

1.

$$p_X(1) = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

[1,1],[1,2],[1,3],[1,4],[1,5],[1,6],

$$p_X(2) = \frac{2}{36} + \frac{1}{36} = \frac{3}{36} = \frac{1}{12}$$

[2,1],[2,2],[2,3],[2,4],[2,5],[2,6],

[3,1],[3,2],[3,3],[3,4],[3,5],[3,6],

$$p_X(3) = \frac{5}{36}$$

[4,1],[4,2],[4,3],[4,4],[4,5],[4,6],

[5,1],[5,2],[5,3],[5,4],[5,5],[5,6],

[6,1],[6,2],[6,3],[6,4],[6,5],[6,6]

$$p_X(4) = \frac{7}{36}$$

$$p_X(5) = \frac{9}{36} = \frac{1}{4}$$

$$p_X(6) = \frac{11}{36}$$

$$E = \left(\frac{1}{36} * 1\right) + \left(\frac{1}{12} * 2\right) + \left(\frac{5}{36} * 3\right) + \left(\frac{7}{36} * 4\right) + \left(\frac{1}{4} * 5\right) + \left(\frac{11}{36} * 6\right) = \frac{161}{36}$$

2.

$$p_X(-1) = 1/5, p_X(0) = 1/10, p_X(1/2) = 1/5, p_X(1) = 3/10, p_X(4) = 1/5$$

Mean:

$$E = \left(\frac{1}{5} * -1\right) + \left(\frac{1}{10} * 0\right) + \left(\frac{1}{5} * 1/2\right) + \left(\frac{3}{10} * 1\right) + \left(\frac{1}{5} * 4\right) = 1$$

Variance:

$$\text{var}(X) = E[X^2] - (E[X])^2$$

$$\sum_k k^2 p_X(k) - 1^2$$

$$(-1^2)\left(\frac{1}{5}\right) + (0^2)\left(\frac{1}{10}\right) + \left(\frac{1^2}{2}\right)\left(\frac{1}{5}\right) + (1^2)\left(\frac{3}{10}\right) + (4^2)\left(\frac{1}{5}\right)$$

$$\frac{1}{5} + 0 + \frac{1}{20} + \frac{3}{10} + 3\frac{1}{5} = 3.75$$

$$3.75 - 1 = 2.75$$

3.

$$p_X(0) = 1 - p$$

$$p_X(a) = p$$

$$E[X] = 1$$

$$\text{var}(X) = 1$$

$$\text{var}(X) = [E(X^2)] - E[X]^2$$

$$1 = [E(X^2)] - 1$$

$$2 = [E(X^2)]$$

$$1 = 0(1 - p) + a(p)$$

$$1 = ap$$

$$p = \frac{1}{a}$$

$$2 = [E(X^2)]$$

$$2 = ap^2$$

$$2 = a\left(\frac{1}{a}\right)^2$$

$$p = \frac{1}{2}$$

$$a = 2$$

4.

$$P(X \geq c \cdot E[X]) \leq \frac{1}{c}$$

avg rainfall = 0.119890411

at least 0.30

$$\frac{0.119890411}{0.30} \approx 0.39 \text{ probability of more than 0.30" of rain}$$

5.

Game 1:

$$p_X(1) = \frac{1}{9} \quad [1,1],[1,2],[1,3],$$

$$p_X(2) = \frac{3}{9} = \frac{1}{3} \quad [2,1],[2,2],[2,3],$$

$$p_X(3) = \frac{5}{9} \quad [3,1],[3,2],[3,3]$$

$$p_X(3) = \frac{5}{9}$$

$$E = \left(\frac{1}{9} * \$10 \right) + \left(\frac{1}{3} * \$20 \right) + \left(\frac{5}{9} * \$30 \right) = \$24.44$$

Game 2:

$$p_X(1) = \frac{1}{3} - \frac{2}{9} = \frac{1}{9}$$

$$p_X(2) = \frac{1}{3} + \frac{1}{9} = \frac{4}{9}$$

$$p_X(3) = \frac{1}{3} + \frac{1}{9} = \frac{4}{9}$$

$$E = \left(\frac{1}{9} * \$11 \right) + \left(\frac{4}{9} * \$22 \right) + \left(\frac{4}{9} * \$33 \right) = \$25.66$$

Game 2 has a higher expected prize