

A: $a_{ij} = P(X_{t+1} = j \mid X_t = i)$

$X_t \mid X_{t+1}$	A	B	H	S
A	0.6	0.1	0.1	0.2
B	0.0	0.3	0.2	0.5
H	0.8	0.1	0.0	0.1
S	0.2	0.0	0.1	0.7

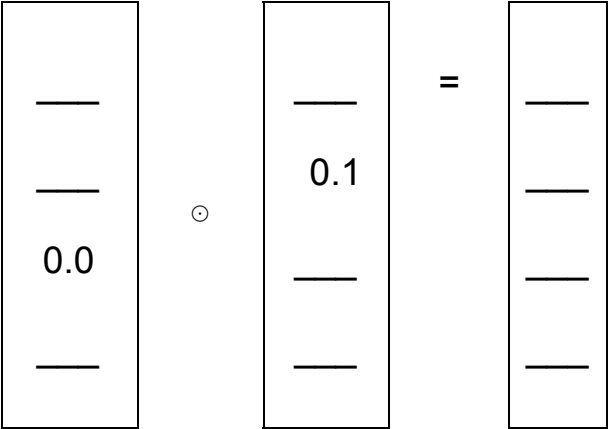
B: $b_{ik} = P(O_t = k \mid X_t = i)$

$X_t \mid O_t$	p	e	b	l
A	0.6	0.2	0.1	0.1
B	0.1	0.4	0.1	0.4
H	0.0	0.0	0.7	0.3
S	0.0	0.0	0.1	0.9

Step 1: Initialize $\delta_1(i)$

$O_1 =$ _____

$\delta_1(i) =$



$\pi = P(X_1 = i) :$

A	B	H	S
0.5	0.0	0.0	0.5

Observations:

$O_{1:4} = \{b,p,l,e\}$

Find:

Most likely hidden state sequence:

$X^*_{1:4}$

$O_2 = \underline{\hspace{1cm}}$

$\delta_2(i) =$

$\max (0.05 \times 0.6 \times 0.6 , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times 0.8 \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.6)$
$\max (0.05 \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.1 , \quad 0 \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad 0.05 \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}})$
$\max (\underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad 0 \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad 0.05 \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}})$
$\max (\underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad 0 \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times 0.7 \times \underline{\hspace{0.5cm}})$

=

max probability	argmax state
$\underline{\hspace{1cm}}$	($\underline{\hspace{0.5cm}}$)
0.0005	($\underline{\hspace{0.5cm}}$)
$\underline{\hspace{1cm}}$	($\underline{\hspace{0.5cm}}$)
0	($\underline{\hspace{0.5cm}}$)

$O_3 = \underline{\hspace{1cm}}$

$\delta_3(i) =$

$\max (0.018 \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.1 , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.1 , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.1)$
$\max (\underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.4 , \quad 0.0005 \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times 0 \times \underline{\hspace{0.5cm}})$
$\max (\underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.3 , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.3)$
$\max (\underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times 0.5 \times \underline{\hspace{0.5cm}} , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times 0.9 , \quad \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}} \times \underline{\hspace{0.5cm}})$

=

max. probability	argmax state
$\underline{\hspace{1cm}}$	($\underline{\hspace{0.5cm}}$)
$\underline{\hspace{1cm}}$	($\underline{\hspace{0.5cm}}$)
$\underline{\hspace{1cm}}$	(A)
$\underline{\hspace{1cm}}$	(A)

$O_4 = \underline{\hspace{1cm}}$
 $\delta_4(i) =$

$$\max (0.00108 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times 0.2 , 0.00054 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}})$$
$$\max (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times 0.4 , \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \times 0.1 \times \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}})$$
$$\max (\underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times 0.0 , \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}})$$
$$\max (\underline{\hspace{1cm}} \times 0.2 \times \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} , 0.00054 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} , 0.00324 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}})$$

=

max. probability	argmax state
<u> </u>	(<u> </u> , <u> </u>)
<u> </u>	(<u> </u>)
<u> </u>	(<u> </u>)
<u> </u>	(<u> </u>)

States and deltas over time:

$\delta_1(i)$	state
<u> </u>	<u> </u>
<u> </u>	<u> </u>
<u> </u>	<u> </u>
<u> </u>	<u> </u>

$\delta_2(i)$	state
<u> </u>	<u> </u>
<u> </u>	<u> </u>
<u> </u>	<u> </u>
<u> </u>	<u> </u>

$\delta_3(i)$	state
<u> </u>	<u> </u>
<u> </u>	<u> </u>
<u> </u>	<u> </u>
<u> </u>	<u> </u>

$\delta_4(i)$	state
<u> </u>	<u> </u> , <u> </u>
<u> </u>	<u> </u>
<u> </u>	<u> </u>
<u> </u>	<u> </u>

Backtracking gives two answers: $\mathbf{x}_{1:4}^* = \{ \underline{\hspace{1cm}} , \underline{\hspace{1cm}} , \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \}$ and $\mathbf{x}_{1:4}^* = \{ \underline{\hspace{1cm}} , \underline{\hspace{1cm}} , \underline{\hspace{1cm}} , \underline{\hspace{1cm}} \}$