

Half Rod, Albedo Problem, Isotropic Scattering

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

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Path Setup

Put a file at `~/hitchhikerpath` with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

```
In[19]:= SetDirectory[Import["~/hitchhikerpath"]]
```

Exponential Random Flight

Notation

α - single-scattering albedo

Σt - extinction coefficient

x - position coordinate in rod (source at $x = 0$)

Analytic solutions

Half rod reflectance/albedo (R)

```
In[20]:= Clear[α, g]; R[α_] := 2  $\frac{\left(\frac{-\alpha}{2} - \sqrt{1-\alpha} + 1\right)}{\alpha}$ 
```

```
In[21]:= Series[R[α], {α, 0, 5}]
```

```
Out[21]=  $\frac{\alpha}{4} + \frac{\alpha^2}{8} + \frac{5\alpha^3}{64} + \frac{7\alpha^4}{128} + \frac{21\alpha^5}{512} + O[\alpha]^6$ 
```

```
In[22]:= R[α_, n_] := α^n (SeriesCoefficient[R[A], {A, 0, n}] /. A -> α)
```

Internal distribution, 'radiance'

```
In[23]:= LR[x_, α_, Σt_] :=  $e^{-\sqrt{1-\alpha} \Sigma t x}$ 
```

```
In[24]:= LL[x_, α_, Σt_] :=  $\frac{\alpha \text{Exp}[-\Sigma t \sqrt{1-\alpha} x]}{-\alpha + 2 \sqrt{1-\alpha} + 2}$ 
```

Fluence

```
In[25]:=  $\phi[x_, \alpha_, \Sigma t_] := LR[x, \alpha, \Sigma t] + LL[x, \alpha, \Sigma t]$ 
```

n-th collided fluence

```
In[26]:=  $\phi[x_, \alpha_, \Sigma t_, n_] := \alpha^n \left( \text{SeriesCoefficient}[\phi[x, \mathbf{A}, \Sigma t], \{\mathbf{A}, 0, n\}] /. \mathbf{A} \rightarrow \alpha \right)$ 
```

Moments

```
In[27]:=  $\phi m[\alpha_, \Sigma t_, k_] := \frac{2 (1 - \alpha)^{-1 - \frac{k}{2}} \left( -1 + \sqrt{1 - \alpha} + \alpha \right) \Sigma t^{-1 - k} \text{Gamma}[1 + k]}{\alpha}$ 
```

Only accurate for n even

```
In[28]:=  $\phi m[\alpha_, \Sigma t_, k_, n_] := \alpha^n \left( 2 (-1)^n \Sigma t^{-1 - k} \text{Gamma}[1 + k] \left( - (2 + n) \text{Gamma}\left[\frac{1 - k}{2}\right] - \right. \right. \\ \left. \left( (k + n) \text{Binomial}\left[-\frac{k}{2}, 1 + n\right] + 2 (2 + n) \text{Binomial}\left[-\frac{k}{2}, 2 + n\right] \right) \right. \\ \left. \left. \text{Gamma}\left[-\frac{1}{2} - \frac{k}{2} - n\right] \text{Gamma}[2 + n] \right) \right) / \left( \text{Gamma}\left[-\frac{1}{2} - \frac{k}{2} - n\right] \text{Gamma}[3 + n] \right)$ 
```

load MC data

```
In[29]:= ppoints[xs_, dx_, maxx_,  $\Sigma t$ _] :=
  Table[{dx (i - 1) + 0.5 dx, (1 /  $\Sigma t$ ) xs[[i]]}, {i, 1, Length[xs]}][[1 ;; -2]]

In[30]:= fs = FileNames[
  "code/rod/halfrod/albedoProblem/data/halfrod_albedoProblem_isotropicscatter_
  _exp*"];

In[31]:= index[x_] := Module[{data,  $\alpha$ ,  $\Sigma t$ },
  data = Import[x, "Table"];
   $\Sigma t$  = data[[1, 11]];
   $\alpha$  = data[[2, 3]];
  { $\alpha$ ,  $\Sigma t$ , data};
  simulations = index /@ fs;

In[33]:= alphas = Union[#[[1]] & /@ simulations]

Out[33]= {0.1, 0.3, 0.5, 0.7, 0.9, 0.95, 0.98, 0.99, 0.999}

In[34]:= muts = Union[#[[2]] & /@ simulations]

Out[34]= {1, 3}
```

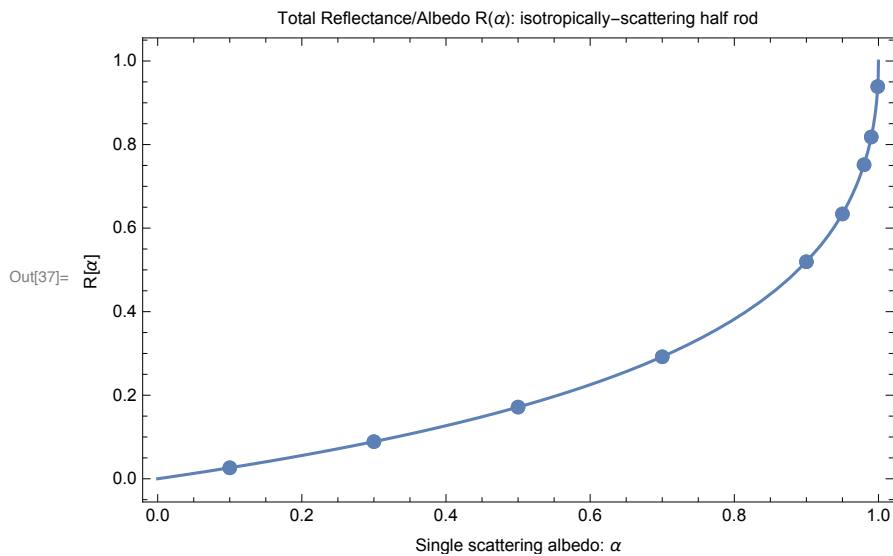
Halfrod Albedo

```
In[35]:= MCalbedo[f_] := Module[{data,  $\alpha$ },
  data = Import[f, "Table"];
   $\alpha$  = data[[2, 3]];
  { $\alpha$ , data[[3, 3]]}
]
```

```
In[36]:= MCalbedos = Table[MCalbedo[f], {f, fs}]
```

```
Out[36]= {{0.1, 0.0262874}, {0.1, 0.0262874}, {0.3, 0.0888128},
          {0.3, 0.0888128}, {0.5, 0.17156}, {0.5, 0.17156}, {0.7, 0.291991},
          {0.7, 0.291991}, {0.95, 0.633904}, {0.95, 0.633904}, {0.98, 0.751703},
          {0.98, 0.751703}, {0.999, 0.938793}, {0.999, 0.938793},
          {0.99, 0.818082}, {0.99, 0.818082}, {0.9, 0.519448}, {0.9, 0.519448}}
```

```
In[37]:= Clear[α]; vizrodalbedoiso = Show[
  Plot[R[c], {c, 0, 1}],
  ListPlot[MCalbedos]
, Frame → True, FrameLabel → {{R[α]}, {"Single scattering albedo: α",
  "Total Reflectance/Albedo R(α): isotropically-scattering half rod"}}
]
```



Compare Deterministic and MC

```

In[38]:= Clear[alpha, Σt];
Manipulate[
  If[Length[simulations] > 0,
    data = SelectFirst[simulations, #[[1]] == α && #[[2]] == Σt &][[3]];
    maxx = data[[2, 5]];
    dx = data[[2, 7]];
    numcollorders = data[[2, 11]];
    nummoments = data[[2, 13]];

    densmom = data[[11]];

    pointsCL = data[[7]];
    (* divide by Σt to convert collision density into L *)
    plotpointsCL = ppoints[pointsCL, dx, maxx, Σt];
    pointsCR = data[[9]];
    plotpointsCR = ppoints[pointsCR, dx, maxx, Σt];
    (* divide by Σt to convert collision density into fluence *)
    plotpointsφ = ppoints[pointsCL + pointsCR, dx, maxx, Σt];

    plotφ = Show[
      ListPlot[plotpointsφ, PlotRange → All, PlotStyle → PointSize[.01]],
      Plot[φ[x, α, Σt], {x, 0, maxx}, PlotRange → All],
      , Frame → True,
      FrameLabel -> {{φ[x]},},
      {x, "Semi-infinite rod, albedo problem, isotropic scattering, fluence
        φ[x], α = "<>ToString[α]<>" , Σt = "<>ToString[Σt]}}
    ];

    plotLL = Show[
      ListPlot[plotpointsCL, PlotRange → All, PlotStyle → PointSize[.01]],
      Plot[LL[x, α, Σt], {x, 0, maxx}, PlotRange → All],
      Plot[φ[x, α, Σt], {x, 0, maxx}, PlotRange → All, PlotStyle → Dashed],
      , Frame → True,
      FrameLabel -> {{L[x]},},
      {x, "Semi-infinite rod, albedo problem, isotropic scattering, L[x], α =
        "<>ToString[α]<>" , Σt = "<>ToString[Σt]}} , PlotRange → All
    ];

    plotLR = Show[
      ListPlot[plotpointsCR, PlotRange → All, PlotStyle → PointSize[.01]],
      Plot[LR[x, α, Σt], {x, 0, maxx}, PlotRange → All],
      Plot[φ[x, α, Σt], {x, 0, maxx}, PlotRange → All, PlotStyle → Dashed],
      , Frame → True,
      FrameLabel -> {{L+[x]},},
      {x, "Semi-infinite rod, albedo problem, isotropic scattering, L+[x], α =
        "<>ToString[α]<>" , Σt = "<>ToString[Σt]}} , PlotRange → All
    ];

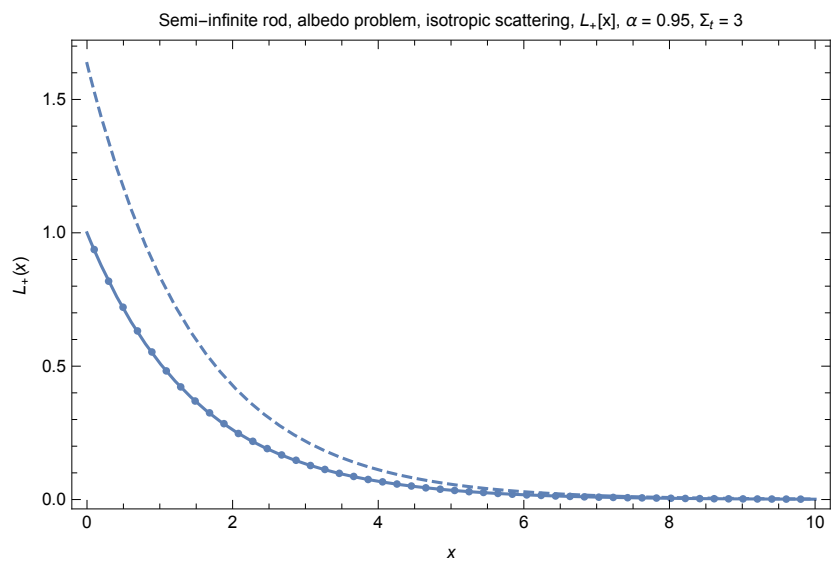
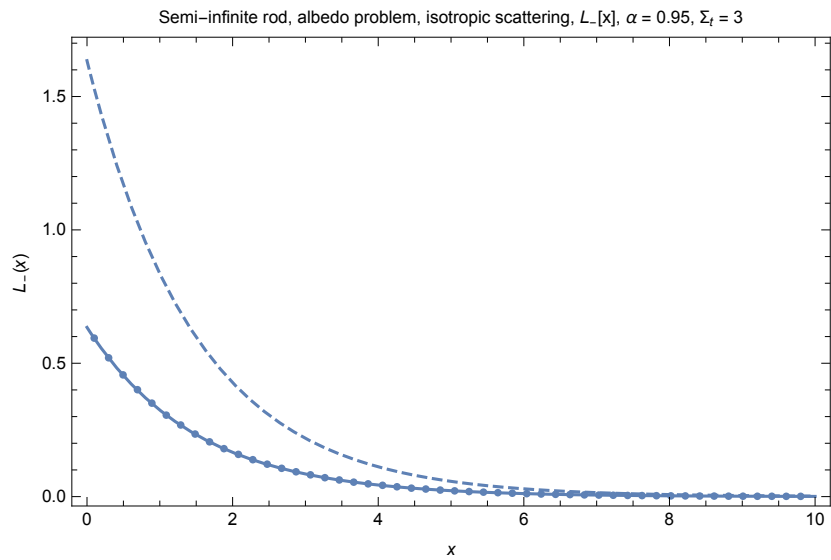
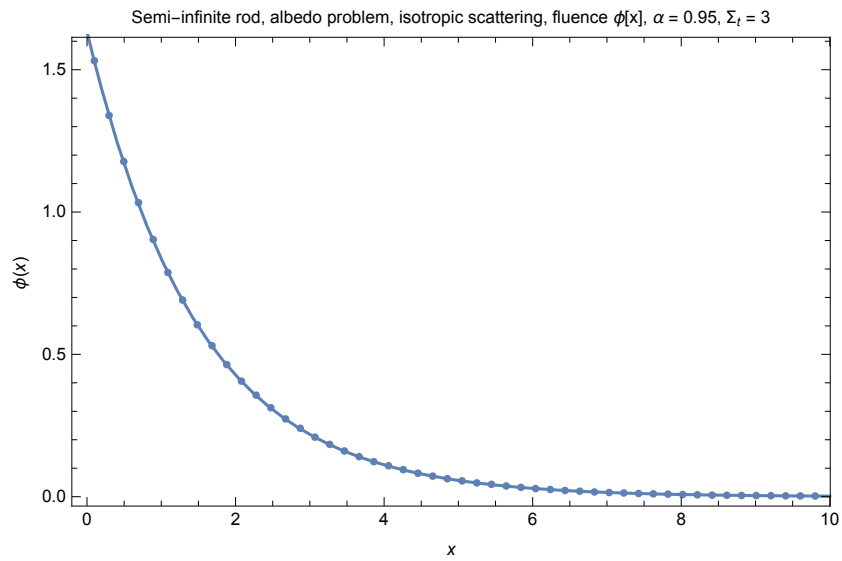
    Show[GraphicsGrid[{{plotφ}, {plotLL}, {plotLR}}], ImageSize → 500]
  ,
  Text[
    "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
    ensure the data path is setup correctly."
  ]
], {α, alphas}, {Σt, muts}]

```

α 0.95



Σt 1 3



Out[39]=

n-th collided albedo

```

In[40]:= Manipulate[
  If[Length[simulations] > 0,
    data = SelectFirst[simulations, #[[1]] ==  $\alpha$  && #[[2]] ==  $\Sigma t$  &][[3]];
    Rs = N[{data[[5]]}];
    ns = Table[n, {n, 0, numcollorders - 1}];
    analytic = Table[R[ $\alpha$ , n], {n, ns}];
    j = Join[{ns}, {analytic}, Rs];
    TableForm[
      Join[{"n", "analytic", "MC"}, Transpose[j]]
    ]
  ,
  Text[
    "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
    ensure the data path is setup correctly."
  ]
, { $\alpha$ , alphas}, { $\Sigma t$ , muts}]

```

Out[40]=

α	0.98	V
Σt	1	3
n	analytic	MC
0	0.	0.
1	0.245	0.244497
2	0.12005	0.120324
3	0.0735306	0.074076
4	0.050442	0.050198
5	0.0370749	0.037007
6	0.0285477	0.028276
7	0.0227311	0.022734
8	0.0185637	0.018692
9	0.0154636	0.015586
10	0.0130878	0.013021
11	0.0112228	0.011227
12	0.0097293	0.00972
13	0.00851313	0.008508
14	0.00750858	0.007559
15	0.00666856	0.006809
16	0.00595856	0.005858
17	0.00535277	0.005461
18	0.00483158	0.004784
19	0.00437983	0.004334

```

In[41]:= Manipulate[
  If[Length[simulations] > 0,
    data = SelectFirst[simulations, #[[1]] ==  $\alpha$  && #[[2]] ==  $\Sigma t$  &][[3]];
     $\phi$ moments = N[ $\frac{\text{data}[[13+n]]}{\Sigma t}$ ];
    ks = {Table[k, {k, 0, nummoments - 1}]};
    analytic = Table[Quiet[N[ $\phi$ m[ $\alpha$ ,  $\Sigma t$ , k, n]]], {k, ks}];
    j = Join[ks, analytic,  $\phi$ moments];
    TableForm[
      Join[{"k", "analytic", "MC"}, Transpose[j]]
    ],
    Text[
      "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
        ensure the data path is setup correctly."
    ]
  ],
  { $\alpha$ , alphas}, { $\Sigma t$ , muts},
  {n, Range[If[NumberQ[numcollorders], numcollorders, 1]]}
]

```

Out[41]=

α	0.9	V
Σt	1	3
n	3	V
k	analytic	MC
0	0.132891	0.132926
1	Indeterminate	0.117654
2	0.157781	0.157646
3	Indeterminate	0.273881
4	0.579375	0.57829
5	Indeterminate	1.42974
6	4.06125	4.0369
7	Indeterminate	12.7794
8	45.6167	44.6977
9	Indeterminate	170.629

Compare moments of ϕ

Divide these results, which are collision density moments, by Σt to produce radiance/fluence moments:

```

In[42]:= Manipulate[
  If[Length[simulations] > 0,
    data = SelectFirst[simulations, #[[1]] ==  $\alpha$  && #[[2]] ==  $\Sigma t$  &][[3]];
     $\phi$ moments = N[ $\frac{\{data[[1]]\}}{\Sigma t}$ ];
    ks = {Table[k, {k, 0, nummoments - 1}]};
    analytic = Table[ $\phi m[\alpha, \Sigma t, k]$ , {k, ks}];
    j = Join[ks, analytic,  $\phi$ moments];
    TableForm[
      Join[{"k", "analytic", "MC"}, Transpose[j]]
    ],
    Text[
      "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
        ensure the data path is setup correctly."
    ]
  ], { $\alpha$ , alphas}, { $\Sigma t$ , muts}]

```

Out[42]=

α	0.9	V
Σt	1	3
k	analytic	MC
0	1.60169	1.60147
1	1.68833	1.68786
2	3.5593	3.55933
3	11.2555	11.2588
4	47.4574	47.4567
5	250.122	249.805
6	1581.91	1576.1
7	11 672.4	11 583.1
8	98 430.2	97 005.
9	933 790.	908 873.

N-th order Radiance/Angular flux

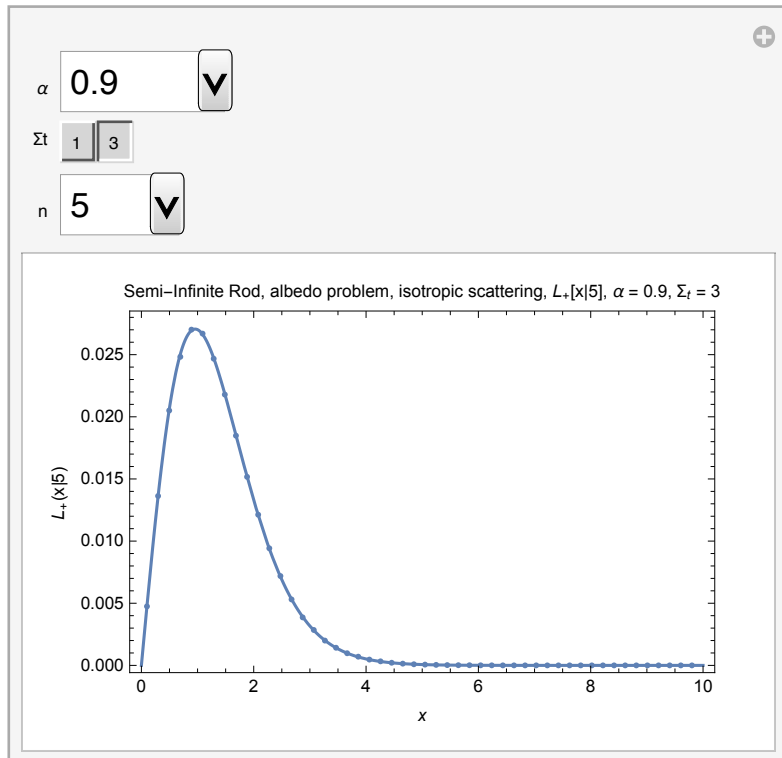
```

In[43]:= Manipulate[
  If[Length[simulations] > 0,
    data = SelectFirst[simulations, #[[1]] ==  $\alpha$  && #[[2]] ==  $\Sigma_t$  &][[3]];
    nthL = data[[13 + numcollorders + 1 ;; 13 + 2 numcollorders]];
    nthR = data[[13 + 2 numcollorders + 2 ;; -1]];

    Clear[c];
    LnR = FullSimplify[SeriesCoefficient[LR[x, c,  $\Sigma_t$ ], {c, 0, n}]  $\alpha^n$ ];
    Show[
      ListPlot[ppoints[nthR[[n + 1]], dx, maxx,  $\Sigma_t$ ],
        PlotRange → All, PlotStyle → PointSize[.01]],
      Plot[LnR, {x, 0, maxx}, PlotRange → All]
    , Frame → True,
      FrameLabel -> {{L+["x"] <> ToString[n]}, {x, "Semi-Infinite Rod, albedo problem, isotropic scattering, L+["x"] <> ToString[n] <> ",  $\alpha$  = " <> ToString[ $\alpha$ ] <> ",  $\Sigma_t$  = " <> ToString[ $\Sigma_t$ ]}}}, PlotRange → All
    ],
    Text[
      "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and ensure the data path is setup correctly."
    ]
  ],
  { $\alpha$ , alphas}, { $\Sigma_t$ , muts},
  {n, Range[If[NumberQ[numcollorders], numcollorders, 1]]}]

```

Out[43]=



N-th order Fluence / scalar flux

```

In[44]:= Manipulate[
  If[Length[simulations] > 0,
    data = SelectFirst[simulations, #[[1]] ==  $\alpha$  && #[[2]] ==  $\Sigma_t$  &][[3]];
    nthL = data[[13 + numcollorders + 1 ;; 13 + 2 numcollorders]];
    nthR = data[[13 + 2 numcollorders + 2 ;; -1]];

    Clear[c];
     $\phi_n$  = FullSimplify[SeriesCoefficient[ $\phi[x, c, \Sigma_t]$ , {c, 0, n}]  $\alpha^n$ ];
    Show[
      ListPlot[ppoints[nthR[[n + 1]] + nthL[[n + 1]], dx, maxx,  $\Sigma_t$ ,
        PlotRange → All, PlotStyle → PointSize[.01]],
      Plot[ $\phi_n$ , {x, 0, maxx}, PlotRange → All]
      , Frame → True,
      FrameLabel -> {{L+[x] <> ToString[n]}, {x, "Semi-Infinite Rod, albedo problem, isotropic scattering,  $\phi[x]$ " <>
        ToString[n] <> "},  $\alpha$  = " <> ToString[ $\alpha$ ] <>
        ",  $\Sigma_t$  = " <> ToString[ $\Sigma_t$ ]}}}, PlotRange → All
    ],
    Text[
      "Uh oh! Couldn't find MC data. Try to evaluate this entire notebook and
        ensure the data path is setup correctly."
    ]
  ],
  { $\alpha$ , alphas}, { $\Sigma_t$ , muts},
  {n, Range[If[NumberQ[numcollorders], numcollorders, 1]]}]

```

Out[44]=

