Infinite 3D medium, Isotropic Plane Source, Isotropic Scattering

Exponential Random Flight

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

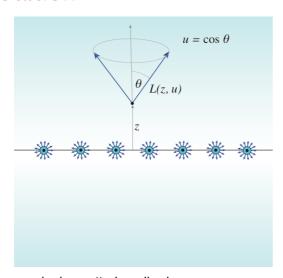
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Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

In[1]:= SetDirectory[Import["~/.hitchhikerpath"]]

Notation



lpha - single-scattering albedo

Σt - extinction coefficient

z - scalar position coordinate in medium (distance from plane source at origin)

 $u = \cos \theta$ - direction cosine

Analytic solutions

Caseology quantities

$$\ln[2] := \text{CaseNO}[c_{,} \text{ v0}_{,}] := \frac{1}{2} \text{ c v0}^{3} \left(\frac{\text{c}}{\text{v0}^{2} - 1} - \frac{1}{\text{v0}^{2}} \right)$$

FindRoot
$$\left[c \text{ v ArcTanh}\left[\frac{1}{v}\right] - 1 = 0, \left\{v, 1.00000000001, 10^{10}\right\}, \text{ Method } \rightarrow \text{"Brent"}\right][[1]][[2]]$$

$$ln[4]:= CaseN[c_, v_] := v \left(Case\lambda[v, c]^2 + \left(\frac{\pi c v}{2}\right)^2\right)$$

$$ln[5]:= Case\lambda[v_, c_] := 1 - c v ArcTanh[v]$$

$$ln[6]:= Case \psi 0 [u_{,}, v0_{,}, c_{,}, z_{,}] := \frac{c}{2} \frac{v0}{v0 - Sign[z] u}$$

Rigorous diffusion approximation

$$\begin{array}{ll} & \inf 3 Disoplane is oscatter \ \phi rigourous Diffusion [z_, \Sigma t_, \alpha_] := \\ & \frac{1}{2} \frac{E^{-Abs[z] \Sigma t/\#}}{CaseN0[\alpha, \#]} \& [Casev0[\alpha]] \end{array}$$

Fluence: exact solution

[Case and Zwiefel 1967]

$$\begin{array}{ll} & \inf 3 D isoplane isoscatter \Big| \phi exact[z_, \Sigma t_, \alpha_] := \\ & \inf 3 D isoplane isoscatter \Big| \phi r igourous D iffusion[z, \Sigma t, \alpha] + \\ & \frac{1}{2} \ N Integrate \Big[\frac{e^{-\Sigma t \, Abs[z] \, / v}}{CaseN[\alpha, \, v]}, \, \{v, \, 0, \, 1\} \Big] \\ & \end{array}$$

Nth-scattered fluence

$$\begin{array}{l} & \inf 3 \text{Disoplaneisoscatter} \ \phi \text{exact1} [x_, \ \Sigma t_, \ \alpha_, \ n_] := \\ & \frac{(\alpha \ \Sigma t)^n}{\pi} \ \text{NIntegrate} \Big[\frac{\text{ArcTan} \Big[\frac{z}{\Sigma t} \Big]^{n+1} \ \text{Cos} [x \ z]}{z^{n+1}}, \\ & \{z, 0, \text{Infinity}\}, \text{Method} \rightarrow \text{"ExtrapolatingOscillatory"} \Big] \end{array}$$

$$\label{eq:local_problem} \begin{split} & \ln[10] = \text{ inf3Disoplaneisoscatter} \\ & \uparrow \phi \text{exact2} \left[\text{x_, } \Sigma \text{t_, } \alpha_, \text{ n_]} \right] := \frac{\alpha^n}{-} \text{ Chop} \left[\text{NIntegrate} \left[\frac{\alpha^n}{\alpha^n} \right] \\ & \vdash \frac{\alpha^n}{\alpha^n} \right] \end{split}$$

$$\dot{\mathbb{1}} \ 2^{-2-n} \ e^{-Abs\left[x\right] \ z \ \Sigma t} \left(\left(\frac{-\dot{\mathbb{1}} \ \pi + Log\left[\frac{z+1}{z-1}\right]}{z} \right)^{1+n} - \left(\frac{\dot{\mathbb{1}} \ \pi + Log\left[\frac{z+1}{z-1}\right]}{z} \right)^{1+n} \right), \ \left\{z, 1, \ Infinity\right\} \right] \right]$$

$$\text{ln[11]:= inf3Disoplaneisoscatter} \hat{\phi} \text{Gaussian}[x_{,} \Sigma t_{,} \alpha_{,} n_{,}] := \frac{e^{-\frac{3 x^2 \Sigma t^2}{4 (1+n)}} \sqrt{\frac{3}{\pi}} \alpha^n}{2 \sqrt{1+n}}$$

Classical Diffusion Approximation

$$|n[12]:= inf3Disoplaneisoscatter^\phi Diffusion[x_, \Sigma t_, \alpha_] := \frac{3 e^{-Abs[x] \sqrt{3-3 \alpha} \Sigma t}}{2 \sqrt{3-3 \alpha}}$$

Grosjean Modified Diffusion Approximation

$$\begin{aligned} & & \inf \text{3Disoplane} \text{isoscatter} \tilde{} \phi \text{Grosjean} \left[\text{x_, } \Sigma \text{t_, } \alpha_ \right] := \\ & & -\frac{1}{2} \text{ ExpIntegralEi[-Abs[x] } \Sigma \text{t]} + \frac{\text{e}^{-\text{Abs[x]} \sqrt{3 + \frac{3}{-2 + \alpha}}} \sum_{\Sigma \text{t}} \sqrt{3 + \frac{3}{-2 + \alpha}}}{2 \; (1 - \alpha)} \end{aligned}$$

Radiance (Angular Flux)

$$\begin{split} &\inf 3 \text{Disoplaneisoscatter} \cdot \text{LrigourousDiffusion}[\textbf{z}_, \textbf{u}_, \Sigma \textbf{t}_, \alpha_] := \\ &\frac{1}{4 \, \text{Pi}} \, \text{Case} \psi \textbf{0} [\textbf{u}, \#, \alpha, \textbf{z}] \, \frac{\textbf{E}^{-\text{Abs}[\textbf{z}] \, \Sigma \textbf{t}/\#}}{\text{CaseNO}[\alpha, \#]} \, \& [\text{CasevO}[\alpha]] \\ &\inf 3 \text{Disoplaneisoscatter} \cdot \text{Lexact}[\textbf{z}_, \textbf{u}_, \Sigma \textbf{t}_, \alpha_] := \\ &\inf 3 \text{Disoplaneisoscatter} \cdot \text{LrigourousDiffusion}[\textbf{z}, \textbf{u}, \Sigma \textbf{t}, \alpha] + \\ &\frac{1}{4 \, \text{Pi}} \left(\text{Case} \lambda [\textbf{u}, \alpha] \, \frac{\textbf{e}^{-\frac{\text{Abs}[\textbf{z}] \, \Sigma \textbf{t}}{\textbf{u}}}}{\text{CaseN}[\alpha, \textbf{u}]} \, \text{HeavisideTheta}[\textbf{1} - \textbf{u}] \, \text{HeavisideTheta}[\textbf{u}] \\ &+ \text{NIntegrate} \Big[\frac{\textbf{e}^{-\frac{\text{Abs}[\textbf{z}] \, \Sigma \textbf{t}}{\textbf{v}}}}{\text{CaseN}[\alpha, \textbf{v}]} \, \frac{\alpha}{2} \, \frac{\textbf{v}}{\textbf{v} - \textbf{u}} \\ &, \, \{ \textbf{v}, \, \textbf{0}, \, \textbf{u}, \, \textbf{1} \} \, , \, \text{Method} \rightarrow \text{"PrincipalValue"}, \, \text{PrecisionGoal} \rightarrow 5 \Big] \\ \end{split}$$

load MC data

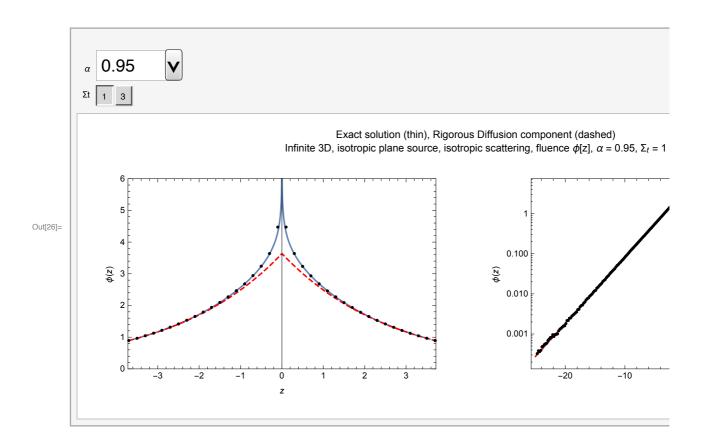
```
inf3Disoplaneisoscatter`ppoints[zs_, dz_, maxz_, \St_] :=
       Table [\{dz(i) - 0.5 dz - maxz, zs[[i]] / \Sigma t\}, \{i, 1, Length[zs]\}][[2;; -2]]
| In[17]:= inf3Disoplaneisoscatter`ppointsu[xs_, du_, Σt_] :=
       Table [\{-1.0 + du (i) - 0.5 du, xs[[i]] / (2 \Sigma t)\}, \{i, 1, Length[xs]\}][[1;; -1]]
In[18]:= inf3Disoplaneisoscatter`fs =
        FileNames["code/3D_medium/infinite3Dmedium/Isotropicplanesource/data/
            inf3D_isotropicplane_isotropicscatter*"];
log_{[19]} = inf3Disoplaneisoscatter index[x] := Module[{data, <math>\alpha, \Sigma t},
         data = Import[x, "Table"];
         Σt = data[[1, 13]];
         \alpha = data[[2, 3]];
         {α, Σt, data}];
     inf3Disoplaneisoscatter`simulations =
        inf3Disoplaneisoscatter`index /@ inf3Disoplaneisoscatter`fs;
     inf3Disoplaneisoscatter`alphas =
      Union[#[[1]] & /@ inf3Disoplaneisoscatter`simulations]
Out[21]= \{0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999\}
```

```
In[22]:= inf3Disoplaneisoscatter`muts =
        Union[#[[2]] & /@ inf3Disoplaneisoscatter`simulations]
Out[22]:= {1, 3}
In[23]:= inf3Disoplaneisoscatter`numcollorders =
        inf3Disoplaneisoscatter`simulations[[1]][[3]][[2, 13]];
    inf3Disoplaneisoscatter`maxz =
        inf3Disoplaneisoscatter`simulations[[1]][[3]][[2, 5]];
    inf3Disoplaneisoscatter`dz =
        inf3Disoplaneisoscatter`simulations[[1]][[3]][[2, 7]];
    inf3Disoplaneisoscatter`numz =
        Floor[2 inf3Disoplaneisoscatter`maxz / inf3Disoplaneisoscatter`dz];
```

Compare Deterministic and MC

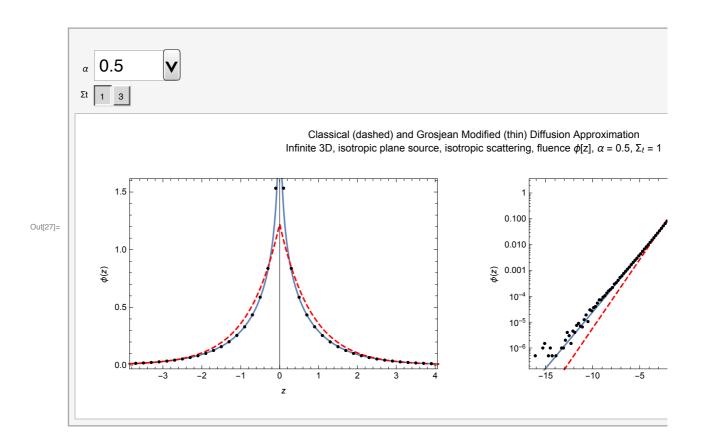
Fluence - Exact solution comparison to MC

```
In[26]:= Manipulate
       If [Length[inf3Disoplaneisoscatter`simulations] > 0,
        Module [\{data, maxz, dz, points\phi, plotpoints\phi, logplot\phi, plot\phi, exactlpoints\}, \}
          data = SelectFirst[
              inf3Disoplaneisoscatter`simulations, \#[[1]] = \alpha \&\& \#[[2]] = \Sigma t \&][[3]];
          maxz = data[[2, 5]];
          dz = data[[2, 7]];
          points\phi = data[[4]];
          (* divide by \Sigmat to convert collision density into fluence *)
          plotpoints\phi = inf3Disoplaneisoscatter`ppoints[points\phi, dz, maxz, \Sigmat];
          exact1points =
           Quiet[{\#[[1]], inf3Disoplaneisoscatter`\phiexact[\#[[1]], \Sigmat, \alpha]}] & /@
             plotpoints\phi;
          numpoints = Length[plotpoints\phi];
          skip = Floor [numpoints \frac{6}{7} \frac{1}{3}];
          plot \phi = Quiet[Show[
               (*ListPlot[exact1points[[skip;;-skip]],PlotRange \rightarrow \{0,6\},Joined \rightarrow True],*)
              Plot[inf3Disoplaneisoscatter\phiexact[z, \Sigmat, \alpha],
                \left\{z, -\frac{\max z}{7}, \frac{\max z}{7}\right\}, PlotRange \rightarrow \{0, 6\}],
              Plot[inf3Disoplaneisoscatter\phirigourousDiffusion[z, \Sigmat, \alpha],
                \{z, -maxz, maxz\}, PlotRange \rightarrow All, PlotStyle \rightarrow \{Red, Dashed\}],
              ListPlot[plotpoints\phi[[skip;;-skip]], PlotRange \rightarrow All,
               PlotStyle → {Black, PointSize[.01]}],
              Frame → True,
              \texttt{FrameLabel} \mathrel{->} \{ \{ \phi [\mathtt{z}] \,, \} \,, \ \{ \mathtt{z} \,, \} \}
          logplot \phi = Quiet[Show[
              ListLogPlot[exact1points, PlotRange → All, Joined → True],
              {\tt LogPlot[inf3Disoplaneisoscatter`$\phi$ rigourousDiffusion[$z$, $\Sigma t$, $\alpha$],}
                \{z, -maxz, maxz\}, PlotRange \rightarrow All, PlotStyle \rightarrow \{Red, Dashed\}],
              ListLogPlot[plotpoints\phi, PlotRange \rightarrow All,
               PlotStyle → {Black, PointSize[.01]}],
              Frame → True,
              \texttt{FrameLabel} \mathrel{->} \{ \{ \phi[\mathtt{z}], \}, \ \{\mathtt{z}, \} \}
          Show[GraphicsGrid[{{plot\phi, logplot\phi}}, ImageSize \rightarrow 800], PlotLabel ->
             "Exact solution (thin), Rigorous Diffusion component (dashed)\nInfinite
                 3D, isotropic plane source, isotropic scattering, fluence
                 \phi[z], \alpha = " \Leftrightarrow ToString[\alpha] \Leftrightarrow ", \Sigma_t = " \Leftrightarrow ToString[\Sigma t]
        Text[
          "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
             ensure the data path is setup correctly."]
       , \{\{\alpha, 0.95\}, inf3Disoplaneisoscatter`alphas\},
       {{Σt, 1}, inf3Disoplaneisoscatter`muts}]
```



Fluence - Diffusion approximations (Classical and Grosjean) comparison to MC

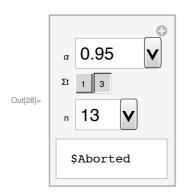
```
In[27]:= Manipulate
      If [Length[inf3Disoplaneisoscatter`simulations] > 0,
        Module \lceil \{data, maxz, dz, points \phi, \}
          plotpoints\phi, logplot\phi, plot\phi, exact1points, numpoints, skip\},
         data = SelectFirst[inf3Disoplaneisoscatter`simulations,
             #[[1]] = \alpha \&\& #[[2]] = \Sigma t \&][[3]];
         maxz = data[[2, 5]];
         dz = data[[2, 7]];
         points\phi = data[[4]];
         (* divide by Σt to convert collision density into fluence *)
         plotpoints\phi = inf3Disoplaneisoscatter`ppoints[points\phi, dz, maxz, \Sigmat];
         numpoints = Length [plotpoints\phi];
         skip = Floor [numpoints \frac{6}{7} \frac{1}{3}];
         plot \phi = Quiet[Show[
             ListPlot[plotpoints\phi[[skip;;-skip]],
              PlotRange → All, PlotStyle → {PointSize[.01], Black}],
             Plot[inf3Disoplaneisoscatter\phiGrosjean[z, \Sigmat, \alpha],
               \{z, -maxz, maxz\}, PlotRange \rightarrow All],
             Plot[inf3Disoplaneisoscatter\phiDiffusion[z, \Sigmat, \alpha],
               \{z, -maxz, maxz\}, PlotRange \rightarrow All, PlotStyle \rightarrow \{Red, Dashed\}],
             ListPlot[plotpoints\phi[[skip;;-skip]], PlotRange \rightarrow All,
              PlotStyle → {PointSize[.01], Black}],
             Frame → True,
             FrameLabel \rightarrow {{ \phi[z],}, {z,}}
            ]];
         logplot \phi = Quiet[Show[
             ListLogPlot[plotpoints\phi,
              PlotRange → All, PlotStyle → {PointSize[.01], Black}],
             LogPlot[inf3Disoplaneisoscatter\phiGrosjean[z, \Sigmat, \alpha],
               \{z, -maxz, maxz\}, PlotRange \rightarrow All],
             LogPlot[inf3Disoplaneisoscatter\phiDiffusion[z, \Sigmat, \alpha],
               \{z, -maxz, maxz\}, PlotRange \rightarrow All, PlotStyle \rightarrow \{Red, Dashed\}\}
             ListLogPlot[plotpoints\phi, PlotRange \rightarrow All,
              PlotStyle → {PointSize[.01], Black}],
             Frame → True,
             FrameLabel -> {{ \phi[z],}, \{z,\}}
         Show[GraphicsGrid[{{plot\phi, logplot\phi}}, ImageSize \rightarrow 800],
          PlotLabel -> "Classical (dashed) and Grosjean Modified (thin)
                Diffusion Approximation\nInfinite 3D, isotropic plane
                source, isotropic scattering, fluence \phi[z], \alpha = "<>
             ToString[\alpha] \iff ", \Sigma_t = " \iff ToString[\Sigma t]]
        Text[
         "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
            ensure the data path is setup correctly."]
       , \{\{\alpha, 0.5\}, inf3Disoplaneisoscatter`alphas\},
       {{Σt, 1}, inf3Disoplaneisoscatter`muts}]
```



N-th order fluence / scalar flux

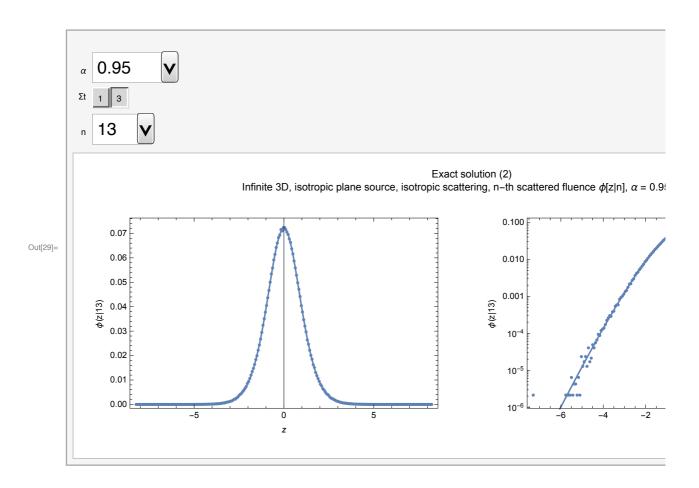
N-th collided Fluence - Exact solution (1) comparison to MC

```
In[28]:= Manipulate[
       If[Length[inf3Disoplaneisoscatter`simulations] > 0,
        \texttt{Module[\{data, maxz, dz, points}\phi,
           plotpoints\phi, logplot\phi, plot\phi, exact1points, numorders\},
         data = SelectFirst[inf3Disoplaneisoscatter`simulations,
             #[[1]] = \alpha \&\& #[[2]] = \Sigma t \&][[3]];
         maxz = data[[2, 5]];
         dz = data[[2, 7]];
         numorders = data[[2, 13]];
         points\phi = data[[9 + numorders + n + 1]];
          (* divide by \Sigmat to convert collision density into fluence *)
         plotpoints\phi = inf3Disoplaneisoscatter`ppoints[points\phi, dz, maxz, \Sigmat];
         exact1points =
          Quiet[\{\#[[1]], \inf 3Disoplaneisoscatter \phiexact1[\#[[1]], \Sigma t, \alpha, n]\}] & /@
            plotpoints\phi;
         plot \phi = Quiet[Show[
             ListPlot[plotpoints\phi, PlotRange \rightarrow All, PlotStyle \rightarrow PointSize[.01]],
             ListPlot[exact1points, PlotRange → All, Joined → True],
             Frame \rightarrow True,
             \label \rightarrow \{\{ \phi [\ "z | \ " <> \texttt{ToString}[n]] \ , \} \ , \ \{z \ , \} \}
            ]];
         logplot \phi = Quiet[Show[
             ListLogPlot[plotpoints\phi, PlotRange \rightarrow All, PlotStyle \rightarrow PointSize[.01]],
             ListLogPlot[exact1points, PlotRange → All, Joined → True],
             Frame → True,
             FrameLabel \rightarrow {{ \phi["z|" \leftarrow ToString[n]],}, {z,}}
         Show[GraphicsGrid[{{plot\phi, logplot\phi}}, ImageSize \rightarrow 800],
           PlotLabel -> "Exact solution (1) \nInfinite 3D, isotropic plane source,
                isotropic scattering, n-th scattered fluence \phi[\mathbf{z} \mid \mathbf{n}], \alpha = " <>
             ToString[\alpha] \Leftrightarrow ", \Sigma_t = " \Leftrightarrow ToString[\Sigma t]]
        ]
        Text[
         "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
            ensure the data path is setup correctly."]
       , \{\{\alpha, 0.99\}, inf3Disoplaneisoscatter`alphas\},
       {{Σt, 3}, inf3Disoplaneisoscatter`muts},
       {{n, 13}, Range[If[NumberQ[inf3Disoplaneisoscatter`numcollorders],
           inf3Disoplaneisoscatter`numcollorders, 1]]}
     1
```



N-th collided Fluence - Exact solution (2) comparison to MC

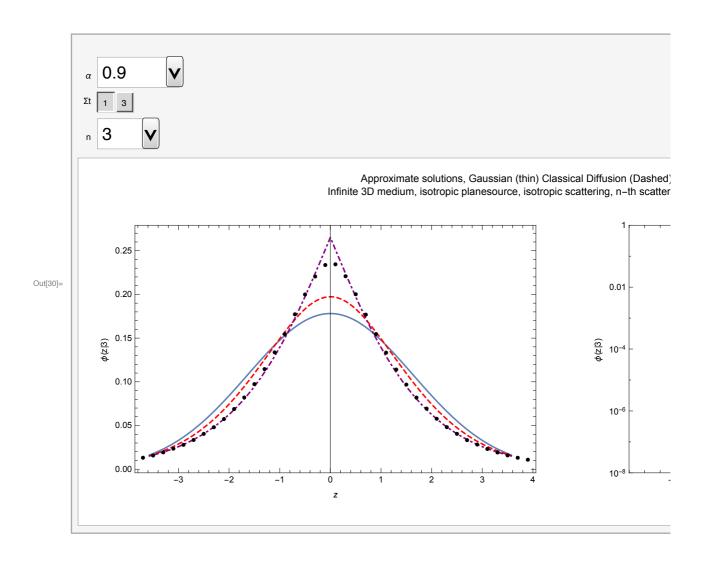
```
In[29]:= Manipulate[
      If[Length[inf3Disoplaneisoscatter`simulations] > 0,
       Module[{data, maxz, dz, points\phi,
          plotpoints\phi, logplot\phi, plot\phi, exact1points, numorders},
         data = SelectFirst[inf3Disoplaneisoscatter`simulations,
             \#[[1]] = \alpha \&\& \#[[2]] = \Sigma t \&][[3]];
         maxz = data[[2, 5]];
         dz = data[[2, 7]];
         numorders = data[[2, 13]];
         points\phi = data[[9 + numorders + n + 1]];
         (* divide by Σt to convert collision density into fluence *)
         plotpoints\phi = inf3Disoplaneisoscatter`ppoints[points\phi, dz, maxz, \Sigmat];
         exact1points =
          Quiet[\{\#[[1]], \inf 3Disoplaneisoscatter \phiexact2[\#[[1]], \Sigma t, \alpha, n]\}] & /@
           plotpoints\phi;
         plot \phi = Quiet[Show[
             ListPlot[plotpoints\phi, PlotRange \rightarrow All, PlotStyle \rightarrow PointSize[.01]],
             ListPlot[exact1points, PlotRange → All, Joined → True],
             Frame → True,
             FrameLabel \rightarrow {{ \phi["z|" <> ToString[n]],}, {z,}}
         logplot \phi = Quiet[Show[
             \texttt{ListLogPlot[plotpoints} \phi, \ \texttt{PlotRange} \rightarrow \texttt{All}, \ \texttt{PlotStyle} \rightarrow \texttt{PointSize[.01]]},
             ListLogPlot[exact1points, PlotRange → All, Joined → True],
             Frame → True,
             FrameLabel -> {{ \phi["z|" <> ToString[n]],}, \{z,\}}
         Show[GraphicsGrid[\{\{plot\phi, logplot\phi\}\}\}, ImageSize \rightarrow 800],
          PlotLabel -> "Exact solution (2) \nInfinite 3D, isotropic plane source,
                isotropic scattering, n-th scattered fluence \phi[z|n], \alpha = "<>
             ToString[\alpha] \iff ", \Sigma_t = " \iff ToString[\Sigma t]
        ]
       Text[
         "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
            ensure the data path is setup correctly."]
      , \{\{\alpha, 0.99\}, inf3Disoplaneisoscatter`alphas\},
      {{Σt, 3}, inf3Disoplaneisoscatter`muts},
      {{n, 13}, Range[If[NumberQ[inf3Disoplaneisoscatter`numcollorders],
          inf3Disoplaneisoscatter`numcollorders, 1]]}
     ]
```



N-th collided Fluence - Approximations

```
In[30]:= Manipulate
       If [Length[inf3Disoplaneisoscatter`simulations] > 0,
        Module \int \{data, maxz, dz, points\phi,
           plotpoints\phi, logplot\phi, plot\phi, exact1points, numorders\},
         data = SelectFirst[inf3Disoplaneisoscatter`simulations,
              #[[1]] = \alpha \&\& #[[2]] = \Sigma t \&][[3]];
         maxz = data[[2, 5]];
         dz = data[[2, 7]];
         numorders = data[[2, 13]];
         points\phi = data[[9 + numorders + n + 1]];
          (* divide by \Sigmat to convert collision density into fluence *)
         plotpoints\phi = inf3Disoplaneisoscatter`ppoints[points\phi, dz, maxz, \Sigmat];
         seriesclassical = \alpha^n SeriesCoefficient[
              inf3Disoplaneisoscatter\phiDiffusion[z, \Sigmat, C], {C, 0, n}];
         \texttt{seriesG} = \alpha^{\texttt{n}} \; \texttt{SeriesCoefficient[inf3Disoplaneisoscatter} \\ \texttt{`$\phi$Grosjean[z, \Sigma t, C]$,} \\
              {C, 0, n}];
         numpoints = Length[plotpoints\phi];
         skip = Floor [numpoints \frac{6}{7} \frac{1}{2}];
         plot \phi = Quiet[Show]
              Plot[inf3Disoplaneisoscatter\phiGaussian[z, \Sigmat, \alpha, n],
```

```
\left\{z, -\frac{\max z}{7}, \frac{\max z}{7}\right\}, PlotRange \rightarrow All],
      Plot[seriesclassical, \{z, -\frac{\max z}{7}, \frac{\max z}{7}\}, PlotRange \rightarrow All,
        PlotStyle → {Red, Dashed}],
      Plot[seriesG, \{z, -\frac{\max z}{7}, \frac{\max z}{7}\}, PlotRange \rightarrow All,
        PlotStyle → {DotDashed, Purple}],
      ListPlot[plotpoints\phi[[skip;;-skip]], PlotRange \rightarrow All,
        PlotStyle → {Black, PointSize[.01]}],
      Frame → True,
      FrameLabel -> {{ \phi["z|" <> ToString[n]],}, \{z,\}}
     ||;
  logplot \phi = Quiet[Show]
      LogPlot[inf3Disoplaneisoscatter\phiGaussian[z, \Sigmat, \alpha, n],
        \{z, -maxz, maxz\}, PlotRange \rightarrow \{10^{-8}, 1\}],
      LogPlot[seriesclassical, {z, -maxz, maxz},
        PlotRange \rightarrow \{10^{-8}, 1\}, PlotStyle \rightarrow \{\text{Red, Dashed}\}\],
      LogPlot[seriesG, {z, -maxz, maxz}, PlotRange \rightarrow {10<sup>-8</sup>, 1},
        PlotStyle → {DotDashed, Purple}],
      ListLogPlot[plotpoints\phi, PlotRange \rightarrow \{10^{-8}, 1\},
        PlotStyle → {Black, PointSize[.01]}],
      Frame → True,
      FrameLabel -> \{ \{ \phi["z|" \iff ToString[n]], \}, \{z, \} \}
     ]];
  Show[GraphicsGrid[{{plot\phi, logplot\phi}}, ImageSize \rightarrow 1000],
    PlotLabel -> "Approximate solutions, Gaussian (thin) Classical
         Diffusion (Dashed) Grosjean (Dot-Dashed) \nInfinite
         3D medium, isotropic planesource, isotropic
         scattering, n-th scattered fluence \phi[z] " <>
      ToString[n] \Leftrightarrow "], \alpha = " \Leftrightarrow ToString[\alpha] \Leftrightarrow ", \Sigma_t = " \Leftrightarrow ToString[\Sigma t]]
 ]
  "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
, \{\{\alpha, 0.9\}, inf3Disoplaneisoscatter`alphas\},
{\Sigmath{\Sigma}t, inf3Disoplaneisoscatter\muts},
{{n, 3}, Range[If[NumberQ[inf3Disoplaneisoscatter`numcollorders],
    inf3Disoplaneisoscatter`numcollorders, 1]]}
```



Angular Distributions

```
In[31]:= Manipulate[
       If[Length[inf3Disoplaneisoscatter`simulations] > 0,
        Module[{data, numorders, pointsu, plotpointsu, du, r, dz, maxz, zsim},
          data = SelectFirst[
              inf3Disoplaneisoscatter`simulations, \#[[1]] = \alpha \&\& \#[[2]] = \Sigma t \&][[3]];
         numorders = data[[2, 13]];
         du = data[[2, 9]];
         dz = data[[2, 7]];
         maxz = data[[2, 5]];
         pointsu = data[[9 + 2 numorders + n]];
         zsim = dz * n - 0.5 dz - maxz;
         plotpointsu = inf3Disoplaneisoscatter`ppointsu[pointsu, du, Σt];
         pp = Show[
            ListPlot[plotpointsu, PlotRange → All, PlotStyle → Black,
              Frame → True,
              \label{eq:frameLabel} \texttt{FrameLabel} \rightarrow \{\{\, \texttt{PiL}[\, \texttt{zsim},\, u]\,, \}\,,\, \{u\,,\}\,\}\,]\,,
            Plot[Pi inf3Disoplaneisoscatter`LrigourousDiffusion[zsim, u, \Sigma t, \alpha],
              \{u, -1, 1\}, PlotStyle \rightarrow \{\text{Red, Dashed}\}\
            ],
            Plot[Pi inf3Disoplaneisoscatter Lexact[zsim, u, \Sigmat, \alpha], {u, -1, 1}
            ],
            PlotLabel -> "Infinite 3D Medium, isotropic plane
                  source, isotropic scattering\nAngular distribution:
                  rigorous diffusion approximation\n\pi L[z,u], z = " <>
               \textbf{ToString[zsim]} <> \texttt{", } \alpha \texttt{ = "} <> \textbf{ToString[}\alpha\texttt{]} <> \texttt{", } \Sigma_{\texttt{t}} \texttt{ = "} <> \textbf{ToString[}\Sigma\texttt{t}\texttt{]}
           ]
        ],
        Text[
          "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
            ensure the data path is setup correctly."]
       , \{\{\alpha, 0.9\}, inf3Disoplaneisoscatter`alphas\},
       \{\{\Sigma t, 1\}, inf3Disoplaneisoscatter`muts\}, \{\{n, 127\}, Range[If[
           NumberQ[inf3Disoplaneisoscatter`numz], inf3Disoplaneisoscatter`numz, 1]]}]
```

