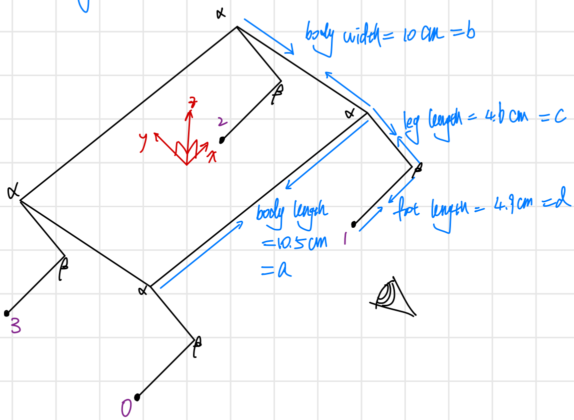
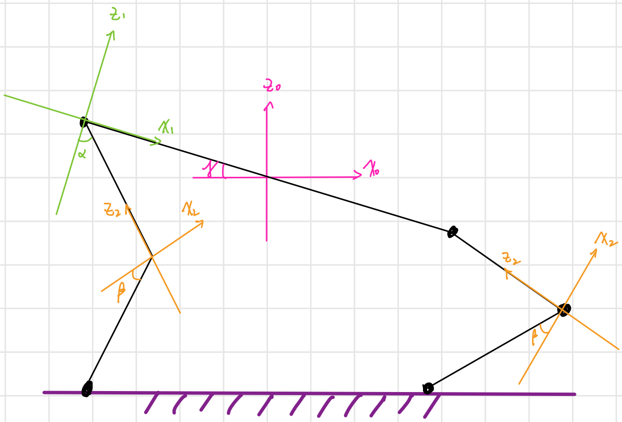
# Frame Setting




Since end-effector (foot) cannot move along y axis,
we mainly focus on x-z plane

- end-effector index

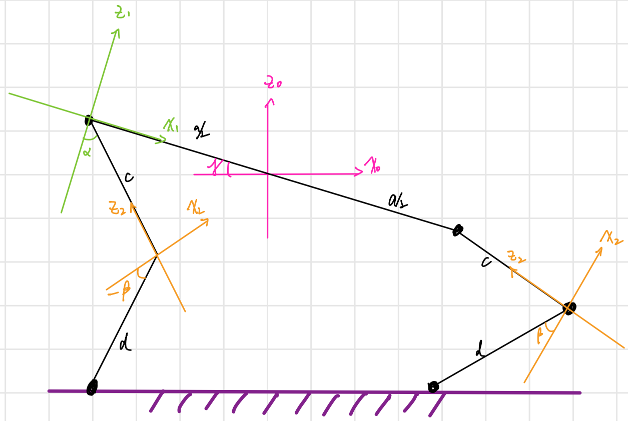



body frame $\begin{cases} x \text{ is horizontal, passing through the center of body} \\ z \text{ is vertical, passing through the center of body} \end{cases}$

shoulder frame $\begin{cases} x \text{ is parallel to body, across the joint} \\ z \text{ is vertical to body, across the joint} \end{cases}$

knee frame

- Forward kinematics




**body-shoulder transformation**

front shoulder

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix} & \begin{bmatrix} -\frac{a}{2}\cos\gamma \\ \frac{a}{2}\sin\gamma \end{bmatrix} \\ 0 \quad 0 & 1 \end{bmatrix}$$

rear shoulder

$$T_{01} = \begin{bmatrix} R_{01} & p_{01} \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \cos\gamma & \sin\gamma \\ -\sin\gamma & \cos\gamma \end{bmatrix} & \begin{bmatrix} \frac{a}{2}\cos\gamma \\ -\frac{a}{2}\sin\gamma \end{bmatrix} \\ 0 \quad 0 & 1 \end{bmatrix}$$

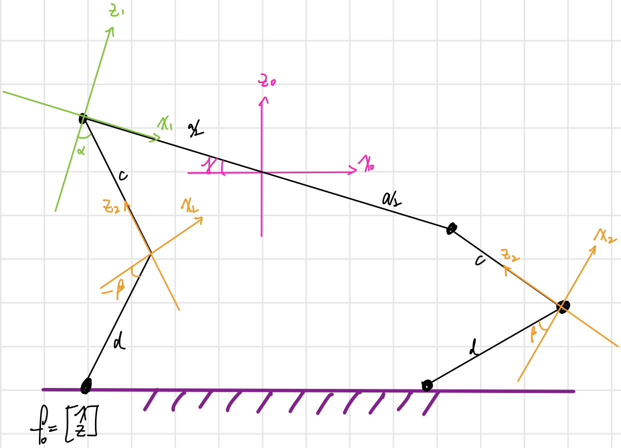
**shoulder-knee transformation**

$$T_{12} = \begin{bmatrix} R_{12} & p_{12} \\ 0^T & 1 \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} & \begin{bmatrix} c\cdot\sin\alpha \\ -c\cdot\cos\alpha \end{bmatrix} \\ 0 \quad 0 & 1 \end{bmatrix}$$

**knee-foot transformation**

foot coordinate in knee frame is $\begin{bmatrix} -d\cdot\cos\beta \\ d\cdot\sin\beta \end{bmatrix}$

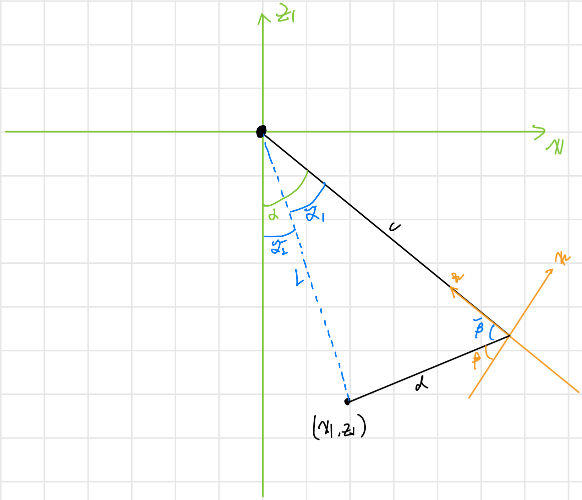
· Inverse Kinematics




$f_0 = \begin{bmatrix} x_0 \\ z_0 \\ 1 \end{bmatrix}$ is the augmented cordinate of end-effector in body frame

$f_1 = \begin{bmatrix} x_1 \\ z_1 \\ 1 \end{bmatrix}$ is the augmented cordinate of end-effector in shoulder frame

$f_0 = T_{01} f_1 \qquad f_1 = T_{01}^{-1} \cdot f_0$




$L = \sqrt{x_1^2 + z_1^2}$

① $d^2 = c^2 + L^2 - 2c \cdot L \cdot \cos\tilde{\alpha}_1$

$\tilde{\alpha}_1 = \arccos\left(\frac{c^2 + L^2 - d^2}{2cL}\right)$

$\tilde{\alpha}_2 = \arcsin\left(x_1 / L\right)$

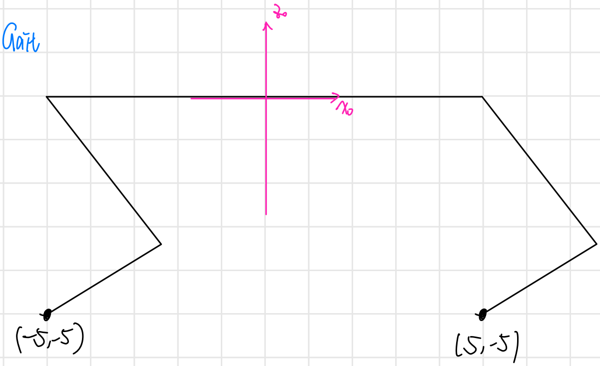
$\alpha = \tilde{\alpha}_1 + \tilde{\alpha}_2$

② $L^2 = c^2 + d^2 - 2cd\cos\tilde{\beta}$

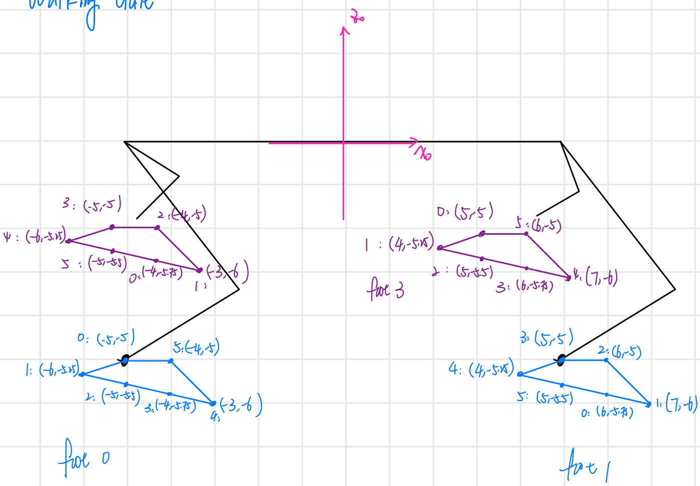
$\tilde{\beta} = \arccos\left(\frac{c^2 + d^2 - L^2}{2cd}\right)$

$\beta = \frac{\pi}{2} - \tilde{\beta}$

- Standing Gait




- Walking Gait




- Stepping Gait