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CE6305-501

Homework2

1. For this problem, we use  when  and  otherwise.

a) given in the problem.

b) 110 + 00101

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | 1 |  |  | 0 | 18 |
|  | 0 | 0 | 1 |  | 0 | 1 | 5 |
|  | 1 | 0 | 1 | 1 | 0 | 0 |  |
|  | 1 |  | 0 | 0 |  |  |  |
|  | 0 | 1 |  | 0 | 1 | 0 |  |
|  | 1 | 0 |  | 0 | 0 |  | 23 |

comment: The result is consistent with decimal arithmetic (18+5=23).

c) 100110 + 01110

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | 0 | 0 | 1 | 1 | 0 | 38 |
|  | 0 | 1 | 1 |  | 1 | 0 | 22 |
|  | 0 | 0 | 0 | 0 | 0 | 0 |  |
|  | 1 |  |  | 0 | 0 | 0 |  |
|  | 1 | 1 | 0 | 1 | 0 | 0 |  |
|  | (1)0 | 0 |  | 1 | 0 | 0 | -4 |

comment: The result is inconsistent with the decimal arithmetic (38+22=-4 instead of 60). However, notice that last column produces carry out (shown in parenthesis), which corresponds to value of 64. So the answer can be modified to be 60+(-4)=60, which does agree with decimal arithmetic.

2. Designing a unlimited carry-free system with radix = 3 and digit set 

a) finding the values for  and ,

Since , . Also , .

So,  is 2 and  is 3. (i.e. )

b) constructing the table for 

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Option 1 | | | Option 2 | | |  |
|  |  |  |  |  |  |  |  |  |
| -8 | [-10,-5] | [-4,1] | -2 | -2 |  |  |  | 11000 |
| -7 | [-9,-4] | [-3,2] | -1 | -2 |  |  |  | 11001 |
| -6 | [-8,-3] | [-2,3] | 0 | -2 |  |  |  | 11010 |
| -5 | [-7,-2] | [-1,4] | 1 | -2 | [-4,1] | -2 | -1 | 11011 |
| -4 | [-6,-1] | [0,5] | 2 | -2 | [-3,2] | -1 | -1 | 11100 |
| -3 | [-5,0] |  |  |  | [-2,3] | 0 | -1 | 11101 |
| -2 | [-4,1] | [-4,1] | -2 | 0 | [-1,4] | 1 | -1 | 11110 |
| -1 | [-3,2] | [-3,2] | -1 | 0 | [0,5] | 2 | -1 | 11111 |
| 0 | [-2,3] | [-2,3] | 0 | 0 |  |  |  | 00000 |
| 1 | [-1,4] | [-1,4] | 1 | 0 | [-4,1] | -2 | 1 | 00001 |
| 2 | [0,5] | [0,5] | 2 | 0 | [-3,2] | -1 | 1 | 00010 |
| 3 | [1,6] |  |  |  | [-2,3] | 0 | 1 | 00011 |
| 4 | [2,7] | [-4,1] | -2 | 2 | [-1,4] | 1 | 1 | 00100 |
| 5 | [3,8] | [-3,2] | -1 | 2 | [0,5] | 2 | 1 | 00101 |
| 6 | [4,9] | [-2,3] | 0 | 2 |  |  |  | 00110 |
| 7 | [5,10] | [-1,4] | 1 | 2 | [-4,1] | -2 | 3 | 00111 |
| 8 | [6,11] | [0,5] | 2 | 2 | [-3,2] | -1 | 3 | 01000 |
| 9 | [7,12] |  |  |  | [-2,3] | 0 | 3 | 01001 |
| 10 | [8,13] |  |  |  | [-1,4] | 1 | 3 | 01010 |

\* Chosen value of  is shown in red.

c) If there are eight digits in the system, we can find the range by looking at the lowest possible value and the greatest possible value.

First of all, the lowest possible value can be obtained when all digits are , which is -4.

So, ,where  means -4, has value of 

Then, finding the greatest value possible, which occurs when all digits are , the value will be. So the greatest value is .

The range of this system is [-13120, 16400]

d) 4 4 + 5 4

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  | 4 | 4 |  | -34 |
|  | 5 | 4 |  |  | 161 |
|  | 2 | 8 | 2 |  |  |
|  |  |  |  | 1 |  |
|  | 3 | 1 |  | 0 |  |
|  | (1)2 | 0 |  | 1 | 46 |

Comment: once again, this result is inconsistent with decimal arithmetic (-34+161=127≠46). However, notice that the most significant column produces carry out (shown in parenthesis), which corresponds to the value of . So if you include this carry out, the value becomes 81+46 = 127, which does agree with decimal arithmetic.

3. From,

a) 

b) 

c) 

d) 

-> Since none of the above has redundancy index  greater than 2 (or at least 2 with  and ) none of the systems can have carry-free addition system.