1.a $\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} \sin(2\pi\alpha_1 x)\cos(2\pi\alpha_2 y)dxdy$

$$\Rightarrow \int\limits_{-\infty}^{+\infty} \sin(2\pi\alpha_1 x)dx \cdot \int\limits_{-\infty}^{+\infty} \cos(2\pi\alpha_2 y)dy$$

$$\Rightarrow \int\limits_{-\infty}^{+\infty} \frac{(e^{i2\pi\alpha_1 x} - e^{-i2\pi\alpha_1 x})}{2i} e^{-i2\pi ux}dx \cdot \int\limits_{-\infty}^{+\infty} \frac{(e^{i2\pi\alpha_2 y} + e^{-i2\pi\alpha_2 y})}{2} e^{-i2\pi vy}dy$$

$$\Rightarrow \frac{1}{2i}\left[\int\limits_{-\infty}^{+\infty} e^{i2\pi\alpha_1 x - i2\pi ux}dx - \int\limits_{-\infty}^{+\infty} e^{-i2\pi\alpha_1 x - i2\pi ux}dx\right] \cdot \frac{1}{2}\left[\int\limits_{-\infty}^{+\infty} e^{i2\pi\alpha_2 y - i2\pi vy}dy + \int\limits_{-\infty}^{+\infty} e^{-i2\pi\alpha_2 y - i2\pi vy}dy\right]$$

$$\Rightarrow \frac{1}{2i}\left[\delta(u-\alpha_1) - \delta(u+\alpha_1)\right] \cdot \frac{1}{2}\left[\delta(v-\alpha_2) + \delta(v+\alpha_2)\right]$$

$$\Rightarrow \frac{1}{4i}\left[\delta(u-\alpha_1) - \delta(u+\alpha_1)\right] \cdot \left[\delta(v-\alpha_2) + \delta(v+\alpha_2)\right]$$

1.b $\int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} cos(2\pi(\alpha_1 x + \alpha_2 y))dxdy$

$$\Rightarrow \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} cos(2\pi\alpha_1 x + 2\pi\alpha_2 y)dxdy$$

$$\Rightarrow \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} cos(2\pi\alpha_1 x) \cdot cos(2\pi\alpha_2 y) - sin(2\pi\alpha_1 x) \cdot sin(2\pi\alpha_2 y)\, dxdy$$

$$\Rightarrow \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} cos(2\pi\alpha_1 x) \cdot cos(2\pi\alpha_2 y)dxdy - \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} sin(2\pi\alpha_1 x) \cdot sin(2\pi\alpha_2 y)\, dxdy$$

$$\Rightarrow \int\limits_{-\infty}^{+\infty} cos(2\pi\alpha_1 x)dx \cdot \int\limits_{-\infty}^{+\infty} cos(2\pi\alpha_2 y)dy - \int\limits_{-\infty}^{+\infty} sin(2\pi\alpha_1 x)dx \cdot \int\limits_{-\infty}^{+\infty} sin(2\pi\alpha_2 y)dy$$

$$\Rightarrow \frac{1}{2}\left[\delta(u-\alpha_1) + \delta(u+\alpha_1)\right] \times \frac{1}{2}\left[\delta(v-\alpha_2) + \delta(v+\alpha_2)\right] - \frac{1}{2i}\left[\delta(u-\alpha_1) - \delta(u+\alpha_1)\right] \times \frac{1}{2i}\left[\delta(v-\alpha_2) - \delta(v+\alpha_2)\right]$$

$$\Rightarrow \frac{1}{4}\left[\delta(u-\alpha_1) + \delta(u+\alpha_1)\right] \times \left[\delta(v-\alpha_2) + \delta(v+\alpha_2)\right] + \frac{1}{4}\left[\delta(u-\alpha_1) - \delta(u+\alpha_1)\right] \times \left[\delta(v-\alpha_2) - \delta(v+\alpha_2)\right]$$

$$\Rightarrow \frac{1}{4}\left[\left[\delta(u-\alpha_1) + \delta(u+\alpha_1)\right] \times \left[\delta(v-\alpha_2) + \delta(v+\alpha_2)\right] + \left[\delta(u-\alpha_1) - \delta(u+\alpha_1)\right] \times \left[\delta(v-\alpha_2) - \delta(v+\alpha_2)\right]\right]$$

$$\Rightarrow \frac{1}{4}\Big[\delta(u-\alpha_1)\cdot\delta(v-\alpha_2) + \delta(u-\alpha_1)\cdot\delta(v+\alpha_2) + \delta(u+\alpha_1)\cdot\delta(v-\alpha_2) + \delta(u+\alpha_1)\cdot\delta(v+\alpha_2)$$

$$+ \delta(u-\alpha_1)\cdot\delta(v-\alpha_2) - \delta(u-\alpha_1)\cdot\delta(v+\alpha_2) - \delta(u+\alpha_1)\cdot\delta(v-\alpha_2) + \delta(u+\alpha_1)\cdot\delta(v+\alpha_2)\Big]$$

$$\Rightarrow \frac{1}{2}\left[\delta(u-\alpha_1)\cdot\delta(v-\alpha_2) + \delta(u+\alpha_1)\cdot\delta(v+\alpha_2)\right]$$

2. $x[n] = \{-1, 2, -1\}$

$h[n] = \{1, 2, 3, 4\}$

From the equation of convolution, the output signal y[n] will be

$$y[n] = \sum_k x[k] \cdot h[n-k]$$

Let's compute the values of $y[0],\ y[1],\ y[2], y[2],\ y[3],\ y[4],\ y[5] \ldots.$

$$\begin{aligned} y[0] &= \sum_{k=-\infty}^{+\infty} x[k] \cdot h[0-k] \\ &= x[0] \cdot h[0-0] + x[1] \cdot h[0-1] + x[2] \cdot h[0-2] + x[3] \cdot h[0-3] + ... \\ &= x[0] \cdot h[0] \\ &= -1 \cdot 1 \\ &= -1 \end{aligned}$$

$$\begin{aligned} y[1] &= \sum_{k=-\infty}^{+\infty} x[k] \cdot h[1-k] \\ &= x[0] \cdot h[1-0] + x[1] \cdot h[1-1] + x[2] \cdot h[1-2] + x[3] \cdot h[1-3] +. ... \\ &= x[0] \cdot h[1] + x[1] \cdot h[0] \\ &= -1 \cdot 2 + 2 \cdot 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y[2] &= \sum_{k=-\infty}^{+\infty} x[k] \cdot h[2-k] \\ &= x[0] \cdot h[1-0] + x[1] \cdot h[1-1] + x[2] \cdot h[1-2] + x[3] \cdot h[1-3] + ... \\ &= x[0] \cdot h[2] + x[1] \cdot h[1] + x[2] \cdot h[0] \\ &= -1 \cdot 3 + 2 \cdot 2 - 1 \cdot 1 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y[3] &= \sum_{k=-\infty}^{+\infty} x[k] \cdot h[3-k] \\ &= x[0] \cdot h[3-0] + x[1] \cdot h[3-1] + x[2] \cdot h[3-2] + x[3] \cdot h[3-3] + ... \\ &= x[0] \cdot h[3] + x[1] \cdot h[2] + x[2] \cdot h[1] \\ &= -1 \cdot 4 + 2 \cdot 3 - 1 \cdot 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} y[4] &= \sum_{k=-\infty}^{+\infty} x[k] \cdot h[4-k] \\ &= x[0] \cdot h[4-0] + x[1] \cdot h[4-1] + x[2] \cdot h[4-2] + x[3] \cdot h[4-3] + ... \\ &= x[1] \cdot h[3] + x[2] \cdot h[2] \\ &= 2 \cdot 4 - 1 \cdot 3 \\ &= 5 \end{aligned}$$

$$\begin{aligned} y[5] &= \sum_{k=-\infty}^{+\infty} x[k] \cdot h[3-k] \\ &= x[0] \cdot h[5-0] + x[1] \cdot h[5-1] + x[2] \cdot h[5-2] + x[3] \cdot h[5-3] + ... \\ &= x[2] \cdot h[3] \\ &= -1 \cdot 4 \\ &= -4 \end{aligned}$$

$$y[n] = \{-1, 0, 0, 0, 5, -4\}$$

3.a

The correct answer is **option (B)**, since in this plot of spectral output *(y-axis)* against wavelength *(x-axis)*, the value of spectral output is non-zero for a single wavelength – corresponding to the wavelength of the monochromatic light – and zero for all other wavelengths.

3.b

Given $P(\lambda) = 1$

$$X = \int_{\lambda=0}^{+\infty} P(\lambda)\bar{x}(\lambda)dx$$
$$= \int_{\lambda=0}^{+\infty} 1 \cdot \bar{x}(\lambda)dx$$
$$= 1$$
$$Y = \int_{\lambda=0}^{+\infty} P(\lambda)\bar{y}(\lambda)dx$$
$$= \int_{\lambda=0}^{+\infty} 1 \cdot \bar{y}(\lambda)dx$$
$$= 1$$
$$Z = \int_{\lambda=0}^{+\infty} P(\lambda)\bar{z}(\lambda)dx$$
$$= \int_{\lambda=0}^{+\infty} 1 \cdot \bar{z}(\lambda)dx$$
$$= 1$$

Now calculating x, y, z from X, Y, Z

$$x = \frac{X}{X+Y+Z} = \frac{1}{1+1+1} = \frac{1}{3}$$
$$y = \frac{Y}{X+Y+Z} = \frac{1}{1+1+1} = \frac{1}{3}$$
$$z = \frac{Z}{X+Y+Z} = \frac{1}{1+1+1} = \frac{1}{3}$$

$(x, y, z) \Rightarrow (0.33, 0.33, 0.33)$

3.c

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.412453 & 0.357580 & 0.180423 \\ 0.212671 & 0.715160 & 0.072169 \\ 0.019334 & 0.119193 & 0.950227 \end{bmatrix} \cdot \begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix}$$

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 3.240481 & -1.537151 & -0.498536 \\ -0.969255 & 1.875990 & 0.041556 \\ 0.055646 & -0.204041 & 1.057311 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 3.240481 & -1.537151 & -0.498536 \\ -0.969255 & 1.875990 & 0.041556 \\ 0.055646 & -0.204041 & 1.057311 \end{bmatrix} \cdot \begin{bmatrix} 0.64 \\ 0.33 \\ 0.03 \end{bmatrix}$$

$$\begin{bmatrix} R_{709} \\ G_{709} \\ B_{709} \end{bmatrix} = \begin{bmatrix} 1.551692 \\ 0.00000021 \\ -0.00000046 \end{bmatrix}$$

4.

Color contrast in terms of luminance is the relationship between the luminance of a brighter area of interest and that of an adjacent darker area. *Luminance* is the intensity of light emitted from a unit area surface in a given direction. Mathematically, it can be formulated as:

$$C_w = \frac{L - L_b}{L}$$

where,

$$\begin{aligned} C_w &\Rightarrow Weber's\,contrast \\ L &\Rightarrow luminance\,of\,the\,object \\ L_b &\Rightarrow luminance\,of\,the\,background \end{aligned}$$

This definition is also called **Weber Contrast**, and is the most commonly useful one in the context of lighting.

5.

$$s(n_1, n_2) = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 & 1 \\ 1 & 1 & 1 & \dots & 1 & 1 \\ 1 & 1 & 1 & \dots & 1 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & 1 & \dots & 1 & 1 \\ 1 & 1 & 1 & \dots & 1 & 1 \end{bmatrix}_{N1 \times N2}$$

$$h(n_1, n_2) = \frac{1}{9} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}_{3\times 3}$$

Convolving the filter / kernel $h(n1, n2)$ over input $s(n1, n2)$ gives us the following output:

$$y(n_1, n_2) = s(n_1, n_2) \, * \, h(n_1, n_2)$$

$$y(n_1, n_2) = \begin{bmatrix} 0.444 & 0.666 & 0.666 & \dots & 0.666 & 0.444 \\ 0.666 & 1 & 1 & \dots & 1 & 0.666 \\ 0.666 & 1 & 1 & \dots & 1 & 0.666 \\ 0.666 & 1 & 1 & \dots & 1 & 0.666 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.444 & 0.666 & 0.666 & \dots & 0.666 & 0.444 \end{bmatrix}_{N1 \times N2}$$

Plotting the frequency response of filter $h(n_1, n_2)$ using *freqz2*