A: $a_{ij} = P(X_{t+1} = j \mid X_t = i)$

$X_t \mid X_{t+1}$	A	B	H	S
A	0.6	0.1	0.1	0.2
B	0.0	0.3	0.2	0.5
H	0.8	0.1	0.0	0.1
S	0.2	0.0	0.1	0.7

B: $b_{ik} = P(O_t = k \mid X_t = i)$

$X_t \mid O_t$	p	e	b	l
A	0.6	0.2	0.1	0.1
B	0.1	0.4	0.1	0.4
H	0.0	0.0	0.7	0.3
S	0.0	0.0	0.1	0.9

$P(X_t = i)$:

$\bar{q} =$

A	B	H	S
0.4	0.2	0.1	0.3

Find:

$P(O_t \mid A, B, P(X_t)) = ?$

$= \bar{q}' B =$

Compute:

$P(O_t = p) = 0.4 \times 0.6 + 0.2 \times 0.1 + 0.1 \times 0.0 + 0.3 \times 0.0 = 0{,}24 + 0{,}02 = 0{,}26$

$P(O_t = e) = 0{,}4 \times 0.2 + 0{,}2 \times 0.4 + 0.1 \times 0{,}0 + 0.3 \times 0{,}0 = 0{,}08 + 0{,}08 = 0{,}16$

$P(O_t = b) = 0.4 \times 0{,}1 + 0{,}2 \times 0{,}1 + 0.1 \times 0{,}7 + 0.3 \times 0.1 = 0{,}04 + 0{,}02 + 0{,}07 + 0{,}03 = 0{,}16$

$P(O_t = l) = 0{,}4 \times 0{,}1 + 0{,}2 \times 0{,}4 + 0{,}1 \times 0{,}3 + 0{,}3 \times 0{,}9 = 0{,}04 + 0{,}08 + 0{,}03 + 0{,}27$
$= 0{,}42$

$P(O_t) =$

0,26
0,16
0,16
0,42

most likely O_t = l

$\pi = P(X_1 = i)$:

A	B	H	S
0.5	0.0	0.0	0.5

observations / emissions: $o_{1:4} = \{ l, p, p, b \}$

Find:

$P(o_{1:4} \mid A, B, \pi) = ?$

Element-wise product:

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \odot \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} ad \\ be \\ cf \end{bmatrix}$$

Compute:

O_1 = l

$$\alpha_1(i) = \begin{bmatrix} 0{,}5 \\ 0{,}0 \\ 0{,}0 \\ 0.5 \end{bmatrix} \odot \begin{bmatrix} 0{,}1 \\ 0.4 \\ 0{,}3 \\ 0{,}9 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0 \\ 0 \\ 0.45 \end{bmatrix}$$

O_2 = p

$\leftarrow$ part framtom $\rightarrow$ p-kolumnen i B

alla kolumner i A $\cdot \alpha_{t-1}(i)$

$$\alpha_2(i) = \begin{bmatrix} 0.05 \times 0.6 + 0.0 \times 0{,}0 + 0.0 \times 0{,}8 + 0.45 \times 0.2 \\ 0{,}05 \times 0.1 + 0{,}0 \times 0.3 + 0.0 \times 0.1 + 0.45 \times 0{,}0 \\ 0.05 \times 0.1 + 0{,}0 \times 0.2 + 0.0 \times 0{,}0 + 0{,}45 \times 0.1 \\ 0.05 \times 0.2 + 0{,}0 \times 0.5 + 0.0 \times 0.1 + 0{,}45 \times 0.7 \end{bmatrix} \odot \begin{bmatrix} 0.6 \\ 0{,}1 \\ 0{,}0 \\ 0{,}0 \end{bmatrix} = \begin{bmatrix} 0.072 \\ 0{,}0005 \\ 0{,}0 \\ 0{,}0 \end{bmatrix}$$

$O_3 =$ p

$$\alpha_3(i) = \begin{bmatrix} 0.072 \times 0{,}6 + 0{,}0005 \times 0.0 + 0.0 \times 0.8 + 0.0 \times 0.2 \\ 0.072 \times 0{,}1 + 0{,}0005 \times 0.3 + 0.0 \times 0{,}1 + 0.0 \times 0.0 \\ 0.072 \times 0{,}1 + 0{,}0005 \times 0.2 + 0.0 \times 0{,}0 + 0.0 \times 0.1 \\ 0.072 \times 0{,}2 + 0{,}0005 \times 0.5 + 0.0 \times 0.1 + 0{,}0 \times 0.7 \end{bmatrix} \odot \begin{bmatrix} 0.6 \\ 0.1 \\ 0.0 \\ 0.0 \end{bmatrix} = \begin{bmatrix} 0{,}02592 \\ 0{,}000735 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$O_4 =$ b

$$\alpha_4(i) = \begin{bmatrix} 0{,}02592 \times 0.6 + 0{,}000735 \times 0.0 + 0.0 \times 0.8 + 0.0 \times 0.2 \\ 0{,}02592 \times 0.1 + 0{,}000735 \times 0.3 + 0.0 \times 0.1 + 0.0 \times 0.0 \\ 0{,}02592 \times 0.1 + 0{,}000735 \times 0.2 + 0.0 \times 0.0 + 0.0 \times 0.1 \\ 0.0259 \times 0.2 + 0{,}000735 \times 0.5 + 0.0 \times 0.1 + 0.0 \times 0.7 \end{bmatrix} \odot \begin{bmatrix} 0{,}1 \\ 0{,}1 \\ 0{,}7 \\ 0{,}1 \end{bmatrix} = \begin{bmatrix} 0{,}00156 \\ 0{,}000281 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$\alpha_4(i) = \begin{bmatrix} 0.00156 \\ 0.000281 \\ 0.0019173 \\ 0.00055515 \end{bmatrix} \qquad P(o_{1:4} \mid A, B, \pi) = \sum_i \alpha_4(i) = 0{,}00431$$

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B	0.0	0.3	0.2	0.5
H	0.8	0.1	0.0	0.1
S	0.2	0.0	0.1	0.7

B: $b_{ik} = P(O_t = k \mid X_t = i)$

$X_t \mid O_t$	p	e	b	l
A	0.6	0.2	0.1	0.1
B	0.1	0.4	0.1	0.4
H	0.0	0.0	0.7	0.3
S	0.0	0.0	0.1	0.9

Step 1: Initialize $\delta_1(i)$

$O_1 =$ b

$$\delta_1(i) = \begin{bmatrix} 0.5 \\ 0.0 \\ 0.0 \\ 0.5 \end{bmatrix} \odot \begin{bmatrix} 0.1 \\ 0.1 \\ 0.7 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 0.05 \\ 0 \\ 0 \\ 0.05 \end{bmatrix}$$

$\pi = P(X_1 = i)$:

A	B	H	S
0.5	0.0	0.0	0.5

Observations:

$O_{1:4} = \{b, p, l, e\}$

Find:
Most likely hidden state sequence:

$X^*_{1:4}$

O_2 = P $\qquad 0.36 \cdot 5 \cdot 10^{-2} =$

$\delta_2(i)$ =

A→A	B→A	H→A	S→A
max (0.05 x 0.6 x 0.6 ,	0 x 0.0 x 0.6 ,	0 x 0.8 x 0.6 ,	0.05 x 0.2 x 0.6)
max (0.05 x 0.1 x 0.1 ,	0 x 0.3 x 0.1 ,	0 x 0.1 x 0.1 ,	0.05 x 0.0 x 0.1)
max (0.05 x 0.1 x 0.0 ,	0 x 0.2 x 0.0 ,	0 x 0 x 0.0 ,	0.05 x 0.1 x 0)
max (0.05 x 0.2 x 0.0 ,	0 x 0.5 x 0.0 ,	0 x 0.1 x 0.0 ,	0.05 x 0.7 x 0.0)

=

max probability	argmax state
0.018	(A)
0.0005	(A)
0	(∅)
0	(∅)

most likely previous state

most likely of most state

O_3 = 2

$\delta_3(i)$ =

max (0.018 x 0.6 x 0.1 ,	0.0005 x 0 x 0.1 ,	0.0005 x 0.8 x 0.1 ,	0.0005 x 0.2 x 0.1)
max (0.018 x 0.1 x 0.4 ,	0.0005 x 0.3 x 0.4 ,	0.0005 x 0.1 x 0.4 ,	0 x 0 x 0.4)
max (0.018 x 0.1 x 0.3 ,	0.0005 x 0.2 x 0.3 ,	0.0005 x 0 x 0.3 ,	0 x 0.1 x 0.3)
max (0.018 x 0.2 x 0.9 ,	0.0005 x 0.5 x 0.9 ,	0.0005 x 0.1 x 0.9 ,	0 x 0.7 x 0.9)

=

max. probability	argmax state	
0.00108	(A)	A
$7.2 \cdot 10^{-4}$	(A)	B
$5.4 \cdot 10^{-4}$	(A)	H
0.00324	(A)	S

Probability of state being and observing bp)

$O_4 =$ e

$\delta_4(i) =$

max (0.00108 x 0.6 x 0.2 , 0.00072 x 0 x 0.2 , 0.00054 x 0.8 x 0.2 , 0.00324 x 0.3 x 0.2)	=	1.296·10⁻⁴ …
max (0.00108 x 0.1 x 0.4 , 0.00072 x 0.3 x 0.4 , 0.00054 x 0.1 x 0.4 , 0.00324 x 0 x 0.4)		
max (0.00108 x 0.1 x 0 , 0.00072 x 0.2 x 0 , 0.00054 x 0 x 0.0 , 0.00324 x 0.1 x 0)		
max (0.00108 x 0.2 x 0 , 0.00072 x 0.5 x 0 , 0.00054 x 0.1 x 0 , 0.00324 x 0.7 x 0)		

max. probability	argmax state
$1.296 \cdot 10^{-4}$	(A,S)
8.64e-5	(B)
0	($\cancel{B}$)
0	($\cancel{A}$)

States and deltas over time:

$\delta_1(i)$	state
0.05	–
0	–
0	–
0.05	–

$\delta_2(i)$	state
0.018	A
0.0005	A
0	–
0	–

$\delta_3(i)$	state
0.00108	A
0.00072	A
0.00054	A
0.00324	A

$\delta_4(i)$	state
$1.296 \cdot 10^{-4}$	A, S
$8.64 \cdot 10^{-5}$	B
0	–
0	–

Backtracking gives two answers: $X^*_{1:4} = \{A, A, A, A\}$ and $X^*_{1:4} = \{A, A, \cancel{B}^{S}, \cancel{B}^{A}\}$