Infinite 3D medium, Isotropic Plane Source, Isotropic Scattering

Exponential Random Flight

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

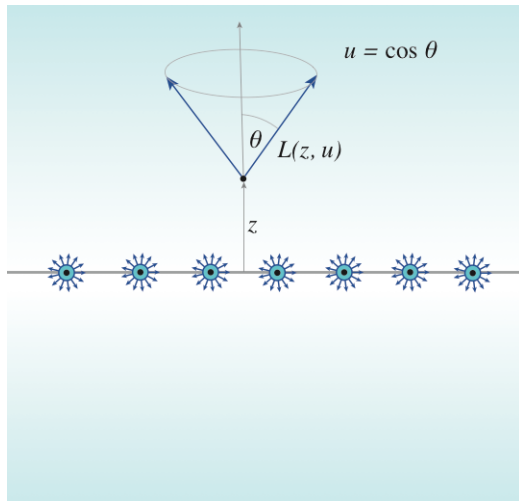## Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

In[4]:= **SetDirectory[Import["~/.hitchhikerpath"]]**

Notation

α - single-scattering albedo

Σt - extinction coefficient

z - scalar position coordinate in medium (distance from plane source at origin)

u = cos θ - direction cosine

Analytic solutions

Caseology quantities

In[5]:= $\texttt{CaseN0[c\_, v0\_]} := \frac{1}{2} \texttt{c v0}^3 \left(\frac{c}{\texttt{v0}^2 - 1} - \frac{1}{\texttt{v0}^2} \right)$

In[6]:= $\texttt{Casev0[c\_?NumericQ]} :=$
$\quad \texttt{FindRoot}\left[\texttt{c v ArcTanh}\left[\frac{1}{v}\right] - 1 == 0, \{v, 1.00000000001, 10^{10}\}, \texttt{Method} \to \texttt{"Brent"}\right][[1]][[2]]$

In[7]:= $\texttt{CaseN[c\_, v\_]} := v \left(\texttt{Case}\lambda[\texttt{v, c}]^2 + \left(\frac{\pi \texttt{ c v}}{2}\right)^2 \right)$

In[8]:= $\texttt{Case}\lambda\texttt{[v\_, c\_]} := 1 - \texttt{c v ArcTanh[v]}$

In[9]:= $\texttt{Case}\psi\texttt{0[u\_, v0\_, c\_, z\_]} := \frac{c}{2} \frac{\texttt{v0}}{\texttt{v0} - \texttt{Sign[z] u}}$

Rigorous diffusion approximation

In[10]:= $\texttt{inf3Disoplaneisoscatter`}\phi\texttt{rigourousDiffusion[z\_, \Sigma t\_, }\alpha\_] :=$
$\quad \frac{1}{2} \frac{\texttt{E}^{-\texttt{Abs[z] \Sigma t/\#}}}{\texttt{CaseN0}[\alpha, \#]} \texttt{ \&[Casev0[}\alpha\texttt{]]}$

Fluence: exact solution

Fourier Transform:

In[191]:= $\texttt{inf3Disoplaneisoscatter`}\phi\texttt{unscattered[z\_, \Sigma t\_]} := \frac{\texttt{Gamma[0, Abs[z \Sigma t]]}}{2}$

In[192]:= $\texttt{inf3Disoplaneisoscatter`}\phi\texttt{exact1[z\_, \Sigma t\_, }\alpha\_] :=$
$\quad \texttt{inf3Disoplaneisoscatter`}\phi\texttt{unscattered[z, \Sigma t]} +$
$\quad \texttt{NIntegrate}\left[\frac{\alpha \texttt{ ArcTan[k]}^2 \texttt{ Cos[k z \Sigma t]}}{\pi \left(k^2 - \alpha \texttt{ k ArcTan[k]}\right)}, \{k, 0, \texttt{Infinity}\}\right]$

[Case 1960, Case and Zwiefel 1967]

In[193]:= $\texttt{inf3Disoplaneisoscatter`}\phi\texttt{exact2[z\_, \Sigma t\_, }\alpha\_] :=$
$\quad \texttt{inf3Disoplaneisoscatter`}\phi\texttt{rigourousDiffusion[z, \Sigma t, }\alpha\texttt{]} +$
$\quad \frac{1}{2} \texttt{NIntegrate}\left[\frac{\texttt{e}^{-\Sigma t \texttt{ Abs[z] /v}}}{\texttt{CaseN}[\alpha, \texttt{v}]}, \{v, 0, 1\}\right]$

Nth-scattered fluence

$\quad \texttt{inf3Disoplaneisoscatter`}\phi\texttt{exact1[x\_, \Sigma t\_, }\alpha\_, \texttt{n\_]} :=$
$\quad \frac{(\alpha \Sigma t)^n}{\pi} \texttt{NIntegrate}\left[\frac{\texttt{ArcTan}\left[\frac{z}{\Sigma t}\right]^{n+1} \texttt{ Cos[x z]}}{z^{n+1}}, \right.$
$\quad \left. \{z, 0, \texttt{Infinity}\}, \texttt{Method} \to \texttt{"ExtrapolatingOscillatory"}\right]$

In[13]:= `inf3Disoplaneisoscatter`ϕexact2[x_, Σt_, α_, n_] := $\frac{\alpha^n}{\pi}$ Chop[NIntegrate[

$\dot{\mathbb{i}}$ 2^{-2-n} $e^{-\text{Abs}[x]\, z\, \Sigma t}$ $\left(\left(\frac{-\dot{\mathbb{i}}\,\pi + \text{Log}\left[\frac{z+1}{z-1}\right]}{z}\right)^{1+n} - \left(\frac{\dot{\mathbb{i}}\,\pi + \text{Log}\left[\frac{z+1}{z-1}\right]}{z}\right)^{1+n}\right)$, {z, 1, Infinity}]]

In[14]:= `inf3Disoplaneisoscatter`ϕGaussian[x_, Σt_, α_, n_] := $\frac{e^{-\frac{3\,x^2\,\Sigma t^2}{4\,(1+n)}}\sqrt{\frac{3}{\pi}}\,\alpha^n}{2\sqrt{1+n}}$

Classical Diffusion Approximation

In[15]:= `inf3Disoplaneisoscatter`ϕDiffusion[x_, Σt_, α_] := $\frac{3\,e^{-\text{Abs}[x]\sqrt{3-3\,\alpha}\,\Sigma t}}{2\sqrt{3-3\,\alpha}}$

Grosjean Modified Diffusion Approximation

In[16]:= `inf3Disoplaneisoscatter`ϕGrosjean[x_, Σt_, α_] :=

$-\frac{1}{2}$ ExpIntegralEi[-Abs[x] Σt] + $\frac{e^{-\text{Abs}[x]\sqrt{3+\frac{3}{-2+\alpha}}\,\Sigma t}\sqrt{3+\frac{3}{-2+\alpha}}\,\alpha}{2\,(1-\alpha)}$

Radiance (Angular Flux)

Caseology - asymptotic solution:

In[17]:= `inf3Disoplaneisoscatter`LrigourousDiffusion[z_, u_, Σt_, α_] :=
$\frac{1}{4\,\text{Pi}}$ Caseψ0[u, #, α, z] $\frac{E^{-\text{Abs}[z]\,\Sigma t/\#}}{\text{CaseN0}[\alpha, \#]}$ &[Casev0[α]]

Caseology - exact solution:

In[18]:= `inf3Disoplaneisoscatter`Lexact[z_, u_, Σt_, α_] :=
  `inf3Disoplaneisoscatter`LrigourousDiffusion[z, u, Σt, α] +

  $\frac{1}{4\,\text{Pi}}$ $\left(\right.$Caseλ[u, α] $\frac{e^{-\frac{\text{Abs}[z]\,\Sigma t}{u}}}{\text{CaseN}[\alpha, u]}$ HeavisideTheta[1 - u] HeavisideTheta[u]

   + NIntegrate$\left[\frac{e^{-\frac{\text{Abs}[z]\,\Sigma t}{v}}}{\text{CaseN}[\alpha, v]}\,\frac{\alpha}{2}\,\frac{v}{v - u}\right.$

   , {v, 0, u, 1}, Method $\rightarrow$ "PrincipalValue", PrecisionGoal $\rightarrow$ 5$\left.\right]$ $\left.\right)$

Fourier transform exact solution [Beach et al. 1959]:

In[140]:= `inf3Disoplaneisoscatter`FourierLunscattered[z_, u_, Σt_] :=
$\frac{e^{-\frac{z\,\Sigma t}{u}}}{4\,\text{Pi}\,u}$ Sign[z] HeavisideTheta[u Sign[z]]
  `inf3Disoplaneisoscatter`FourierLscattered[z_, c_, u_, Σt_] :=
$\frac{1}{4\,\text{Pi}}\,\frac{c}{\text{Pi}}$ NIntegrate$\left[\frac{\text{ArcTan}[k]\,(\text{Cos}[k\,z\,\Sigma t] + k\,u\,\text{Sin}[k\,z\,\Sigma t])}{(1 + k^2\,u^2)\,(k - c\,\text{ArcTan}[k])}, \{k, 0, \text{Infinity}\}\right]$

In[142]:= `inf3Disoplaneisoscatter`LexactFourier[z_, u_, Σt_, α_] :=
  `inf3Disoplaneisoscatter`FourierLunscattered[z, u, Σt] +
    `inf3Disoplaneisoscatter`FourierLscattered[z, α, u, Σt]

load MC data

```
In[21]:= inf3Disoplaneisoscatter`ppoints[zs_, dz_, maxz_, Σt_] :=
          Table[{dz (i) - 0.5 dz - maxz, zs[[i]] / Σt}, {i, 1, Length[zs]}][[2 ;; -2]]
```

```
In[22]:= inf3Disoplaneisoscatter`ppointsu[xs_, du_, Σt_] :=
          Table[{-1.0 + du (i) - 0.5 du, xs[[i]] / (2 Σt)}, {i, 1, Length[xs]}][[1 ;; -1]]
```

```
In[23]:= inf3Disoplaneisoscatter`fs =
          FileNames["code/3D_medium/infinite3Dmedium/Isotropicplanesource/data/
             inf3D_isotropicplane_isotropicscatter*"];
```

```
In[24]:= inf3Disoplaneisoscatter`index[x_] := Module[{data, α, Σt},
             data = Import[x, "Table"];
             Σt = data[[1, 13]];
             α = data[[2, 3]];
             {α, Σt, data}];
          inf3Disoplaneisoscatter`simulations =
            inf3Disoplaneisoscatter`index /@ inf3Disoplaneisoscatter`fs;
          inf3Disoplaneisoscatter`alphas =
          Union[#[[1]] & /@ inf3Disoplaneisoscatter`simulations]
```

```
Out[26]= {0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999}
```

```
In[27]:= inf3Disoplaneisoscatter`muts =
          Union[#[[2]] & /@ inf3Disoplaneisoscatter`simulations]
```

```
Out[27]= {1, 3}
```

```
In[28]:= inf3Disoplaneisoscatter`numcollorders =
             inf3Disoplaneisoscatter`simulations[[1]][[3]][[2, 13]];
          inf3Disoplaneisoscatter`maxz =
           inf3Disoplaneisoscatter`simulations[[1]][[3]][[2, 5]];
          inf3Disoplaneisoscatter`dz =
           inf3Disoplaneisoscatter`simulations[[1]][[3]][[2, 7]];
          inf3Disoplaneisoscatter`numz =
             Floor[2 inf3Disoplaneisoscatter`maxz / inf3Disoplaneisoscatter`dz];
```

Compare Deterministic and MC

Fluence - Exact solution 1 (Fourier Transform) comparison to MC

```
In[199]:= Manipulate[
    If[Length[inf3Disoplaneisoscatter`simulations] > 0,
     Module[{data, maxz, dz, points𝜙, plotpoints𝜙, logplot𝜙, plot𝜙, exact1points},
      data = SelectFirst[
         inf3Disoplaneisoscatter`simulations, #[[1]] == α && #[[2]] == Σt &][[3]];
      maxz = data[[2, 5]];
      dz = data[[2, 7]];

      points𝜙 = data[[4]];

      (* divide by Σt to convert collision density into fluence *)
      plotpoints𝜙 = inf3Disoplaneisoscatter`ppoints[points𝜙, dz, maxz, Σt];

      exact1points =
       Quiet[{#[[1]], inf3Disoplaneisoscatter`𝜙exact[#[[1]], Σt, α]}] & /@
        plotpoints𝜙;

      numpoints = Length[plotpoints𝜙];
      skip = Floor[numpoints 6/7 1/2];


      plot𝜙 = Quiet[Show[
         Plot[inf3Disoplaneisoscatter`𝜙exact1[z, Σt, α], {z, -maxz/7, maxz/7}],
         Plot[inf3Disoplaneisoscatter`𝜙unscattered[z, Σt],
          {z, -maxz, maxz}, PlotRange → All, PlotStyle → {Red, Dashed}],
         ListPlot[plotpoints𝜙[[skip ;; -skip]], PlotRange → All,
          PlotStyle → {Black, PointSize[.01]}],
         Frame → True,
         FrameLabel -> {{ 𝜙[z],}, {z,}}
        ]];
      logplot𝜙 = Quiet[Show[
         ListLogPlot[exact1points, PlotRange → All, Joined → True],
         ListLogPlot[plotpoints𝜙[[1 ;; -1 ;; 3]],
          PlotRange → All, PlotStyle → {Black, PointSize[.01]}],
         Frame → True,
         FrameLabel -> {{ 𝜙[z],}, {z,}}
        ]];
      Show[GraphicsGrid[{{plot𝜙, logplot𝜙}}, ImageSize → 800], PlotLabel ->
        "Exact solution 1 (thin), Unscattered component (dashed)\nInfinite
           3D, isotropic plane source, isotropic scattering, fluence
           𝜙[z], α = " <> ToString[α] <> ", Σt = " <> ToString[Σt]]
     ]
     ,
     Text[
      "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
        ensure the data path is setup correctly."]
    ]
    , {{α, 0.5}, inf3Disoplaneisoscatter`alphas},
    {{Σt, 1}, inf3Disoplaneisoscatter`muts}]
```
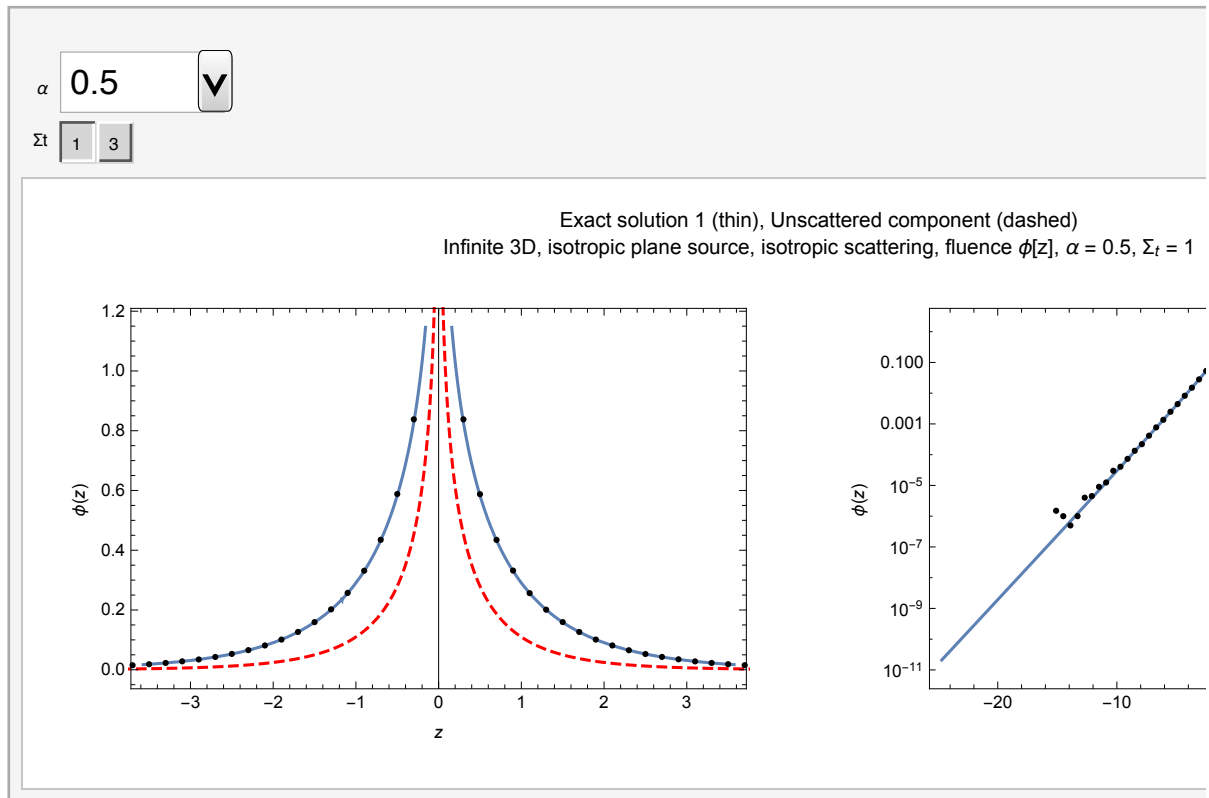
Exact solution 1 (thin), Unscattered component (dashed)
Infinite 3D, isotropic plane source, isotropic scattering, fluence $\phi[z]$, $\alpha = 0.5$, $\Sigma_t = 1$

Fluence - Exact solution 2 (Caseology) comparison to MC

```mathematica
In[252]:= Manipulate[
   If[Length[inf3Disoplaneisoscatter`simulations] > 0,
    Module[{data, maxz, dz, pointsϕ, plotpointsϕ, logplotϕ, plotϕ, exact1points},
      data = SelectFirst[
          inf3Disoplaneisoscatter`simulations, #[[1]] == α && #[[2]] == Σt &][[3]];
      maxz = data[[2, 5]];
      dz = data[[2, 7]];

      pointsϕ = data[[4]];

      (* divide by Σt to convert collision density into fluence *)
      plotpointsϕ = inf3Disoplaneisoscatter`ppoints[pointsϕ, dz, maxz, Σt];

      exact1points =
       Quiet[{#[[1]], inf3Disoplaneisoscatter`ϕexact2[#[[1]], Σt, α]}] & /@
         plotpointsϕ;

      numpoints = Length[plotpointsϕ];
      skip = Floor[numpoints 6/7 1/2];


      plotϕ = Quiet[Show[
          (*ListPlot[exact1points[[skip;;-skip]],PlotRange→{0,6},Joined→True],*)
          Plot[inf3Disoplaneisoscatter`ϕexact2[z, Σt, α],
           {z, - maxz/7, maxz/7}, PlotRange → {0, 6}],
          Plot[inf3Disoplaneisoscatter`ϕrigourousDiffusion[z, Σt, α],
           {z, -maxz, maxz}, PlotRange → All, PlotStyle → {Red, Dashed}],
          ListPlot[plotpointsϕ[[skip ;; -skip]], PlotRange → All,
           PlotStyle → {Black, PointSize[.01]}],
          Frame → True,
          FrameLabel -> {{ ϕ[z],}, {z,}}
         ]];
      logplotϕ = Quiet[Show[
          ListLogPlot[exact1points, PlotRange → All, Joined → True],
          LogPlot[inf3Disoplaneisoscatter`ϕrigourousDiffusion[z, Σt, α],
           {z, -maxz, maxz}, PlotRange → All, PlotStyle → {Red, Dashed}],
          ListLogPlot[plotpointsϕ[[1 ;; -1 ;; 3]], PlotRange → All,
           PlotStyle → {Black, PointSize[.01]}],
          Frame → True,
          FrameLabel -> {{ ϕ[z],}, {z,}}
         ]];
      Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 800],
       PlotLabel -> "Exact solution 2 (thin), Rigorous Diffusion
          component (dashed)\nInfinite 3D, isotropic plane
          source, isotropic scattering, fluence ϕ[z], α = " <>
         ToString[α] <> ", Σt = " <> ToString[Σt]]
    ]
    ,
    Text[
     "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
        ensure the data path is setup correctly."]
   ]
   , {{α, 0.95}, inf3Disoplaneisoscatter`alphas},
   {{Σt, 1}, inf3Disoplaneisoscatter`muts}]
```
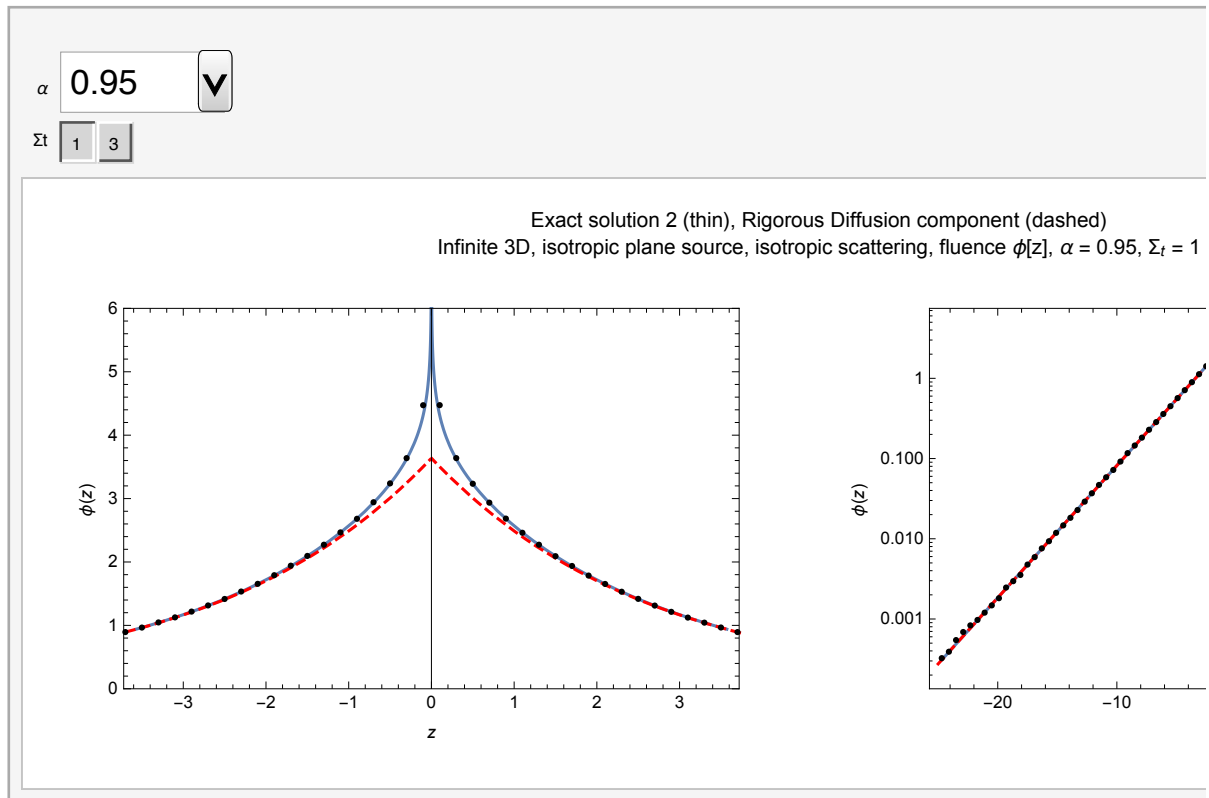
Out[252]=

α 0.95

Σt 1 3

Exact solution 2 (thin), Rigorous Diffusion component (dashed)
Infinite 3D, isotropic plane source, isotropic scattering, fluence $\phi[z]$, $\alpha = 0.95$, $\Sigma_t = 1$

Fluence - Diffusion approximations (Classical and Grosjean) comparison to MC

```
Manipulate[
 If[Length[inf3Disoplaneisoscatter`simulations] > 0,
  Module[{data, maxz, dz, pointsϕ,
     plotpointsϕ, logplotϕ, plotϕ, exact1points, numpoints, skip},
   data = SelectFirst[inf3Disoplaneisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxz = data[[2, 5]];
   dz = data[[2, 7]];

   pointsϕ = data[[4]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = inf3Disoplaneisoscatter`ppoints[pointsϕ, dz, maxz, Σt];

   numpoints = Length[plotpointsϕ];
   skip = Floor[numpoints 6/7 1/2];


   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ[[skip ;; -skip]],
       PlotRange → All, PlotStyle → {PointSize[.01], Black}],
      Plot[inf3Disoplaneisoscatter`ϕGrosjean[z, Σt, α],
       {z, -maxz, maxz}, PlotRange → All],
      Plot[inf3Disoplaneisoscatter`ϕDiffusion[z, Σt, α],
       {z, -maxz, maxz}, PlotRange → All, PlotStyle → {Red, Dashed}],
      ListPlot[plotpointsϕ[[skip ;; -skip]], PlotRange → All,
       PlotStyle → {PointSize[.01], Black}],
      Frame → True,
      FrameLabel -> {{ ϕ[z],}, {z,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ,
       PlotRange → All, PlotStyle → {PointSize[.01], Black}],
      LogPlot[inf3Disoplaneisoscatter`ϕGrosjean[z, Σt, α],
       {z, -maxz, maxz}, PlotRange → All],
      LogPlot[inf3Disoplaneisoscatter`ϕDiffusion[z, Σt, α],
       {z, -maxz, maxz}, PlotRange → All, PlotStyle → {Red, Dashed}],
      ListLogPlot[plotpointsϕ, PlotRange → All,
       PlotStyle → {PointSize[.01], Black}],
      Frame → True,
      FrameLabel -> {{ ϕ[z],}, {z,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 800],
    PlotLabel -> "Classical (dashed) and Grosjean Modified (thin)
        Diffusion Approximation\nInfinite 3D, isotropic plane
        source, isotropic scattering, fluence ϕ[z], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
  ]
  ,
  Text[
   "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
 ]
 , {{α, 0.5}, inf3Disoplaneisoscatter`alphas},
 {{Σt, 1}, inf3Disoplaneisoscatter`muts}]
```
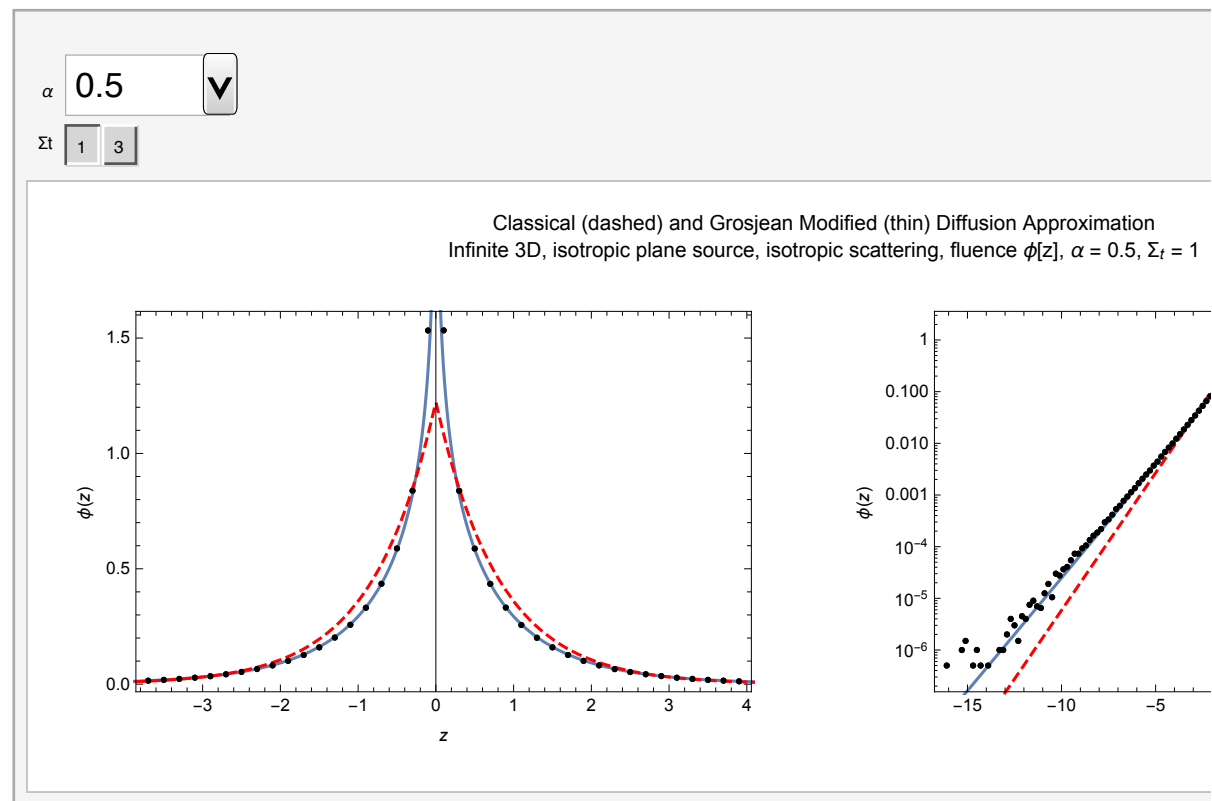
α 0.5   ⌄

Σt 1   3

Classical (dashed) and Grosjean Modified (thin) Diffusion Approximation
Infinite 3D, isotropic plane source, isotropic scattering, fluence $\phi[z]$, $\alpha = 0.5$, $\Sigma_t = 1$

N-th order fluence / scalar flux

N-th collided Fluence - Exact solution (1) comparison to MC

```
Manipulate[
 If[Length[inf3Disoplaneisoscatter`simulations] > 0,
  Module[{data, maxz, dz, pointsϕ,
    plotpointsϕ, logplotϕ, plotϕ, exact1points, numorders},
   data = SelectFirst[inf3Disoplaneisoscatter`simulations,
      #[[1]] == α && #[[2]] == Σt &][[3]];
   maxz = data[[2, 5]];
   dz = data[[2, 7]];

   numorders = data[[2, 13]];

   pointsϕ = data[[9 + numorders + n + 1]];

   (* divide by Σt to convert collision density into fluence *)
   plotpointsϕ = inf3Disoplaneisoscatter`ppoints[pointsϕ, dz, maxz, Σt];

   exact1points =
    Quiet[{#[[1]], inf3Disoplaneisoscatter`ϕexact1[#[[1]], Σt, α, n]}] & /@
     plotpointsϕ;

   plotϕ = Quiet[Show[
      ListPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{ ϕ["z|" <> ToString[n]],}, {z,}}
     ]];
   logplotϕ = Quiet[Show[
      ListLogPlot[plotpointsϕ, PlotRange → All, PlotStyle → PointSize[.01]],
      ListLogPlot[exact1points, PlotRange → All, Joined → True],
      Frame → True,
      FrameLabel -> {{ ϕ["z|" <> ToString[n]],}, {z,}}
     ]];
   Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 800],
    PlotLabel -> "Exact solution (1)\nInfinite 3D, isotropic plane source,
       isotropic scattering, n-th scattered fluence ϕ[z|n], α = " <>
      ToString[α] <> ", Σt = " <> ToString[Σt]]
  ]
  ,
  Text[
   "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
 ]
 , {{α, 0.99}, inf3Disoplaneisoscatter`alphas},
 {{Σt, 3}, inf3Disoplaneisoscatter`muts},
 {{n, 13}, Range[If[NumberQ[inf3Disoplaneisoscatter`numcollorders],
    inf3Disoplaneisoscatter`numcollorders, 1]]}

]
```

α 0.95 ∨

Σt [1] [3]

n 13 ∨

$Aborted

N-th collided Fluence - Exact solution (2) comparison to MC

```
Manipulate[
 If[Length[inf3Disoplaneisoscatter`simulations] > 0,
  Module[{data, maxz, dz, points𝜙,
     plotpoints𝜙, logplot𝜙, plot𝜙, exact1points, numorders},
   data = SelectFirst[inf3Disoplaneisoscatter`simulations,
       #[[1]] == α && #[[2]] == Σt &][[3]];
   maxz = data[[2, 5]];
   dz = data[[2, 7]];

   numorders = data[[2, 13]];

   points𝜙 = data[[9 + numorders + n + 1]];

   (* divide by Σt to convert collision density into fluence *)
   plotpoints𝜙 = inf3Disoplaneisoscatter`ppoints[points𝜙, dz, maxz, Σt];

   exact1points =
    Quiet[{#[[1]], inf3Disoplaneisoscatter`𝜙exact2[#[[1]], Σt, α, n]}] & /@
     plotpoints𝜙;

   plot𝜙 = Quiet[Show[
       ListPlot[plotpoints𝜙, PlotRange → All, PlotStyle → PointSize[.01]],
       ListPlot[exact1points, PlotRange → All, Joined → True],
       Frame → True,
       FrameLabel -> {{ 𝜙["z|" <> ToString[n]],}, {z,}}
      ]];
   logplot𝜙 = Quiet[Show[
       ListLogPlot[plotpoints𝜙, PlotRange → All, PlotStyle → PointSize[.01]],
       ListLogPlot[exact1points, PlotRange → All, Joined → True],
       Frame → True,
       FrameLabel -> {{ 𝜙["z|" <> ToString[n]],}, {z,}}
      ]];
   Show[GraphicsGrid[{{plot𝜙, logplot𝜙}}, ImageSize → 800],
    PlotLabel -> "Exact solution (2)\nInfinite 3D, isotropic plane source,
        isotropic scattering, n-th scattered fluence 𝜙[z|n], α = " <>
       ToString[α] <> ", Σt = " <> ToString[Σt]]
  ]
  ,
  Text[
   "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
     ensure the data path is setup correctly."]
 ]
 , {{α, 0.99}, inf3Disoplaneisoscatter`alphas},
 {{Σt, 3}, inf3Disoplaneisoscatter`muts},
 {{n, 13}, Range[If[NumberQ[inf3Disoplaneisoscatter`numcollorders],
    inf3Disoplaneisoscatter`numcollorders, 1]]}

 ]
```
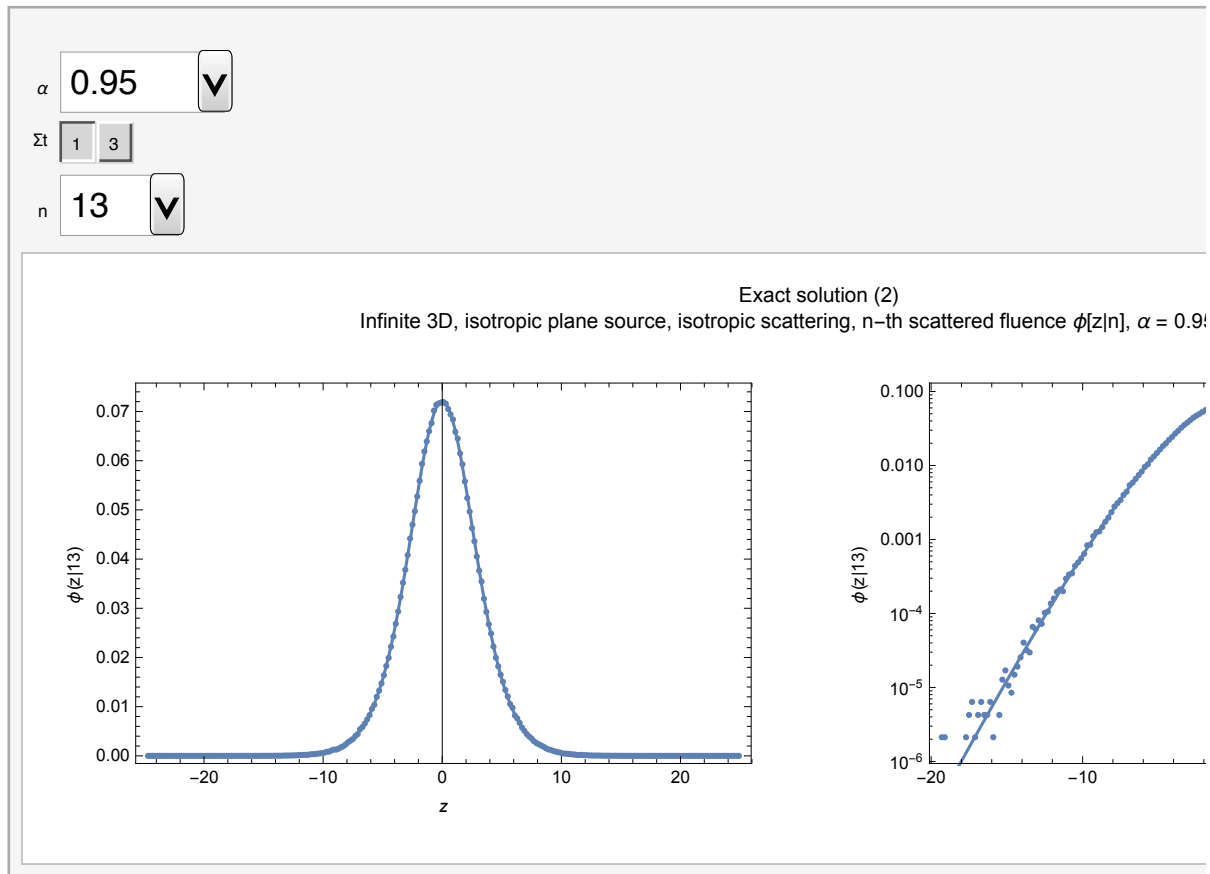
α 0.95 ⌄

Σt [1] [3]

n 13 ⌄

Exact solution (2)
Infinite 3D, isotropic plane source, isotropic scattering, n–th scattered fluence $\phi[z|n]$, α = 0.95

N-th collided Fluence - Approximations

```
Manipulate[
 If[Length[inf3Disoplaneisoscatter`simulations] > 0,
  Module[{data, maxz, dz, points$\phi$,
    plotpoints$\phi$, logplot$\phi$, plot$\phi$, exact1points, numorders},
   data = SelectFirst[inf3Disoplaneisoscatter`simulations,
       #[[1]] == $\alpha$ && #[[2]] == $\Sigma$t &][[3]];
   maxz = data[[2, 5]];
   dz = data[[2, 7]];
   numorders = data[[2, 13]];

   points$\phi$ = data[[9 + numorders + n + 1]];

   (* divide by $\Sigma$t to convert collision density into fluence *)
   plotpoints$\phi$ = inf3Disoplaneisoscatter`ppoints[points$\phi$, dz, maxz, $\Sigma$t];

   seriesclassical = $\alpha^n$ SeriesCoefficient[
     inf3Disoplaneisoscatter`$\phi$Diffusion[z, $\Sigma$t, C], {C, 0, n}];
   seriesG = $\alpha^n$ SeriesCoefficient[inf3Disoplaneisoscatter`$\phi$Grosjean[z, $\Sigma$t, C],
     {C, 0, n}];

   numpoints = Length[plotpoints$\phi$];
   skip = Floor[numpoints $\frac{6}{7}$ $\frac{1}{2}$];

   plot$\phi$ = Quiet[Show[
     Plot[inf3Disoplaneisoscatter`$\phi$Gaussian[z, $\Sigma$t, $\alpha$, n],
```

```mathematica
                  {z, - maxz/7, maxz/7}, PlotRange → All],
         Plot[seriesclassical, {z, - maxz/7, maxz/7}, PlotRange → All,
           PlotStyle → {Red, Dashed}],
         Plot[seriesG, {z, - maxz/7, maxz/7}, PlotRange → All,
           PlotStyle → {DotDashed, Purple}],
         ListPlot[plotpointsϕ[[skip ;; -skip]], PlotRange → All,
           PlotStyle → {Black, PointSize[.01]}],
         Frame → True,
         FrameLabel -> {{ ϕ["z|" <> ToString[n]],}, {z,}}
        ]];
     logplotϕ = Quiet[Show[
        LogPlot[inf3Disoplaneisoscatter`ϕGaussian[z, Σt, α, n],
          {z, -maxz, maxz}, PlotRange → {10^-8, 1}],
        LogPlot[seriesclassical, {z, -maxz, maxz},
          PlotRange → {10^-8, 1}, PlotStyle → {Red, Dashed}],
        LogPlot[seriesG, {z, -maxz, maxz}, PlotRange → {10^-8, 1},
          PlotStyle → {DotDashed, Purple}],
        ListLogPlot[plotpointsϕ, PlotRange → {10^-8, 1},
          PlotStyle → {Black, PointSize[.01]}],
        Frame → True,
        FrameLabel -> {{ ϕ["z|" <> ToString[n]],}, {z,}}
       ]];
     Show[GraphicsGrid[{{plotϕ, logplotϕ}}, ImageSize → 1000],
      PlotLabel -> "Approximate solutions, Gaussian (thin) Classical
          Diffusion (Dashed) Grosjean (Dot-Dashed)\nInfinite
          3D medium, isotropic planesource, isotropic
          scattering, n-th scattered fluence ϕ[z|" <>
        ToString[n] <> "], α = " <> ToString[α] <> ", Σt = " <> ToString[Σt]]
   ]
   ,
   Text[
     "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
       ensure the data path is setup correctly."]
  ]
  , {{α, 0.9}, inf3Disoplaneisoscatter`alphas},
  {Σt, inf3Disoplaneisoscatter`muts},
  {{n, 3}, Range[If[NumberQ[inf3Disoplaneisoscatter`numcollorders],
      inf3Disoplaneisoscatter`numcollorders, 1]]}]
```

Approximate solutions, Gaussian (thin) Classical Diffusion (Dashed)
Infinite 3D medium, isotropic planesource, isotropic scattering, n–th scatter
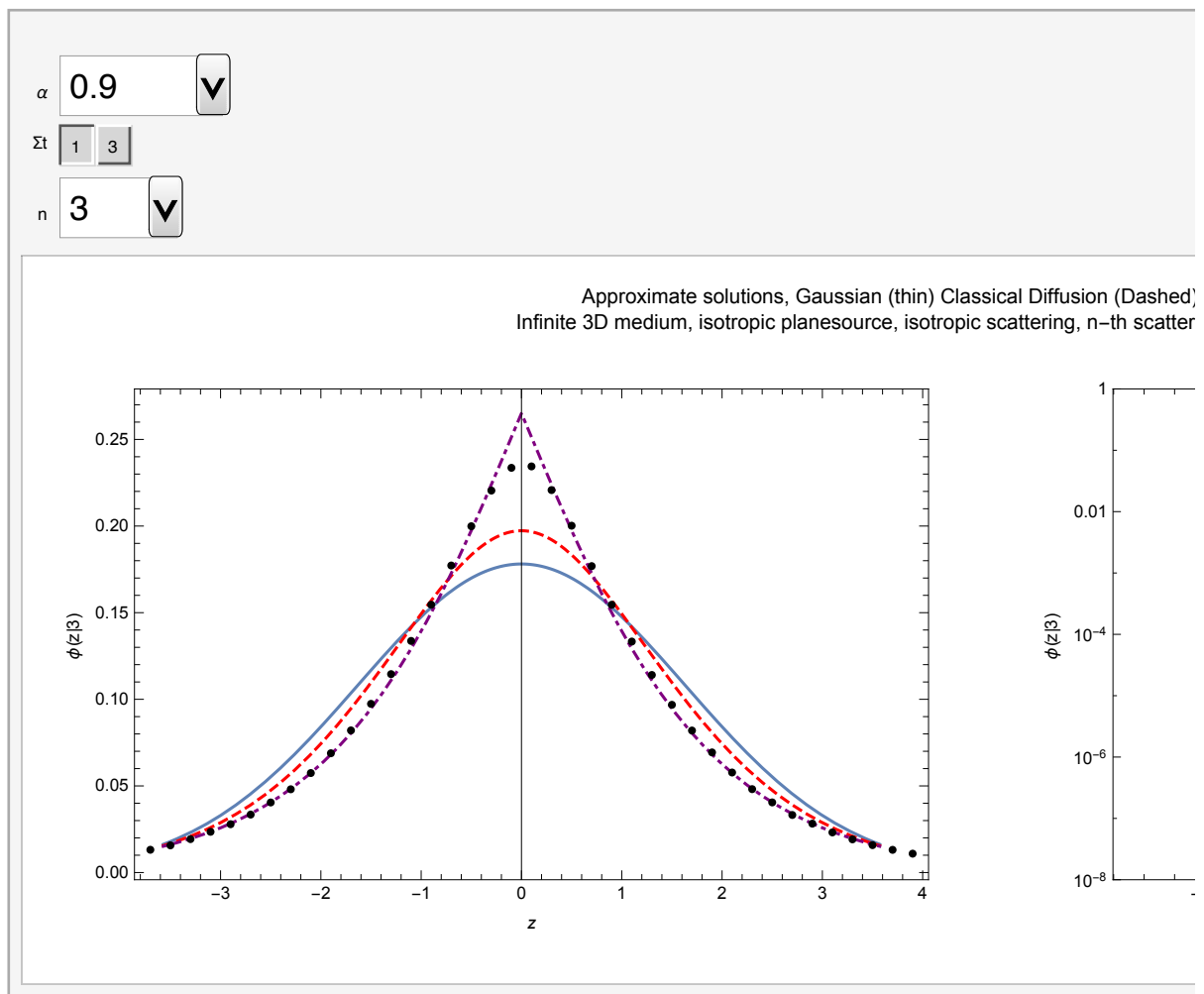
Angular Distributions

Exact solution (Caseology)

```mathematica
In[378]:= Manipulate[
     If[Length[inf3Disoplaneisoscatter`simulations] > 0,
      Module[{data, numorders, pointsu, plotpointsu, du, r, dz, maxz, zsim},
       data = SelectFirst[
           inf3Disoplaneisoscatter`simulations, #[[1]] == α && #[[2]] == Σt &][[3]];
       numorders = data[[2, 13]];
       du = data[[2, 9]];
       dz = data[[2, 7]];
       maxz = data[[2, 5]];

       pointsu = data[[9 + 2 numorders + n]];

       zsim = dz * n - 0.5 dz - maxz;

       plotpointsu = inf3Disoplaneisoscatter`ppointsu[pointsu, du, Σt];
       pp = Show[
         ListPlot[plotpointsu, PlotRange → All, PlotStyle → Black,
          Frame → True,
          FrameLabel -> {{ Pi L[zsim, u],}, {u,}}],
         Plot[Pi inf3Disoplaneisoscatter`LrigourousDiffusion[zsim, u, Σt, α],
          {u, -1, 1}, PlotStyle → {Red, Dashed}
          ],
         Plot[Pi inf3Disoplaneisoscatter`Lexact[zsim, u, Σt, α], {u, -1, 1}
          ],
         PlotLabel -> "Infinite 3D Medium, isotropic plane
             source, isotropic scattering\nAngular distribution:
             rigorous diffusion approximation\nπ L[z,u], z = " <>
          ToString[zsim] <> ", α = " <> ToString[α] <> ", Σt = " <> ToString[Σt]
         ]

       ],
      Text[
       "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
         ensure the data path is setup correctly."]
      ]
     , {{α, 0.9}, inf3Disoplaneisoscatter`alphas},
     {{Σt, 1}, inf3Disoplaneisoscatter`muts}, {{n, 127}, Range[If[
        NumberQ[inf3Disoplaneisoscatter`numz], inf3Disoplaneisoscatter`numz, 1]]}]
```
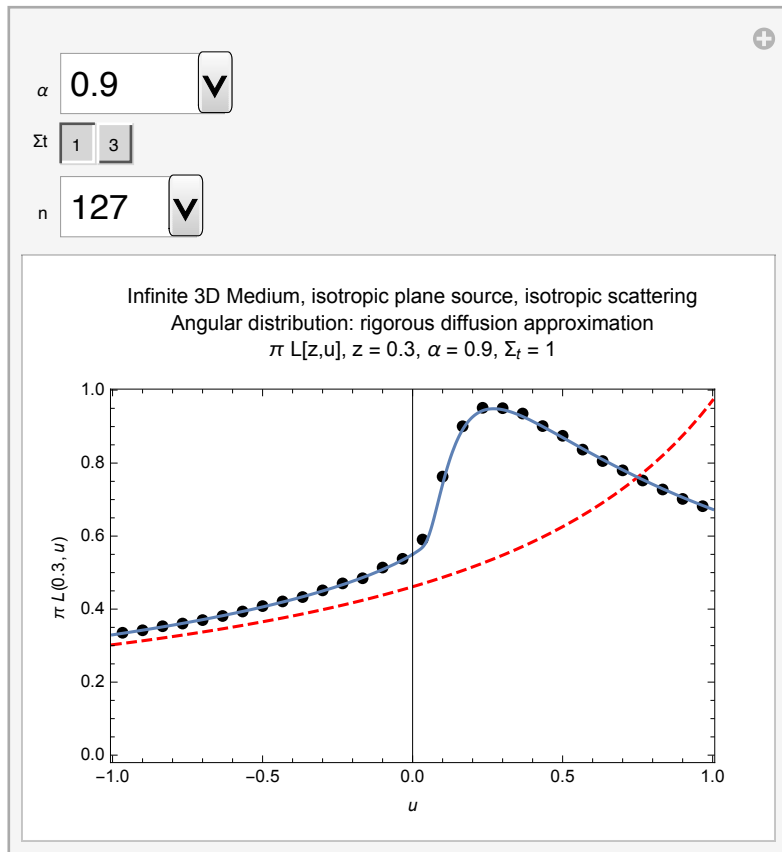
Out[378]=

Infinite 3D Medium, isotropic plane source, isotropic scattering
Angular distribution: rigorous diffusion approximation
π L[z,u], z = 0.3, α = 0.9, Σ_t = 1

Exact solution (Fourier Transform)

```
In[42]:= Manipulate[
      If[Length[inf3Disoplaneisoscatter`simulations] > 0,
       Module[{data, numorders, pointsu, plotpointsu, du, r, dz, maxz, zsim},
        data = SelectFirst[
           inf3Disoplaneisoscatter`simulations, #[[1]] == α && #[[2]] == Σt &][[3]];
        numorders = data[[2, 13]];
        du = data[[2, 9]];
        dz = data[[2, 7]];
        maxz = data[[2, 5]];

        pointsu = data[[9 + 2 numorders + n]];

        zsim = dz * n - 0.5 dz - maxz;

        plotpointsu = inf3Disoplaneisoscatter`ppointsu[pointsu, du, Σt];
        Show[
         ListPlot[plotpointsu, PlotRange → All, PlotStyle → Black,
          Frame → True,
          FrameLabel -> {{ Pi L[zsim, u],}, {u,}}],
         Plot[Pi inf3Disoplaneisoscatter`LexactFourier[zsim, u, Σt, α], {u, -1, 1}],
         PlotLabel -> "Infinite 3D Medium, isotropic plane source,
             isotropic scattering\nAngular distribution: Exact
             Fourier Transform solution\nπ L[z,u], z = " <>
           ToString[zsim] <> ", α = " <> ToString[α] <> ", Σt = " <> ToString[Σt]
        ]

       ],
      Text["Uh oh!  Couldn't find MC data.
          Try to evaluate this entire notebook and ensure the data path is setup
           correctly."]
     ]
     , {{α, 0.5}, inf3Disoplaneisoscatter`alphas},
     {{Σt, 3}, inf3Disoplaneisoscatter`muts}, {{n, 122}, Range[If[
        NumberQ[inf3Disoplaneisoscatter`numz], inf3Disoplaneisoscatter`numz, 1]]}]
```

α 0.5 ⌄

Σt 1 ‖ 3

n 121 ⌄

Out[42]=

Infinite 3D Medium, isotropic plane source, isotropic scattering
Angular distribution: Exact Fourier Transform solution
π L[z,u], z = −0.299993, α = 0.5, Σ_t = 3