Dynamics of pulsed-ultrasound driven gas-liquid interfaces

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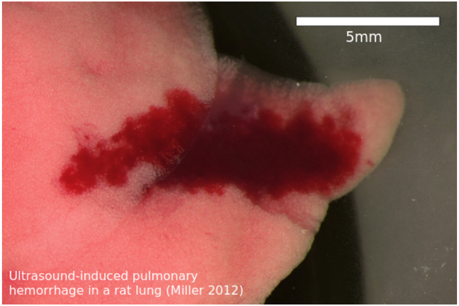
Diagnostic ultrasound can trigger lung hemorrhage in mammals

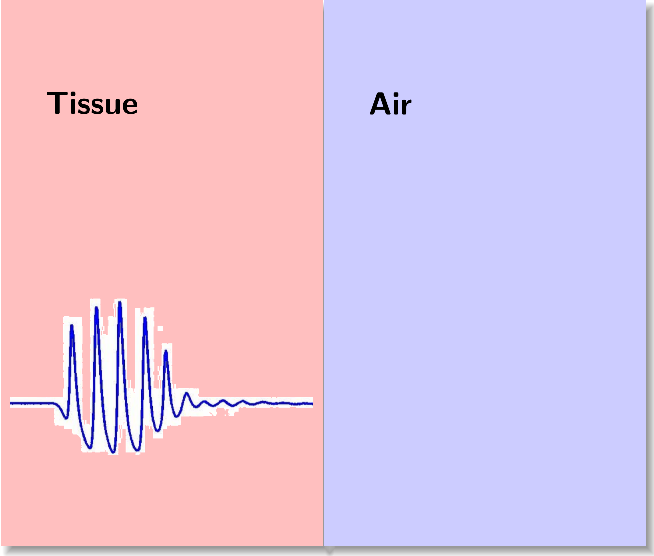
- Lung Hemorrhage (LH) is the only known bioeffect of non-contrast DUS
- Has been shown to occur in mice, rats, pigs, rabbits, monkeys (Child *et al.*, 1990; O'Brien & Zachary, 1997; Tarantal & Canfield, 1994).
- The underlying physical damage mechanism is not understood.



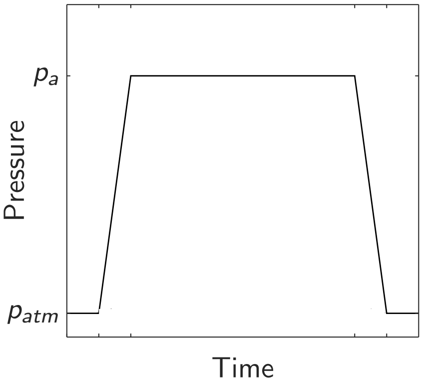
Ultrasound-induced pulmonary hemorrhage in a rat lung (Miller 2012)

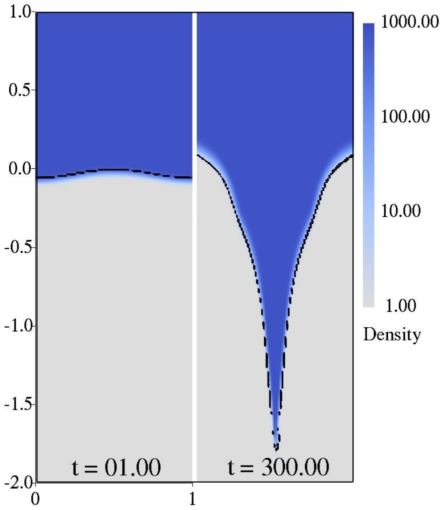
Fundamental physical problem:

An acoustic wave interacting with a tissue-air interface



Past work: Acoustic waves are capable of deforming gas-liquid interfaces





A trapezoidal acoustic waves caused significant deformations of an almost flat air-water interfaces. Linear acoustics couldn't explain this.

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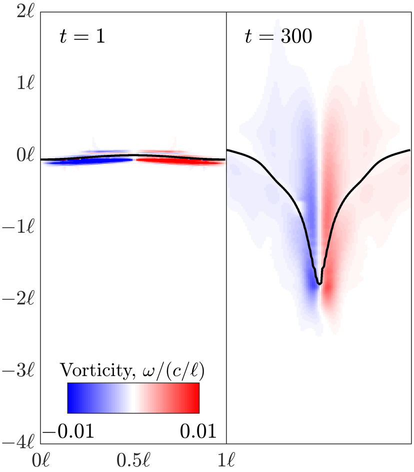

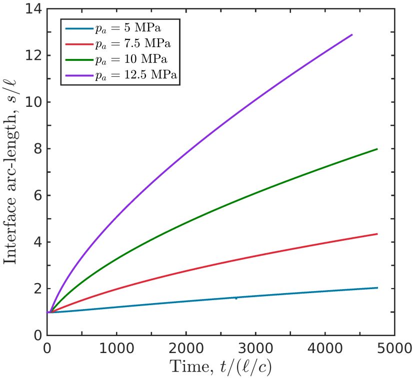

Interface arc length: $s(t) \sim \Gamma(t)t^{1/2}$

Vorticity drives deformation in a mathematically describable wave.

Hypothesis:
Ultrasound waves generate baroclinic vorticity at gas-liquid interfaces, driving the interface deformation

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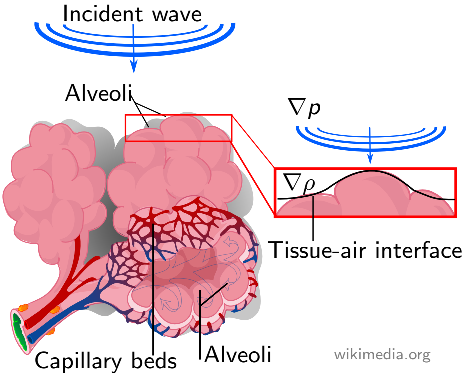
Aims:

1. Test hypothesis
2. Calculate approximate interface stresses and strains for clinically relevant ultrasound waves
3. Compare calculated stresses and strains with expected alveolar failure thresholds

We hypothesize that US waves generate baroclinic vorticity at gas-liquid interfaces, driving deformation.

- Air-tissue interfaces have sharp density gradients
- US has strong pressure gradients
- US-induced baroclinic vorticity may cause strain, similar to shock-driven interfaces
- Linear acoustics does not capture this.



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The vorticity generation equation

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega} \cdot \nabla)\,\mathbf{u} \quad - \boldsymbol{\omega}\,(\nabla \cdot \mathbf{u}) \;+ \frac{\nabla\rho \times \nabla p}{\rho^2} \;- \nabla \times \left(\frac{\nabla \cdot \boldsymbol{\tau}}{\rho}\right) \quad + \nabla \times \mathbf{B}$$

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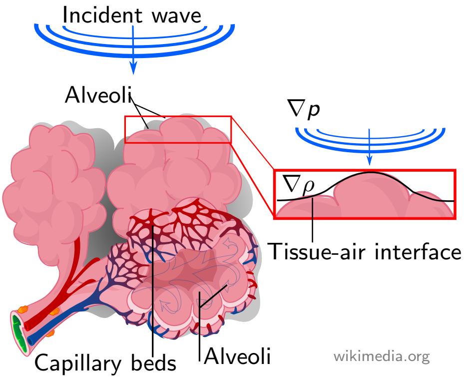

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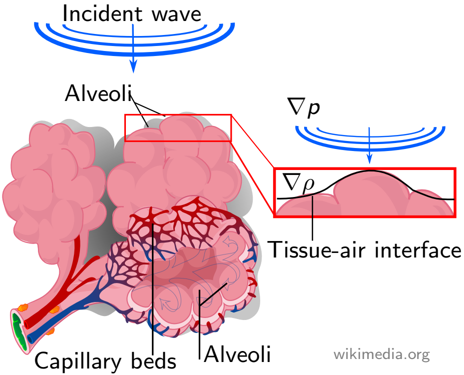
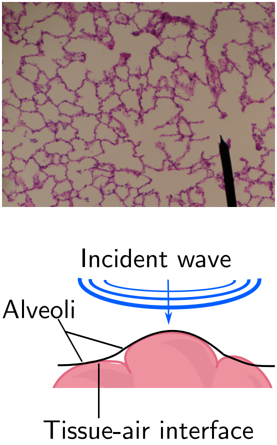
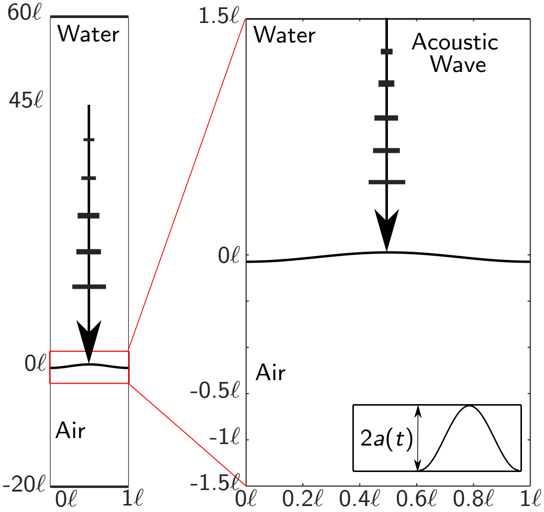


Ultrasound driven alveoli are modeled as compressible fluids.



(a) Physical problem schematic



(b) Domain and model problem schematic.

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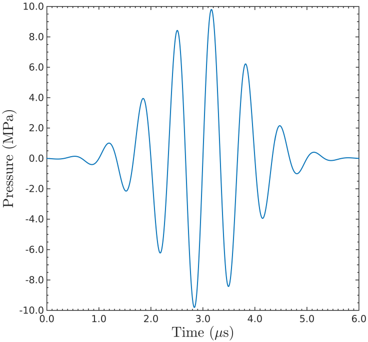
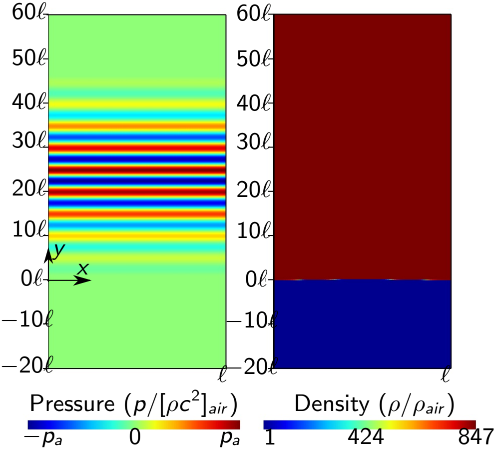


Ultrasound pulse waveform



Initial Condition

Governing Equations

Euler equations of fluid motion

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0,$$

$$\frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x}\left(\rho u^2 + p\right) + \frac{\partial}{\partial y}(\rho u v) = 0,$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial}{\partial x}(\rho u v) + \frac{\partial}{\partial y}\left(\rho v^2 + p\right) = 0,$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}\left[u(E + p)\right] + \frac{\partial}{\partial y}\left[v(E + p)\right] = 0,$$

Stiffened equation of state

$$E = \frac{\rho\left(u^2 + v^2\right)}{2} + \frac{p + \gamma B}{\gamma - 1}.$$

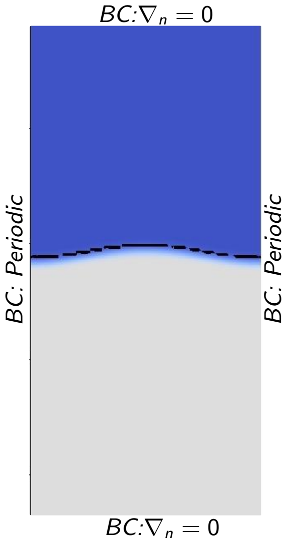
Advection equations for γ, B prevent interface pressure oscillations.

$$\frac{\partial}{\partial t}\left(\frac{\gamma B}{\gamma - 1}\right) + u\frac{\partial}{\partial x}\left(\frac{\gamma B}{\gamma - 1}\right) + v\frac{\partial}{\partial y}\left(\frac{\gamma B}{\gamma - 1}\right) = 0,$$

$$\frac{\partial}{\partial t}\left(\frac{1}{\gamma - 1}\right) + u\frac{\partial}{\partial x}\left(\frac{1}{\gamma - 1}\right) + v\frac{\partial}{\partial y}\left(\frac{1}{\gamma - 1}\right) = 0$$

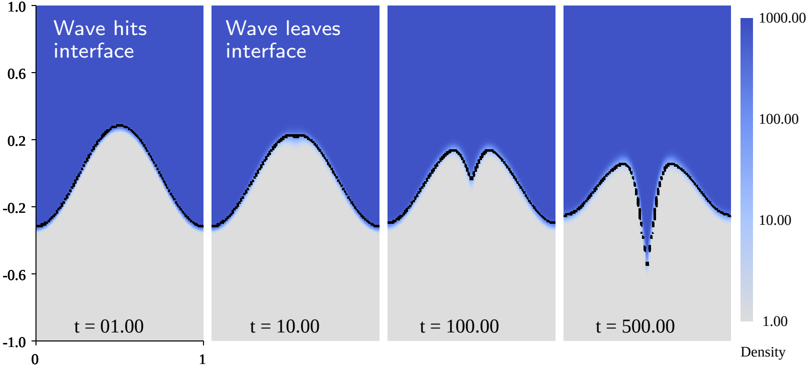
A high-order accurate computational solution strategy is invoked

- An in-house developed code is used to solve the Euler equations.
- Numerical methods
  - 3^{rd} order Discontinuous Galerkin method is used in space
  - 4^{th} order Runge-Kutta time marching
  - Roe Solver used to handle discontinuities
- Acoustic waves are prescribed within the domain.
- Grid stretching reduces reflections.
- Grid size: $\ell \times 80\ell\,(L_x \times L_y)$



The theoretical interface dynamics are simulated

Ex: Ultrasound pulse: $f = 1.5$ MHz, $p_a = 5$ MPa; Interface:$a_0 = 0.3\ell$.



The interface evolves long after the wave has passed.

Linear acoustics can't explain this.

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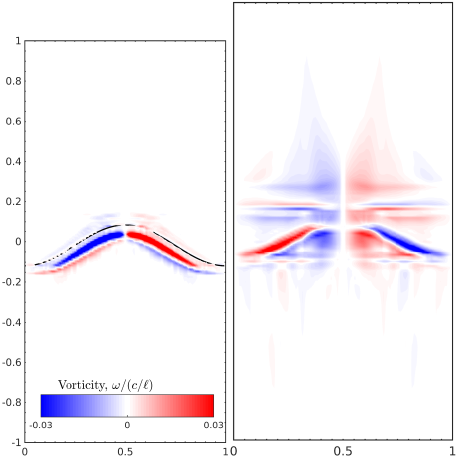
The ultrasound pulse deposits vorticity at the interface



- Vorticity snapshot

Baroclinic vorticity is driving the deformation after the passage of the wave

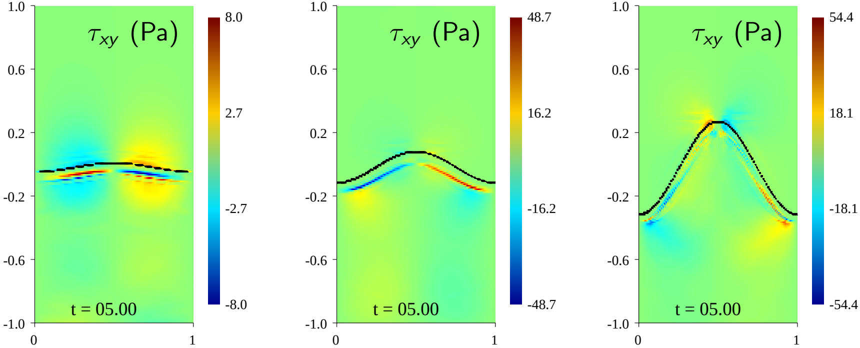
- Plot of circulation vs time for 10 MPa wave
- Plot of s(t)/Gamma(t) for both trapezoidal and US pulse waves

The specific case of interest

- $a_0 = 0.03, 0.1, 0.3\ell$
- $p_a = 1, 2.5, 5$ MPa

The approximate viscous stress is calculated

$$\tau_{xy}(x, y, t) = \mu\left(\frac{du}{dy} + \frac{dv}{dx}\right), \qquad p_a = 5 \text{ MPa}$$



(a) $a_0 = 0.03\ell$ (b) $a_0 = 0.1\ell$ (c) $a_0 = 0.3\ell$

Maximum calculated viscous stresses are far below expected stress failure thresholds 80 kPa for alveolar walls, but the comparison isn't perfect.

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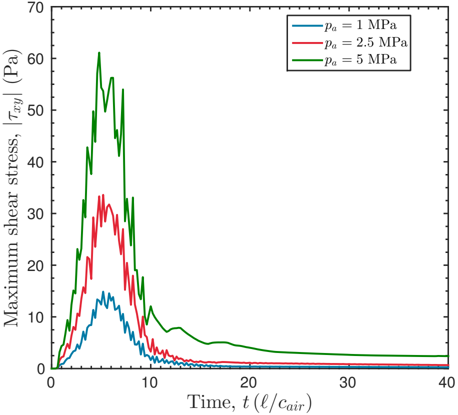

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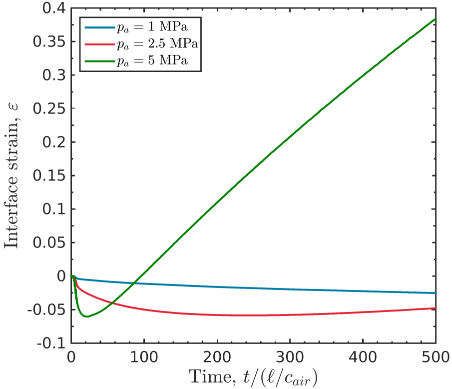
The interface strain is calculated



(a) $a_0 = 0.3\ell$

- $s(t)$ for $p_a = 1, 2.5, 5$ MPa for $a_0 = 0.3\ell$
- $s(t)$ for $p_a = 2.5$ MPa for $a_0 = 0.03\ell, 0.1\ell, 0.3\ell$

Discussion of results

- A single pulse is sufficient to appreciably strain an air-water interface and may similarly strain air-tissue interfaces
- Circulation from subsequent pulses could add up.
- Physical effects such as elasticity and viscosity that don't effect the dynamics here, may plan an important role, so a better description of these properties would be useful.

Beyer, Robert T. 1974. *Nonlinear Acoustics*.

Child, S.Z., Hartman, C.L., Schery, L.A., & Carstensen, E.L. 1990. Lung damage from exposure to pulsed ultrasound. *Ultrasound med. biol.*, **16**(8), 817–825.

Miller, DL. 2012. Induction of Pulmonary Hemorrhage in Rats During Diagnostic Ultrasound. *Ultrasound med. biol.*, **38**(8), 1476–1482.

O'Brien, William D. W.D., & Zachary, J.F. 1997. Lung damage assessment from exposure to pulsed-wave ultrasound in the rabbit, mouse, and pig. *Ieee trans. ultrason. ferroelectr. freq. control*, **44**(2), 473–485.

Tarantal, Alice F., & Canfield, Don R. 1994. Ultrasound-induced lung hemorrhage in the monkey. *Ultrasound med. biol.*, **20**(1), 65–72.

West, J B, Tsukimoto, K, Mathieu-Costello, O, & Prediletto, R. 1991. Stress failure in pulmonary capillaries. *J. appl. physiol.*, **70**(4), 1731–1742.

BACKUP SLIDES

Argument against viscosity - viscous length scales

$\nu_w = 0.7\ \mu\text{m}^2/\text{s}, \qquad \nu_a = 16.6\mu\text{m}^2/\text{s}, \qquad f_c = \mathcal{O}\left(10^6\right)\text{Hz}$

$\sqrt{\nu_{air}/f_c} = 4\mu\ \text{m} = \mathcal{O}\left(10^{-6}\right) << L_{\text{alveolus}} = \mathcal{O}\left(10^{-4}\right)$

$\sqrt{\nu_{air,ND} t} \approx 0.5 < a(t) - a_0 \approx 4$ at $t = 1000$

Therefore the scale of the viscous effect is smaller than the scale of the problem we are looking at, but may be important at late times.

Dimensional Numbers

- Let $\lambda_{alveolus} = 100\mu$ m, $u_0 = c_{air} = 343$ m, $v_0 =< \dot{a}(t) >\approx 0.65$ m/s, $u_{intf}(t = 20) = 12.8$ m/s, $G = 1$ kPa
- $\lambda_{alveolus} = 100\mu$ m, $u_0 = c_{air} = 343$ m, $v_0 =< \dot{a}(t) >\approx 0.65$ m/s
- $t = 1 \rightarrow t_{dim} = 0.292\mu$ s

Dimensionless Numbers

- $Fr = \frac{u_0}{\sqrt{g_0 \lambda}} \approx 11000$
- $Fr = \frac{v_0}{\sqrt{g_0 \lambda}} \approx 21$
- $Ca = \frac{\rho u_{intf}^2}{G_{Alv}} = 163$

Interface treatment

Interface thickness parameter:

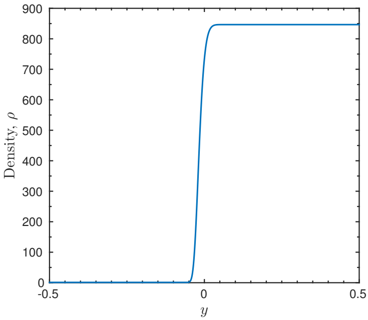
$$\delta = 0.08\lambda$$

Normalized distance from interface:

$$d = \frac{\delta + y(x)_{interface} - y}{2\delta}$$

Volume fraction:

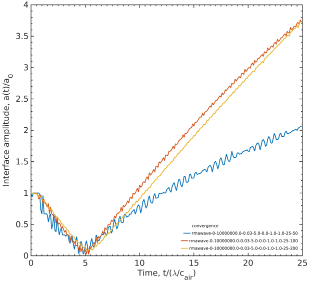
$$y_0 = \begin{cases} 1 \\ exp\left(log\left(10^{-16}\right)|d|^8\right) \\ 0 \end{cases}$$

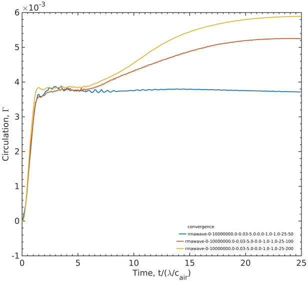


Radiation Pressure

$P_{net} = \frac{\Delta p_a}{2}\left[1 - \frac{c_w}{c_a} + \frac{(\rho c)_a - (\rho c)_w}{(\rho c)_a (\rho c)_w}\right]$ Beyer (1974)
Stress failure in the lungs:
Rabbit lungs under transmural pressure: $\approx 5.2kPa$ (West *et al.*, 1991);

Convergence tests: Compression wave





50 pts / λ,   100 pts / λ 200 pts / λ