Infinite Flatland medium, Isotropic Point Source, Isotropic Scattering

Exponential Random Flight

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

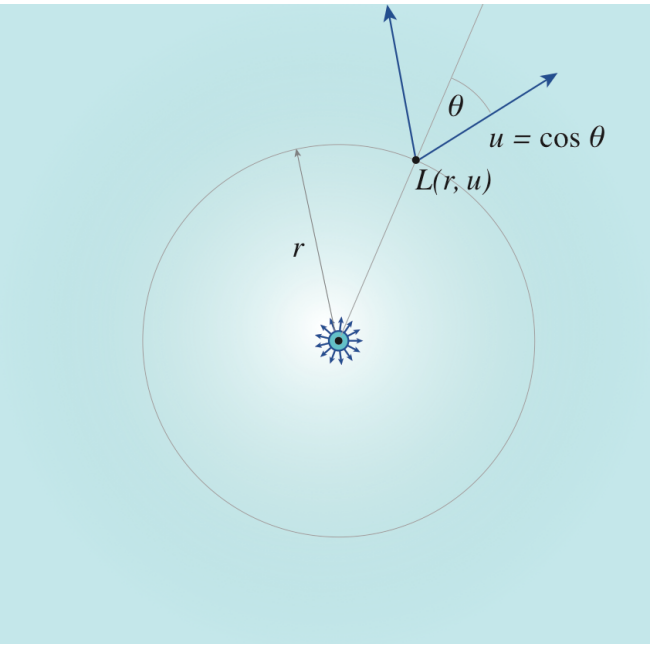
www.eugenedeon.com

Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

In[46]:=
```
SetDirectory[Import["~/.hitchhikerpath"]]
```

Notation



α - single-scattering albedo
Σt - extinction coefficient
r - radial position coordinate in medium (distance from point source at origin)
u = cos θ - direction cosine

Analytic solutions

Fluence: exact solution (1)

In[47]:=
$$\text{infflatlandisopointisoscatter`}\phi\text{exact1}[r\_, \Sigma t\_, \alpha\_] := \frac{\text{Exp}[-\Sigma t\, r]}{2\,\text{Pi}\, r} + \frac{\Sigma t}{2\,\text{Pi}}\,\text{NIntegrate}\left[\frac{\alpha\, z\, \text{BesselJ}[0, r\, z\, \Sigma t]}{1 + z^2 - \alpha\sqrt{1+z^2}}, \{z, 0, \text{Infinity}\}, \text{Method} \to \text{"LevinRule"}\right]$$

Fluence: exact solution (2)

In[48]:=
$$\text{infflatlandisopointisoscatter`}\phi\text{exact2}[r\_, \Sigma t\_, \alpha\_] := \Sigma t\,\alpha\,\frac{\text{BesselK}\left[0, r\,\Sigma t\sqrt{1-\alpha^2}\right]}{\text{Pi}} + \frac{\Sigma t}{2\,\text{Pi}}\,\text{NIntegrate}\left[\frac{z\,\text{BesselJ}[0, \Sigma t\, r\, z]}{\sqrt{1+z^2}+\alpha}, \{z, 0, \text{Infinity}\}, \text{Method} \to \text{"LevinRule"}\right]$$

Fluence: exact solution (3)

In[49]:=
$$\text{infflatlandisopointisoscatter`}\phi\text{exact3a}[r\_, \Sigma t\_, \alpha\_] := \text{NIntegrate}\left[\frac{\Sigma t}{2\,\text{Pi}}\,\frac{k\,\text{BesselJ}[0, k\,\Sigma t\, r]}{\sqrt{k^2+1}-\alpha}, \{k, 0, \text{Infinity}\}, \text{Method} \to \text{"LevinRule"}\right]$$

In[50]:=
$$\text{besselk}[n\_, x\_] := \sqrt{\frac{2}{\text{Pi}\, x}}\,\text{BesselK}[n+1/2, x];$$
$$\text{infflatlandisopointisoscatter`}\phi\text{exact3b}[r\_, \Sigma t\_, \alpha\_, M\_] := \frac{\text{Exp}[-\Sigma t\, r]}{2\,\text{Pi}\, r} + \frac{\alpha\,\Sigma t}{2\,\text{Pi}}\,\text{BesselK}\left[0, \Sigma t\sqrt{1-\alpha^2}\, r\right] + \frac{\Sigma t}{2\,\text{Pi}}\,\text{Sum}\left[\frac{\alpha^{2n}\, n!}{(2n)!}\,(2\,\Sigma t\, r)^n\,\text{besselk}[n-1, \Sigma t\, r], \{n, 1, M\}\right]$$

Classical diffusion approximation

In[52]:=
$$\text{infflatlandisopointisoscatter`}\phi\text{Diffusion}[r\_, \Sigma t\_, \alpha\_] := \frac{\Sigma t\,\text{BesselK}\left[0, \sqrt{2-2\alpha}\, r\,\Sigma t\right]}{\pi}$$

Rigorous diffusion approximation

In[53]:=
$$\text{infflatlandisopointisoscatter`}\phi\text{RigorousDiffusion}[r\_, \Sigma t\_, \alpha\_] := \frac{\Sigma t\,\alpha\,\text{BesselK}\left[0, \sqrt{1-\alpha^2}\, r\,\Sigma t\right]}{\pi}$$

Grosjean-style diffusion approximation

In[54]:=
$$\text{infflatlandisopointisoscatter`}\phi\text{Grosjean}[r\_, \Sigma t\_, \alpha\_] := \frac{\text{Exp}[-r\,\Sigma t]}{2\,\text{Pi}\, r} + \frac{\alpha\,\Sigma t}{(2-\alpha)\,\text{Pi}}\,\text{BesselK}\left[0, r\,\Sigma t\left(\sqrt{2}\,\frac{\sqrt{1-\alpha}}{\sqrt{2-\alpha}}\right)\right]$$

n-th scattered fluence

In[55]:=
$$\text{infflatlandisopointisoscatter`}\phi[r\_, \Sigma t\_, \alpha\_, n\_] := \frac{2^{\frac{1}{2}(-1-n)}\,\alpha^n\, r^{\frac{1}{2}(-1+n)}\,\Sigma t^{\frac{1+n}{2}}\,\text{BesselK}\left[\frac{1}{2}(-1+n), r\,\Sigma t\right]}{\pi\,\text{Gamma}\left[\frac{1+n}{2}\right]}$$

Moments

In[56]:=
$$\text{infflatlandisopointisoscatter`}\phi m[\Sigma t\_, \alpha\_, m\_] := \left(1-\alpha^2\right)^{-1-\frac{m}{2}}\Sigma t^{-1-m}\left(2^m\,\alpha\,\text{Gamma}\left[1+\frac{m}{2}\right]^2 + \text{Gamma}[1+m]\,\text{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{m}{2}, \frac{1}{2}, \alpha^2\right]\right)$$

In[57]:=
$$\text{infflatlandisopointisoscatter`}\phi m[\Sigma t\_, \alpha\_, n\_, m\_] := \frac{2^m\,\alpha^n\,\Sigma t^{-1-m}\,\text{Gamma}\left[1+\frac{m}{2}\right]\,\text{Gamma}\left[\frac{1}{2}(1+m+n)\right]}{\text{Gamma}\left[\frac{1+n}{2}\right]}$$

Angular phi integral

In[58]:=
$$\text{infflatlandisopointisoscatter`Lintegral}[r\_, u\_, \Sigma t\_, \alpha\_, \phi\_] := \frac{\alpha\,\Sigma t}{2\,\text{Pi}}\,\text{NIntegrate}\left[\phi\left[\sqrt{r^2+t^2-2\,r\,t\,u}, \Sigma t, \alpha\right]\text{Exp}[-\Sigma t\,t], \{t, 0, \text{Infinity}\}\right]$$

Angular Classical diffusion approximation

In[59]:=
$$\text{infflatlandisopointisoscatter`Ldiffusion}[r\_, u\_, \Sigma t\_, \alpha\_] := \frac{1}{2\,\pi^2}\,\Sigma t\left(\text{BesselK}\left[0, r\sqrt{2-2\,\alpha}\,\Sigma t\right] + u\sqrt{2-2\,\alpha}\,\text{BesselK}\left[1, r\sqrt{2-2\,\alpha}\,\Sigma t\right]\right)$$

load MC data

In[60]:=
```
infflatlandisopointisoscatter`ppoints[xs_, dr_, maxx_, Σt_] :=
 Table[{dr (i) - 0.5 dr, xs[[i]] / Σt}, {i, 1, Length[xs]}][[1 ;; -2]]
```

In[61]:=
```
infflatlandisopointisoscatter`ppointsu[xs_, du_, Σt_] :=
 Table[{-1.0 + du (i) - 0.5 du, Sqrt[1 - (-1.0 + du (i) - 0.5 du)^2] xs[[i]] / (2 Σt)},
   {i, 1, Length[xs]}][[1 ;; -1]]
```

In[62]:=
```
infflatlandisopointisoscatter`fs =
  FileNames["code/flatland/infiniteFlatland/Isotropicpointsource/data/
     infflatland_isotropicpoint_isotropicscatter*"];
```

In[63]:=
```
infflatlandisopointisoscatter`index[x_] := Module[{data, α, Σt},
    data = Import[x, "Table"];
    Σt = data[[1, 13]];
    α = data[[2, 3]];
    {α, Σt, data}];
infflatlandisopointisoscatter`simulations =
  infflatlandisopointisoscatter`index /@ infflatlandisopointisoscatter`fs;
infflatlandisopointisoscatter`alphas =
 Union[#[[1]] & /@ infflatlandisopointisoscatter`simulations]
```

Out[65]=
$$\{0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999\}$$

In[66]:=
```
infflatlandisopointisoscatter`muts =
 Union[#[[2]] & /@ infflatlandisopointisoscatter`simulations]
```

Out[66]=
```
{1, 3}
```

In[67]:=
```
infflatlandisopointisoscatter`numcollorders =
  infflatlandisopointisoscatter`simulations[[1]][[3]][[2, 13]];
infflatlandisopointisoscatter`maxr =
 infflatlandisopointisoscatter`simulations[[1]][[3]][[2, 5]];
infflatlandisopointisoscatter`dr =
 infflatlandisopointisoscatter`simulations[[1]][[3]][[2, 7]];
infflatlandisopointisoscatter`numr =
  Floor[infflatlandisopointisoscatter`maxr / infflatlandisopointisoscatter`dr];
```

Compare Deterministic and MC

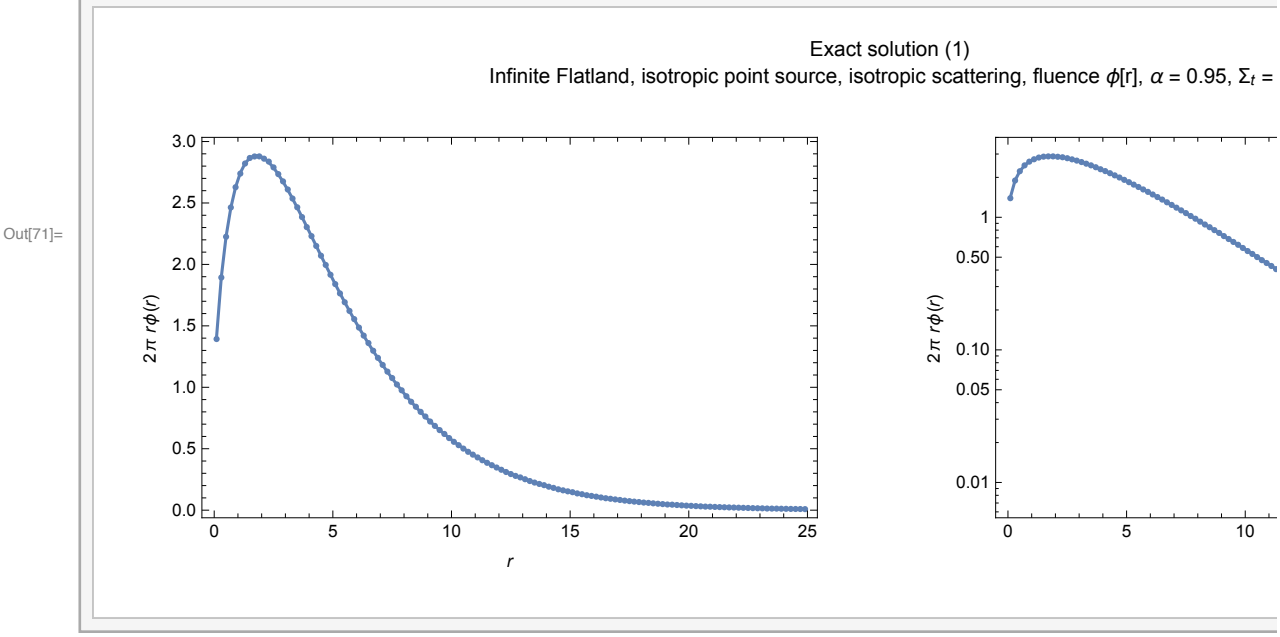
Fluence - Exact solution (1) comparison to MC

In[70]:=
```
Clear[alpha, Σt];
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, maxr, dr, pointsϕ, plotpointsϕ, logplotϕ, plotϕ, exact1points},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxr = data[[2, 5]];
   dr = data[[2, 7]];

   pointsϕ = data[[4]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = infflatlandisopointisoscatter`ppoints[pointsϕ, dr, maxr, Σt];

   exact1points =
    Quiet[{#[[1]], 2 Pi #[[1]] infflatlandisopointisoscatter`ϕexact1[
          #[[1]], Σt, α]}] & /@ plotpointsϕ;

   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListLogPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 800],
    PlotLabel -> "Exact solution (1)\nInfinite Flatland, isotropic point
       source, isotropic scattering, fluence ϕ[r], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
 ]
 ,
 Text[
  "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
    ensure the data path is setup correctly."]
]
, {{α, 0.95}, infflatlandisopointisoscatter`alphas},
{Σt, infflatlandisopointisoscatter`muts}]
```

Out[71]=

α 0.95

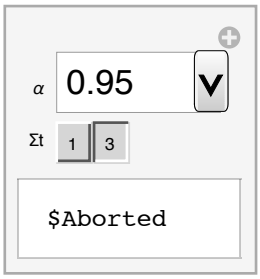
Σt 1 3



Fluence - Exact solution (2) comparison to MC

In[72]:=
```
Clear[alpha, Σt];
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, maxr, dr, pointsϕ, plotpointsϕ, logplotϕ, plotϕ, exact1points},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxr = data[[2, 5]];
   dr = data[[2, 7]];

   pointsϕ = data[[4]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = infflatlandisopointisoscatter`ppoints[pointsϕ, dr, maxr, Σt];

   exact1points =
    Quiet[{#[[1]], 2 Pi #[[1]] infflatlandisopointisoscatter`ϕexact2[
          #[[1]], Σt, α]}] & /@ plotpointsϕ;

   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListLogPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 800],
    PlotLabel -> "Exact solution (2)\nInfinite Flatland, isotropic point
       source, isotropic scattering, fluence ϕ[r], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
 ]
 ,
 Text[
  "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
    ensure the data path is setup correctly."]
]
, {{α, 0.99}, infflatlandisopointisoscatter`alphas},
{{Σt, 3}, infflatlandisopointisoscatter`muts}]
```

Out[73]=



Fluence - Exact solution (3a) comparison to MC

In[74]:=
```
Clear[alpha, Σt];
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, maxr, dr, pointsϕ, plotpointsϕ, logplotϕ, plotϕ, exact1points},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxr = data[[2, 5]];
   dr = data[[2, 7]];

   pointsϕ = data[[4]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = infflatlandisopointisoscatter`ppoints[pointsϕ, dr, maxr, Σt];

   exact1points =
    Quiet[{#[[1]], 2 Pi #[[1]] infflatlandisopointisoscatter`ϕexact3a[
          #[[1]], Σt, α]}] & /@ plotpointsϕ;

   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListLogPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 800],
    PlotLabel -> "Exact solution (3a)\nInfinite Flatland, isotropic point
        source, isotropic scattering, fluence ϕ[r], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
 ]
 ,
 Text[
  "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
    ensure the data path is setup correctly."]
]
, {{α, 0.99}, infflatlandisopointisoscatter`alphas},
{{Σt, 3}, infflatlandisopointisoscatter`muts}]
```

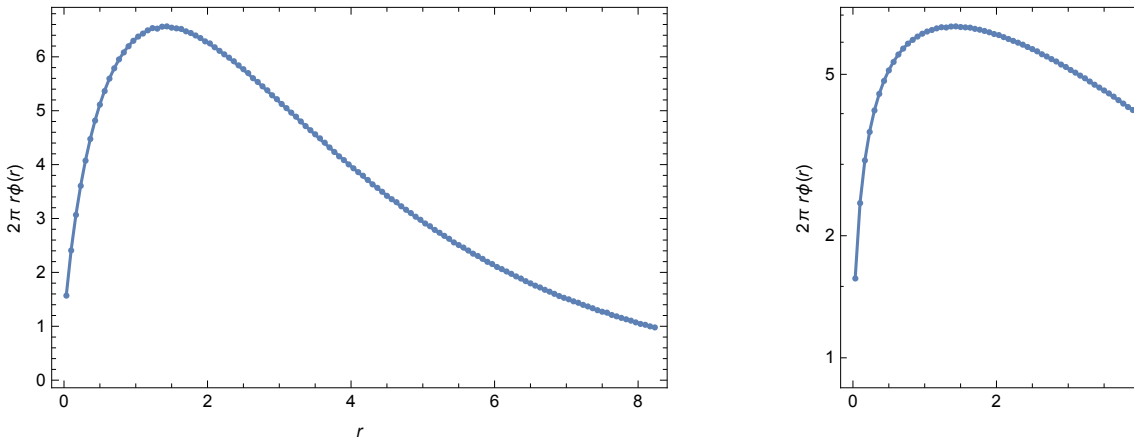
Out[75]=

α 0.99

Σt 1 3

Exact solution (3a)

Infinite Flatland, isotropic point source, isotropic scattering, fluence ϕ[r], α = 0.99, Σ_t =



Fluence - Exact solution (3b) comparison to MC

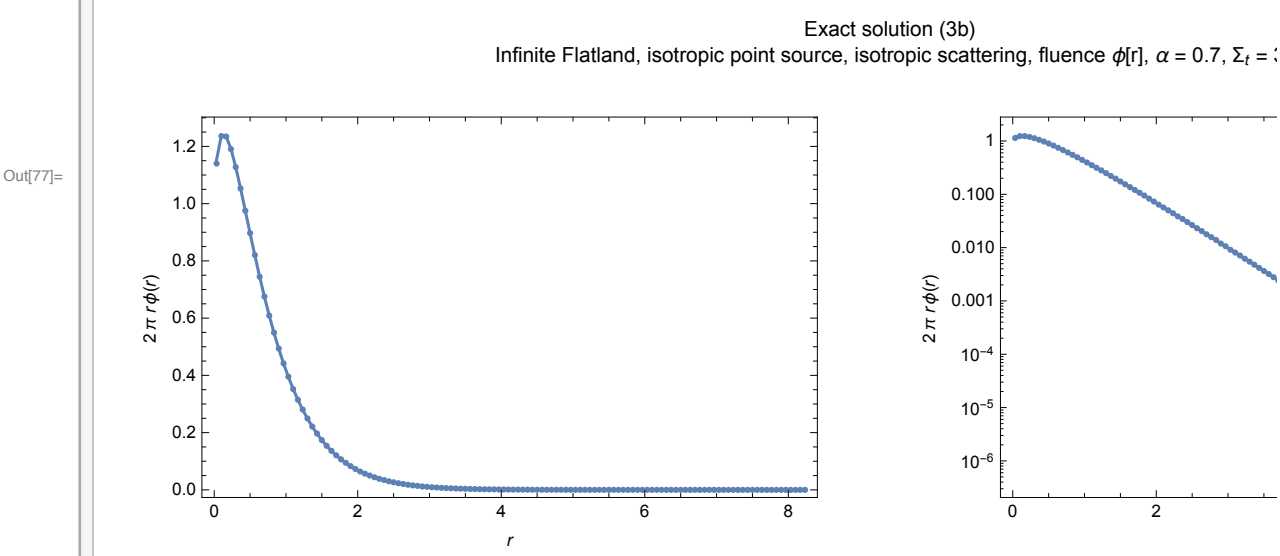
In[76]:=
```
Clear[alpha, Σt];
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, maxr, dr, pointsϕ, plotpointsϕ, logplotϕ, plotϕ, exact1points},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxr = data[[2, 5]];
   dr = data[[2, 7]];

   pointsϕ = data[[4]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = infflatlandisopointisoscatter`ppoints[pointsϕ, dr, maxr, Σt];

   exact1points =
    Quiet[{#[[1]], 2 Pi #[[1]] infflatlandisopointisoscatter`ϕexact3b[
          #[[1]], Σt, α, M]}] & /@ plotpointsϕ;

   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListLogPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 800],
    PlotLabel -> "Exact solution (3b)\nInfinite Flatland, isotropic point
       source, isotropic scattering, fluence ϕ[r], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
 ]
 ,
 Text[
  "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
    ensure the data path is setup correctly."]
]
, {{α, 0.7}, infflatlandisopointisoscatter`alphas},
{{Σt, 3}, infflatlandisopointisoscatter`muts}, {{M, 10}, Range[20]}]
```

Out[77]=

α 0.7

Σt 1 3

M 10



Fluence - Diffusion approximations (Classical and Grosjean) comparison to MC

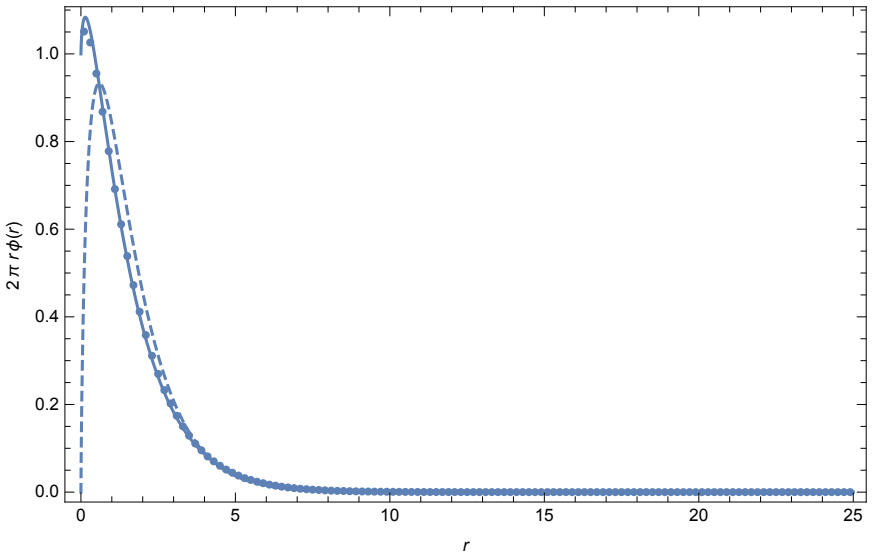
In[78]:=
```
Clear[alpha, Σt];
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, maxr, dr, pointsϕ, plotpointsϕ, logplotϕ, plotϕ, exact1points},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxr = data[[2, 5]];
   dr = data[[2, 7]];

   pointsϕ = data[[4]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = infflatlandisopointisoscatter`ppoints[pointsϕ, dr, maxr, Σt];

   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      Plot[2 Pi r infflatlandisopointisoscatter`ϕGrosjean[r, Σt, α],
       {r, 0, maxr}, PlotRange → All],
      Plot[2 Pi r infflatlandisopointisoscatter`ϕDiffusion[r, Σt, α],
       {r, 0, maxr}, PlotRange → All, PlotStyle → Dashed],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      LogPlot[2 Pi r infflatlandisopointisoscatter`ϕGrosjean[r, Σt, α],
       {r, 0, maxr}, PlotRange → All],
      LogPlot[2 Pi r infflatlandisopointisoscatter`ϕDiffusion[r, Σt, α],
       {r, 0, maxr}, PlotRange → All, PlotStyle → Dashed],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 1000],
    PlotLabel -> "Classical (dashed) and Grosjean Modified (thin)
        Diffusion Approximation\nInfinite Flatland, isotropic
        point source, isotropic scattering, fluence ϕ[r], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
  ]
  ,
  Text[
   "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
 ]
 , {{α, 0.5}, infflatlandisopointisoscatter`alphas},
 {Σt, infflatlandisopointisoscatter`muts}]
```

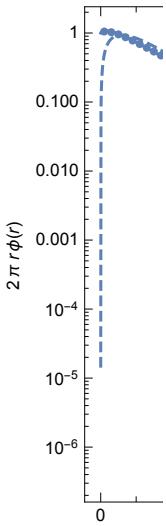
Out[79]=

α 0.5

Σt 1 3

Classical (dashed) and Grosjean Modified (thin) Diffusion
Infinite Flatland, isotropic point source, isotropic scattering, fluen





Fluence - Diffusion approximation (Rigorous) comparison to MC

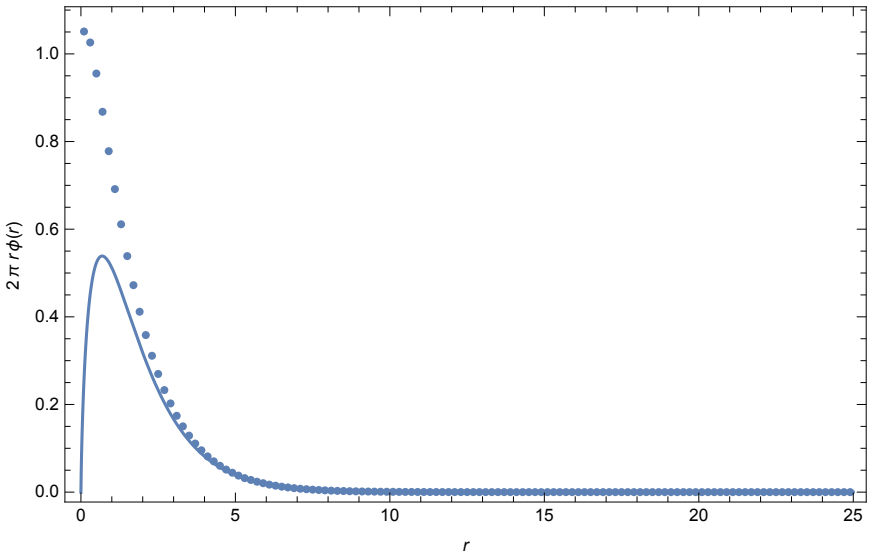
In[80]:=
```
Clear[alpha, Σt];
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, maxr, dr, pointsϕ, plotpointsϕ, logplotϕ, plotϕ, exact1points},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxr = data[[2, 5]];
   dr = data[[2, 7]];

   pointsϕ = data[[4]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = infflatlandisopointisoscatter`ppoints[pointsϕ, dr, maxr, Σt];

   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      Plot[2 Pi r infflatlandisopointisoscatter`ϕRigorousDiffusion[r, Σt, α],
       {r, 0, maxr}, PlotRange → All],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      LogPlot[2 Pi r infflatlandisopointisoscatter`ϕRigorousDiffusion[r, Σt, α],
       {r, 0, maxr}, PlotRange → All],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r],}, {r,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 1000], PlotLabel ->
     "Rigorous Diffusion Approximation\nInfinite Flatland, isotropic
        point source, isotropic scattering, fluence ϕ[r], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
 ]
 ,
 Text[
  "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
    ensure the data path is setup correctly."]
]
, {{α, 0.5}, infflatlandisopointisoscatter`alphas},
{Σt, infflatlandisopointisoscatter`muts}]
```

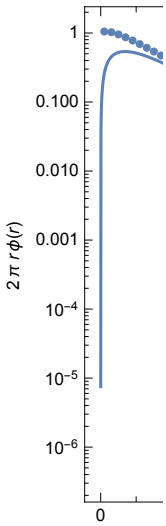
Out[81]=

α 0.5

Σt 1 3

Rigorous Diffusion Approximation
Infinite Flatland, isotropic point source, isotropic scattering, fluen





N-th order fluence / scalar flux

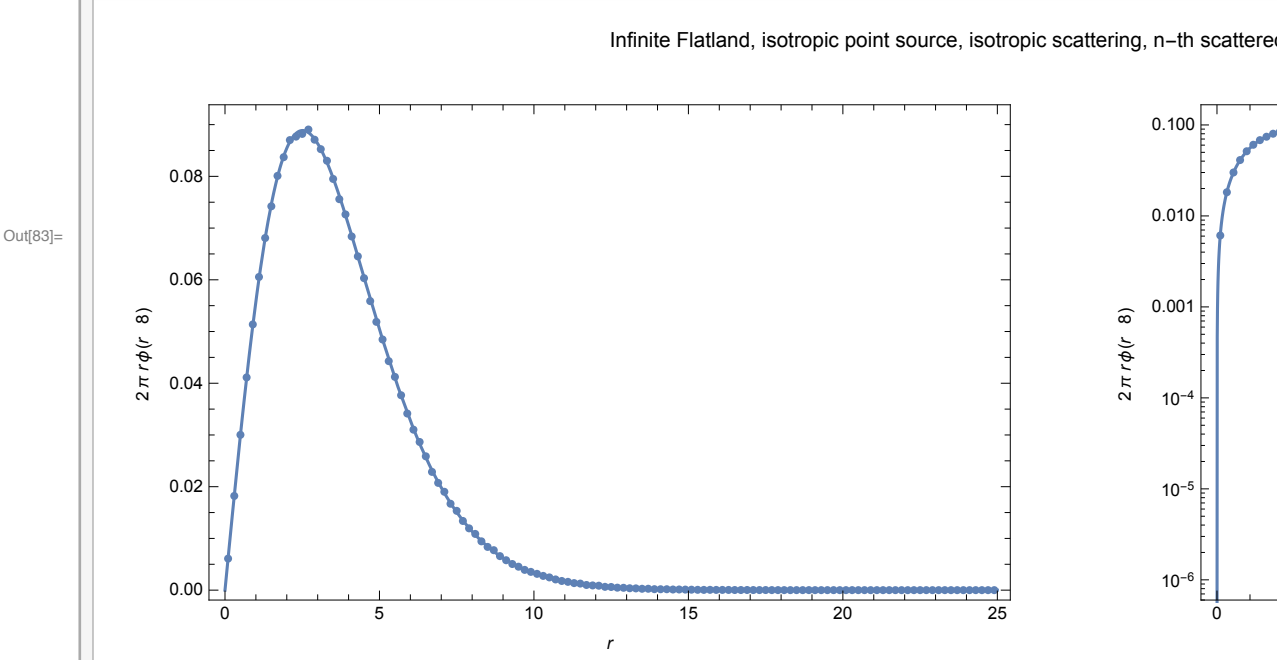
In[82]:=
```
Clear[alpha, Σt];
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, maxr, dr, pointsϕ,
    plotpointsϕ, logplotϕ, plotϕ, exact1points, numorders},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxr = data[[2, 5]];
   dr = data[[2, 7]];
   numorders = data[[2, 13]];

   pointsϕ = data[[9 + numorders + n + 1]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = infflatlandisopointisoscatter`ppoints[pointsϕ, dr, maxr, Σt];

   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      Plot[2 Pi r infflatlandisopointisoscatter`ϕ[r, Σt, α, n],
       {r, 0, maxr}, PlotRange → All],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r | n],}, {r,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      LogPlot[2 Pi r infflatlandisopointisoscatter`ϕ[r, Σt, α, n],
       {r, 0, maxr}, PlotRange → All],
      Frame → True,
      FrameLabel -> {{2 Pi r ϕ[r | n],}, {r,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 1000],
    PlotLabel -> "Infinite Flatland, isotropic point source, isotropic
        scattering, n-th scattered fluence ϕ[r|n], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
 ]
 ,
 Text[
  "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
    ensure the data path is setup correctly."]
]
, {{α, 0.9}, infflatlandisopointisoscatter`alphas},
{Σt, infflatlandisopointisoscatter`muts},
{{n, 8}, Range[If[NumberQ[infflatlandisopointisoscatter`numcollorders],
   infflatlandisopointisoscatter`numcollorders, 1]]}]
```

Out[83]=

α 0.9

Σt 1 3

n 8



Compare moments of ϕ

In[84]:=
```
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, nummoments, ϕmoments, ks, analytic, j},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   nummoments = data[[2, 15]];
   ϕmoments = N[{data[[6]]/Σt}];
   ks = {Table[k, {k, 0, nummoments - 1}]};
   analytic = Table[infflatlandisopointisoscatter`ϕm[Σt, α, k], {k, ks}];
   j = Join[ks, analytic, ϕmoments];
   TableForm[
    Join[{{"k", "analytic", "MC"}}, Transpose[j]]
   ]
  ],
  Text[
   "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
 ]
 , {{α, 0.95}, infflatlandisopointisoscatter`alphas},
 {{Σt, 3}, infflatlandisopointisoscatter`muts}]
```

Out[84]=

α 0.95

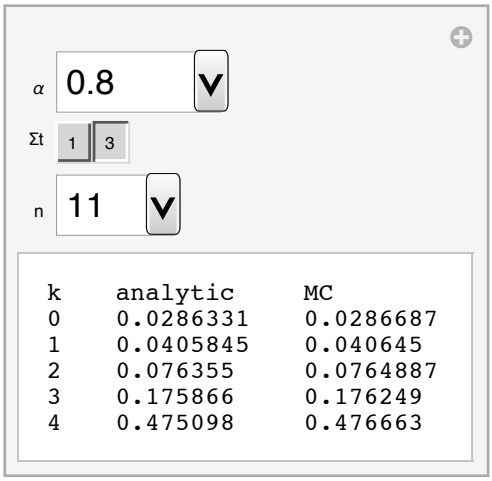
Σt 1 3

k	analytic	MC
0	6.66667	6.65803
1	10.931	10.9132
2	29.6296	29.5695
3	111.733	111.445
4	539.918	538.247

n-th collided moments of ϕ

In[85]:=
```
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, ϕmoments, ks, analytic, j, nummoments},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   nummoments = data[[2, 15]];
   ϕmoments = N[{data[[9 + n]]}/Σt];
   ks = {Table[k, {k, 0, nummoments - 1}]};
   analytic =
    Table[Quiet[N[infflatlandisopointisoscatter`ϕm[Σt, α, n, k]]], {k, ks}];
   j = Join[ks, analytic, ϕmoments];
   TableForm[
    Join[{{"k", "analytic", "MC"}}, Transpose[j]]
   ]
  ],
  Text[
   "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
 ]
 , {{α, 0.8}, infflatlandisopointisoscatter`alphas},
 {{Σt, 3}, infflatlandisopointisoscatter`muts},
 {{n, 11}, Range[If[NumberQ[infflatlandisopointisoscatter`numcollorders],
    infflatlandisopointisoscatter`numcollorders, 1]]}]
```

Out[85]=

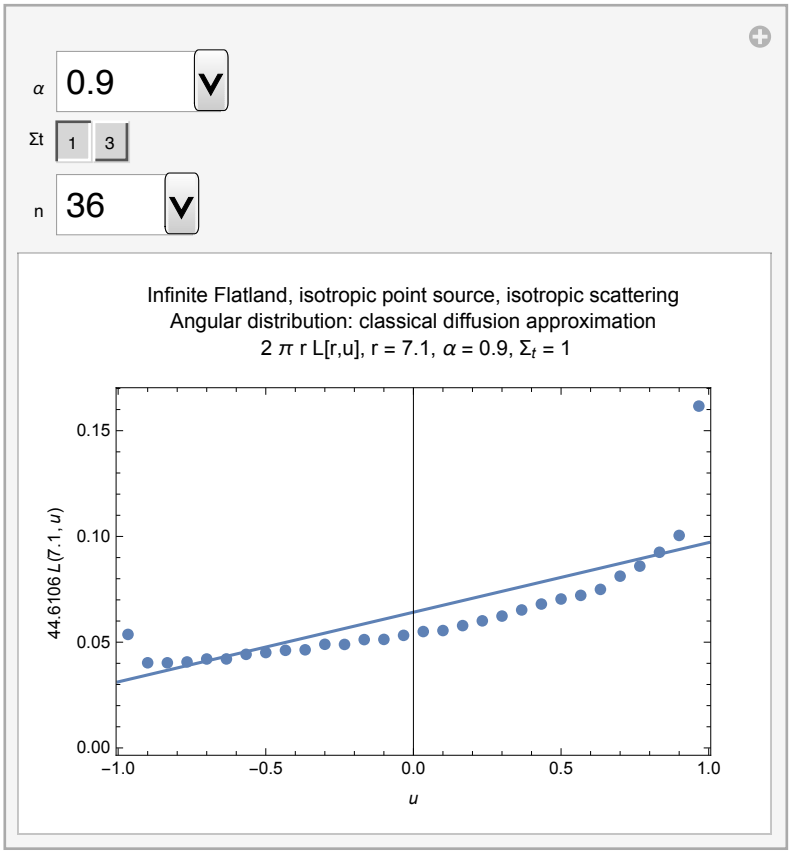


k	analytic	MC
0	0.0286331	0.0286687
1	0.0405845	0.040645
2	0.076355	0.0764887
3	0.175866	0.176249
4	0.475098	0.476663

Angular distributions

In[86]:=
```
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, numorders, pointsu, plotpointsu, du, r, dr},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   numorders = data[[2, 13]];
   du = data[[2, 9]];
   dr = data[[2, 7]];

   pointsu = data[[9 + 2 numorders + n]];

   r = dr * n - 0.5 dr;

   (* divide by Σt to convert collision density into fluence *)
   plotpointsu = infflatlandisopointisoscatter`ppointsu[pointsu, du, Σt];
   Show[
    ListPlot[plotpointsu, PlotRange → All,
     Frame → True,
     FrameLabel -> {{2 Pi r L[r, u],}, {u,}}],
    Plot[2 Pi r infflatlandisopointisoscatter`Ldiffusion[r, u, Σt, α],
     {u, -1, 1}, PlotRange → All
    ],
    PlotLabel -> "Infinite Flatland, isotropic point source,
        isotropic scattering\nAngular distribution: classical
        diffusion approximation\n 2 π r L[r,u], r = " <>
      ToString[r] <> ", α = " <> ToString[α] <> ", Σt = " <> ToString[Σt]
   ]

  ],
  Text[
   "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
 ]
 , {{α, 0.9}, infflatlandisopointisoscatter`alphas},
 {{Σt, 1}, infflatlandisopointisoscatter`muts},
 {{n, 36}, Range[If[NumberQ[infflatlandisopointisoscatter`numr],
    infflatlandisopointisoscatter`numr, 1]]}]
```

Out[86]=


α
0.9
Σt
1
3
n
36
Infinite Flatland, isotropic point source, isotropic scattering
Angular distribution: classical diffusion approximation
2 π r L[r,u], r = 7.1, α = 0.9, Σt = 1
44.6106 L(7.1, u)
0.15
0.10
0.05
0.00
−1.0
−0.5
0.0
0.5
1.0
u

In[87]:=
```
Manipulate[
 If[Length[infflatlandisopointisoscatter`simulations] > 0,
  Module[{data, numorders, pointsu, plotpointsu, du, r, dr},
   data = SelectFirst[infflatlandisopointisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   numorders = data[[2, 13]];
   du = data[[2, 9]];
   dr = data[[2, 7]];

   pointsu = data[[9 + 2 numorders + n]];

   r = dr * n - 0.5 dr;

   (* divide by Σt to convert collision density into fluence *)
   plotpointsu = infflatlandisopointisoscatter`ppointsu[pointsu, du, Σt];
   Show[
    ListPlot[plotpointsu, PlotRange → All,
     Frame → True,
     FrameLabel -> {{2 Pi r L[r, u],}, {u,}}],
    Plot[2 Pi r infflatlandisopointisoscatter`Lintegral[r, u, Σt, α,
       infflatlandisopointisoscatter`ϕGrosjean], {u, -1, 1}, PlotRange → All],
    PlotLabel -> "Infinite Flatland, isotropic point source,
        isotropic scattering\nAngular distribution:
        Fluence integral Grosjean\n 2 π r L[r,u], r = " <>
      ToString[r] <> ", α = " <> ToString[α] <> ", Σt = " <> ToString[Σt]
   ]

  ],
  Text[
   "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
 ]
 , {{α, 0.9}, infflatlandisopointisoscatter`alphas},
 {{Σt, 1}, infflatlandisopointisoscatter`muts},
 {{n, 53}, Range[If[NumberQ[infflatlandisopointisoscatter`numr],
    infflatlandisopointisoscatter`numr, 1]]}]
```

Out[87]=

α 0.9

Σt 1 3

n 53