ISE3230 Project Model - LinkNYC Optimization

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Optimization Model

Problem Description

We aim to optimize the installation of LinkNYC kiosks across census tracts in New York City. The objective is to minimize installation costs while ensuring adequate kiosk coverage for the population and meeting specific constraints.

Model Formulation

Constants

- **P**: Vector of populations for each census tract (P_i for tract i).
- Cost: Vector of installation costs per kiosk for each census tract (c_i for tract i, determined by the borough it belongs to).
- POI: Vector of points of interest for each census tract (POI_i for tract i).
- **K**: Vector of current kiosks for each census tract (K_i for tract i).
- ρ: Minimum population per kiosk.
- MinPOI: Minimum number of points of interest required to consider installing kiosks.
- MinPop: Minimum population required to consider installing kiosks.
- M: A large constant for binary variable z_i activation.

Decision Variables

- x_i: Number of new kiosks to install in census tract i ($x_i \geq 0, x_i \in \mathbb{Z}$).
- z_i: Binary variable indicating whether kiosks are installed in tract i ($z_i \in \{0, 1\}$).

Objective Function

$$\text{Minimize } Z = \sum_{i}^{2327} c_i \cdot x_i$$

Where c_i is the cost assigned to the borough where tract i is located.

Constraints

1. **Population Coverage:**
$$\frac{P_i}{\rho} \leq K_i + x_i \quad \forall i$$
2. **Binary Activation Link:**
$$x_i \leq M \cdot z_i \quad \forall i$$
3. **Thresholds for POIs and Population:**
$$z_i \leq \frac{\text{POI}_i}{\text{MinPOI}}, \quad z_i \leq \frac{P_i}{\text{MinPop}} \quad \forall i$$
4. **Non-Negativity and Integer Constraints:**
$$x_i \geq 0, \quad x_i \in \mathbb{Z}, \quad z_i \in \{0, 1\}$$

Complete Formulation

Objective:
$$\text{Minimize } Z = \sum_i c_i \cdot x_i$$

Subject to:
$$\begin{aligned}
&P_i \leq \rho \cdot (K_i + x_i) \quad \forall i \\
&x_i \leq M \cdot z_i \quad \forall i \\
&z_i \leq \frac{\text{POI}_i}{\text{MinPOI}} \forall i \\
&z_i \leq \frac{P_i}{\text{MinPop}} \quad \forall i \\
&x_i \geq 0, \quad x_i \in \mathbb{Z}, \quad z_i \in \{0, 1\}
\end{aligned}$$