# Neural network derivation

Machine Learning I (2023-2024)
UMONS

Let $x \in \mathbb{R}^2$ denote the input of a neural network and $y \in \mathbb{R}$ the target value. We define the parameters of the neural network, the forward pass and the loss function below. Can you determine the derivative of each parameter with respect to the loss function for the backward pass?

## Parameters

- $u \in \mathbb{R}^{3 \times 2}$: Weight matrix for the input-to-hidden layer.

- $b \in \mathbb{R}^3$: Bias vector for the hidden layer.

- $w \in \mathbb{R}^3$: Weight vector for the hidden-to-output layer.

- $c \in \mathbb{R}$: Bias for the output.

## Forward pass

$$g_j = \sum_{k=1}^{2} u_{jk} x_k + b_j \quad \text{for } j = 1, 2, 3$$

$$h_j = \sigma(g_j) \quad \text{for } j = 1, 2, 3$$

$$z = \sum_{j=1}^{3} w_j h_j + c$$

Note that $\sigma$ is the logistic function defined as

$$\sigma(x) = \frac{1}{1 + \exp(-x)}$$

and that its derivative is

$$\frac{\partial \sigma(x)}{\partial x} = \sigma(x)(1 - \sigma(x)).$$

## Loss Function

$$L = (z - y)^2$$

## Backward Pass