

# Decomposing Changes in Inequality and Welfare Between EU Regions: The Roles of Population Change, Re-Ranking and Income Growth

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**Abstract** This paper explores the changes in inequality and welfare between EU regions at the NUTS 3 level over the 2003–2011 period. Changes in absolute and relative inequalities are broken down into components explaining the effects of population change, re-ranking of regions and income growth between regional per capita incomes. Each component of inequality change is further decomposed by subgroup, revealing the contributions arising from changes within subgroups and from changes between subgroups. The decomposition of the change in absolute inequality is used to develop a decomposition of the change in welfare between EU regions.

**Keywords** Inequality · Decomposition · EU regions · Gini index · Welfare

## 1 Introduction

In recent years, the change in inequality between countries has received considerable attention in the literature on inequality measurement (Bosmans et al. 2014; Fredriksen 2012), while the trend in inequality between regions has been extensively investigated within countries because between-region inequality plays an important role in explaining overall inequality between individuals within a country (Kanbur and Venables 2005; Lessmann 2014). However, less attention has been paid to the analysis of the trend in inequality between regions in the same international organization (Piacentini 2014). In the EU, monitoring the evolution of inequality between EU regions is relevant to policy-making, since inclusive growth and disparity reduction are among the main objectives of EU regional policies and the EU allocates a substantial share of its total budget to regional

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policy objectives.<sup>1</sup> A major issue in exploring the change in inequality between EU regions is the detection of the roles played by the various causes of inequality change. A second issue concerns the choice of the concept of inequality used to measure changes between regions. Inequality can be measured in relative and absolute terms. This paper breaks down both the change in relative inequality and the change in absolute inequality between EU regions, explaining the roles of population change, re-ranking and per capita income growth. Moreover, using the link between the change in absolute inequality and the change in welfare, a decomposition of welfare change is developed.

The decomposition of the change in relative inequality over time was introduced by Jenkins and Van Kerm (2006) and applied to panel data on individual incomes in a given country. More specifically, the Jenkins and Van Kerm decomposition isolates two components of the change in the Gini index of relative inequality: one component due to the re-ranking of individuals in the income distribution, a second component caused by the differences in income growth between initially poorer and richer individuals when moving from an initial time to a final time. When measuring inequality between EU regions instead of between individuals in a country, one common approach consists of considering regional per capita incomes and weighting the per capita income of each region by its population size.<sup>2</sup> Since regional population sizes vary over time, the population change plays a role in explaining the change in inequality. Thus, decomposing the change in relative inequality between regions requires separating the components due to population change, re-ranking of regions, and different growth in per capita income between initially poorer and richer regions. This three-term decomposition of the change in relative inequality is obtained by using the Mussini and Grossi approach to the decomposition of the change in carbon dioxide emission inequality between countries (Mussini and Grossi 2015). Moreover, the Mussini and Grossi (2015) decomposition technique is adjusted in order to break down the change in absolute inequality. The change in the Gini index of absolute inequality is broken down into three components explaining the roles played by population change, re-ranking and changes in absolute income disparities between regions. The components of changes in absolute and relative inequalities are further broken down by subgroup, separating the contribution of changes within subgroups and the contribution of changes between subgroups.

The decomposition of the change in absolute inequality is used to break down a measure of welfare change suggested by Silber and Weber (2005). Silber and Weber (2005) measured the change in welfare on the basis of the change in the Gini index of absolute inequality, suggesting three alternative measures of welfare change. The first measure apparently neglects the effect of re-ranking between regions, while the other two measures are derived from the former by adding a component measuring re-ranking. We decompose the first measure of welfare change, showing that it implicitly includes the effect of re-ranking, along with those of population change and income growth. This decomposition

<sup>1</sup> The European Regional Development Fund, the Cohesion Fund and the European Social Fund provide the necessary investments to meet EU regional policies aimed at reducing economic and social disparities and promoting inclusive growth. Together these Funds will allocate almost a third of the total EU budget over the 2014–2020 period (The European Commission 2015).

<sup>2</sup> As noted by Milanovic (2006), the population-weighted approach is often used for measuring international inequality since it requires only information on per capita incomes and population sizes. An alternative approach measures international inequality without weighting per capita incomes by the respective population sizes. If this approach were used in our study, all regions would be treated as equally important irrespective of their population sizes.

also provides additional details on the interpretation of the second and third measures of welfare change.

The decompositions are applied to explore the changes in inequality and welfare between EU regions over the 2003–2011 period. Regions at the NUTS 3 level are considered since this is the finest level of spatial disaggregation. Results show that welfare increases over the period considered, especially in 2003–2007. The change in welfare is small during 2007–2011 as the effects of re-ranking and population change offset the reduction in absolute income disparities between regions. Changes in absolute and relative inequalities are different on the basis of the concept of inequality adopted. Absolute income differences between regions increase in 2003–2007, especially between poorer and richer regions. Relative income disparities decrease over the entire 2003–2011 period, suggesting that per capita incomes of initially poorer regions increase proportionally more than those of initially richer regions. Even though relative income differences decrease, re-ranking between poorer and richer regions is negligible. This suggests that initial disparities between richer and poorer regions are large enough to prevent these regions from swapping their positions in the income distribution, at least over the medium term.

The remainder of the paper is organized as follows: Sect. 2 introduces notation and preliminary notions. Section 3 shows the decompositions of the changes in welfare, absolute inequality and relative inequality. In Sect. 4, the decompositions are used to break down the changes in inequality and welfare between EU regions over the 2003–2011 period. Section 5 concludes.

## 2 Notation and Preliminaries

The trend in inequality among individuals included in a panel survey is commonly measured by using longitudinal sample weights in calculating an inequality index each year. When inequality among EU regions is measured, regional per capita incomes are weighted by the respective regional population sizes in calculating an inequality index. However, regional population sizes vary over years and play a role in determining the change in inequality. In Sect. 3, the change in inequality is broken down, explaining its link with the change in regional population sizes, re-ranking of regions and regional per capita income growth. Here notation is introduced and the change in inequality is defined in terms of the variation in an inequality index over time. Inequality is measured in relative and absolute terms by using the Gini index of relative inequality (henceforth, the Gini index) and the Gini index of absolute inequality (hereafter, the absolute Gini index).

Let us consider  $k$  regions, whose per capita incomes  $y_1, \dots, y_k$  and population sizes  $n_1, \dots, n_k$  are known.  $p_i = n_i / \sum_{i=1}^k n_i$  being the population share of the region with per capita income  $y_i$ , the Gini index is:<sup>3</sup>

$$G = \frac{1}{2\bar{y}(p)} \sum_{i=1}^k \sum_{j=1}^k p_i p_j |y_i - y_j|, \quad (1)$$

where  $\bar{y}(p)$  is the weighted average of per capita incomes where the weights are the population shares. An equivalent expression for the Gini index in Eq. 1 can be obtained by following the matrix approach suggested by Mussini (2013a) and recently extended by

<sup>3</sup> A complete review of the various approaches to the calculation of the Gini index is provided by Xu (2004).

Mussini and Grossi (2015). Let  $\mathbf{y} = (y_1, \dots, y_k)^T$  be the  $k \times 1$  vector of per capita incomes sorted in decreasing order, and  $\mathbf{p} = (p_1, \dots, p_k)^T$  be the  $k \times 1$  vector of the corresponding population shares. Let  $\mathbf{1}_k$  stand for the  $k \times 1$  vector with each element equal to 1, and  $\mathbf{E}$  denote the  $k \times k$  skew-symmetric matrix:

$$\mathbf{E} = \frac{1}{\bar{y}(p)} (\mathbf{1}_k \mathbf{y}^T - \mathbf{y} \mathbf{1}_k^T) = \begin{bmatrix} \frac{y_1 - y_1}{\bar{y}(p)} & \dots & \frac{y_k - y_1}{\bar{y}(p)} \\ \vdots & \ddots & \vdots \\ \frac{y_1 - y_k}{\bar{y}(p)} & \dots & \frac{y_k - y_k}{\bar{y}(p)} \end{bmatrix}. \quad (2)$$

The elements of  $\mathbf{E}$  are the  $k^2$  relative pairwise differences between the per capita incomes as ordered in  $\mathbf{y}$ . Let  $\mathbf{P} = \text{diag}\{\mathbf{p}\}$  be the  $k \times k$  diagonal matrix with the diagonal elements equal to the population shares in  $\mathbf{p}$ , and  $\mathbf{G}$  be a  $k \times k$   $G$ -matrix (a skew-symmetric matrix whose diagonal elements are equal to 0 and the upper diagonal elements are equal to -1 and the lower diagonal elements are equal to 1) (Silber 1989). The Gini index can be expressed in matrix form:

$$G(\mathbf{p}, \mathbf{y}) = \frac{1}{2} \text{tr}(\tilde{\mathbf{G}} \mathbf{E}^T), \quad (3)$$

where the matrix  $\tilde{\mathbf{G}} = \mathbf{P} \mathbf{G} \mathbf{P}$  is the weighting  $G$ -matrix, a generalization of the Silber  $G$ -matrix introduced by Mussini and Grossi (2015) in order to use the weighting vector  $\mathbf{p}$  in calculating the Gini index.  $\tilde{g}_{ij}$  denoting the  $(i, j)$ -th element of  $\tilde{\mathbf{G}}$  with  $i > j$ ,  $\tilde{g}_{ij}$  equals  $p_i p_j$  whereas  $\tilde{g}_{ji}$  equals  $-p_i p_j$  since  $\tilde{\mathbf{G}}$  is skew-symmetric. Thus, the elements of  $\tilde{\mathbf{G}}$  are the weights of the relative pairwise differences in  $\mathbf{E}$ . The absolute Gini index is

$$G_a(\mathbf{p}, \mathbf{y}) = \bar{y}(p) G(\mathbf{p}, \mathbf{y}) = \frac{1}{2} \text{tr}(\tilde{\mathbf{G}} \mathbf{A}^T), \quad (4)$$

where  $\mathbf{A} = \bar{y}(p) \mathbf{E}$  is the matrix containing the  $k^2$  pairwise differences between the per capita incomes as ordered in  $\mathbf{y}$ .

Suppose that the per capita incomes and population shares of  $k$  regions are observed at times  $t$  and  $t + 1$ . Let  $\mathbf{y}_t$  be the  $k \times 1$  vector of the  $t$  per capita incomes sorted in decreasing order and  $\mathbf{p}_t$  be the  $k \times 1$  vector of the corresponding population shares. Let  $\mathbf{y}_{t+1}$  be the  $k \times 1$  vector of the  $t + 1$  per capita incomes sorted in decreasing order and  $\mathbf{p}_{t+1}$  be the  $k \times 1$  vector of the corresponding population shares. The change in relative inequality from  $t$  to  $t + 1$  is measured by the difference between the Gini index in  $t + 1$  and the Gini index in  $t$  (Duro 2013):

$$\Delta G = G(\mathbf{p}_{t+1}, \mathbf{y}_{t+1}) - G(\mathbf{p}_t, \mathbf{y}_t) = \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_{t+1} \mathbf{E}_{t+1}^T) - \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_t \mathbf{E}_t^T). \quad (5)$$

$\bar{y}_{t+1}(p_{t+1})$  and  $\bar{y}_t(p_t)$  being respectively the weighted averages of per capita incomes in  $t + 1$  and  $t$ , the change in absolute inequality is the difference between the absolute Gini index in  $t + 1$  and the absolute Gini index in  $t$ :

$$\begin{aligned}
\Delta G_a &= \bar{y}_{t+1}(p_{t+1})G(\mathbf{p}_{t+1}, \mathbf{y}_{t+1}) - \bar{y}_t(p_t)G(\mathbf{p}_t, \mathbf{y}_t) \\
&= G_a(\mathbf{p}_{t+1}, \mathbf{y}_{t+1}) - G_a(\mathbf{p}_t, \mathbf{y}_t) \\
&= \frac{1}{2}tr(\tilde{\mathbf{G}}_{t+1}\mathbf{A}_{t+1}^T) - \frac{1}{2}tr(\tilde{\mathbf{G}}_t\mathbf{A}_t^T),
\end{aligned} \tag{6}$$

where  $\mathbf{A}_{t+1} = \bar{y}_{t+1}(p_{t+1})\mathbf{E}_{t+1}$  and  $\mathbf{A}_t = \bar{y}_t(p_t)\mathbf{E}_t$ . In the next section  $\Delta G$  and  $\Delta G_a$  are broken down by separating the population change, re-ranking and per capita income growth components.<sup>4</sup>

### 3 Methods

In this section,  $\Delta G_a$  and  $\Delta G$  are broken down into three components explaining the effects of population change, re-ranking and income growth. The decompositions of  $\Delta G_a$  and  $\Delta G$  are obtained by following the matrix decomposition approach recently suggested by Mussini and Grossi (2015). The Mussini and Grossi decomposition breaks down the relative inequality in carbon dioxide emissions between countries. In this paper, the decomposition of  $\Delta G$  becomes an adaptation of the Mussini and Grossi decomposition to the context of the change in regional inequality and is shown in Sect. 3.2. The decomposition of  $\Delta G_a$  requires a more detailed presentation for two reasons. First, the use of absolute inequality instead of relative inequality entails that the expressions and interpretations of some components change. Second, using the decomposition of  $\Delta G_a$ , a new decomposition of the change in welfare is obtained. This decomposition explains the link between three measures of welfare change suggested by Silber and Weber (2005). The decompositions of changes in absolute inequality and welfare are shown in Sect. 3.1. Section 3.3 shows that the components of  $\Delta G_a$  and  $\Delta G$  can be further broken down by subgroup.

#### 3.1 Decomposing Changes in Absolute Inequality and Welfare

Measuring the change in absolute inequality is important per se because it shows a view of inequality change which is alternative to that of the change in relative inequality. Moreover, the change in absolute inequality is relevant to policy-makers since it has an impact on welfare change (Silber and Weber 2005). Using the Atkinson (1970) concept of '*equally distributed equivalent level of income*', Silber and Weber (2005) expressed the change in welfare as the sum of the change in average income and the change in the absolute Gini index. More specifically, the equally distributed equivalent level of income is equal to the product of the average income and the complement to 1 of the Gini index (Silber and Weber 2005):

$$x = \bar{y}(p)[1 - G(\mathbf{p}, \mathbf{y})] = \bar{y}(p) - G_a(\mathbf{p}, \mathbf{y}). \tag{7}$$

$x_{t+1}$  and  $x_t$  being respectively the the equally distributed equivalent levels of incomes in  $t + 1$  and  $t$ , the welfare change from  $t$  to  $t + 1$  is

<sup>4</sup> The change in the Gini index between two points in time is decomposed by Bönke et al. (2010) to explain the effects of changes in various income sources on the change in overall income inequality.

$$\begin{aligned}
\Delta W_a &= x_{t+1} - x_t \\
&= [\bar{y}_{t+1}(p_{t+1}) - \bar{y}_t(p_t)] - [G_a(\mathbf{p}_{t+1}, \mathbf{y}_{t+1}) - G_a(\mathbf{p}_t, \mathbf{y}_t)] \\
&= M_a - \Delta G_a
\end{aligned} \tag{8}$$

where  $M_a = \bar{y}_{t+1}(p_{t+1}) - \bar{y}_t(p_t)$  is the component measuring the change in average income and  $\Delta G_a$  is the component measuring the change in absolute inequality (Silber and Weber 2005). We show that the change in absolute inequality in Eq. 8 can be decomposed into three terms. One component measures the effect of the change in population shares on the change in absolute inequality. A second component measures the contribution caused by the re-ranking of regions in the per capita income distribution when moving from  $t$  to  $t + 1$ . A third component measures the effect of different growth among regional per capita incomes from  $t$  to  $t + 1$ .

Initially, we define two indices useful for deriving the decomposition: one is the absolute Gini index of  $t + 1$  per capita incomes calculated by using the  $t$  population shares; the second is the absolute concentration index of  $t + 1$  per capita incomes sorted by the  $t$  per capita incomes, calculated by using the  $t$  population shares. Let  $\mathbf{p}_{t|t+1}$  be the  $k \times 1$  vector of  $t$  population shares arranged by the decreasing order of the corresponding  $t + 1$  per capita incomes, with  $\mathbf{P}_{t|t+1} = \text{diag}\{\mathbf{p}_{t|t+1}\}$ .  $\tilde{\mathbf{G}}_{t|t+1} = \mathbf{P}_{t|t+1} \mathbf{G} \mathbf{P}_{t|t+1}$  being the weighting  $G$ -matrix obtained by using the population shares in  $t$  instead of those in  $t + 1$ , the absolute Gini index of  $t + 1$  per capita incomes calculated by using the  $t$  population shares is

$$G_a(\mathbf{p}_{t|t+1}, \mathbf{y}_{t+1}) = \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_{t|t+1} \mathbf{A}_{t+1}^T). \tag{9}$$

Let  $\mathbf{y}_{t+1|t}$  be the  $k \times 1$  vector of  $t + 1$  per capita incomes sorted in decreasing order of the respective  $t$  per capita incomes. The absolute concentration index of  $t + 1$  per capita incomes sorted by the  $t$  per capita incomes, calculated by using the  $t$  population shares, is defined as follows:

$$C_a(\mathbf{p}_t, \mathbf{y}_{t+1|t}) = \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_t \mathbf{A}_{t+1|t}^T), \tag{10}$$

where  $\mathbf{A}_{t+1|t} = (\mathbf{1}_k \mathbf{y}_{t+1|t}^T - \mathbf{y}_{t+1|t} \mathbf{1}_k^T)$  is the  $k \times k$  skew-symmetric matrix with the  $(i, j)$ -th element equal to the difference between  $y_{j,t+1|t}$  and  $y_{i,t+1|t}$ ; that is,  $\mathbf{A}_{t+1|t}$  contains the  $k^2$  pairwise differences between the per capita incomes as arranged in  $\mathbf{y}_{t+1|t}$ . The absolute concentration index  $C_a(\mathbf{p}_t, \mathbf{y}_{t+1|t})$  can be re-written as a function of  $\mathbf{A}_{t+1}$  instead of  $\mathbf{A}_{t+1|t}$ :

$$C_a(\mathbf{p}_t, \mathbf{y}_{t+1|t}) = \frac{1}{2} \text{tr}(\mathbf{B}^T \tilde{\mathbf{G}}_t \mathbf{B} \mathbf{A}_{t+1}^T), \tag{11}$$

where  $\mathbf{B}$  is the  $k \times k$  permutation matrix re-arranging the elements of  $\mathbf{y}_{t+1}$  to obtain  $\mathbf{y}_{t+1|t}$ , that is  $\mathbf{y}_{t+1|t} = \mathbf{B} \mathbf{y}_{t+1}$  (Mussini and Grossi 2015).

If  $G_a(\mathbf{p}_{t|t+1}, \mathbf{y}_{t+1})$  and  $C_a(\mathbf{p}_t, \mathbf{y}_{t+1|t})$  are added and subtracted to the right-hand side of Eq. 6,  $\Delta G_a$  can be broken down into three terms:

$$\Delta G_a = \left[ G_a(\mathbf{p}_{t+1}, \mathbf{y}_{t+1}) - G_a(\mathbf{p}_{t|t+1}, \mathbf{y}_{t+1}) \right] + \left[ G_a(\mathbf{p}_{t|t+1}, \mathbf{y}_{t+1}) - C_a(\mathbf{p}_t, \mathbf{y}_{t+1|t}) \right] - \left[ G_a(\mathbf{p}_t, \mathbf{y}_t) - C_a(\mathbf{p}_t, \mathbf{y}_{t+1|t}) \right]. \quad (12)$$

Given Eqs. 9, 10 and 11, the matrix decomposition of the change in absolute inequality is

$$\begin{aligned} \Delta G_a &= \left[ \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_{t+1} \mathbf{A}_{t+1}^T) - \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_{t|t+1} \mathbf{A}_{t+1}^T) \right] \\ &\quad + \left[ \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_{t|t+1} \mathbf{A}_{t+1}^T) - \frac{1}{2} \text{tr}(\mathbf{B}^T \tilde{\mathbf{G}}_t \mathbf{B} \mathbf{A}_{t+1}^T) \right] \\ &\quad - \left[ \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_t \mathbf{A}_t^T) - \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_t \mathbf{A}_{t+1|t}^T) \right] \\ &= \frac{1}{2} \text{tr}[(\tilde{\mathbf{G}}_{t+1} - \tilde{\mathbf{G}}_{t|t+1}) \mathbf{A}_{t+1}^T] + \frac{1}{2} \text{tr}[(\tilde{\mathbf{G}}_{t|t+1} - \mathbf{B}^T \tilde{\mathbf{G}}_t \mathbf{B}) \mathbf{A}_{t+1}^T] \\ &\quad - \frac{1}{2} \text{tr}[\tilde{\mathbf{G}}_t (\mathbf{A}_t^T - \mathbf{A}_{t+1|t}^T)] \\ &= \frac{1}{2} \text{tr}(\mathbf{S}_a \mathbf{A}_{t+1}^T) + \frac{1}{2} \text{tr}(\mathbf{R}_a \mathbf{A}_{t+1}^T) - \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_t \mathbf{D}_a^T) \\ &= S_a + R_a - D_a, \end{aligned} \quad (13)$$

where  $\mathbf{S}_a = \tilde{\mathbf{G}}_{t+1} - \tilde{\mathbf{G}}_{t|t+1}$ ,  $\mathbf{R}_a = \tilde{\mathbf{G}}_{t|t+1} - \mathbf{B}^T \tilde{\mathbf{G}}_t \mathbf{B}$  and  $\mathbf{D}_a = \mathbf{A}_t - \mathbf{A}_{t+1|t}$ .

In the following, each component and the respective matrix are explained in detail.  $S_a$  is the component measuring the effect of the change in population shares from  $t$  to  $t+1$ . Its matrix expression shows that matrix  $\mathbf{S}_a$  captures the population change causing the change in inequality, irrespective of other causes since the disparities between per capita incomes and the ranking of regions are fixed in  $t+1$ . The  $(i, j)$ -th element of  $\mathbf{S}_a$  is  $s_{a,ij} = p_{i,t+1}p_{j,t+1} - p_{i,t|t+1}p_{j,t|t+1}$ .  $s_{a,ij}$  measures the effect of population changes in the regions arranged in positions  $i$  and  $j$  in vector  $\mathbf{y}_{t+1}$ . If  $s_{a,ij} > 0$  with  $i > j$ ,<sup>5</sup> the population change increases absolute inequality as the disparity between the  $t+1$  per capita incomes of the two regions has a larger weight at time  $t+1$  than at time  $t$ . If  $s_{a,ij} < 0$  with  $i > j$ , the population change reduces absolute inequality as the disparity between the  $t+1$  per capita incomes of the two regions has a smaller weight at time  $t+1$  than at time  $t$ .<sup>6</sup> When  $S_a > 0$ , the change in population sizes increases absolute inequality from  $t$  to  $t+1$ . When  $S_a < 0$ , the change in population sizes reduces absolute inequality from  $t$  to  $t+1$ . If all region

<sup>5</sup> If  $s_{a,ij}$  is positive and  $i > j$ ,  $s_{a,ji}$  is negative and equal to  $-s_{a,ij}$  as matrix  $\mathbf{S}_a$  is skew-symmetric.

<sup>6</sup> Suppose that  $\mathbf{y}_t = (y_{1,t} = 5.5, y_{2,t} = 5, y_{3,t} = 3.5, y_{4,t} = 1)^T$  contains the per capita incomes of four regions in  $t$  sorted in decreasing order and that  $\mathbf{p}_t = (p_{1,t} = 0.4, p_{2,t} = 0.2, p_{3,t} = 0.1, p_{4,t} = 0.3)^T$  includes the corresponding population shares in  $t$ . Let  $\mathbf{y}_{t+1|t} = (y_{1,t+1|t} = 6, y_{2,t+1|t} = 8, y_{3,t+1|t} = 4, y_{4,t+1|t} = 2)^T$  be the per capita incomes in  $t+1$  sorted by the decreasing order of the respective per capita incomes in  $t$ . The vector of the  $t+1$  per capita incomes sorted in decreasing order is  $\mathbf{y}_{t+1} = (y_{1,t+1} = 8, y_{2,t+1} = 6, y_{3,t+1} = 4, y_{4,t+1} = 2)^T$  and the vector of the corresponding population shares is  $\mathbf{p}_{t+1} = (p_{1,t+1} = 0.25, p_{2,t+1} = 0.35, p_{3,t+1} = 0.15, p_{4,t+1} = 0.25)^T$ . Consequently,  $\mathbf{p}_{t|t+1} = (p_{1,t|t+1} = 0.2, p_{2,t|t+1} = 0.4, p_{3,t|t+1} = 0.1, p_{4,t|t+1} = 0.3)^T$ . For example, when considering the two richest regions by per capita income in  $t+1$ , we obtain  $s_{a,21} = 0.0075$  (and obviously  $s_{a,12} = -0.0075$ ) in the  $4 \times 4$  matrix  $\mathbf{S}_a$ . This suggests that the population change increases the weight of the absolute inequality between the two regions.

population sizes vary by the same proportion from  $t$  to  $t + 1$ , then  $S_a = 0$  since  $\tilde{\mathbf{G}}_{t+1} = \tilde{\mathbf{G}}_{t|t+1}$ ; i.e., the change in region population sizes does not affect the change in absolute inequality.

$R_a$  measures the re-ranking of regions from  $t$  to  $t + 1$ . It yields a non-negative contribution to the change in absolute inequality as  $0 \leq R_a \leq 2G_a(\mathbf{p}_{t|t+1}, \mathbf{y}_{t+1})$ . Matrix  $\mathbf{R}_a$  tracks the movements of regions within the per capita income distribution by identifying each pair of regions where the regions re-rank when moving from  $t$  to  $t + 1$ .  $r_{a,ij}$  being the generic  $(i, j)$ -th element of  $\mathbf{R}_a$  and assuming that  $i > j$ ,  $r_{a,ij}$  is equal to  $2p_{i,t|t+1}p_{j,t|t+1}$  if the  $(i, j)$ -th element of  $\mathbf{A}_{t+1}$  is the difference between the per capita incomes of two re-ranking regions;<sup>7</sup> otherwise  $r_{a,ij}$  is equal to 0.<sup>8</sup> If the ranking of regions does not change from  $t$  to  $t + 1$ , then  $R_a = 0$  as  $\mathbf{B} = \mathbf{I}_k$  and  $\tilde{\mathbf{G}}_{t|t+1} = \tilde{\mathbf{G}}_t$ .

$D_a$  explains the role played by the change in absolute disparities between regional per capita incomes in determining  $\Delta G_a$ . Component  $D_a$  is obtained by keeping the population shares and the ranking of regions in  $t$  fixed. The generic  $(i, j)$ -th element of  $\mathbf{D}_a$ , written as  $d_{a,ij}$ , compares the absolute difference between the  $t$  per capita incomes of the regions in positions  $j$  and  $i$  in  $\mathbf{y}_t$  with the absolute difference between the  $t + 1$  per capita incomes of the same two regions in  $\mathbf{y}_{t+1|t}$ , since  $t + 1$  per capita incomes are arranged in  $\mathbf{y}_{t+1|t}$  in decreasing order of the respective per capita incomes in  $t$ .<sup>9</sup> If  $d_{a,ij} > 0$  with  $i > j$  (and  $d_{a,ji} < 0$  since  $\mathbf{D}_a$  is skew-symmetric), the absolute disparity between the  $t$  per capita incomes of two regions is greater than the absolute disparity between the  $t + 1$  per capita incomes of the same regions; thus, the change in per capita income disparities reduces absolute inequality from  $t$  to  $t + 1$ . If  $d_{a,ij} < 0$  with  $i > j$ , the absolute disparity between the  $t$  per capita incomes of two regions is less than the absolute disparity between the  $t + 1$  per capita incomes of the same regions; thus, the change in per capita income disparities increases absolute inequality from  $t$  to  $t + 1$ .<sup>10</sup> When  $D_a > 0$ , the absolute disparities in regional per capita incomes globally decrease from  $t$  to  $t + 1$ ; that is, the absolute disparities between poorer and richer regions in  $t$  diminish when moving to  $t + 1$ , reducing absolute inequality. When  $D_a < 0$ , the absolute disparities in regional per capita incomes globally increase from  $t$  and  $t + 1$ ; that is, the absolute disparities between initially poorer and richer regions increase when moving from  $t$  to  $t + 1$ , augmenting absolute inequality. If all regional per capita incomes vary by the same amount from  $t$  to  $t + 1$ , then  $D_a = 0$  as the absolute difference between regional per capita incomes is unchanged for each pair of regions.

Given the decomposition in Eq. 13, the welfare change is broken down into four components

<sup>7</sup> Since  $\mathbf{R}_a$  is skew-symmetric,  $r_{a,ji}$  is equal to  $-2p_{i,t|t+1}p_{j,t|t+1}$ .

<sup>8</sup> Referring to the numerical example shown in footnote 6, re-ranking occurs between the two regions with the highest per capita incomes in  $t$  when moving to  $t + 1$ . The  $4 \times 4$  matrix  $\mathbf{R}_a$  detects the re-ranking by showing  $r_{a,21} = 2p_{2,t|t+1}p_{1,t|t+1}$  and  $r_{a,12} = -2p_{2,t|t+1}p_{1,t|t+1}$ , whereas its remaining elements are equal to zero; that is,  $r_{a,21} = 0.16$  and  $r_{a,12} = -0.16$ .

<sup>9</sup> Both  $\mathbf{A}_t$  and  $\mathbf{A}_{t+1|t}$  are obtained by keeping regions sorted in decreasing order of their per capita incomes in  $t$ . The difference in the  $(i, j)$ -th entry of  $\mathbf{A}_t$  and the difference in the  $(i, j)$ -th entry of  $\mathbf{A}_{t+1|t}$  are calculated between the per capita incomes of the same two regions at times  $t$  and  $t + 1$ , respectively.

<sup>10</sup> In the numerical example cited in footnote 6, the  $4 \times 4$  matrix  $\mathbf{D}_a$  has  $d_{a,41} = 0.5$  (and obviously  $d_{a,14} = -0.5$ ). This indicates that the absolute inequality between the regions with the highest and lowest per capita incomes in  $t$  decreases from  $t$  to  $t + 1$ .



$$\Delta W_a = M_a - S_a - R_a + D_a. \quad (14)$$

The expression of  $\Delta W_a$  in Eq. 14 provides additional information compared to that in Eq. 8, since the roles of the population change, re-ranking and income growth are separated. It explains that a decrease in income disparities (i.e.,  $D_a > 0$ ) increases welfare, whereas welfare decreases when income disparities increase (i.e.,  $D_a < 0$ ). Since the re-ranking component is non-negative, re-ranking decreases welfare. The population change increases welfare when  $S_a$  is negative, that is when the weights assigned to more unequal pairs of regions decrease from  $t$  to  $t + 1$ . In addition, this decomposition explains the link between  $\Delta W_a$  and another measure of welfare change proposed by Silber and Weber (2005). This second measure is

$$\begin{aligned} \Delta W_b &= M_a - \Delta G_a + \left[ G_a(\mathbf{p}_{t+1}, \mathbf{y}_{t+1}) - C_a(\mathbf{p}_{t+1|t}, \mathbf{y}_{t+1|t}) \right] \\ &= M_a - \Delta G_a + R_b \\ &= \Delta W_a + R_b. \end{aligned} \quad (15)$$

In the Silber and Weber (2005) formulation,  $R_b$  is a measure of re-ranking added to  $\Delta W_a$  in order to account for the exchange of positions within the income distribution from  $t$  to  $t + 1$ . Considering the expression of  $\Delta W_a$  in Eq. 14,  $\Delta W_b$  can be written as

$$\Delta W_b = M_a - S_a - R_a + D_a + R_b. \quad (16)$$

The decomposition of  $\Delta W_b$  shows that the re-ranking component is already included in  $\Delta W_a$ . The only difference between  $R_a$  and  $R_b$  is that the former is calculated by fixing the population shares at time  $t$ , as the decomposition of  $\Delta W_a$  in Eq. 14 detects also the population change component. Since  $R_a$  enters negatively in the expression of  $\Delta W_b$  whereas  $R_b$  enters positively, they neutralize each other. This implies that the value of  $\Delta W_b$  does not reflect the impact of re-ranking.

Silber and Weber (2005) suggested a third measure of welfare change. This third measure is

$$\begin{aligned} \Delta W_c &= M_a - \Delta G_a - \left[ G_a(\mathbf{p}_t, \mathbf{y}_t) - C_a(\mathbf{p}_{t|t+1}, \mathbf{y}_{t|t+1}) \right] \\ &= M_a - \Delta G_a - R_c \\ &= \Delta W_a - R_c, \end{aligned} \quad (17)$$

where  $C_a(\mathbf{p}_{t|t+1}, \mathbf{y}_{t|t+1})$  is the absolute concentration index of  $t$  per capita incomes sorted by the  $t + 1$  per capita incomes.  $R_c$  measures re-ranking by using incomes in  $t$  instead of those in  $t + 1$ . Nevertheless, subtracting  $R_c$  to  $\Delta W_a$  as shown in Eq. 17, the re-ranking effect is taken into account two times in  $\Delta W_c$  since it can be re-written as follows:

$$\Delta W_c = M_a - S_a - R_a + D_a - R_c. \quad (18)$$

To sum up,  $\Delta W_a$  already includes the re-ranking effect, which is revealed by the decomposition developed in Eq. 14. Thus, adding (or subtracting) a further re-ranking component to  $\Delta W_a$  implies that the welfare change is overestimated (or underestimated).

### 3.2 Decomposing Change in Relative Inequality

The decomposition of  $\Delta G$  is based on the Mussini and Grossi (2015) three-term decomposition of the change in carbon dioxide emission inequality. The Mussini and Grossi decomposition is adjusted in order to break down the change in relative inequality between regions. One component measures the effect of the change in population shares on the change in relative inequality. A second component measures the contribution caused by the re-ranking of regions in the per capita income distribution from  $t$  to  $t + 1$ . A third component measures the effect of disproportionate growth among regional per capita incomes from  $t$  to  $t + 1$ . Disproportionate growth between regional per capita incomes means that the  $t + 1$  per capita incomes are not equal to the  $t$  per capita incomes changed by the same proportion (Mussini 2014). For instance, if the per capita incomes of two regions change by the same proportion, the relative inequality between them is unaltered. The relative inequality between the two regions changes if the income growth rate of one region is greater than that of the other region; that is, when the per capita income of one region grows proportionately more than the per capita income of the other region.

The re-ranking and disproportionate income growth components were originally isolated by Jenkins and Van Kerm in their two-term decomposition of  $\Delta G$  (Jenkins and Van Kerm 2006). More recently, Mussini (2013a, b) suggested a matrix decomposition of  $\Delta G$  which also enables the decomposition of the re-ranking and disproportionate income growth components by subgroup. This matrix decomposition was further developed by Mussini and Grossi (2015) to obtain a three-term decomposition of the change in carbon dioxide emission inequality between countries. The Mussini and Grossi decomposition links the Mussini decomposition and the Duro decomposition of  $\Delta G$  (Duro 2013), with the latter detecting the effect of the change in population shares on the change in carbon dioxide emission inequality.<sup>11</sup>  $\lambda = \bar{y}_{t+1}(p_{t+1})/\bar{y}_{t+1}(p_t)$  being the ratio of the actual  $t + 1$  average per capita income to the fictitious  $t + 1$  average per capita income calculated by using the population shares in  $t$ , the Mussini and Grossi decomposition of the change in relative inequality between regions is

$$\begin{aligned}\Delta G &= \frac{1}{2} \text{tr}(\mathbf{S}\mathbf{E}_{t+1}^T) + \frac{1}{2} \text{tr}(\mathbf{R}\lambda\mathbf{E}_{t+1}^T) - \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_t\mathbf{D}^T) \\ &= \mathbf{S} + \mathbf{R} - \mathbf{D},\end{aligned}\quad (19)$$

where  $\mathbf{S} = \tilde{\mathbf{G}}_{t+1} - \lambda\tilde{\mathbf{G}}_{t|t+1}$ ,  $\mathbf{R} = \tilde{\mathbf{G}}_{t|t+1} - \mathbf{B}^T\tilde{\mathbf{G}}_t\mathbf{B}$  and  $\mathbf{D} = \mathbf{E}_t - \mathbf{E}_{t+1|t}$ . Starting with the expression of the population change component, we see that the elements of matrix  $\mathbf{S}$  differ from those of  $\mathbf{S}_a$ .  $s_{ij}$  being the generic element of  $\mathbf{S}$ , it is the outcome of a region-specific population change effect combined with an overall population change effect. The region-specific population change effect is the change in the population shares of the two regions from  $t$  to  $t + 1$ . The overall population change effect, captured by  $\lambda$ , is the change in the average per capita income caused by the change in the population shares of all regions. The combination of the region-specific population change effect and the overall population change effect determines the effect of the population change on relative inequality. The interpretation of  $\mathbf{S}$  is the same as that of  $\mathbf{S}_a$ . When  $\mathbf{S}$  is positive, the weights assigned to more unequal pairs of regions increase, augmenting relative inequality from  $t$  to  $t + 1$ ; the opposite occurs when  $\mathbf{S}$  is negative.

<sup>11</sup> Duro isolated the population change component of  $\Delta G$  by using a general decomposition approach suggested by Esteban (1994) to measure the effect of the change in population on an inequality index.

$R$  measures the re-ranking of regions from  $t$  to  $t + 1$ , yielding a non-negative contribution to the change in relative inequality as  $0 \leq R \leq 2G(\mathbf{p}_{t|t+1}, \mathbf{y}_{t+1})$ .<sup>12</sup> Since matrix  $\mathbf{R}$  plays the same role of  $\mathbf{R}_a$  (i.e., identifying each pair of regions where the regions re-rank), the two matrices are identical.

Component  $D$  measures the effect of disproportionate growth between regional per capita incomes. The generic  $(i, j)$ -th element of  $\mathbf{D}$ , written as  $d_{ij}$ , compares the relative difference between the  $t$  per capita incomes of the regions in positions  $j$  and  $i$  in  $\mathbf{y}_t$  with the relative difference between the  $t + 1$  per capita incomes of the same two regions in  $\mathbf{y}_{t+1|t}$ . If  $d_{ij} > 0$  with  $i > j$ , the relative disparity between the  $t$  per capita incomes of two regions is greater than the relative disparity between the  $t + 1$  per capita incomes of the same regions; that is, the poorest region gains proportionately more than the richest region, reducing relative inequality between the two regions. If  $d_{ij} < 0$  with  $i > j$ , the relative disparity between the  $t$  per capita incomes of two regions is less than the relative disparity between the  $t + 1$  per capita incomes of the same regions; that is, the richest region gains proportionately more than the poorest region, increasing relative inequality between the two regions. When  $D > 0$ , the relative disparities in regional per capita incomes globally decrease from  $t$  to  $t + 1$ ; that is, the relative disparities between poorer and richer regions in  $t$  diminish when moving to  $t + 1$ , reducing relative inequality. When  $D < 0$ , the relative disparities in regional per capita incomes globally increase from  $t$  and  $t + 1$ ; that is, the relative disparities between initially poorer and richer regions increase, augmenting relative inequality. If all regional per capita incomes vary by the same proportion from  $t$  to  $t + 1$ , then  $D = 0$  as the relative inequality between regional per capita incomes is unchanged for each pair of regions.

### 3.3 Decomposing inequality change by subgroup

Using the matrix decomposition approach, each component of  $\Delta G_a$  and  $\Delta G$  can be broken down by subgroup,<sup>13</sup> enabling analysts to deepen the investigation of the inequality change. Only the subgroup decomposition of the components of  $\Delta G_a$  is shown in detail since that of the components of  $\Delta G$  can be obtained in the same way (Mussini and Grossi 2015).

Suppose that regions are split into  $r$  subgroups according to a given criterion (e.g., per capita income level, geographical area). Let  $\mathbf{A}_{h,t+1}$  be the  $n \times n$  matrix whose  $(i, j)$ -th element is equal to the  $(i, j)$ -th element of  $\mathbf{A}_{t+1}$  if the  $(i, j)$ -th element of  $\mathbf{A}_{t+1}$  is the difference between the per capita incomes of two regions in subgroup  $h$ ; otherwise, the  $(i, j)$ -th element of  $\mathbf{A}_{h,t+1}$  is 0. Let  $\mathbf{D}_{a,h}$  be the  $n \times n$  matrix whose  $(i, j)$ -th element is equal to the  $(i, j)$ -th element of  $\mathbf{D}_a$  if the  $(i, j)$ -th element of  $\mathbf{D}_a$  involves two regions of subgroup  $h$ ; otherwise, the  $(i, j)$ -th element of  $\mathbf{D}_{a,h}$  is 0.<sup>14</sup> By replacing  $\mathbf{A}_{t+1}$  and  $\mathbf{D}_a$  in Eq. 13 with

<sup>12</sup>  $R$  coincides with the Atkinson-Plotnick re-ranking coefficient used to measure re-ranking between income receivers in income distribution (Jenkins and Van Kerm 2006). The Atkinson-Plotnick re-ranking coefficient equals 0 if the ranking of income receivers is unchanged from  $t$  to  $t + 1$ , whereas it is equal to two times the Gini index in  $t + 1$  if the ranking in  $t + 1$  is the inverse of the ranking in  $t$ .

<sup>13</sup> The Gini index, expressed as the half of the Gini relative mean difference in Eq. 1, was broken down by subgroup in Dagum (1997) and in Costa (2008). More recently, Mornet et al. (2013) suggested a subgroup decomposition of the  $\alpha$ -Gini, an extended version of the Gini index in Eq. 1 including a parameter of inequality aversion.

<sup>14</sup> "Appendix 1" shows that  $\mathbf{A}_{h,t+1}$  and  $\mathbf{D}_{a,h}$  are obtained from  $\mathbf{A}_{t+1}$  and  $\mathbf{D}_a$ , respectively.

$\mathbf{A}_{h,t+1}$  and  $\mathbf{D}_{a,h}$ , respectively, we obtain the decomposition of the change in the inequality contribution of subgroup  $h$ :

$$\begin{aligned}\Delta G_a^h &= \frac{1}{2} \text{tr}(\mathbf{S}\mathbf{A}_{h,t+1}^T) + \frac{1}{2} \text{tr}(\mathbf{R}\mathbf{A}_{h,t+1}^T) - \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_t \mathbf{D}_{a,h}^T) \\ &= S_a^h + R_a^h - D_a^h.\end{aligned}\quad (20)$$

Let  $\mathbf{A}_{gh,t+1}$  be the  $n \times n$  matrix whose  $(i,j)$ -th element is equal to the  $(i,j)$ -th element of  $\mathbf{A}_{t+1}$  if the  $(i,j)$ -th element of  $\mathbf{A}_{t+1}$  is the difference between the per capita income of a region in subgroup  $g$  and the per capita income of a region in subgroup  $h$ ; otherwise, the  $(i,j)$ -th element of  $\mathbf{A}_{gh,t+1}$  is 0. Let  $\mathbf{D}_{a,gh}$  be the  $n \times n$  matrix whose  $(i,j)$ -th element is equal to the  $(i,j)$ -th element of  $\mathbf{D}_a$  if the  $(i,j)$ -th element of  $\mathbf{D}_a$  involves a region of subgroup  $g$  and a region of subgroup  $h$ ; otherwise, the  $(i,j)$ -th element of  $\mathbf{D}_{a,gh}$  is 0.<sup>15</sup> By replacing  $\mathbf{A}_{t+1}$  and  $\mathbf{D}_a$  in Eq. 13 with  $\mathbf{A}_{gh,t+1}$  and  $\mathbf{D}_{a,gh}$ , respectively, we obtain the decomposition of the change in the contribution of inequality between subgroups  $g$  and  $h$ :

$$\begin{aligned}\Delta G_a^{gh} &= \frac{1}{2} \text{tr}(\mathbf{S}\mathbf{A}_{gh,t+1}^T) + \frac{1}{2} \text{tr}(\mathbf{R}\mathbf{A}_{gh,t+1}^T) - \frac{1}{2} \text{tr}(\tilde{\mathbf{G}}_t \mathbf{D}_{a,gh}^T) \\ &= S_a^{gh} + R_a^{gh} - D_a^{gh},\end{aligned}\quad (21)$$

where  $\Delta G_a^{gh} = \Delta G_a^{hg}$  by construction.

Given Eqs. 20 and 21, the subgroup decomposition of the components of absolute inequality change is

$$\begin{aligned}\Delta G_a &= \sum_{h=1}^r (S_a^h + R_a^h - D_a^h) + \sum_{h=2}^r \sum_{g=1}^{h-1} (S_a^{gh} + R_a^{gh} - D_a^{gh}) \\ &= \Delta G_a^{WG} + \Delta G_a^{BG},\end{aligned}\quad (22)$$

where  $\Delta G_a^{WG}$  and  $\Delta G_a^{BG}$  are respectively the within-group and between-group components of absolute inequality change.

#### 4 A Decomposition of the Changes in Inequality and Welfare Between EU Regions

Inequality and welfare changes across the EU-28 at the regional NUTS 3 level are broken down by using the decompositions shown in Sect. 3. Income is represented by per capita gross domestic product (hereafter, GDP) adjusted by purchasing power parities (henceforth, PPPs). There are two reasons for this choice. First, per capita GDP is considered a broad indicator of standard of living and is often used for monitoring convergence across the EU. Second, per capita GDP can be obtained from the Eurostat database in the finest detail (i.e., the NUTS 3 level). The changes in inequality (absolute and relative) and welfare over the 2003–2011 period are broken down to detect the roles played by the various components separated in Sect. 3. The entire period is split into two 4-year sub-periods because of two important events during the 2003–2011 period. In 2004, the EU membership grew as never before, with the accession of ten countries. In the 2003–2007 sub-period, changes in inequality and welfare are explored from 1 year before the

<sup>15</sup> In “Appendix 1”,  $\mathbf{A}_{gh,t+1}$  and  $\mathbf{D}_{a,gh}$  are obtained respectively from  $\mathbf{A}_{t+1}$  and  $\mathbf{D}_a$ .

**Table 1** Decomposition of the change in welfare over the 2003–2011 period

Period	$\Delta W_a$	$M_a$	$S_a$	$R_a$	$D_a$
03–07	3.516376	4.248815	−0.006909	0.089565	−0.649784
07–11	0.192254	0.168765	0.010405	0.145104	0.178998

enlargement to 3 years later. Second, the financial crisis of 2008 caused a severe economic downturn which affected the EU along with the rest of the world in the subsequent years. Therefore, it seems worth analyzing the changes in inequality and welfare from 2007 to 2011 in detail.

Table 1 shows the decomposition of  $\Delta W_a$  in the two sub-periods under consideration. Welfare increases from 2003 to 2007. Average income increases from 2003 to 2007, since the average income is equal to 20.553751 (thousands of euro) in 2003 and to 24.802566 in 2007. The increase in average income is measured by component  $M_a$  (4.248815), which plays the most important role in determining the gain in welfare from 2003 to 2007. The rise in absolute disparities between regional per capita incomes (−0.649784) reduces welfare from 2003 to 2007. Re-ranking component (0.089565) reduces welfare, however the impact of re-ranking is far less important than that of the increase in absolute disparities between regional per capita incomes. The effect of population change is small (−0.006909), but increases welfare.<sup>16</sup> This means that the weights assigned to more unequal pairs of regions slightly decrease from 2003 to 2007, reducing absolute inequality and consequently increasing welfare.

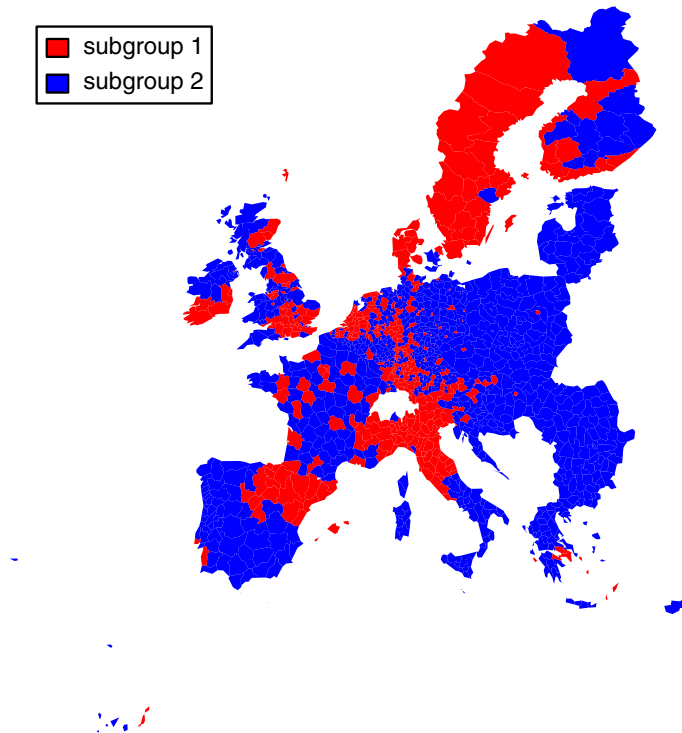
The increase in welfare from 2007 to 2011 is much lower than that over the 2003–2007 sub-period. Moreover, the effects of the various components change when compared with those shown in the previous sub-period. The absolute disparities between regional per capita incomes decrease (0.178998), playing a major role in determining the increase in welfare. The increase in average income (0.168765) is small and increases welfare. However, the positive effects of  $M_a$  and  $D_a$  are partially offset by the effect of re-ranking (0.145104), which is greater than the re-ranking effect over the 2003–2007 period. The population change component (0.010405) reduces welfare, suggesting that the weights assigned to more unequal pairs in 2011 are greater than those in 2007.

To clarify the trend in inequality between EU regions, they are divided into subgroups. Subgroups are identified by using the algorithm proposed by Davies and Shorrocks (1989) in order to find the optimal partition of income distribution when the Gini index is used. For any fixed number  $k$ , the Davies and Shorrocks procedure divides the distribution into  $k$  non-overlapping income ranges, each of which represents a subgroup including the regions whose per capita incomes fall between the income boundaries of the subgroup. The boundaries of the  $k$  subgroups are set in order to maximize the between-group inequality component of the Gini index. This enables to find the  $k$  most unequal subgroups.<sup>17</sup>

Figure 1 shows the partition of regions for  $k = 2$  in 2003. Subgroup 1 comprises richer regions whereas poorer regions are included in subgroup 2. Subgroup 1 comprises almost all regions of Sweden, Denmark, Belgium and Netherlands. The regions in North-Eastern

<sup>16</sup> Since  $S_a$  enters negatively in the decomposition of  $\Delta W_a$ , a negative value of  $S_a$  increases welfare.

<sup>17</sup> Esteban et al. (2007) applied the Davies and Shorrocks procedure to divide the income distribution into subgroups in their study of polarization in five OECD countries. They found the optimal partitions for  $k = 2, 3, 4$ .



**Fig. 1** Map of NUTS 3 regions divided in two subgroups in 2003

Spain, North-Central Italy and Southern Ireland are included in subgroup 1. Several regions of Austria, England, France and Germany are within subgroup 1. Figure 1 shows that most of regions in subgroup 2 belong to countries which joined the EU since 2004.<sup>18</sup> Since the differences in income and employment opportunities between the EU-15 member countries, which joined the EU before 2004, and new EU member countries from Central and Eastern Europe were large, it is not surprising that the regions of these new EU member countries are included in the poorest subgroup. Almost all of Greek and Portuguese regions are included in subgroup 2. Table 2 shows the components of the change in absolute inequality broken down by subgroup, when regions are divided in two subgroups. Since the partition of regions into subgroups in 2007 is similar to the partition shown in Fig. 1, the map of EU regions divided in two subgroups in 2007 is shown in “Appendix 2”. Absolute inequality increases over the 2003–2007 sub-period whereas decreases over the 2007–2011 sub-period, but the intensity of inequality change from 2003 to 2007 is greater than that from 2007 to 2011. Most of the increase in absolute income disparities during 2003–2007 ( $-0.649784$ ) is due to the increase of absolute differences between per capita incomes of regions belonging to different subgroups ( $-0.479324$ ). This means that absolute disparities in income between poorer and richer regions increase from 2003 to 2007. The subgroup decomposition of the re-ranking component shows that re-ranking

<sup>18</sup> Ten new countries joined the EU in May 2004 : Czech Republic, Estonia, Cyprus, Latvia, Lithuania, Hungary, Malta, Poland, Slovakia and Slovenia. Two more countries from Eastern Europe, Bulgaria and Romania, joined the EU in January 2007. Croatia joined the EU in July 2013.

**Table 2** Decomposition of the change in absolute inequality over the 2003–2011 period, with  $k = 2$  subgroups

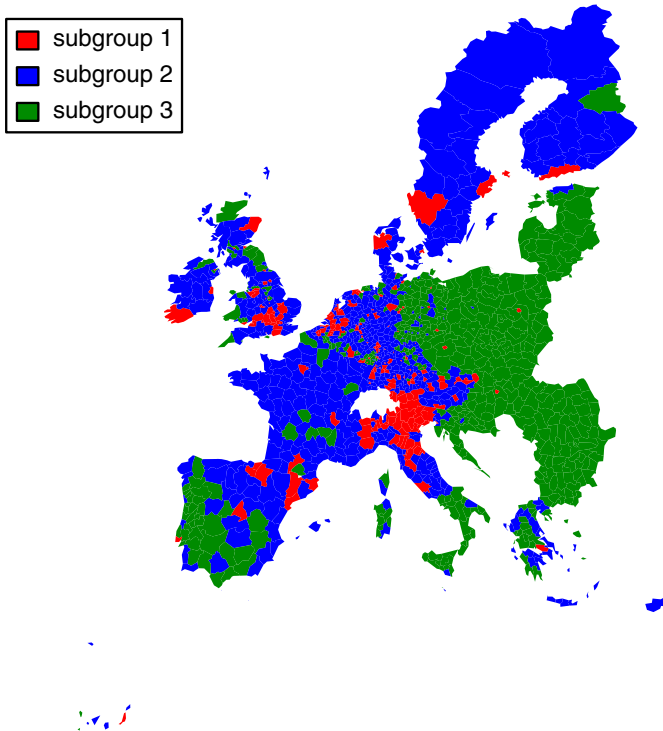
Period	Subgroup component	$G_a(\mathbf{p}_t, \mathbf{y}_t)$	$G_a(\mathbf{p}_{t+4}, \mathbf{y}_{t+4})$	$\Delta G_a$	$S_a$	$R_a$	$D_a$
03–07	1	0.848988	1.024914	0.175927	0.022067	0.032782	−0.121077
	2	0.811434	0.892430	0.080996	−0.021376	0.052989	−0.049382
	12	3.542066	4.017583	0.475517	−0.007601	0.003793	−0.479324
	Total	5.202488	5.934927	0.732440	−0.006909	0.089565	−0.649784
07–11	1	0.984520	1.111340	0.126820	0.025696	0.055576	−0.045780
	2	0.896422	0.847343	−0.049078	−0.019641	0.060522	0.089708
	12	4.053986	3.952755	−0.101230	0.004350	0.029612	0.135070
	Total	5.934927	5.911439	−0.023489	0.010405	0.145104	0.178998

mainly occurs between regions in the same subgroup, whereas the impact of re-ranking between poorer and richer regions is very small (0.003793). The largest effect of re-ranking is within subgroup 2 (0.052989), suggesting that poorer regions grow differently and exchange their positions in subgroup 2 from 2003 to 2007. The decomposition of  $S_a$  explains that the weights assigned to inequalities between poorer and richer regions decrease from 2003 to 2007 (−0.007601). There is also a decrease in the weights of inequalities between poorer regions (−0.021376), whereas inequalities between richer regions are more weighted in 2007 than in 2003 (0.022067). These population change effects are partially caused by the intra-EU migration from new Central and Eastern EU member countries to old Western EU member countries. The regime of free movement of individuals within the EU fosters mobility in most of the new EU member countries, leading to outflows of the population in new EU member countries and inflows of immigrants from new EU member countries to old EU member countries (Coleman 2008; Zaiceva and Zimmermann 2008).<sup>19</sup> Migration flows from outside the EU also have an impact on the unbalanced population change across EU regions, since richer regions of old EU member countries are the most attractive destinations for migrants from countries outside the EU (Abel and Sander 2014).

The decrease in the weights of inequalities between poorer regions is confirmed over the 2007–2011 period (−0.019641), while the weights of inequalities between richer regions still increase from 2007 to 2011 (0.025696). Re-ranking between poorer and richer regions in 2007–2011 (0.029612) is a non-negligible component of overall re-ranking, suggesting that poorer and richer regions swap their positions more frequently than during 2003–2007. The positive value of the between-group component of  $D_a$  (0.135070) indicates that absolute income differences between poorer and richer regions decrease over the 2007–2011 period. This means that poorer regions fare better than richer regions during 2007–2011. Absolute income differences diminish between poorer regions (0.089708) whereas increase between richer regions (−0.045780).

Figure 2 shows the partition of regions when the number of subgroups is  $k = 3$ . Subgroup 3, including poorer regions, is mainly composed of Central and Eastern EU regions.

<sup>19</sup> UK, Ireland and Sweden implemented a free movement policy for citizens of new EU member countries since 2004, while other EU-15 member countries retained some temporary restrictions (Kahanec and Zimmermann 2009).



**Fig. 2** Map of NUTS 3 regions divided in three subgroups in 2003

**Table 3** Decomposition of the change in absolute inequality over the 2003–2011 period, with  $k = 3$  subgroups

Period	Subgroup component	$G_a(\mathbf{p}_t; \mathbf{y}_t)$	$G_a(\mathbf{p}_{t+4}; \mathbf{y}_{t+4})$	$\Delta G_a$	$S_a$	$R_a$	$D_a$
03–07	1	0.289128	0.359257	0.070129	0.010100	0.015709	−0.044320
	2	0.246211	0.339494	0.093283	0.004484	0.032700	−0.056100
	3	0.243453	0.280278	0.036825	−0.011885	0.023137	−0.025573
	12	1.284622	1.508442	0.223820	0.029072	0.004971	−0.189778
	13	1.786833	1.989562	0.202730	−0.014130	0.000000	−0.216859
	23	1.352241	1.457894	0.105653	−0.024549	0.013047	−0.117156
	Total	5.202488	5.934927	0.732440	−0.006909	0.089565	−0.649784
07–11	1	0.332940	0.388945	0.056005	0.012096	0.018822	−0.025087
	2	0.298065	0.374710	0.076645	0.001636	0.079408	0.004399
	3	0.271509	0.245584	−0.025925	−0.008958	0.021702	0.038668
	12	1.486944	1.561619	0.074674	0.029951	0.011442	−0.033282
	13	2.024418	1.971452	−0.052967	0.000015	0.000001	0.052983
	23	1.521051	1.369130	−0.151922	−0.024334	0.013729	0.141317
	Total	5.934927	5.911439	−0.023489	0.010405	0.145104	0.178998



Subgroup 3 comprises Southern Italy and South-Western Iberian Peninsula. Only few regions of North-Western EU are included in subgroup 3. Subgroup 2 includes middle-income regions and is mainly formed by Scandinavian, French, German, Ireland and UK regions. Subgroup 1 includes richer regions, which in general are not spatially adjacent. Only some regions of South-Eastern France, Northern Italy, Western Austria and Southern Germany constitute a spatial cluster.<sup>20</sup> Since the partition of regions in three subgroups in 2007 is similar to that in 2003 shown in Fig. 2, the map showing the partition of regions in 2007 is shown in “Appendix 2”.

Table 3 shows the subgroup decomposition of components of absolute inequality change when regions are divided into three subgroups. Absolute income disparities increase within and between the three subgroups over the 2003–2007 period. Re-ranking between the regions in the richest subgroup and those in the poorest subgroup is negligible (0.000000),<sup>21</sup> suggesting that disparities between richer and poorer regions are large enough to prevent these regions from swapping their positions, at least over a 4-year period. Population change reduces the impact of inequality within the poorest subgroup (−0.011885) and those of the inequalities between this subgroup and the other two subgroups, suggesting that the population sizes of poorer regions are relatively lower in 2007. Since the poorest subgroup comprises most of regions of new EU member countries, the outflows in the population of these Central and Eastern EU countries explain at least in part the negative effect of population change on the inequality components involving subgroup 3.

Table 3 shows that absolute disparities between poorer regions and other regions decrease over the 2007–2011 period. In addition, absolute income differences diminish in the poorest subgroup (0.038668) and in the middle-income subgroup (0.004399). The largest reduction in absolute income differences occurs between the poorest subgroup and the middle-income subgroup (0.141317). Re-ranking between poorer and richer regions is negligible (0.000001) also in 2007–2011. Population change reduces the weights of inequality within the poorest subgroup (−0.008958) and of inequality between the poorest subgroup and the middle-income subgroup (−0.024334), while the impact of population change on absolute inequality between poorer and richer regions is almost negligible (0.000015) over the 2007–2011 period.

We now turn to explore the decomposition of the change in relative inequality over the 2003–2011 period. Table 4 shows the decomposition when regions are divided in two subgroups. The disproportionate income growth component (0.015951) plays a major role in the reduction in relative inequality from 2003 to 2007. This suggests that the per capita incomes of regions in the lower part of the distribution grow proportionally more than those of regions in the upper part. The equalizing effect of  $D$  is reinforced by the population change effect (−0.001508), even though  $S$  is far less important than  $D$ . The re-ranking component (0.003630) increases relative inequality from 2003 to 2007; however, this disequalizing effect is more than offset by the equalizing effects of  $S$  and  $D$ . The within-group and between-group components of  $\Delta G$  play different roles in the reduction of relative inequality from 2003 to 2007. The inequality contribution of the richest subgroup slightly increases (0.000017) whereas that of the poorest subgroup decreases (−0.003497). The reduction in the between-group inequality component (−0.010349) plays the most important role in the overall inequality reduction. The causes of the changes in the

<sup>20</sup> In particular, subgroup 1 includes several regions of the Four Motors for Europe: Baden-Württemberg (Germany), Catalonia (Spain), Lombardy (Italy), and Rhône-Alpes (France).

<sup>21</sup> A value of 0.000000 shown in tables means that the observed value is <0.000001.

**Table 4** Decomposition of the change in relative inequality over the 2003–2011 period, with  $k = 2$  subgroups

Period	Subgroup component	$G(\mathbf{p}_t, \mathbf{y}_t)$	$G(\mathbf{p}_{t+4}, \mathbf{y}_{t+4})$	$\Delta G$	$S$	$R$	$D$
03–07	1	0.041306	0.041323	0.000017	0.000682	0.001329	0.001994
	2	0.039479	0.035981	−0.003497	−0.001051	0.002147	0.004594
	12	0.172332	0.161983	−0.010349	−0.001139	0.000154	0.009364
	Total	0.253116	0.239287	−0.013829	−0.001508	0.003630	0.015951
07–11	1	0.039694	0.044505	0.004810	0.000847	0.002226	−0.001737
	2	0.036142	0.033933	−0.002210	−0.000932	0.002424	0.003702
	12	0.163450	0.158292	−0.005159	−0.000487	0.001186	0.005858
	Total	0.239287	0.236729	−0.002558	−0.000571	0.005835	0.007822

subgroup inequality components are explained by observing the subgroup components of  $S$ ,  $R$  and  $D$ . The re-ranking within subgroup 2 (0.002147) causes most of the overall re-ranking effect over the 2003–2007 period. Within subgroup 2, the re-ranking effect partially offsets the disproportionate income growth effect (0.004594) which causes the reduction in the inequality contribution of subgroup 2. Disproportionate growth between per capita incomes of richer regions reduces the relative income disparities among richer regions from 2003 to 2007 (0.001994), however this equalizing effect is more than offset by the disequalizing effects of re-ranking (0.001329) and population change (0.000682). The decrease in relative income disparities between poorer and richer regions (0.009364) is crucial for the reduction in the between-group inequality component (−0.010349), but also

**Table 5** Decomposition of the change in relative inequality over the 2003–2011 period, with  $k = 3$  subgroups

Period	Subgroup component	$G(\mathbf{p}_t, \mathbf{y}_t)$	$G(\mathbf{p}_{t+4}, \mathbf{y}_{t+4})$	$\Delta G$	$S$	$R$	$D$
03–07	1	0.014067	0.014485	0.000418	0.000335	0.000637	0.000554
	2	0.011979	0.013688	0.001709	0.000111	0.001325	−0.000272
	3	0.011845	0.011300	−0.000544	−0.000540	0.000938	0.000942
	12	0.062501	0.060818	−0.001683	0.000866	0.000201	0.002750
	13	0.086935	0.080216	−0.006719	−0.000984	0.000000	0.005734
	23	0.065790	0.058780	−0.007010	−0.001296	0.000529	0.006243
	Total	0.253116	0.239287	−0.013829	−0.001508	0.003630	0.015951
07–11	1	0.013424	0.015576	0.002152	0.000421	0.000757	−0.000974
	2	0.012018	0.015006	0.002988	0.000003	0.003193	0.000208
	3	0.010947	0.009835	−0.001112	−0.000401	0.000873	0.001584
	12	0.059951	0.062536	0.002585	0.000943	0.000460	−0.001182
	13	0.081621	0.078949	−0.002673	−0.000329	0.000000	0.002343
	23	0.061326	0.054828	−0.006498	−0.001208	0.000552	0.005843
	Total	0.239287	0.236729	−0.002558	−0.000571	0.005835	0.007822

the between-group population change component ( $-0.001139$ ) plays a non-negligible role by overcoming the disequalizing effect of the between-group re-ranking component ( $0.000154$ ). It is worth mentioning that relative disparities between per capita incomes generally decrease in 2003–2007 whereas the opposite occurs when considering absolute disparities between per capita incomes (see Table 2). This highlights that absolute and relative inequalities are based on different concepts of inequality, whose measures give different results since the same distributional changes are analyzed from different perspectives.

The reduction in relative inequality over the 2007–2011 sub-period is small ( $-0.002558$ ), and is the result of the interaction between  $S$ ,  $R$  and  $D$  instead of slight changes in the distributions of income and population among EU regions. The disequalizing effect of  $R$  ( $0.005835$ ) offsets most of the equalizing effect of  $D$  ( $0.007822$ ), while the population change causes a very small reduction in relative inequality ( $-0.000571$ ). Disproportionate income growth causes an increase in relative income disparities between richer regions in subgroup 1 ( $-0.001737$ ), where this disequalizing effect is reinforced by the re-ranking effect ( $0.002226$ ) and the population change effect ( $0.000847$ ); that is, each of the three causes of change in relative inequality increases the inequality contribution of the richest subgroup. In subgroup 2, relative income differences decrease during the 2007–2011 period ( $0.003702$ ), causing a decrease in relative inequality between poorer regions. The population change reduces the weight of inequality within the poorest subgroup ( $-0.000932$ ) whereas augments that of inequality in the richest subgroup ( $0.000847$ ). The decrease in relative income disparities between poorer and richer regions ( $0.005858$ ) is consistent with the decrease in absolute income disparities between the two subgroups shown in 2007–2011. This means that inequality between poorer and richer regions decreases irrespective of the concept of inequality used to measure the changes in income distribution.

Table 5 shows the decomposition of the change in relative inequality when regions are divided in three subgroups. Disproportionate income growth increases inequality in subgroup 2 over the 2003–2007 period, suggesting that relative income disparities between middle-income regions increase from 2003–2007. This finding underlines the role of a finer partition of the distribution, since considering three subgroups instead of two reveals that relative inequality between middle-income regions increases. Re-ranking between the poorest subgroup and the richest subgroup is negligible ( $0.000000$ ) in 2003–2007. This result is obviously in line with that observed in the decomposition of the change in absolute inequality, since the re-ranking component reflects the movements of regions in the distribution and is not influenced by the concept of inequality adopted in the analysis. Population change reduces the impact of relative inequalities within the poorest subgroup ( $-0.000540$ ) over the 2003–2007 period. The weights of relative inequalities between the poorest subgroup and the other two subgroups decrease too. These results explain that the fall in the population shares of poorer regions diminishes the effect of relative inequalities between poorer regions and other regions (i.e., richer and middle-income regions). The role played by population change is the same over the 2007–2011 period, confirming the reduction in the weight of poorer regions in determining overall inequality in both relative and absolute terms. Disproportionate income growth increases inequalities among richer regions ( $-0.000974$ ) and inequalities between richer and middle-income regions ( $-0.001182$ ) during 2007–2011. This means that income disparities between regions in the upper part of the distribution increase not only in absolute terms (see Table 3) but also in relative terms.

## 5 Conclusions

This article decomposes the changes in inequality and welfare between EU regions at the NUTS 3 level over the 2003–2011 period. From a methodological point of view, the article contributes to the literature on inequality measurement by adjusting the Mussini and Grossi (2015) decomposition technique in order to break down changes in both absolute and relative inequalities between regions. Broadly speaking, these decompositions can be applied to analyze the inequality trend between countries or territorial units at any level. The decompositions explain the changes in relative and absolute inequalities by separating the roles of population change, re-ranking and per capita income growth. The various components of inequality change can be further broken down by subgroup, showing the extent to which the within-group and between-group contributions are important for changes in absolute and relative inequalities. Moreover, the decomposition of the change in absolute inequality is used to obtain the decomposition of a measure of welfare change suggested by Silber and Weber (2005). This decomposition reveals the effects of changes in per capita income disparities, regional population sizes and ranking of regions.

The above described decomposition tool is applied to explore inequality and welfare changes between EU NUTS 3 regions over the sub-periods 2003–2007 and 2007–2011. Welfare increases over the 2003–2011 period, especially from 2003 to 2007. Most of the increase in welfare is caused by the growth in average income. Changes in absolute inequality play a minor role in explaining welfare changes. This occurs especially during 2007–2011 because of the offsetting interaction between the equalizing effect of per capita income growth and the disequalizing effects of re-ranking and population change. This explains that a small change in inequality may conceal changes in population, income distribution and ranking of regions. While absolute income disparities between regions augment during 2003–2007, relative income disparities decrease. Both absolute and relative income disparities decrease in 2007–2011, suggesting that inequalities between EU regions overall decrease during 2007–2011. However, the decrease in inequality is not homogeneous in the EU. The subgroup decomposition explains that absolute and relative inequalities decrease between poorer regions but increase between richer regions. Poorer regions are mainly those of new EU member countries, the Central and Eastern European countries which joined the EU in 2004 or later. Richer regions are situated in old EU member countries, that is the countries which accessed the EU before 2004. Relative income disparities between poorer and richer regions decrease over the entire 2003–2011 period. This means that per capita incomes of poorer regions in 2003 increase proportionally more than those of richer regions in 2003 during 2003–2011. Policy-wise, EU cohesion actions would be strengthened in reducing inequality if they fostered convergence in per capita income between the regions of new EU member countries and the regions of old EU member countries. Even though poorer regions gain proportionally more than richer regions in 2003–2011, the re-ranking between poorer and richer regions plays a minor role.

The last remark concerns the non-negligible role played by population change in explaining changes in both relative and absolute inequalities. Population change augments the impact of inequality between richer regions on overall inequality, whereas the impact of inequality between poorer regions is reduced. This population change effect is explained in part by migration flows across the EU, from poorer regions of new EU countries to richer regions of old EU countries, and by migration flows from countries outside the EU to richer regions of old EU countries, increasing the population sizes of these regions.

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## Appendix 1: Calculation of $\mathbf{A}_{h,t+1}$ , $\mathbf{D}_{a,h}$ , $\mathbf{A}_{gh,t+1}$ , $\mathbf{D}_{a,gh}$

Let  $\mathbf{w}_{h,t}$  be the  $k \times 1$  vector with nonzero elements equal to 1 in the corresponding places of the  $t$  per capita incomes of the regions belonging to subgroup  $h$  (with  $h = 1, \dots, r$ ) in  $\mathbf{y}_t$ . The  $k \times k$  matrix  $\mathbf{W}_{h,t} = \mathbf{w}_{h,t} \mathbf{w}_{h,t}^T$  has its  $(i, j)$ -th element equal to 1 if and only if the  $(i, j)$ -th element of  $\mathbf{A}_t$  is the difference between the per capita incomes of two regions belonging to subgroup  $h$ , otherwise the  $(i, j)$ -th element of  $\mathbf{W}_{h,t}$  is equal to 0. By using the Hadamard product,<sup>22</sup> the pairwise differences between the per capita incomes of the regions in subgroup  $h$  are drawn from  $\mathbf{A}_t$ :

$$\mathbf{A}_{h,t} = \mathbf{W}_{h,t} \odot \mathbf{A}_t. \quad (23)$$

The pairwise differences between the  $t + 1$  per capita incomes of subgroup  $h$  in  $\mathbf{A}_{t+1|t}$  occupy the same entries in which the pairwise differences between the  $t$  per capita incomes of subgroup  $h$  are arranged in  $\mathbf{A}_t$ . Thus,  $\mathbf{W}_{h,t}$  can also be used to select the pairwise differences between the  $t + 1$  per capita incomes of subgroup  $h$  from  $\mathbf{A}_{t+1|t}$ :

$$\mathbf{A}_{h,t+1|t} = \mathbf{W}_{h,t} \odot \mathbf{A}_{t+1|t}. \quad (24)$$

$\mathbf{w}_{h,t+1}$  being the  $k \times 1$  vector with nonzero elements equal to 1 in the corresponding places of the  $t + 1$  per capita incomes belonging to regions of subgroup  $h$  in  $\mathbf{y}_{t+1}$ , the  $k \times k$  matrix  $\mathbf{W}_{h,t+1} = \mathbf{w}_{h,t+1} \mathbf{w}_{h,t+1}^T$  selects the pairwise differences between the  $t + 1$  per capita incomes of regions within subgroup  $h$  from  $\mathbf{A}_{t+1}$ :

$$\mathbf{A}_{h,t+1} = \mathbf{W}_{h,t+1} \odot \mathbf{A}_{t+1}. \quad (25)$$

Since  $\mathbf{D}_a = \mathbf{A}_t - \mathbf{A}_{t+1|t}$ , the Hadamard product between  $\mathbf{W}_{h,t}$  and  $\mathbf{D}_a$  is the matrix whose nonzero elements are the elements of  $\mathbf{D}_a$  measuring the change in the absolute disparities between the per capita incomes of regions within subgroup  $h$ :

$$\mathbf{D}_{a,h} = \mathbf{A}_{h,t} - \mathbf{A}_{h,t+1|t} = \mathbf{W}_{h,t} \odot (\mathbf{A}_t - \mathbf{A}_{t+1|t}) = \mathbf{W}_{h,t} \odot \mathbf{D}_a. \quad (26)$$

The  $k \times k$  matrix  $\mathbf{W}_{gh,t} = \mathbf{w}_{g,t} \mathbf{w}_{h,t}^T + \mathbf{w}_{h,t} \mathbf{w}_{g,t}^T$  shows nonzero elements equal to 1 in the entries corresponding to the pairwise differences between the per capita incomes of subgroup  $g$  and those of subgroup  $h$  in both  $\mathbf{A}_t$  and  $\mathbf{A}_{t+1|t}$ ; hence,  $\mathbf{W}_{gh,t}$  selects the between-group pairwise differences from both the matrices:

$$\mathbf{A}_{gh,t} = \mathbf{W}_{gh,t} \odot \mathbf{A}_t \quad (27)$$

and

$$\mathbf{A}_{gh,t+1|t} = \mathbf{W}_{gh,t} \odot \mathbf{A}_{t+1|t}. \quad (28)$$

<sup>22</sup> Let  $\mathbf{X}$  and  $\mathbf{Y}$  be  $n \times n$  matrices. The Hadamard product  $\mathbf{X} \odot \mathbf{Y}$  is defined as the  $n \times n$  matrix with the  $(i, j)$ -th element equal to  $x_{ij}y_{ij}$ . The Hadamard product is the element-by-element matrix product (Abadir and Magnus 2005).

The Hadamard product between  $\mathbf{W}_{gh,t}$  and  $\mathbf{D}_a$  selects the elements of  $\mathbf{D}_a$  measuring the change in the disparities between the per capita incomes of regions in subgroup  $g$  and the per capita incomes of regions in subgroup  $h$ :

$$\mathbf{D}_{a,gh} = \mathbf{A}_{gh,t} - \mathbf{A}_{gh,t+1|t} = \mathbf{W}_{gh,t} \odot (\mathbf{A}_t - \mathbf{A}_{t+1|t}) = \mathbf{W}_{gh,t} \odot \mathbf{D}_a. \quad (29)$$

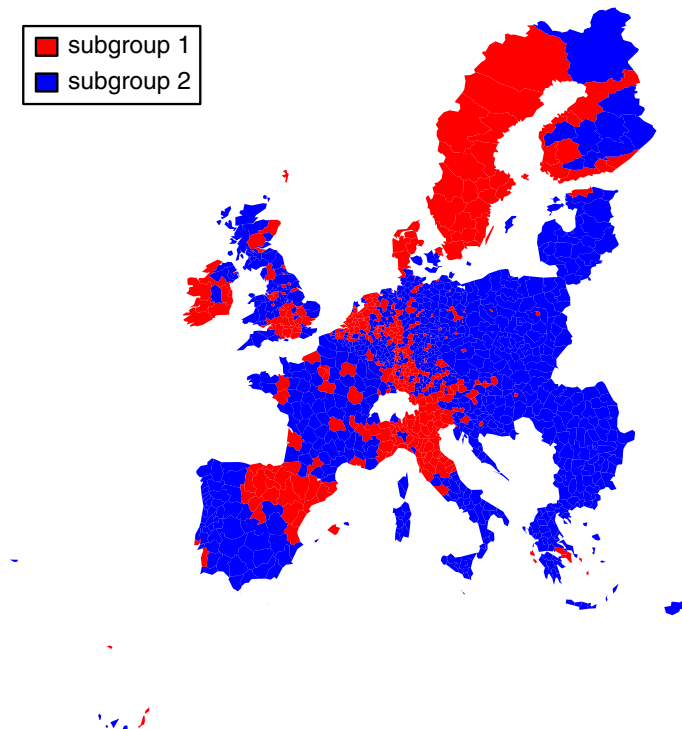
The  $k \times k$  matrix  $\mathbf{W}_{gh,t+1} = \mathbf{w}_{g,t+1} \mathbf{w}_{h,t+1}^T + \mathbf{w}_{h,t+1} \mathbf{w}_{g,t+1}^T$  has nonzero elements equal to 1 in the entries corresponding to the pairwise differences between the per capita incomes of subgroup  $g$  and the per capita incomes of subgroup  $h$  in  $\mathbf{A}_{t+1}$ . Thus, the matrix

$$\mathbf{A}_{gh,t+1} = \mathbf{W}_{gh,t+1} \odot \mathbf{A}_{t+1} \quad (30)$$

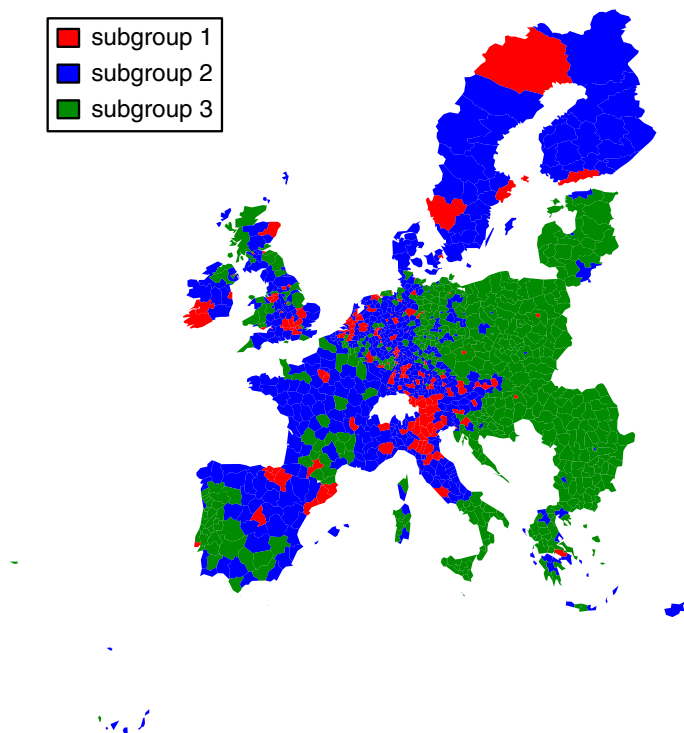
comprises the pairwise differences between the  $t + 1$  per capita incomes of subgroup  $g$  and those of subgroup  $h$ .

## Appendix 2: Maps of EU Regions Divided in Two and Three Subgroups in 2007

Figure 3 shows the partition of regions in 2007 for  $k = 2$ . Figure 4 shows the partition of regions in 2007 for  $k = 3$ .



**Fig. 3** Map of NUTS 3 regions divided in two subgroups in 2007



**Fig. 4** Map of NUTS 3 regions divided in three subgroups in 2007

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