Fresnel Boundaries

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

www.eugenedeon.com

Non-polarized

Reflection and Refraction

In[59]:=

$$\texttt{refract[w\_, n\_, eta1\_, eta2\_] :=} \frac{-\texttt{eta1}}{\texttt{eta2}} (\texttt{w} - (\texttt{w.n})\ \texttt{n}) - \left(\sqrt{1 - \left(\frac{\texttt{eta1}}{\texttt{eta2}}\right)^2 \left(1 - (\texttt{w.n})^2\right)}\right) \texttt{n;}$$

```
reflect[v_, n_] := -v + 2 n n.v;
```

Dielectric Fresnel

Dielectric reflectance for incoming light with incoming cosine costhetai and ratio of internal to external indices of etaratio:

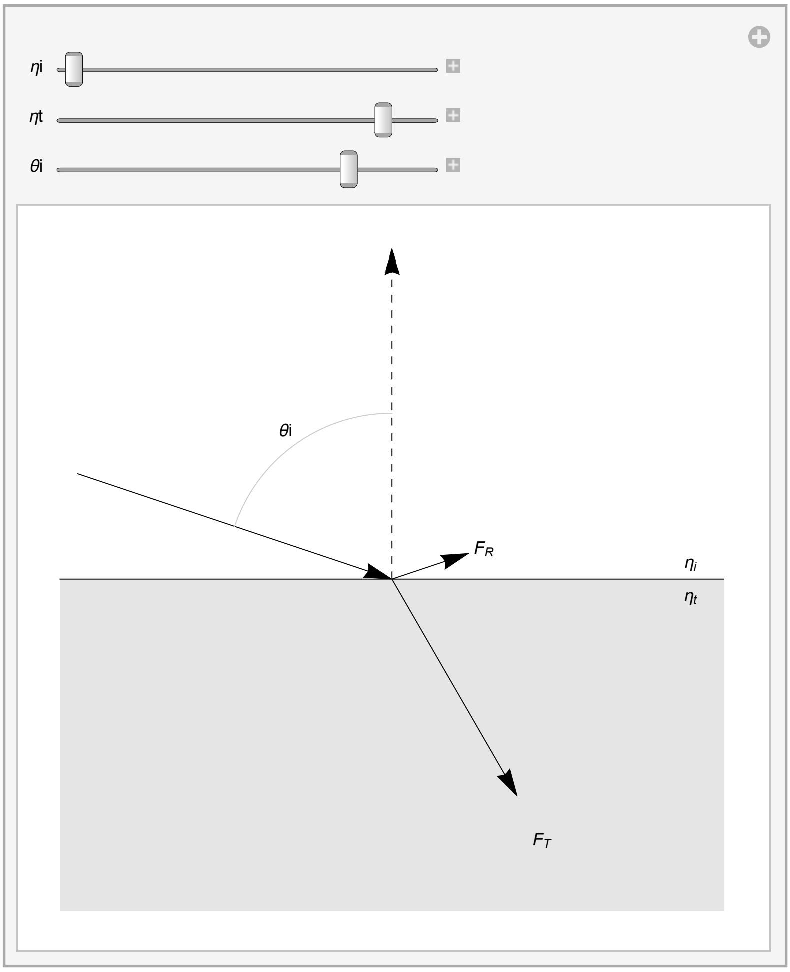
In[61]:=

$$\texttt{evalF[g\_, c\_] :=} \frac{1}{2} \frac{(\texttt{g} - \texttt{c})^2}{(\texttt{g} + \texttt{c})^2} \left(1 + \frac{(\texttt{c}\ (\texttt{g} + \texttt{c}) - 1)^2}{(\texttt{c}\ (\texttt{g} - \texttt{c}) + 1)^2}\right)\texttt{;}$$

$$\begin{aligned}&\texttt{FR[etaratio\_, costhetai\_] := If[}\\ &\quad \texttt{etaratio}^2 - 1 + \texttt{costhetai}^2 \geq 0\texttt{,}\\ &\quad \texttt{evalF}\left[\sqrt{\texttt{etaratio}^2 - 1 + \texttt{costhetai}^2}\texttt{, costhetai}\right]\texttt{,}\\ &\quad \texttt{1]}\end{aligned}$$

```
In[63]:= Manipulate[
  plotFresnel = Graphics[
    {
     Gray,
     GrayLevel[.9],
     Rectangle[{-1, -1}, {1, 0}],
     Black,
     Line[{{-1, 0}, {1, 0}}],
     {Dashed,
      Arrow[{{0, 0}, {0, 1}}]},
     Arrow[{{-Sin[θi], Cos[θi]}, {0, 0}}],
     Arrow[{{0, 0}, FR[ηt/ηi, Cos[θi]] {Sin[θi], Cos[θi]}}],
     Arrow[{{0, 0},
        (1 - FR[ηt/ηi, Cos[θi]]) refract[{-Sin[θi], Cos[θi]}, {0, 1}, ηi, ηt]}],
     Text["ηi", {.9, 0.05}],
     Text["ηt", {.9, -0.05}],
     Text["FR", 1.2 FR[ηt/ηi, Cos[θi]] {Sin[θi], Cos[θi]}],
     Text["FT",
      1.2 (1 - FR[ηt/ηi, Cos[θi]]) refract[{-Sin[θi], Cos[θi]}, {0, 1}, ηi, ηt]],
     GrayLevel[.8],
     Circle[{0, 0}, 0.5, {Pi/2, Pi/2 + θi}],
     Black,
     Text["θi", .55 {-Sin[θi/2], Cos[θi/2]}]
    }
   ], {ηi, 1, 2}, {ηt, 1, 2}, {θi, 0, Pi/2}]
```

Out[63]=



Benchmark data

In[64]:=

```
ns = {0.5, 0.7, 0.9, 0.99, 1.01, 1.1, 1.4, 2};
FRdata = Table[FR[n, Cos[t]], {n, ns}, {t, {0., 0.2, 0.5, 1., 1.2, 1.5}}];
Transpose[Join[{Table[n, {n, ns}]}, Transpose[FRdata]]] // Grid
```

Out[66]=

0.5	0.111111	0.111752	0.283268	1	1	1
0.7	0.0311419	0.0312467	0.0387004	1	1	1
0.9	0.00277008	0.00277588	0.00309559	0.0452967	1	1
0.99	0.0000252519	0.0000252954	0.0000275606	0.000184621	0.00132454	1
1.01	0.0000247519	0.0000247928	0.0000269003	0.000160946	0.000962525	0.143225
1.1	0.00226757	0.0022707	0.0024255	0.00987357	0.0376234	0.507226
1.4	0.0277778	0.0277999	0.0288139	0.0611806	0.135736	0.657913
2	0.111111	0.111145	0.112621	0.15019	0.222786	0.6834

In[67]:=

Conductor Fresnel

Exact

In[68]:=
```
Clear[p, q, rhoPerp, rhoPar, Rs, Rp];
p[ni_, n_, k_, theta_] :=
```
$$\sqrt{\left(\frac{1}{2}\left(\sqrt{\left(n^2-k^2-ni^2\,\mathrm{Sin}[theta]^2\right)^2+4\,n^2\,k^2}+\left(n^2-k^2-ni^2\,\mathrm{Sin}[theta]^2\right)\right)\right)};$$
```
q[ni_, n_, k_, theta_] :=
```
$$\sqrt{\left(\frac{1}{2}\left(\sqrt{\left(n^2-k^2-ni^2\,\mathrm{Sin}[theta]^2\right)^2+4\,n^2\,k^2}-\left(n^2-k^2-ni^2\,\mathrm{Sin}[theta]^2\right)\right)\right)};$$

In[71]:=
$$\mathrm{rhoPerp}[p\_,\,q\_,\,n1\_,\,t\_] := \frac{(n1\,\mathrm{Cos}[t]-p)^2+q^2}{(n1\,\mathrm{Cos}[t]+p)^2+q^2};$$
$$\mathrm{rhoPar}[p\_,\,q\_,\,n1\_,\,t\_] := \frac{\left(p-n1\,\mathrm{Sin}[t]\,\mathrm{Tan}[t]\right)^2+q^2}{\left(p+n1\,\mathrm{Sin}[t]\,\mathrm{Tan}[t]\right)^2+q^2}\,\mathrm{rhoPerp}[p,\,q,\,n1,\,t]$$

In[73]:=
$$\mathrm{FR}[ni\_,\,n\_,\,k\_,\,t\_] := \frac{1}{2}\left(\mathrm{rhoPerp}[p[ni,n,k,t],\,q[ni,n,k,t],\,ni,\,t]+\mathrm{rhoPar}[p[ni,n,k,t],\,q[ni,n,k,t],\,ni,\,t]\right)$$

Reflectance at normal incidence:

In[74]:=
$$\mathrm{ConductorReflectance}[ni\_,\,n\_,\,k\_] := 1-\frac{4\,ni\,n}{(ni+n)^2+k^2}$$

Schlick's Approximation

In[75]:=
```
mix[a_, b_, t_] := b t + (1 - t) a;
FRSchlickFresnel[ni_, n_, k_, theta_] := mix[
   ConductorReflectance[ni, n, k],
   1,
   (1 - Cos[theta])^5
  ]
```

Additional approximate form

Mentioned in [Pharr and Humphreys - Physically Based Rendering], first edition, [9.1], [9.2] (more accurate for larger k)

In[100]:=
```
FRPharrHumphreys[n_, k_, t_] :=
```
$$\frac{1}{2}\left(\frac{\left(n^2+k^2\right)\mathrm{Cos}[t]^2-2\,n\,\mathrm{Cos}[t]+1}{\left(n^2+k^2\right)\mathrm{Cos}[t]^2+2\,n\,\mathrm{Cos}[t]+1}+\left(\frac{\left(n^2+k^2\right)-2\,n\,\mathrm{Cos}[t]+\mathrm{Cos}[t]^2}{\left(n^2+k^2\right)+2\,n\,\mathrm{Cos}[t]+\mathrm{Cos}[t]^2}\right)\right)$$

Misunderstood form: Substitute complex index (n+ik) into the *dielectric* formula and take the magnitude of the result, Abs[FR]

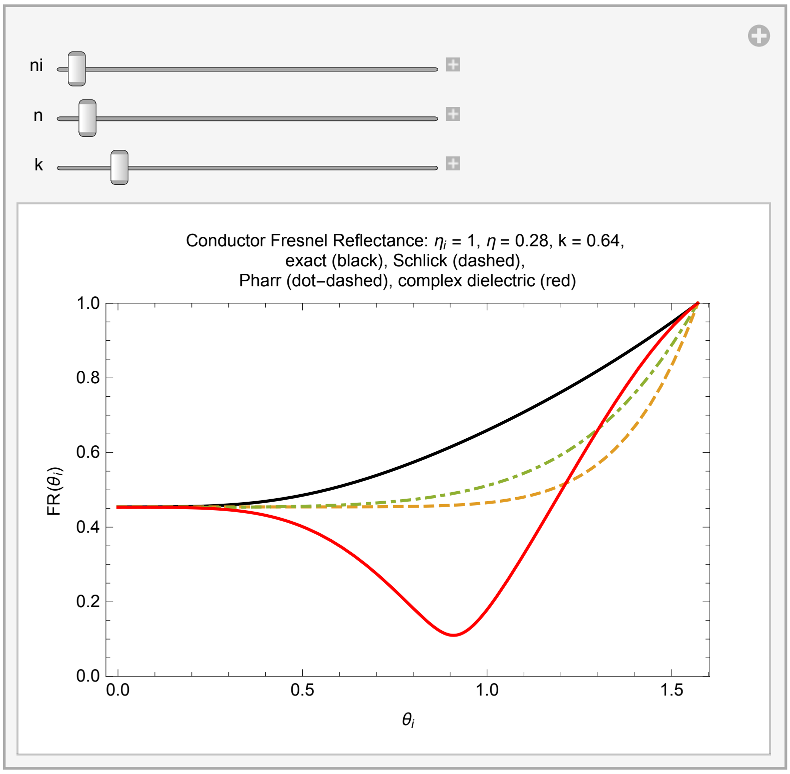
Remove the conditional so that this works with complex numbers:

In[79]:=
$$\mathrm{FR2}[etaratio\_,\,costhetai\_] := \mathrm{evalF}\left[\sqrt{etaratio^2-1+costhetai^2},\,costhetai\right]$$

Conductor Fresnel Reflectance Comparison

In[103]:=
```
Manipulate[
 condFRplot = Show[
   Plot[{
     FR[ni, n, k, t],
     FRSchlickFresnel[ni, n, k, t],
     FRPharrHumphreys[n, k, t],
     Abs[FR2[(n + I k), Cos[t]]]
    }, {t, 0, Pi/2}, PlotRange → {0, 1},
    PlotStyle → {Black, Dashed, DotDashed, Red}], Frame → True,
   FrameLabel -> {{FR[θi],},
     {θi, "Conductor Fresnel Reflectance: ηi = 1, η = " <>
       ToString[n] <> ", k = " <> ToString[k] <>
       ", \nexact (black), Schlick (dashed), \nPharr (dot-dashed), complex
        dielectric (red)"}}], {ni, 1, 2}, {n, 0.1, 5}, {k, 0, 5}]
```

Out[103]=



Benchmark data

In[81]:=
```
ns = {1.01, 1.1, 1.4, 2, 10};
ni = 1;
k = 0.5;
FRdata1 = Table[FR[ni, n, k, t], {n, ns}, {t, {0., 0.2, 0.5, 1., 1.2, 1.5}}];
k = 5;
FRdata2 = Table[FR[ni, n, k, t], {n, ns}, {t, {0., 0.2, 0.5, 1., 1.2, 1.5}}];
Join[Transpose[Join[{Table[n, {n, ns}]}, Transpose[FRdata1]]],
  Transpose[Join[{Table[n, {n, ns}]}, Transpose[FRdata2]]]] // Grid
```

Out[87]=

1.01	0.058297	0.0583739	0.0617829	0.143398	0.270677	0.771216
1.1	0.055794	0.0558562	0.0586152	0.127731	0.243942	0.751578
1.4	0.0682196	0.0682653	0.070265	0.121631	0.216441	0.716803
2	0.135135	0.135172	0.136733	0.174903	0.24661	0.694004
10	0.670103	0.67007	0.66871	0.640179	0.594733	0.500498
1.01	0.860882	0.860862	0.86007	0.845425	0.827007	0.880432
1.1	0.850391	0.85037	0.849536	0.834112	0.814725	0.871181
1.4	0.817945	0.817922	0.816973	0.799379	0.777241	0.841838
2	0.764706	0.764679	0.763584	0.7431	0.716979	0.789545
10	0.726027	0.725995	0.724679	0.696901	0.651431	0.522924