds of target = -24.17 + 0.135 indus + 28.874 nox + 0.015 age + 0.492 rad - 0.017 tax + 0.344 ptratio + 0.1 1stat + 0.079 medv

We can see that indus, nox, age, rad, ptratio, lstat, and medv have positive effects on target, but tax has a negative effect on target.

For this model, the most unexpected result is that medv (median value of owner-occupied homes in \$1000s) has a positive effect on crime. And if we go back to check figure 1.2, we saw the weak negative correlationship (-0.27) between medv and target. Since this model includes indus, nox, age, rad, ptratio, 1stat, medv and tax, multicollinearity could be the root cause.

3.2 Model 2: Non-normalized with derived variables

This model also used backward selection on the non-normalized variables and the probit for the link function, but instead used the derived variables from part 2, instead of their original counterparts.

```
##
## Call:
  glm(formula = target ~ nox + tax + lstat + radN, family = binomial(link = "probit"),
##
##
       data = training)
##
## Deviance Residuals:
##
                   1Q
                         Median
                                       3Q
        Min
                                                 Max
  -2.32446 -0.04097
                        0.00000
                                  0.02465
                                            2.62981
##
  Coefficients:
##
                 Estimate Std. Error z value Pr(>|z|)
## (Intercept) -16.708448
                            2.552283
                                      -6.546 5.89e-11 ***
                                       6.182 6.35e-10 ***
## nox
                32.430637
                            5.246318
                -0.005576
                                      -2.672 0.00754 **
## tax
                            0.002087
                 0.080376
                            0.027954
                                       2.875 0.00404 **
## lstat
                                       5.273 1.34e-07 ***
## radN1
                 2.932473
                            0.556149
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 438.75 on 316 degrees of freedom
## Residual deviance: 107.01 on 312
                                      degrees of freedom
  AIC: 117.01
##
##
## Number of Fisher Scoring iterations: 9
```

From the output, the model is as follows:

```
the log odds of target = -16.71 + 32.43 \text{ nox} - 0.006 \text{ tax} + 0.08 \text{ lstat} + 2.93 \text{ radN1}
```

We can see that nox and lstat have positive effects on the log odds of target, but tax has a negative effect on the log odds of target. In addition, when radN equals to 1, the log odds of target increases by 2.93.

This model only include 4 variables and we transformed rad to categorical variable radN, so multicollinearity became less of an issue to us. Although we saw a positive correlation between tax and target in figure 1.2, both Model 1 and Model 2 show that tax has a negative yet weak effect on the log odds of target, which fits our intuition.

3.3 Model 3: Principle Component Analysis

For this model, we used an orthogonal transformation to convert our variables (normalization applied to all variables) into a set of values of linearly uncorrelated variables, which is called principal components. And then we chose the first two principal components that account for around 95% proportion of variance in the data. Finally, we used those chosen principal components to build a logistic regression model.

pca 25000 Variances 15000 5000 Comp.1 Comp.3 Comp.5 Comp.7 Comp.9 ## target Comp.1 Comp.2 -10.79214 ## 1 1 7.982215 ## 2 1 14.663799 -36.79382 ## 3 1 -237.185595 -109.13296 115.531924 17.45648 ## 5 214.333148 31.11811 ## 6 35.789981 -29.36113 ## ## glm(formula = target ~ ., data = training_pca) ## ## Deviance Residuals: ## Min 1Q Median 3Q Max ## -0.59191 -0.27404 -0.10935 0.83770 0.04818 ## ## Coefficients: ## Estimate Std. Error t value Pr(>|t|) ## (Intercept) 0.5116620 0.0217455 23.530 <2e-16 *** ## Comp.1 -0.0018248 0.0001246 -14.639 <2e-16 *** ## Comp.2 -0.0001726 0.0002618 -0.659 0.51 ## ## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1 ## (Dispersion parameter for gaussian family taken to be 0.1496637) ## ## Null deviance: 79.073 on 316 degrees of freedom ## Residual deviance: 46.994 on 314 degrees of freedom

AIC: 302.49

Number of Fisher Scoring iterations: 2

##

From the output, the formula we got was the following:

```
the log odds of target = 0.512 - 0.00182 \text{ Comp.} 1 - 0.00017 \text{ Comp.} 2
```

For this model, since we did an orthogonal transformation of variables, **multicollinearity** was no longer an issue. Both principal components have negative effects on the log odds of target. And we only chose two principal components, so we keep the model as is, even though the p value of Comp. 2 is not significant.

3.4 Model 4: Normalized Backward Selection

This model used the normalized data, the logit in the link function, and used backward selection using R's step() function with the backward option to arive at the below model. Insignificant variables (those with a P value > .05) were discarded.

```
##
## Call:
   glm(formula = target ~ nox + age + rad + tax + ptratio + black +
       medv, family = binomial, data = training_norm)
##
## Deviance Residuals:
##
                   1Q
        Min
                          Median
                                        3Q
                                                  Max
## -1.85486 -0.18466
                       -0.01769
                                   0.00474
                                              2.84168
##
  Coefficients:
               Estimate Std. Error z value Pr(>|z|)
##
                 3.7989
                             0.9758
                                      3.893 9.90e-05 ***
## (Intercept)
                 3.7313
## nox
                             0.6959
                                      5.362 8.23e-08 ***
## age
                 0.7877
                             0.3430
                                      2.297 0.021636 *
                                      4.182 2.89e-05 ***
## rad
                 6.2349
                             1.4908
## tax
                -1.9056
                             0.5618
                                     -3.392 0.000693 ***
                                      3.264 0.001099 **
## ptratio
                 1.0324
                             0.3163
## black
                -4.5850
                             1.5835
                                     -2.896 0.003785 **
## medv
                 0.8730
                             0.3800
                                      2.297 0.021603 *
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##
                                       degrees of freedom
       Null deviance: 452.80
                               on 326
## Residual deviance: 131.34
                               on 319
                                       degrees of freedom
## AIC: 147.34
##
## Number of Fisher Scoring iterations: 9
```

The backward selection model is below:

```
the log odds of target = 3.80 + 3.73 nox + 0.79 age + 6.23 rad - 1.91 tax 1.03 ptratio - 4.59 black + .87 medy
```

As in model 1, this model predicts a positive effect of 'medv' (median value of owner occupied homes) on target or crime, which is counterintuitive. This is an indication that multicollinearity could be an issue.

The high number of predictors (7) in this model also has the effect of decreasing the interpretive ability of the model.

3.5 Model 5: Normalized Forward Selection

This model used the normalized data, the logit in the link function, and used forward selection using R's step() function with the forward option to arive at the below model.

```
##
## Call:
  glm(formula = target ~ nox + rad + tax + ptratio + black + medv +
       age + dis + zn + lstat, family = binomial, data = data3b)
##
##
##
  Deviance Residuals:
##
       Min
                 1Q
                      Median
                                    30
                                            Max
##
  -1.7582
            -0.1591
                     -0.0017
                                0.0029
                                         3.3169
##
## Coefficients:
##
               Estimate Std. Error z value Pr(>|z|)
                                      3.384 0.000714 ***
## (Intercept)
                 3.4969
                             1.0333
## nox
                 4.4006
                             0.9301
                                      4.731 2.23e-06 ***
## rad
                 6.4586
                             1.6456
                                      3.925 8.68e-05 ***
## tax
                -1.7948
                             0.5993
                                     -2.995 0.002746 **
## ptratio
                 0.9818
                             0.3278
                                      2.995 0.002746 **
## black
                -4.2670
                             1.5925
                                     -2.679 0.007375 **
## medv
                 1.4443
                             0.5357
                                      2.696 0.007017 **
                 0.8985
                             0.3753
                                      2.394 0.016661 *
## age
                 1.0075
                             0.5447
                                      1.850 0.064354 .
## dis
## zn
                -1.6529
                             0.8970
                                     -1.843 0.065360 .
                 0.3301
                                      0.888 0.374665
## lstat
                             0.3718
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
##
       Null deviance: 452.80
                              on 326
                                       degrees of freedom
## Residual deviance: 124.91
                               on 316
                                       degrees of freedom
## AIC: 146.91
##
## Number of Fisher Scoring iterations: 9
```

The normalized forward selection model is below:

```
the log odds of target = 3.50 + 4.40 \text{ nox} + 6.46 \text{ rad} - 1.79 \text{ tax} + 0.98 \text{ ptratio} - 4.27 \text{ black} + 1.44 \text{ medv} + 0.90 \text{ age} + 1.01 \text{ dis} - 1.65 \text{ zn} + 0.33 \text{ lstat}
```

Many of the concerns that appeared in Model 4 (normalized backward selection) reappear in Model 5. Medv again positively predicts a higher crime level, indicating multicollinearity.

This model also includes a large number of predictors (11), even larger than model 4. The inclusion of some insginificant variables was used to compare it's performance against a similar model that did not include insignificant variables (Model 4).

4 Model Selection

During our modeling process, **multicollinearity** was the biggest issue we faced. Model 3 completely took care of this issue; Model 2 minimized this issue to a great extent and Models 1, 4 and 5 partially solved this problem. We used cross validation technique to further study our models and try to find the best model.

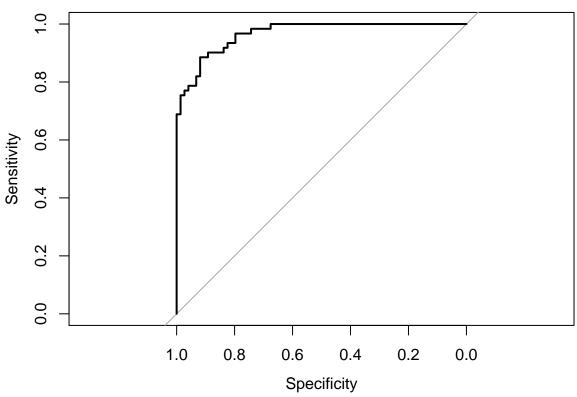
4.1 Confusion Matrices

Table 3: Confusion Matrix

	Model1	Model2	Model3	Model4	Model5
Accuracy	0.8814815	0.9037037	0.8222222	0.8920863	0.8920863
Kappa	0.7621145	0.8075447	0.6315670	0.7840050	0.7840050
AccuracyLower	0.8146770	0.8409602	0.7471282	0.8282642	0.8282642
AccuracyUpper	0.9307163	0.9477237	0.8826473	0.9383301	0.9383301
AccuracyNull	0.5481481	0.5481481	0.5481481	0.5179856	0.5179856
AccuracyPValue	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
McnemarPValue	0.4532547	0.0960923	0.0005202	1.0000000	1.0000000
Sensitivity	0.9016393	0.9508197	0.6557377	0.8888889	0.8888889
Specificity	0.8648649	0.8648649	0.9594595	0.8955224	0.8955224
Pos Pred Value	0.8461538	0.8529412	0.9302326	0.9014085	0.9014085
Neg Pred Value	0.9142857	0.9552239	0.7717391	0.8823529	0.8823529
Prevalence	0.4518519	0.4518519	0.4518519	0.5179856	0.5179856
Detection Rate	0.4074074	0.4296296	0.2962963	0.4604317	0.4604317
Detection Prevalence	0.4814815	0.5037037	0.3185185	0.5107914	0.5107914
Balanced Accuracy	0.8832521	0.9078423	0.8075986	0.8922056	0.8922056

4.2 ROC Curve and Area Under the Curve

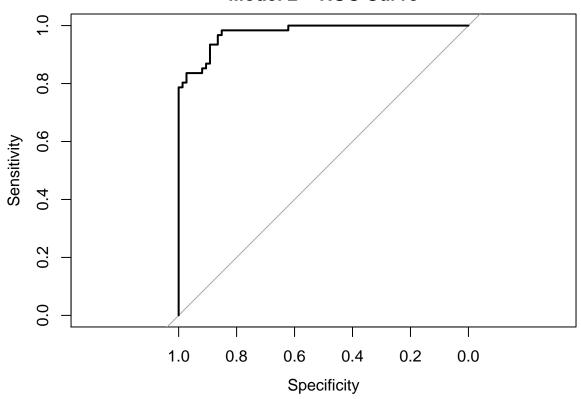




Call:

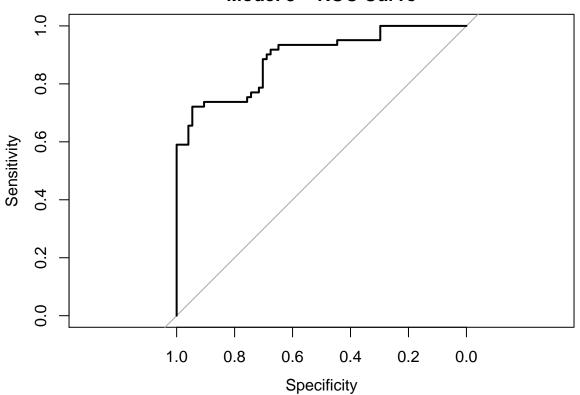
```
## roc.formula(formula = factor(target) ~ predict_1, data = testing)
##
## Data: predict_1 in 74 controls (factor(target) 0) < 61 cases (factor(target) 1).
## Area under the curve: 0.967</pre>
```

Model 2 - ROC Curve



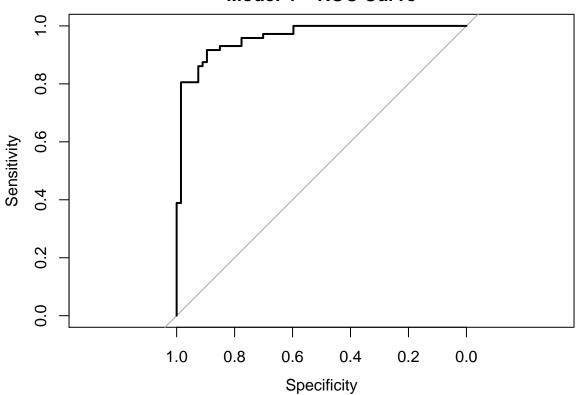
```
##
## Call:
## roc.formula(formula = factor(target) ~ predict_2, data = testing)
##
## Data: predict_2 in 74 controls (factor(target) 0) < 61 cases (factor(target) 1).
## Area under the curve: 0.9759</pre>
```





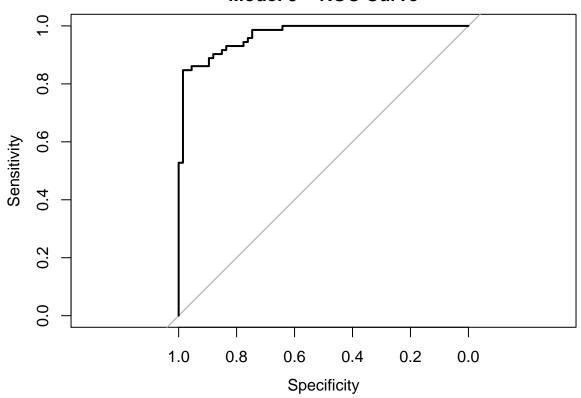
```
##
## Call:
## roc.formula(formula = factor(target) ~ predict_3, data = testing_pca)
##
## Data: predict_3 in 74 controls (factor(target) 0) < 61 cases (factor(target) 1).
## Area under the curve: 0.8903</pre>
```





```
##
## Call:
## roc.formula(formula = factor(target) ~ predict_4, data = testing_norm)
##
## Data: predict_4 in 67 controls (factor(target) 0) < 72 cases (factor(target) 1).
## Area under the curve: 0.9604</pre>
```

Model 5 - ROC Curve



```
##
## Call:
## roc.formula(formula = factor(target) ~ predict_5, data = testing_norm)
##
## Data: predict_5 in 67 controls (factor(target) 0) < 72 cases (factor(target) 1).
## Area under the curve: 0.9672</pre>
```

Table 4: Area under the curve

AUC
0669916
9758529
3903412
9604063
9672471

4 of the 5 area under the curve measurements were above .96, indicating that all but model 3 have excellent predictive power when measured against the test set.

4.3 Log-likelihood/AIC/BIC

Table 5: Log-likelihood/AIC/BIC

	Log Likelihood	AIC	BIC
Model 1	-51.49	120.98	154.81
Model 2	-53.51	117.01	135.81
Model 3	-147.25	302.49	317.53
Model 4	-65.67	147.34	177.66
${\rm Model}\ 5$	-62.45	146.91	188.60

Overall, Model 2 stands out with high Accuracy, high Sensitivity, high Specificity, high AUC, high Log-likelihood number, low AIC and low BIC. Although Model 3 was the best model to deal with **multicollinearity** issues, we still wanted a model with good prediction.

Model 2 not only greatly minimized the **multicollinearity** issue, but also gave the better prediction. We can verify the low level of multicollinearity in Model 2 by looking at variance inflation factors (using the 'car' library's vif() function. VIF measures how much the variance of each variable is inflated due to multicollinearity.

```
## nox tax lstat radN
## 2.009370 1.959690 1.262095 1.970707
```

Since the scores are all below 4, we can conclude multicollinearity has been handled appropriately.

More importantly, Likelihood, AIC and BIC numbers also indicate Model 2 is a better model to fit.

5 Predictions on Evaluation Data

After we applied our chosen model (Model 2) to crime-evaluation-data set, we predicted that for 16 observations target equals 0, and 24 observations, which target equals 1. Please check the file *result.csv* for detailed information.