BACS HW15

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Question 1

a. Show a scree plot of data

```
df <- read.csv("/home/johnbjohn/Documents/git-repos/bacs-hw/hw15/security_questions.csv")
pca <- prcomp(df, scale. = TRUE)
```

Create a noise simulation function

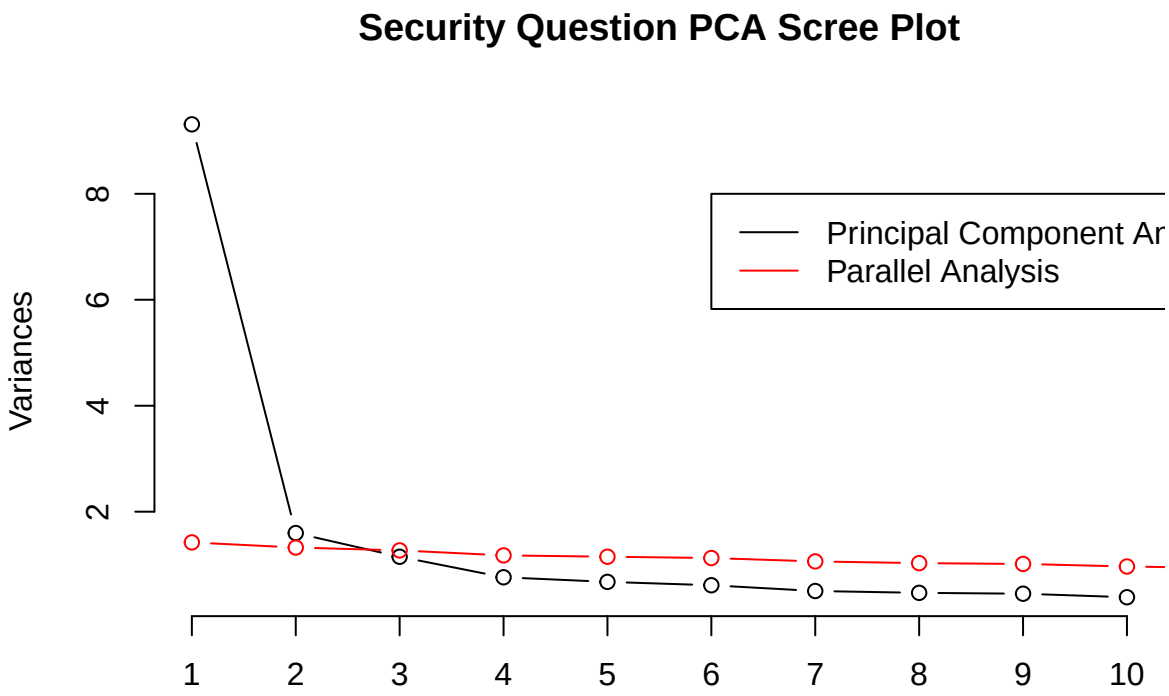
```
noise_simulation <- function(x, y) {
  random_normal_dist <- replicate(y, rnorm(x))
  noise <- data.frame(random_normal_dist)
  eigenvalues <- eigen(cor(noise))$values
  return(eigenvalues)
}
```

```
set.seed(2000)
x <- dim(df)[1]
y <- dim(df)[2]
simulate_noise <- replicate(1, noise_simulation(x, y))
evaluated_mean <- apply(simulate_noise, 1, mean)

screeplot(
  pca,
  type = "line",
  col = "black",
  main = "Security Question PCA Scree Plot"
)

lines(evaluated_mean, type = "b", col = "red")

legend(
  6,
  8,
  c("Principal Component Analysis", "Parallel Analysis"),
  lty=c(1,1),
  col = c("black", "red")
)
```



b. How many dimensions would you retain if we used Parallel Analysis?

I would keep 3 dimensions since the red line intersects the black line at the third column.

Question 2

```
pca_result <- principal(df, nfactors = 3, rotate = "none", scores = TRUE)
pca_result
```

```
## Principal Components Analysis
## Call: principal(r = df, nfactors = 3, rotate = "none", scores = TRUE)
## Standardized loadings (pattern matrix) based upon correlation matrix
##      PC1   PC2   PC3   h2   u2 com
## Q1  0.82 -0.14  0.00 0.69 0.31 1.1
## Q2  0.67 -0.01  0.09 0.46 0.54 1.0
## Q3  0.77 -0.03  0.09 0.60 0.40 1.0
## Q4  0.62  0.64  0.11 0.81 0.19 2.1
## Q5  0.69 -0.03 -0.54 0.77 0.23 1.9
## Q6  0.68 -0.10  0.21 0.52 0.48 1.2
## Q7  0.66 -0.32  0.32 0.64 0.36 2.0
## Q8  0.79  0.04 -0.34 0.74 0.26 1.4
## Q9  0.72 -0.23  0.20 0.62 0.38 1.4
```

```
## Q10 0.69 -0.10 -0.53 0.76 0.24 1.9
## Q11 0.75 -0.26  0.17 0.66 0.34 1.4
## Q12 0.63  0.64  0.12 0.82 0.18 2.1
## Q13 0.71 -0.06  0.08 0.52 0.48 1.0
## Q14 0.81 -0.10  0.16 0.69 0.31 1.1
## Q15 0.70  0.01 -0.33 0.61 0.39 1.4
## Q16 0.76 -0.20  0.18 0.65 0.35 1.3
## Q17 0.62  0.66  0.11 0.83 0.17 2.0
## Q18 0.81 -0.11 -0.07 0.67 0.33 1.1
##
##                         PC1  PC2  PC3
## SS loadings            9.31 1.60 1.15
## Proportion Var         0.52 0.09 0.06
## Cumulative Var         0.52 0.61 0.67
## Proportion Explained   0.77 0.13 0.10
## Cumulative Proportion  0.77 0.90 1.00
##
## Mean item complexity =  1.5
## Test of the hypothesis that 3 components are sufficient.
##
## The root mean square of the residuals (RMSR) is  0.05
##  with the empirical chi square  258.65  with prob <  1.4e-15
##
## Fit based upon off diagonal values = 0.99
```

a. To which principal components does each question seems to best belong?

Let's set a threshold of 0.7, we then have `Q1, Q3, Q8, Q9, Q11, Q13, Q14, Q16, Q18` that belong to `PC1` and the other questions seem to belong to either `PC2` or `PC3`.

b. How much variance captured by `PC1`, `PC2`, and `PC3`.

```
summary(pca)$importance[2, c(1:3)]
```

```
##     PC1     PC2     PC3
## 0.51728 0.08869 0.06386
```

All we need is the first three columns of the second row, which is `Proprotion of Variance`. Thus, the variance explained by the `PC1` is 51%, `PC2` is 8%, and `PC3` is 6%. In total, `PC1`, `PC2`, and `PC3` explain roughly 66% of total variance.

c. Which questions are less than adequately explained by the first 3 principal components?

It's `Q2` since it has 'h2' of 0.4605433.

d. How many measurement items share similar loadings between 2 or more components?

```
evaluate_loadings <- function(df, range) {
  return(
    (
      abs(df[range, 1] - df[range, 2])<0.1 |
      abs(df[range, 2] - df[range, 3])<0.1 |
      abs(df[range, 1] - df[range, 3])<0.1
    ) &
    (
      df[range, 1] < 0.7 &
      df[range, 2] < 0.7 &
      df[range, 3] < 0.7
    )
  )
}

evaluate_loadings(pca_result$loading, 1:18)
```

```
##    Q1    Q2    Q3    Q4    Q5    Q6    Q7    Q8    Q9   Q10   Q11   Q12   Q13
## FALSE FALSE FALSE  TRUE FALSE FALSE FALSE FALSE FALSE FALSE FALSE  TRUE FALSE
##   Q14   Q15   Q16   Q17   Q18
## FALSE FALSE FALSE  TRUE FALSE
```

Only Q4, Q12, and Q17 share similar loadings between 2 or more components.

e. Can you distinguish a 'meaning' behind the first principal component from the items that load best upon it? (see the wording of the questions of those items)

The meaning of PC1 might not be easy to interpret when we compare it with the wording of the questions.

Question 3

a. Individually, does each RC explain the same amount of variance?

```
rc <- principal(df, nfactors = 3, rotate = "varimax", scores = TRUE)$loadings
rc
```

```
##
## Loadings:
##     RC1   RC3   RC2
## Q1  0.660 0.450 0.221
## Q2  0.544 0.286 0.288
## Q3  0.621 0.337 0.311
## Q4  0.218 0.193 0.854
```

```
## Q5  0.244 0.828 0.162
## Q6  0.652 0.199 0.234
## Q7  0.790 0.103
## Q8  0.382 0.706 0.305
## Q9  0.738 0.234 0.138
## Q10 0.277 0.823 0.102
## Q11 0.757 0.278 0.118
## Q12 0.233 0.186 0.854
## Q13 0.593 0.315 0.259
## Q14 0.719 0.310 0.283
## Q15 0.342 0.656 0.244
## Q16 0.740 0.267 0.174
## Q17 0.205 0.187 0.870
## Q18 0.609 0.495 0.227
##
##                  RC1   RC3   RC2
## SS loadings    5.613 3.490 2.954
## Proportion Var 0.312 0.194 0.164
## Cumulative Var 0.312 0.506 0.670
```

Looking at the proportion variance, we can see `RC1` is 30% and `PC1` is 51%. `RC2` is 19% and `PC3` is 8%. While `RC3` is 16% and `PC3` is 6%. Clearly, they are different.

b. Together, do these rotated components explain the same amount of variance like the principal components?

Yes, both the rotated components and the principal components have the same total amount of variances explained.

c. do Q4, Q12, and Q17 have more clearly differentiated loadings among rotated components?

```
rc[c(4,12,17), 1:3]
```

```
##           RC1       RC3       RC2
## Q4  0.2182880 0.1933627 0.8536838
## Q12 0.2327616 0.1861745 0.8542346
## Q17 0.2054021 0.1869028 0.8703910
```

Since these rotate components have loading value above 0.7, clearly they are more clearly differentiated.

d. Can you interpret the meaning of the three rotated components?

```
rc[rc[, 1] > 0.7, 1]
```

```
##        Q7        Q9       Q11       Q14       Q16
## 0.7895344 0.7378148 0.7573493 0.7187578 0.7396241
```

RC1 means data protection.

```
rc[rc[, 2] > 0.7, 2]
```

```
##        Q5        Q8       Q10
## 0.8279850 0.7062018 0.8229206
```

RC2 means transaction processing.

```
rc[rc[, 3] > 0.7, 3]
```

```
##        Q4       Q12       Q17
## 0.8536838 0.8542346 0.8703910
```

RC3 means providing evidence to protect against its denial.

e. If we reduced the number of rotated components, does the meaning of our rotated components change?

```
reduced_rc <- principal(df, nfactors = 2, rotate = "varimax", scores = TRUE)
reduced_rc$loadings[,1][reduced_rc$loadings[,1] > 0.7]
```

```
##        Q1        Q7        Q9       Q11       Q14       Q16       Q18
## 0.7830951 0.7284256 0.7451939 0.7855784 0.7591295 0.7615661 0.7616746
```

Yes. When we decreased `nfactors` by 1, the number of questions belong to ‘RC1‘ would increase. However, the meaning stays the same.