

Prelim - Brandon Patterson

Slide 3

Ref on both images, might not be important

Slide 5

How good is MC techniques in general?

6<sup>th</sup> figure difficult to see where the method is good  $\rightarrow$  better image?

What is shallow? How is that relative to the depth of importance relevant to the application?

Slide 7

Ref to figures? Don't know if they are needed

Slide 8

How many maximum bubble radius are experienced in the extreme cases of negative bioeffects? Once, twice?  
How does that connect w/ frequency?

Slide 12

"Richtmeyer" is misspelled, I believe.

Slide 18

"0.1.00" don't need this digit

# Applications of computation in acoustics: Ultrasound bioeffects and transmission loss uncertainty

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July 8, 2016



Part I

My work focuses on two very different problem areas of acoustics.

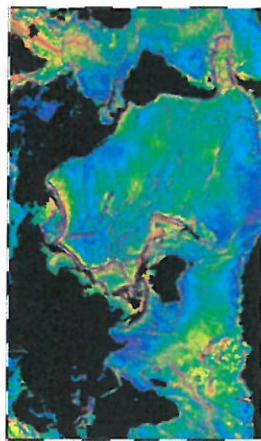
Consequences  
as many  
sound to  
see

Ultrasound bioeffects



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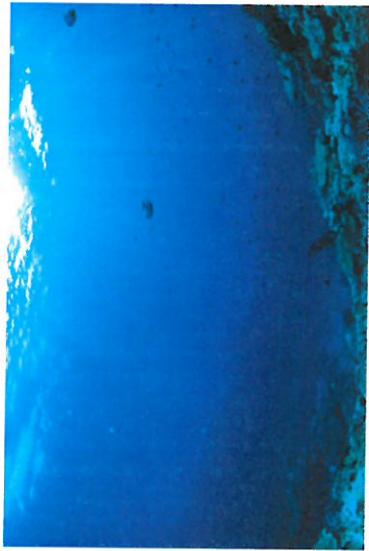
Acoustic uncertainty in the ocean



better explanation  
of migration

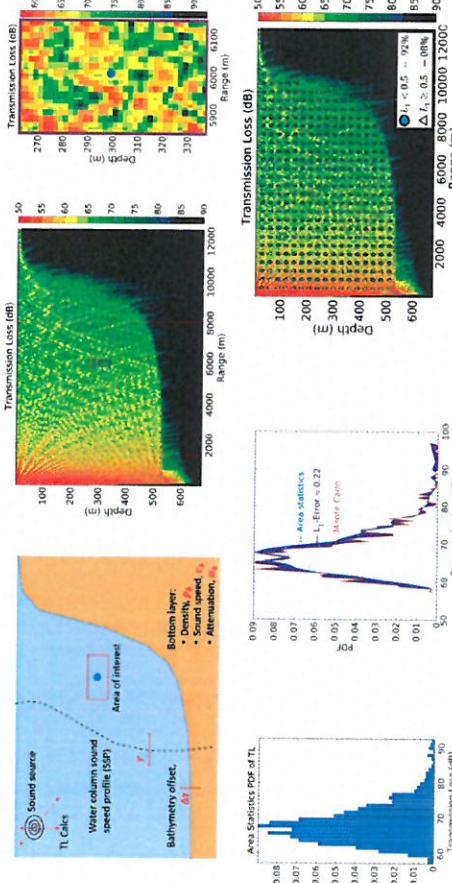
*Past work:* Efficient estimation of the probability density function of transmission loss in uncertain ocean environments

Transmission Loss,  $TL = 20 \log_{10} \left( \frac{P_{source}}{P_{receiver}} \right)$ , is useful for naval applications.



TL uncertainty is important for those making decisions based on TL, but traditional methods are slow and expensive.

*Past work:* We developed a computationally efficient way of computing TL in uncertain environments



- Engineering level accurate ( $L_1$ -error < 0.5) in 93% of test cases in bottom reflecting environments.
- $\approx \mathcal{O}(10^{-6})$  the cost of 1000-sample Monte Carlo Methods.

## Background on medical ultrasound

### Diagnostic



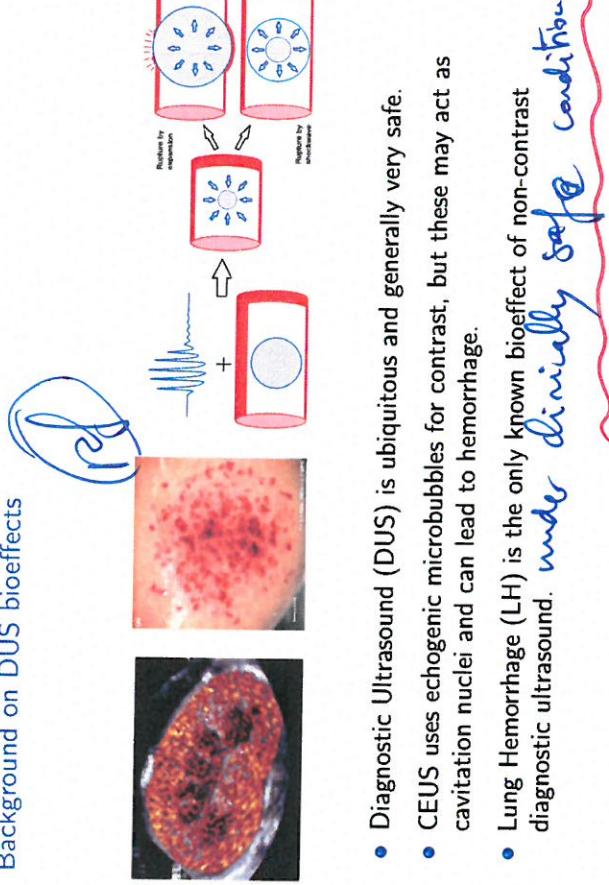
### Therapeutic



- High frequency (MHz) sound waves travel into the body and scatter at material interfaces.
- Acoustic energy is dissipated as heat or through mechanical means.  
*at least*

I focus on Diagnostic Ultrasound (DUS) bioeffects and relevant physics.

## Background on DUS bioeffects



- Diagnostic Ultrasound (DUS) is ubiquitous and generally very safe.
- CEUS uses echogenic microbubbles for contrast, but these may act as cavitation nuclei and can lead to hemorrhage.
- Lung Hemorrhage (LH) is the only known bioeffect of non-contrast diagnostic ultrasound. *under clinically safe conditions*

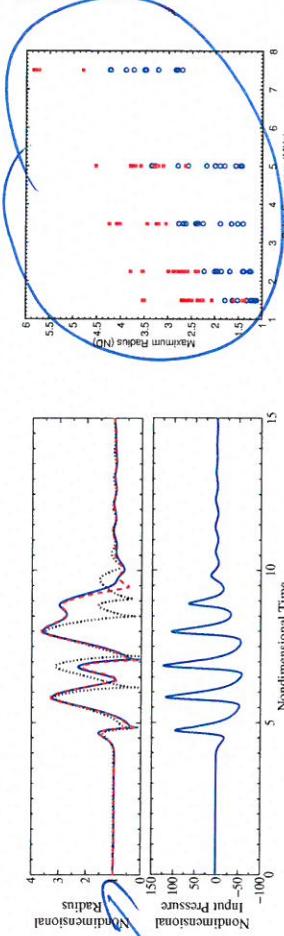
Past work: Theoretical microbubble dynamics in a viscoelastic medium at capillary breaching thresholds



$$\left(1 - \frac{\dot{R}}{R}\right) R \ddot{R} + \frac{3}{2} \left(1 - \frac{\dot{R}}{R}\right) \dot{R}^2 = \left(1 + \frac{\dot{R}}{R}\right) [p_B - 1 - p_a - \frac{\rho}{\xi} \frac{dp_a}{dt}] + \frac{\rho}{\xi} \dot{p}_B,$$

$$p_B = \left(1 + \frac{2}{M_\infty}\right) \frac{1}{R^{3/\gamma}} - \frac{2}{M_\infty R} + \tau_R,$$

Parameter	Dimensional value	Dimensionless number
Viscosity	$\mu = 0.015 \text{ (Pa s)}$	$\text{Re} = \rho u R_0 / \mu = 2/3$
Elasticity	$G = 10^5 \text{ (Pa)}$	$\text{Ca} = \rho u^2 / G = 1.0$
Surface tension	$S = 0.056 \text{ (N/m)}$	$\text{We} = \rho u^2 R_0 / S = 2$
Sound speed	$c = 1570 \text{ (m/s)}$	$C = c/u = 157$



Patterson, B., Miller, D. L., & Johnson, E. (2012). Theoretical microbubble dynamics in a viscoelastic medium at capillary breaching thresholds. *JASA*, 132(6), 3770.

## **Part II: Current Project**

Diagnostic ultrasound-induced lung hemorrhage and acoustic wave interactions with liquid-gas interfaces

### DUS-induced lung hemorrhage is not a new problem

- Lung Hemorrhage (LH) is the only known bioeffect of non-contrast DUS
- Has been shown to occur in mice, rats, pigs, rabbits, monkeys (Child *et al.*, 1990; O'Brien & Zachary, 1997; Tarantal & Canfield, 1994).
- DUS-induced LH does not appear to be a result of cavitation or heating.
- The underlying physical damage mechanism is not understood.

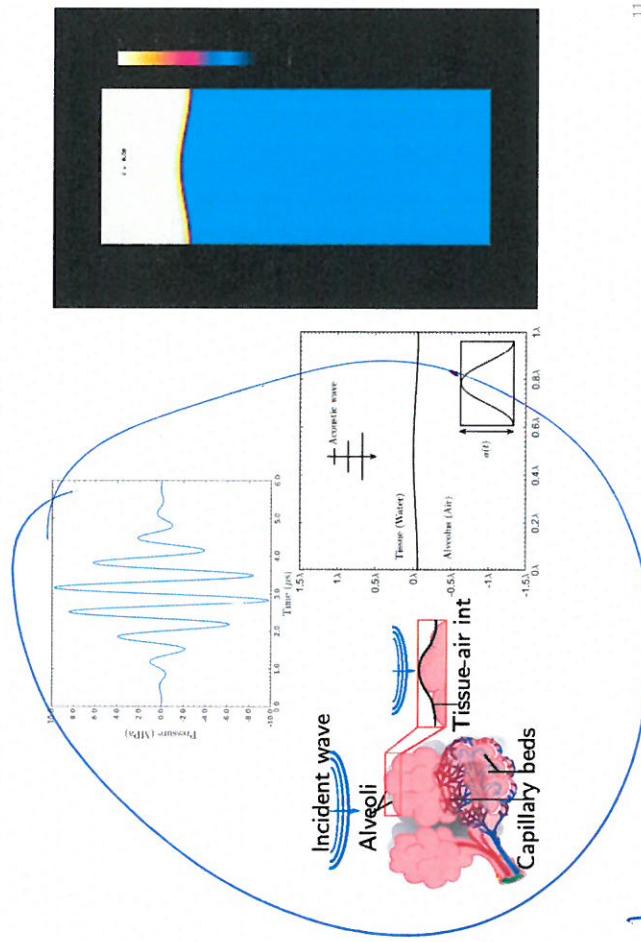


Ultrasound-induced pulmonary hemorrhage in a rat lung (Miller, 2012)

To gain insight into DUS-induced LH, we aim to use computational modeling to investigate DUS-lung interaction.  
+ Simulation →

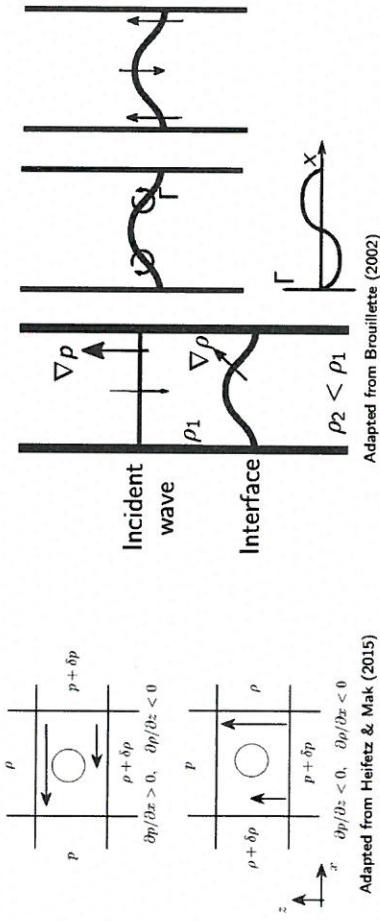
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We simulated and US-pulse impinging on a water-air interface



need  
to explain  
better if you're  
sorry

Shock-driven fluid-fluid interfaces have been studied extensively



Adapted from Brouillet (2002)

- Shocks deposit baroclinic vorticity at perturbed fluid-fluid interfaces (Drake, 2006), which drives the interface perturbation to grow.

• This is the Richtmyer-Meshkov "instability".

• This problem is not well studied for acoustic waves.

So I thought to study it  
~~to~~ not a good way to  
justify studying

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You want  $\nabla p \times \nabla V$  is generally small in acoustic radiation  
 but avoid this is small  
 but not large (so)  
 will  $\nabla p$  effects on  
 b/c here to neglect  
 $\Rightarrow$  larger no  
 we make this  
 vorticity way  
~~not~~ skin  
 growth

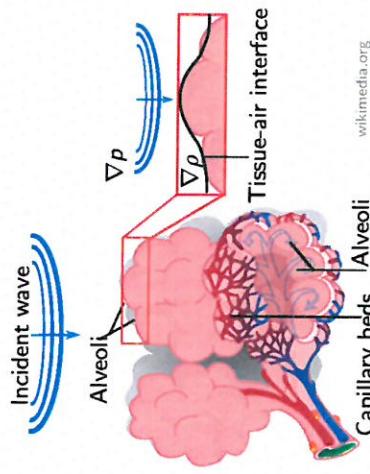
we need fully  
 nonlinear effects

We hypothesize that US waves generate baroclinic vorticity at air-tissue interfaces in the lungs, straining fragile alveolar walls.

The vorticity generation equation

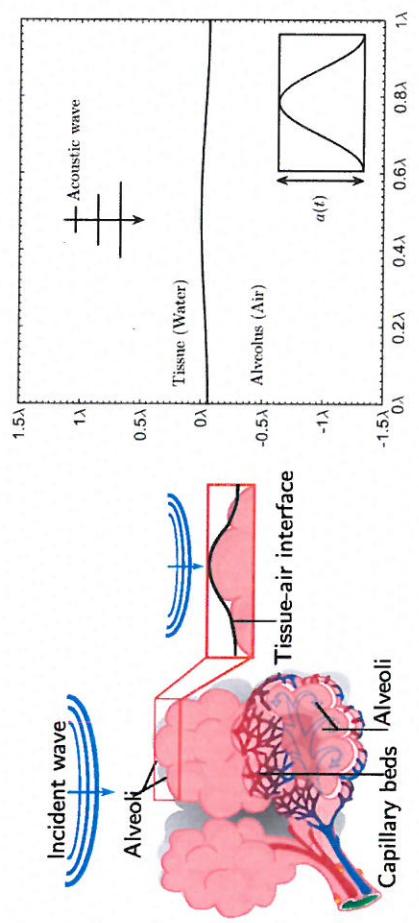
$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla) \omega = (\omega \cdot \nabla) \mathbf{u} - \omega (\nabla \cdot \mathbf{u}) + \frac{\nabla \rho \times \nabla p}{\rho^2} - \nabla \times \left( \frac{\nabla \cdot \tau}{\rho} \right) + \nabla \times \mathbf{B}$$

$$\frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla) \omega = (\omega \cdot \nabla) \mathbf{u}^0 - \omega (\nabla \cdot \mathbf{u}) + \frac{\nabla \rho \times \nabla p}{\rho^2} - \nabla \times \left( \frac{\nabla \cdot \tau^0}{\rho} \right) + \nabla \times \mathbf{B}^0$$



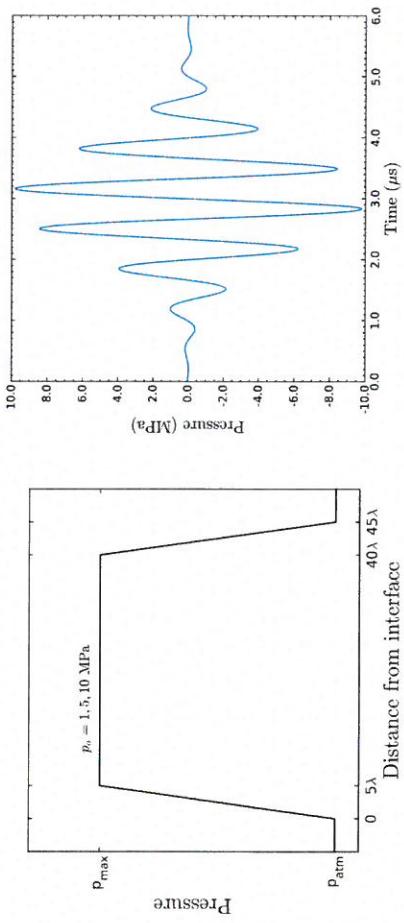
- Linear acoustics does not capture this.

Problem setup: We model the ultrasound-alveolar interaction as a 2D, compressible, inviscid fluid system.



An acoustic wave impinges downward from water toward a perturbed air interface ( $a_0=0.03\lambda$ ).

Trapezoidal and US pulse acoustic waveforms are used.



- The trapezoidal waves is simple for understanding physics and analysis, but able to capture feature of US pulse.
- Pulse waveforms are used to check relevance to DUS.

## Governing Equations

Euler equations of fluid motion

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} &= 0, \\ \frac{\partial \rho u}{\partial t} + \frac{\partial}{\partial x} (\rho u^2 + p) + \frac{\partial}{\partial y} (\rho uv) &= 0, \\ \frac{\partial \rho v}{\partial t} + \frac{\partial}{\partial x} (\rho uv) + \frac{\partial}{\partial y} (\rho v^2 + p) &= 0, \\ \frac{\partial E}{\partial t} + \frac{\partial}{\partial x} [u(E + p)] + \frac{\partial}{\partial y} [v(E + p)] &= 0,\end{aligned}$$

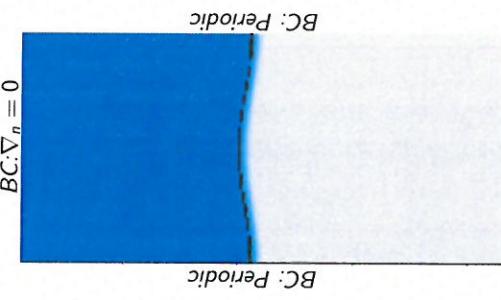
Stiffened equation of state

$$E = \frac{\rho(u^2 + v^2)}{2} + \frac{p + \gamma B}{\gamma - 1}.$$

Advection equations for  $\gamma, B$  prevent interface pressure oscillations.

$$\begin{aligned}\frac{\partial}{\partial t} \left( \frac{\gamma B}{\gamma - 1} \right) + u \frac{\partial}{\partial x} \left( \frac{\gamma B}{\gamma - 1} \right) + v \frac{\partial}{\partial y} \left( \frac{\gamma B}{\gamma - 1} \right) &= 0, \\ \frac{\partial}{\partial t} \left( \frac{1}{\gamma - 1} \right) + u \frac{\partial}{\partial x} \left( \frac{1}{\gamma - 1} \right) + v \frac{\partial}{\partial y} \left( \frac{1}{\gamma - 1} \right) &= 0\end{aligned}$$

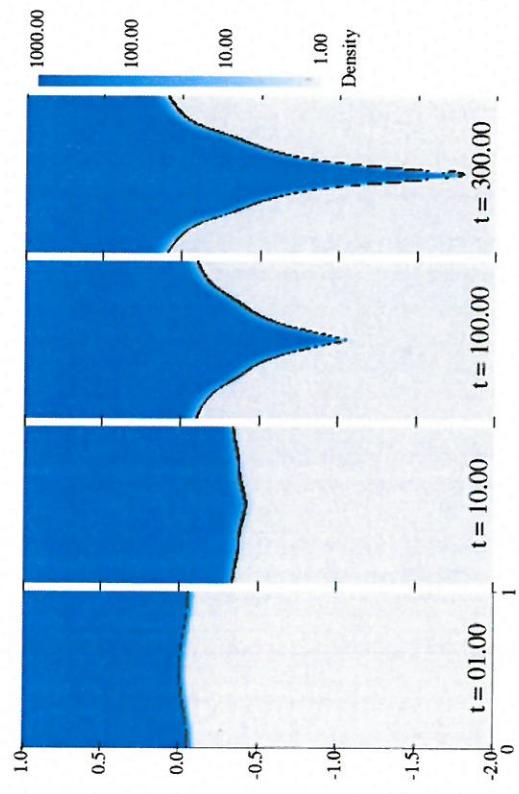
A high-order accurate computational solution strategy is invoked



- An in-house developed code is used to solve the Euler equations.
- Numerical methods
  - 3<sup>rd</sup> order Discontinuous Galerkin method is used in space
  - 4<sup>th</sup> order Runge-Kutta time marching
  - Roe Solver used to handle discontinuities
- Acoustic waves are prescribed within the domain.
- Grid stretching reduces reflections.
- Grid size:  $\lambda \times 70\lambda (L_x \times L_y)$

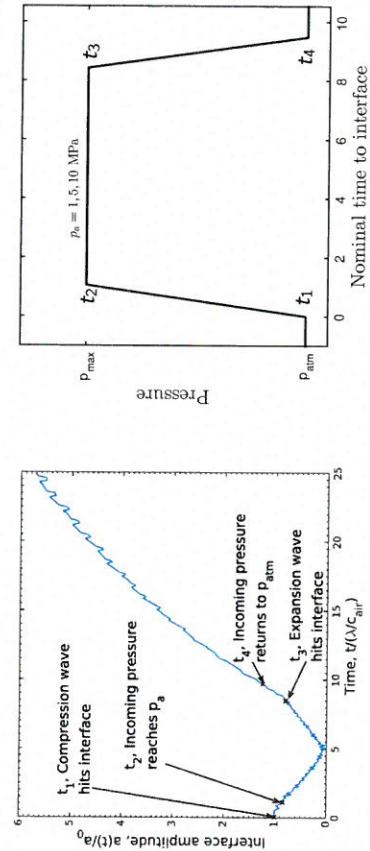
$BC: \nabla_n = 0$

Results: Evolution of the interface after 10 MPa trapezoidal wave



The interface perturbation evolves from a smooth sinusoid into a sharp point.

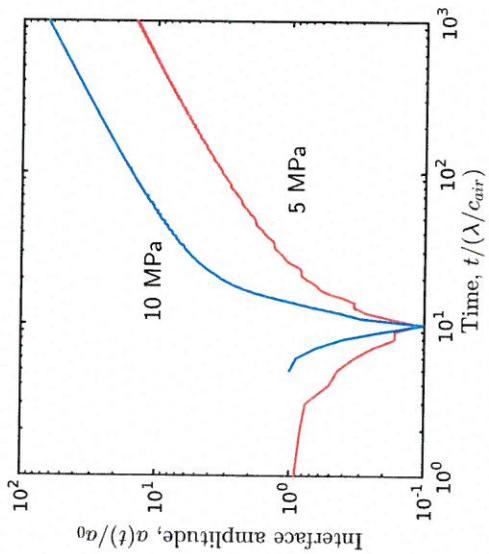
### Results: Evolution of the interface



The interface perturbation is initially compressed ( $0^+ \leq t \leq 5$ ), experiences a phase change ( $t = 5$ ), then grows  $t > 5$ .

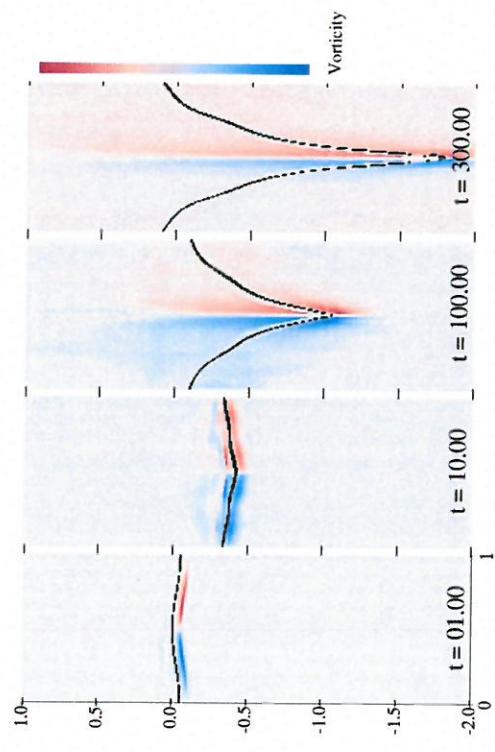
Results: Late-time evolution of the interface

and at least one more  
7.5 MPa?  
(2.5 MPa?)



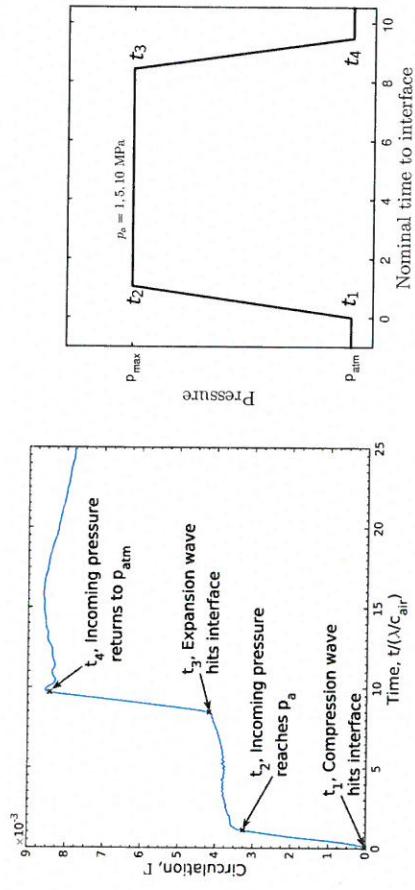
We suspect vorticity is driving this late time growth.

### Results: Vorticity dynamics



Vorticity is initially deposited in the air-dominated portion of the interface region. As the interface evolves, some vorticity advects with it.

Results: A closer look at how circulation is deposited.

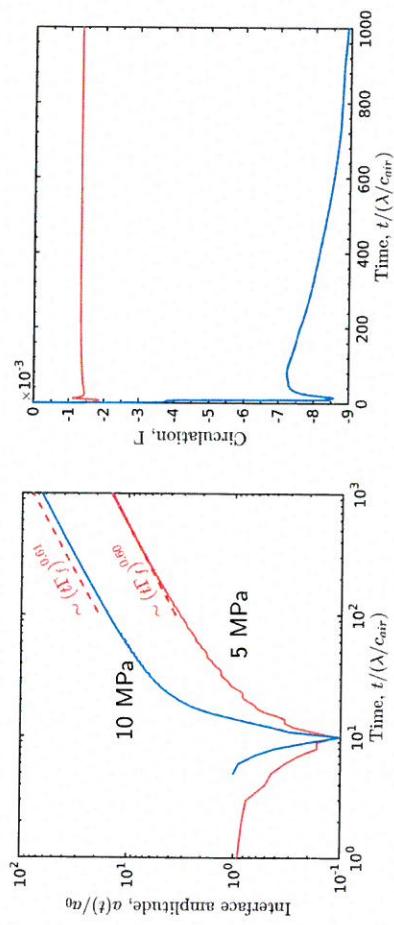


Both the compression and expansion deposit vorticity.

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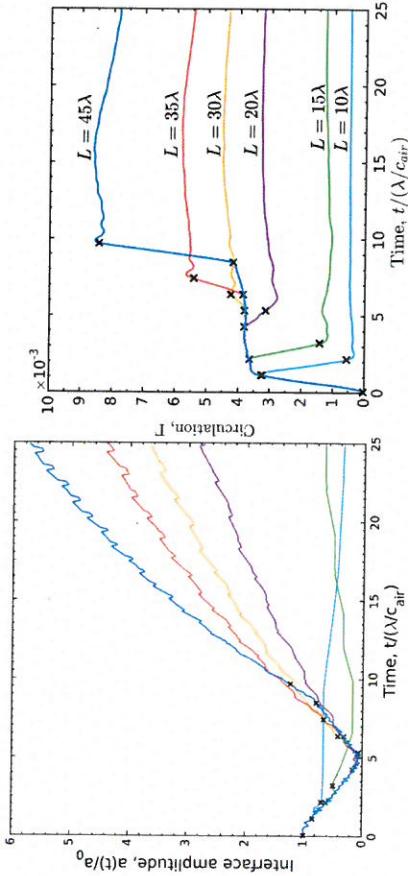
Show contours to support  
your statement

### Results: Late-time evolution of the interface



From dimensional analysis, we expect a purely circulation driven interface to grow as  $a(t) \sim \sqrt{\Gamma t}$ .

Results: Dependence on the length of the wave



The length of the wave strongly effects the vorticity deposited and thus the long-term dynamics of the interface.

~~changing width  $\Rightarrow$  changes time when wave units~~

at first time, interface has enough  $\Rightarrow$  different amount of vorticity deposited

should explain why this term is not including in the vorticity equation (e.g., you're not including viscous terms)

### Order of magnitude analysis of vorticity generation

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{u} \cdot \nabla) \vec{\omega} = -\vec{\omega}(\nabla \cdot \vec{u}) + \frac{\nabla \rho \times \nabla p}{\rho^2}.$$

Acoustic Relations and operator treatments

$$\Delta \rho_a = \pm \Delta u_a \rho c = c^2 \Delta \rho_a,$$

Baroclinic vorticity generation

$$\left\| \frac{\nabla \rho \times \nabla p}{\rho^2} \right\| = O \left( \frac{|\Delta \rho|}{|\Delta L_a|} \frac{|\Delta p_a|}{|\Delta L_a|} \frac{1}{|\rho|^2} |\theta| \right)$$

Advection and compressible vorticity generation -  $\|\vec{\omega}\| = \int_0^{\Delta t_a} (\text{baroclinic term}) dt$

$$\|(\vec{u} \cdot \nabla) \vec{\omega}\| \sim \|-\vec{\omega}(\nabla \cdot \vec{u})\| = O \left( \left[ \frac{|\Delta u_a|}{|\Delta L_a|} \right]^2 \right),$$

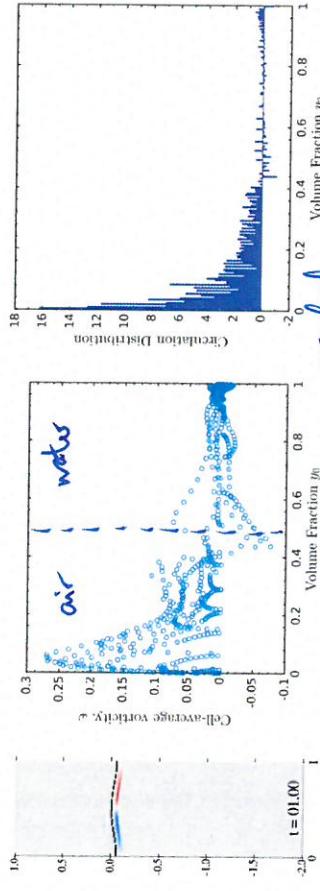
Comparing terms for our problem

$$\begin{aligned} \Delta t_a &\approx 5\lambda/c_w, & \Delta L_a &= 5\lambda, & \Delta \rho_a &= 10 MPa, & \Delta L_I &\approx 0.05\lambda \\ \frac{\|\frac{\nabla \rho \times \nabla p}{\rho^2}\|}{\|-\vec{\omega}(\nabla \cdot \vec{u})\|} &= \frac{c}{|\Delta u_a|} = \frac{\rho}{|\Delta \rho_a|} = O(10^2) \end{aligned}$$

Calculated values at  $t = 1$ :

$$\frac{\nabla \rho \times \nabla p}{\rho^2} = 7.7 \times 10^{-3}, \quad (\vec{u} \cdot \nabla) \vec{\omega} = -5.3 \times 10^{-5}, \quad -\vec{\omega}(\nabla \cdot \vec{u}) = 2.7 \times 10^{-5},$$

Vorticity generation occurs predominantly in gas-dominated fluid



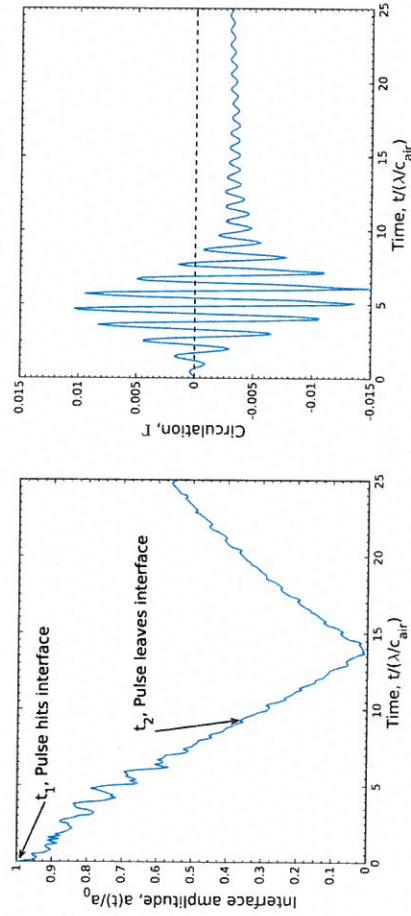
$$\frac{\left\| \frac{\nabla \rho \times \nabla p}{\rho^2} \right\|_{air}}{\left\| \frac{\nabla \rho \times \nabla p}{\rho^2} \right\|_{water}} = O\left(\left|\frac{|\rho^-|}{|\rho^+|}\right|^2\right) \approx 357$$

*define*

*dependence on  $\Delta x$ .*

- 97% of circulation appears in fluid with  $y_0 < 0.5$
  - Computed circulation ratio from air( $y_0 < 0.1$ ) to water( $y_0 > 0.9$ ) is  $\approx 27$
- define*

### Interface response to a 10 MPa US pulse



- Qualitatively, the interface response for the 10 MPa US pulse looks very similar to the 10 MPa trapezoidal wave.
- The circulation deposited is of the same order as the equivalent amplitude trapezoidal wave.

- Interaction of finite duration in wave (acoustic) with interface has density
- ~~baroclinic vorticity~~

Conclusions thus far

- Acoustically-generated baroclinic vorticity is likely capable of significantly deforming perturbed liquid-gas interfaces.

At interface

- Baroclinic vorticity is predominantly deposited in gaseous fluids.

- Changes in the acoustic waveform that have little effect on the interface during the wave-interface interaction can substantially affect the interface over long periods of time, via vorticity.
  - Interface response from an US wave is qualitatively similar to that for a trapezoidal wave.
- Much matters is amount of work, which in these problems strongly depends on morphology of interface

### Part III: Future work

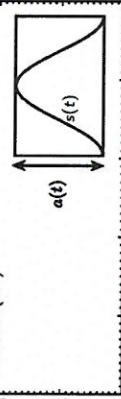
Curious is good  
Your motivate in  
the form of hypothesis  
dram much

I plan to increase the relevance to DUS

1.5 $\lambda$

Tissue (Water) Interface(t) length  $s(t)$

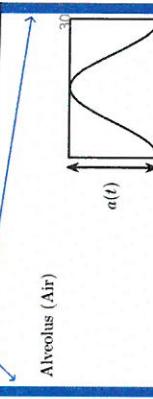
Alveolus (Air)



1.5 $\lambda$

Tissue (Water) Interface(t) length  $s(t)$

Alveolus (Air)



1.5 $\lambda$

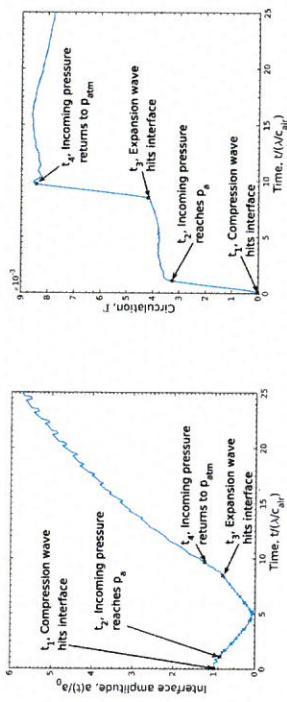
Tissue (Water) Interface(t) length  $s(t)$

Alveolus (Air)



- Calculate interface strain
- Calculate stress at the interface
- Investigate the effects of alveolar side wall structures
- Investigate the propagation of acoustic waves into subsequent layers of alveoli

I aim to further our understanding of the relevant fluid mechanics



- Explain the discrepancies between numerical results and  $a(t) \sim \sqrt{t}$
- Develop a model to approximate the circulation deposited on a slightly perturbed interface by a compression or expansion wave
- Develop a model to predict the interface phase-reversal time for a compression wave
- Design an acoustic waveform to minimize circulation deposited and interface growth.
- Investigate cause of late time circulation growth

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