BACS HW4

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March 26, 2021

Question 1

Given the critical DOI score that Google uses to detect malicious apps (-3.7), what is the probability that a randomly chosen app from Google's app store will turn off the Verify security feature? (report a precise decimal fraction, not a percentage)

The probability of an application from Google App Store would turn off the Verify security feature is $pnorm(-3.7) = 1.0779973 \times 10^{-4}$.

Assuming there were ~2.2 million apps when the article was written, what number of apps on the Play Store did Google expect would maliciously turn off the Verify feature once installed?

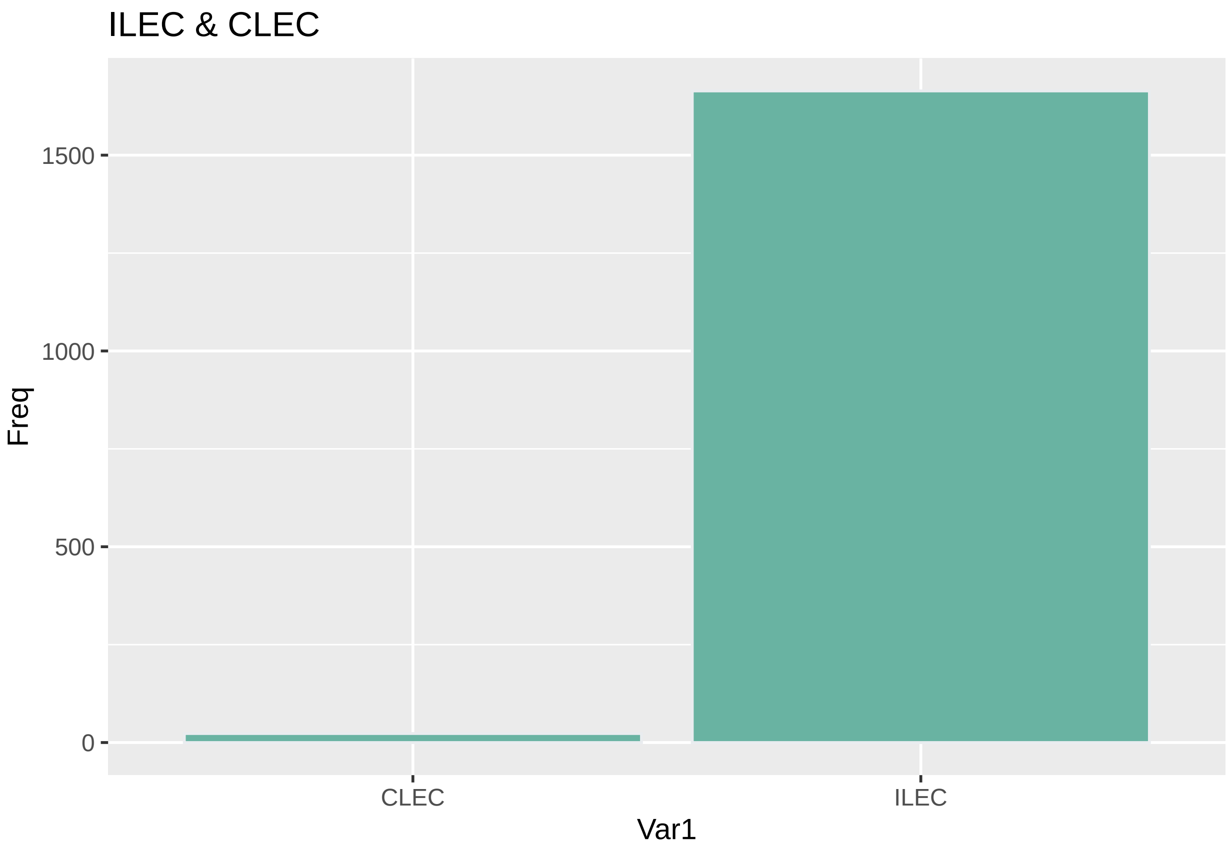
With approx. 2.2 million apps out there, we can expect $2200000 \times pnorm(-3.7) = 237.1594137$ apps would maliciously turn off the Verify feature once installed.

Question 2

A. The Null distribution of t-values:

Before we dive in the questions down below, let's explore to see how the dataset looks like.

```
counts = as.data.frame(table(verizon["Group"]))
ggplot(data=counts, aes(x=Var1, y=Freq)) +
  geom_bar(
    stat="identity",
    fill="#69b3a2",
    color="#e9ecef"
  ) +
  ggtitle("ILEC & CLEC")
```




```
##
## CLEC ILEC
##   23 1664
```

From the graph above, we can see that there are two targets in the data set:

- Competitive Local Exchange Carrier (CLEC) is a company that is characterized by providing local telephone service. It is those companies that have rented space from the Incumbent Local Exchange Carrier.
- Incumbent Local Exchange Carrier (ILEC) is companies that are known to provide an alternative service to the ILEC within its territory.

We can also notice that, there are only 23 observations for CLEC and 1664 for ILEC.

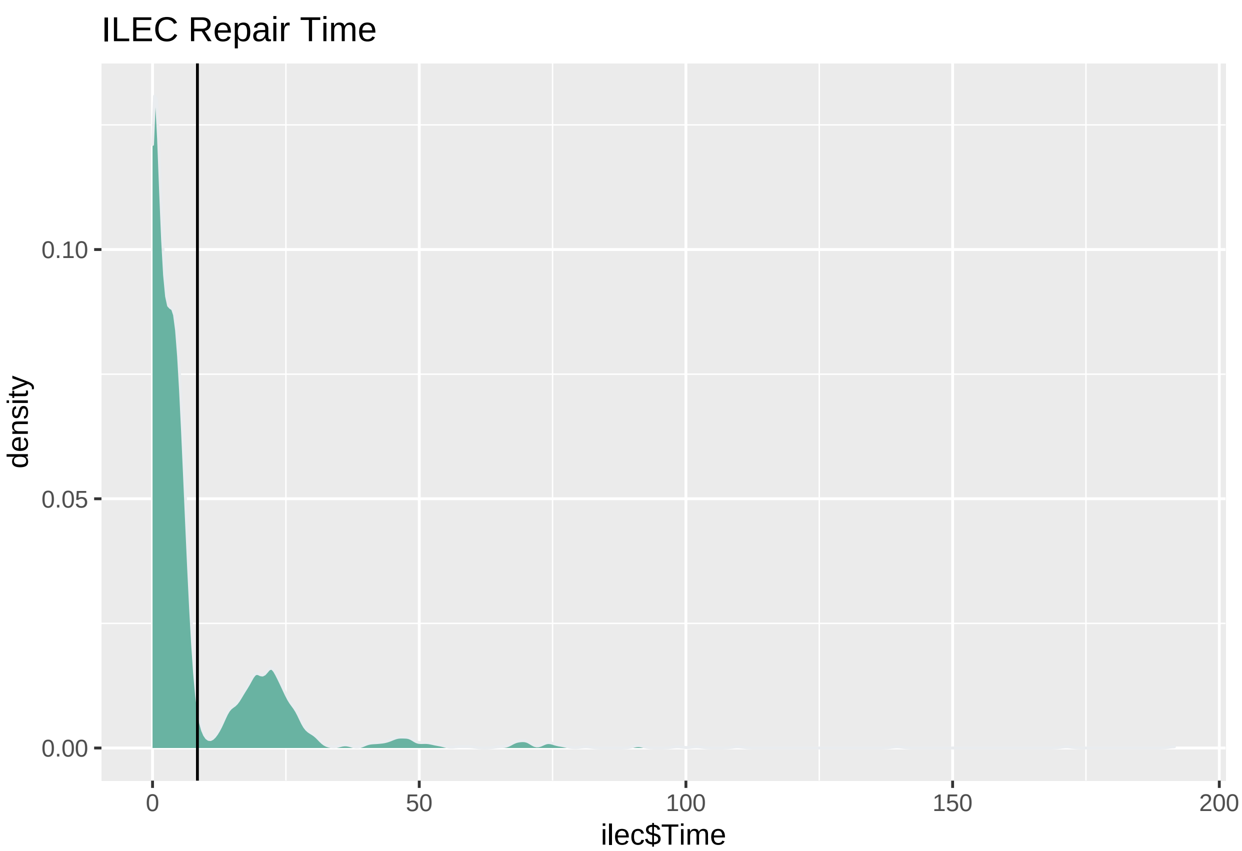
With `max()` and `min()` functions, the maximum and the minimum repair time can be acquired. With the maximum repair time 191.6 minutes and the minimum repair time 0 minute.

1. Visualize the distribution of Verizon's repair times, marking the mean with a vertical line

In visualizing the data set, I am going to split the data set into two parts, ILEC and CLEC.

```
ilec <- as.data.frame(verizon[verizon$Group == "ILEC", ])

mean <- mean(ilec$Time)
```

```
ggplot(mapping = aes(ilec$Time)) +
  geom_density(
    fill="#69b3a2",
    color="#e9ecef"
  ) +
  ggtitle("ILEC Repair Time") +
  coord_cartesian(xlim = c(0, max(ilec$Time))) +
  geom_vline(xintercept = mean)
```

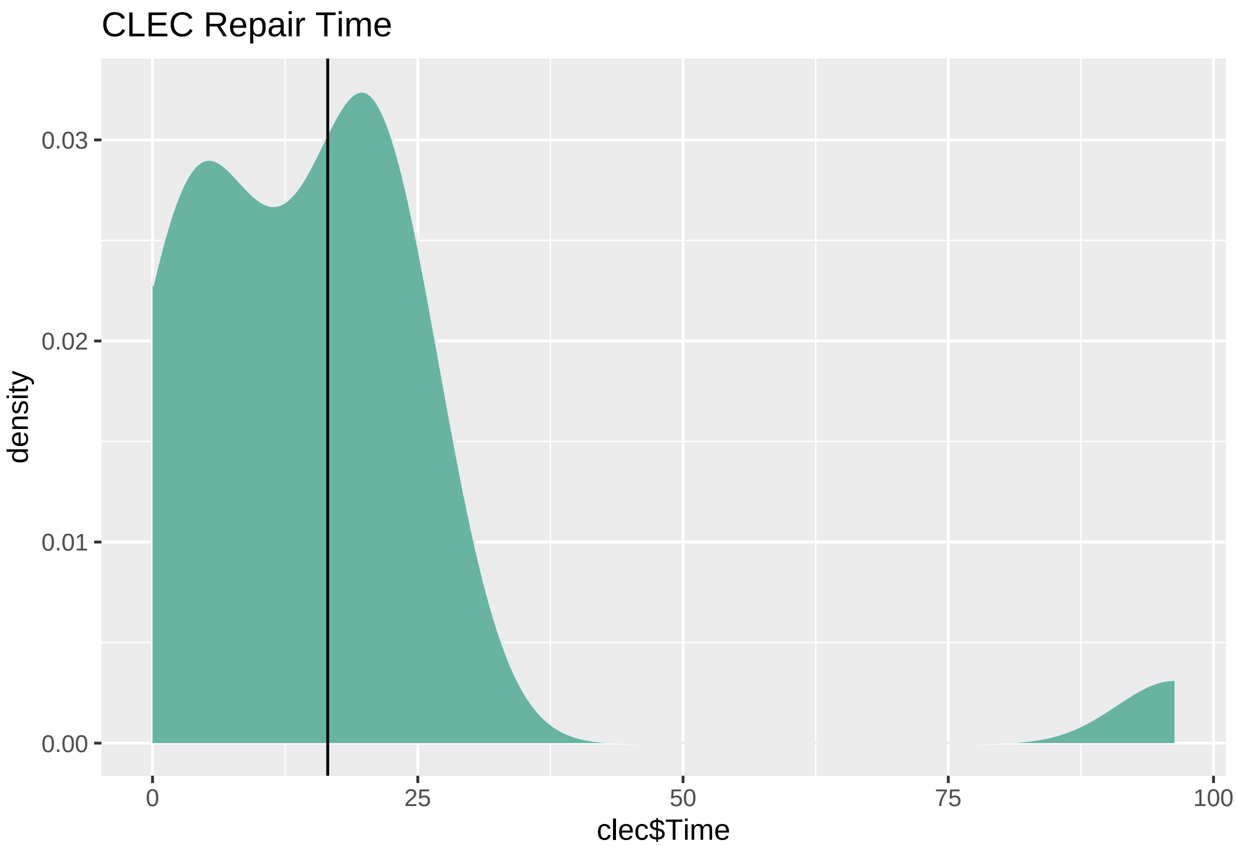


ILEC data set information:

```
##      Number of observation in ILEC -> 1664
##      ILEC minimum repair time -> 0
##      ILEC average repair time -> 8.41161057692308
##      ILEC maximum repair time -> 191.6
```

```
clec <- as.data.frame(verizon[verizon$Group == "CLEC", ])

mean <- mean(clec$Time)

ggplot(mapping = aes(clec$Time)) +
  geom_density(
    fill="#69b3a2",
    color="#e9ecef"
  ) +
  ggtitle("CLEC Repair Time") +
```

```r
  coord_cartesian(xlim = c(0, max(clec$Time))) +
  geom_vline(xintercept = mean)
```



CLEC data set information:

```
##      Number of observation in CLEC -> 23
##      CLEC minimum repair time -> 0
##      CLEC average repair time -> 16.5091304347826
##      CLEC maximum repair time -> 96.32
```

2. Given what PUC wishes to test, how would you write the hypothesis? (not graded)

In this case, the null hypothesis will be "Verizon's average repair phone service time is 7.6 minutes".

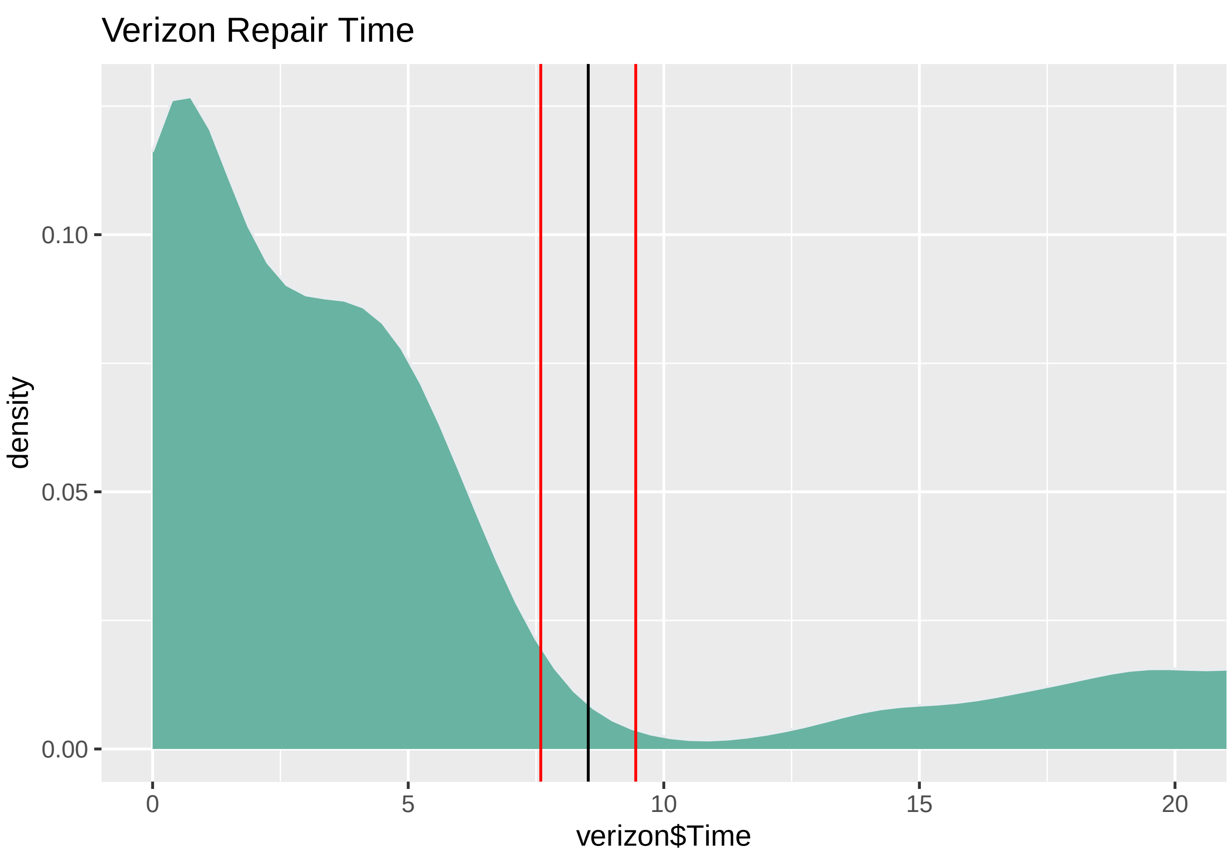
3. Estimate the population mean, and the 99% confidence interval (CI) of this estimate

```r
population_mean <- mean(verizon$Time)
population_sd <- sd(verizon$Time)
population_sderr <- population_sd / sqrt(nrow(verizon))

ci99_low <- population_mean - population_sderr * 2.58
ci99_high <- population_mean + population_sderr * 2.58
```

Thus, the population mean is 8.5220095. While the 99% CI of the population mean is between 7.5930734 and 9.4509455.

Just making sure of it, we can visualize the entire data set and draw the mean (black line) and 99% CI lines (red lines).

```
mean <- mean(verizon$Time)

ggplot(data = verizon, mapping = aes(verizon$Time)) +
  geom_density(
    fill="#69b3a2",
    color="#e9ecef"
  ) +
  ggtitle("Verizon Repair Time") +
  coord_cartesian(xlim = c(0, 20)) +
  geom_vline(xintercept = mean) +
  geom_vline(xintercept = ci99_low, col="red") +
  geom_vline(xintercept = ci99_high, col="red") +
  theme(legend.position = c(1,1))
```



4. Using the traditional statistical testing methods we saw in class, find the t-statistic and p-value of the test

```
t <- (population_mean - hypothesized_mean) / population_sderr
```

T-statistic value is 2.5607623

```
df <- nrow(verizon) - 1
p <- 1 - pt(t, df)
```

P-value is 0.0052653

5. Briefly describe how these values relate to the Null distribution of t (not graded)

The t-statistics or t-value measures the size of the difference relative to the variation in your sample data. Put another way, t-value is simply the calculated difference represented in units of standard error. The greater the magnitude of t-value, the greater the evidence against the null hypothesis.

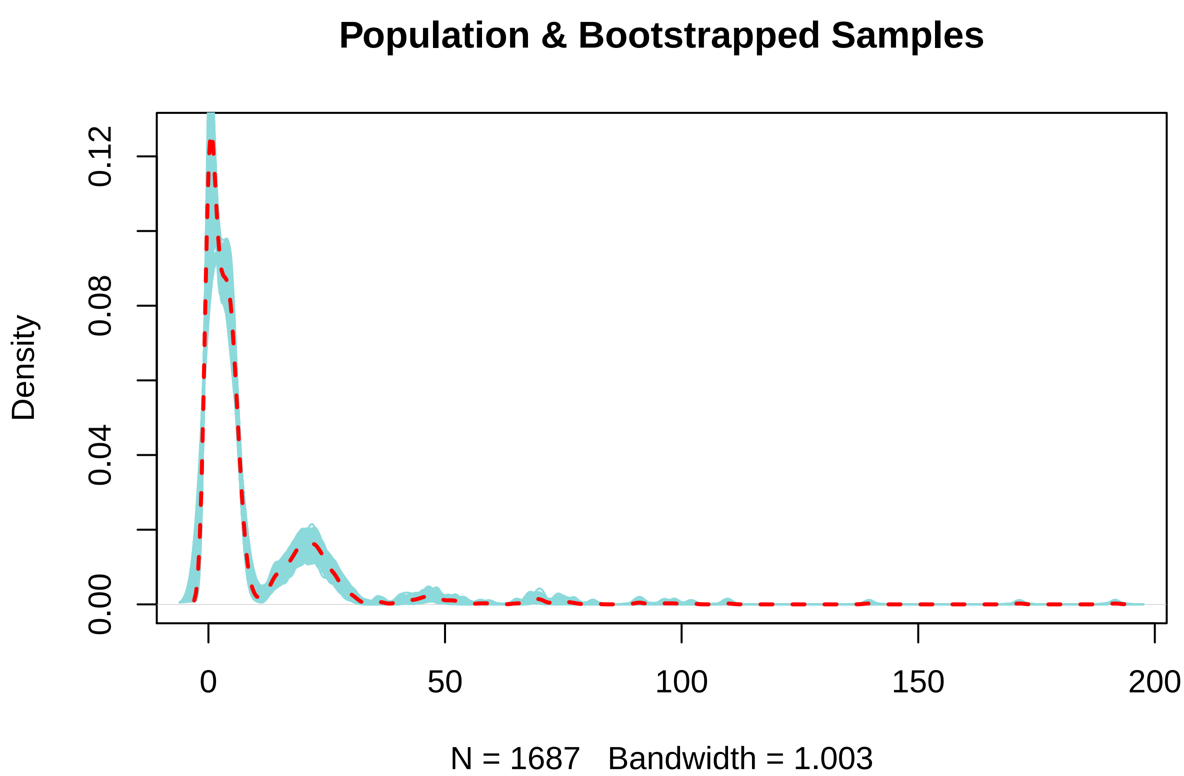
6. What is your conclusion about the advertising claim from this t-statistic, and why?

In this case, we have T-statistics value 2.5607623 which is far away from 0. This means that what Verizon's repair time is not like what Verizon has advertised.

B. Let's use bootstrapping on the sample data to examine this problem:

1. Bootstrapped Percentile: Estimate the bootstrapped 99% CI of the mean

```
bootstrapped_samples <- replicate(1000, sample(verizon$Time, nrow(verizon), replace=TRUE))

plot(
  density(verizon$Time),
  main="Population & Bootstrapped Samples"
)

plot_sample_and_get_mean <- function(sample) {
  lines(
    density(sample),
    col="#8cd9db"
  )
  return(mean(sample))
}

sample_means <- apply(bootstrapped_samples, 2,FUN=plot_sample_and_get_mean)

lines(density(verizon$Time), col="red", lwd=2, lty="dashed")
```




```
average_sample_mean <- mean(sample_means)
sample_mean_sderror <- sd(sample_means) / sqrt(length(sample_means))
mean_ci99_low <- average_sample_mean - 2.58 * sample_mean_sderror
mean_ci99_high <- average_sample_mean + 2.58 * sample_mean_sderror
```

The 99% CI of the mean is between 8.4953493 and 8.5546287.

2. Bootstrapped Difference of Means: What is the 99% CI of the bootstrapped difference between the population mean and the hypothesized mean?

```
diff <- sample_means - hypothesized_mean
diff_means <- mean(abs(diff))
diff_sderror <- sd(diff) / sqrt(length(diff))
diff_ci99_low <- diff_means - 2.58 * diff_sderror
diff_ci99_high <- diff_means + 2.58 * diff_sderror
```

The 99% CI of the mean is between 0.8960371 and 0.9553165.

3. Bootstrapped t-Interval: What is 99% CI of the bootstrapped t-statistic?

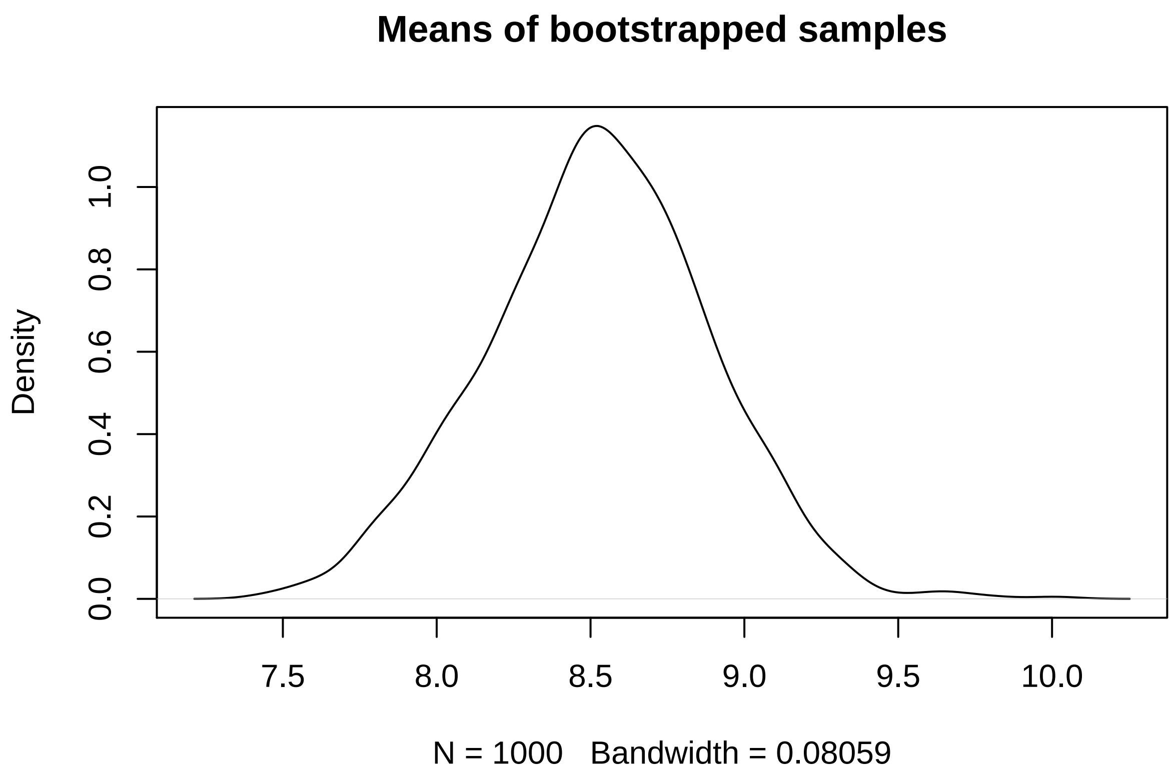
```
compute_t <- function(sample) {
  sample_sderr <- sd(sample) / sqrt(length(sample))
  return (
    (mean(sample) - hypothesized_mean) / sample_sderr
  )
```

```
}

sample_t <- apply(bootstrapped_samples, 2, FUN=compute_t)
average_sample_t <- mean(sample_t)
sample_t_sderr <- sd(sample_t) / sqrt(length(sample_t))
t_ci99_low <- average_sample_t - 2.58 * sample_t_sderr
t_ci99_high <- average_sample_t + 2.58 * sample_t_sderr
```

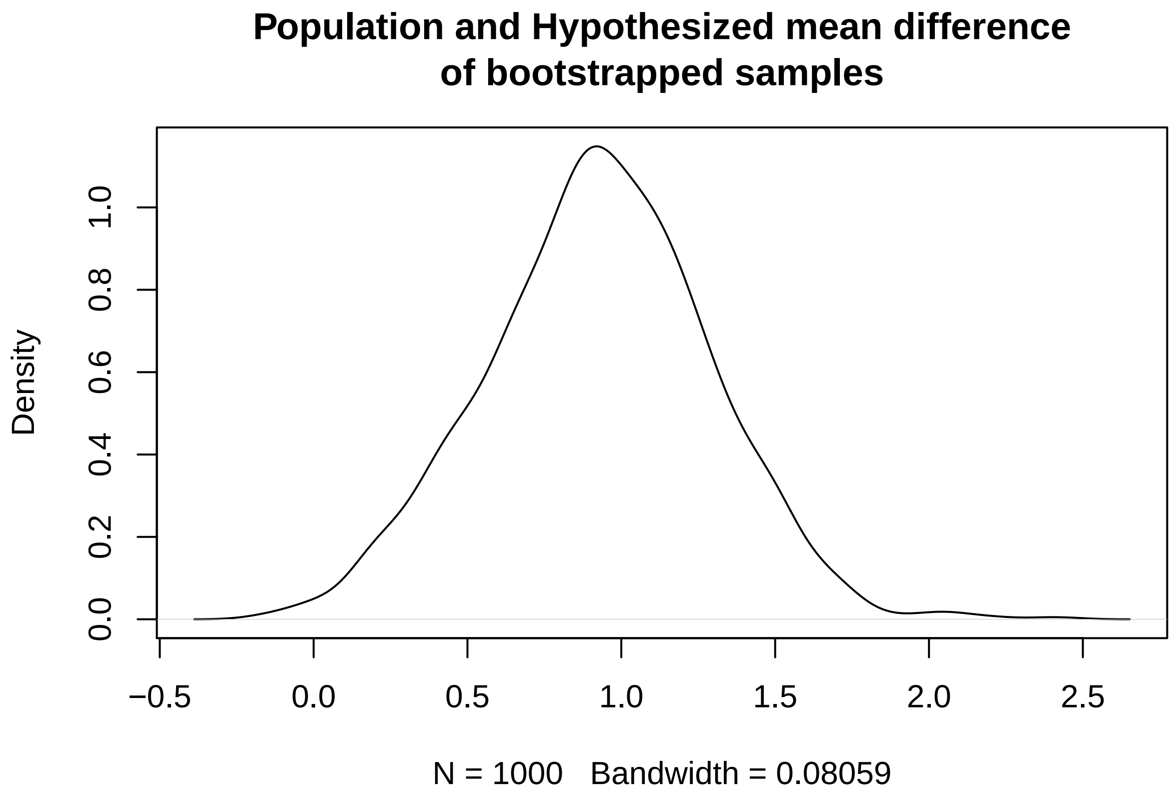
The 99% CI of the bootstrapped t-statistic is between 2.4639013 and 2.6095622.

4. Plot separate distributions of all three bootstraps above (for 2 and 3 make sure to include zero on the x-axis)

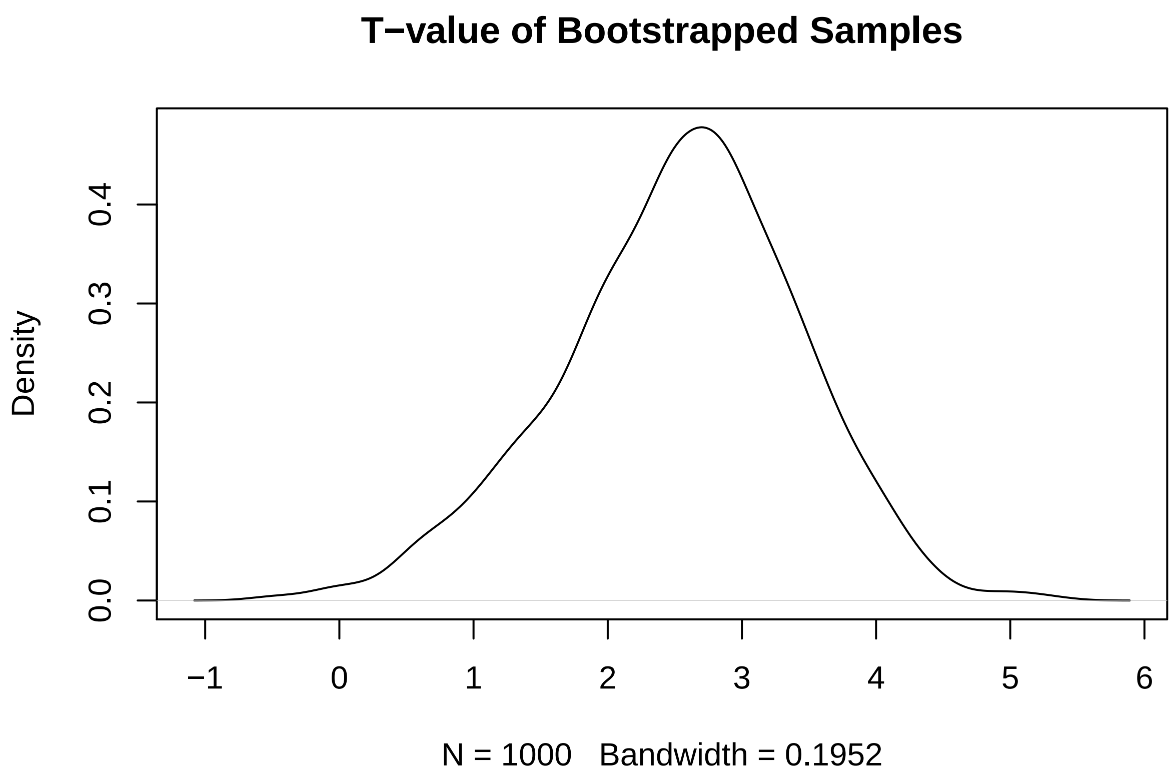
```
plot(density(sample_means), main="Means of bootstrapped samples")
```




```
plot(density(diff), main="Population and Hypothesized mean difference\nof bootstrapped samples")
```




```
plot(density(sample_t), main="T-value of Bootstrapped Samples")
```



C. Do the four methods (traditional test, bootstrapped percentile, bootstrapped difference of means, bootstrapped T-Interval) agree with each other on the test?

Before discussing whether the four methods agree with each other on the test, we should define what 99% Confidence Interval is.

By definition, 99% Confidence Interval is a range where an unknown parameter will fall within the range for 99% of the time.

Mean value

From the traditional test, we have a population mean of 8.5220095. From the bootstrapping test, we have a 99% CI of mean between 8.4953493 and 8.5546287. Clearly, the population mean from the traditional test falls in the 99% CI range.

Difference of population mean and hypothesized mean

From the traditional test, we have a difference of population mean and hypothesized mean of 0.9220095. From the bootstrapping test, we have a 99% CI of difference of population mean and hypothesized mean is between 0.8960371 and 0.9553165. Clearly, in this case, the difference of population mean and hypothesized mean that we get from the population sample falls in the 99% CI range.

T-Interval value

From the traditional test, we have a t-statistics value of 2.5607623. From the bootstrapping test, we have a 99% CI of ‘t-statistics‘ or ‘t-value‘ is between 2.4639013 and 2.6095622. Clearly, in this case, the t-statistics value from the population falls in the 99% CI range.

From the tests and the observations above, we can conclude that these four methods agree on each other. After all, we can reject Verizon’s claim (null hypothesis) because the T-Value both in the population sample and the bootstrapped samples is way further from 0. In the end, we end up having an alternate hypothesis with "Verizon’s average repair phone service time is more than 7.6 minutes."