Homework No. 1 Due date: Sep/5/2022 at 11:59 pm

Each part is worth 20%

1. Compute the Fourier transform of
   $\sin(2\pi\alpha_1 x)\cos(2\pi\alpha_2 y)$, and

   $\cos(2\pi(\alpha_1 x + \alpha_2 y))$

   Hints:

   Fourier Transform (FT)
   $$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt$$
   $$\omega \in (-\infty, +\infty)$$

   Fourier transform of a product is $\mathscr{F}\{g(t)h(t)\} = G(f) * H(f)$

   Based on Euler's formula, an exponential can be expressed in terms of Sin() and cos(); and sin() and cos() can be expressed as exponentials.

   $$\cos x = \mathrm{Re}(e^{ix}) = \frac{e^{ix} + e^{-ix}}{2},$$
   $$\sin x = \mathrm{Im}(e^{ix}) = \frac{e^{ix} - e^{-ix}}{2i}.$$

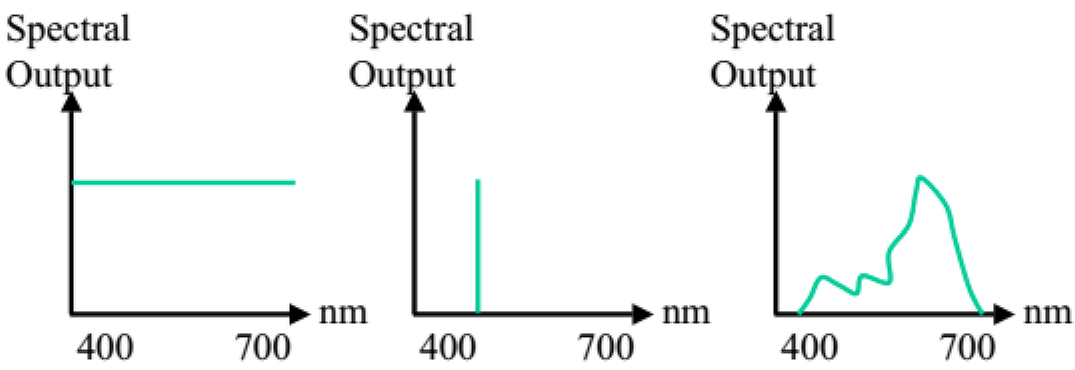
   From the FT table: $e^{j\omega_0 t} \xrightarrow{\mathscr{F}} 2\pi\delta(\omega - \omega_0)$

2. Compute the convolution of the two sequences:
   [-1, 2, -1] and [1, 2, 3, 4]; write it in a matrix-vector multiplication form

   Hint: the size of the Toepliz matrix is M+N-1, where M and N are the sizes of the sequences.

3. Three parts:

   a. Which spectral output corresponds to monochromatic light?



(A) (B) (C)

   b. If $P(\lambda) = 1$ for white light, using the definition of XYZ, compute the coordinates (x,y,z) for white light.

   c. Convert (x,y,z) = (0.64,0.33,..) to RGB_{709}.

4. How would you compute color contrast?

   Hint: you can use luminance of the color (chroma) or the hue angles, or the distance in color space, any of those answers is ok

5. Exercise 1.5 from Tekalp's Ch. 1.

1.5 Convolve

$$s(n_1,n_2)=\begin{cases}1 & 0\le n_1\le N_1-1,\ 0\le n_2\le N_2-1\\ 0 & \text{otherwise}\end{cases}$$

with

$$h(n_1,n_2)=\begin{cases}1/9 & -1\le n_1\le 1,\ -1\le n_2\le 1\\ 0 & \text{otherwise}\end{cases}$$

Find the frequency response of this filter.

Hint:

The output can be computed by convolution, for the frequency response of h(n1,n2) you can use matlab.

h(n1,n2) is a bunch or impulses, and the FT of an impulse is

$$G(f)=\Im\{\delta(t-a)\}=\int_{-\infty}^{\infty}\delta(t-a)e^{-i2\pi ft}\,dt$$
$$=e^{-i2\pi fa}$$

The FT of h is its frequency response, you can use the function *freqz()* to visualize the magnitude and phase and cut & paste the graph in your report.

If you do the FT of the signal s and the filter h, then the output is just the multiplication of them. Doing the inverse FT you can get the output in time (space) domain.