# DSP

*** Course Content:**

1. Introduction to Digital Signal Processing.
2. Z-Transform (Z.T) & (I.Z.T).
3. Realization of D.T Systems.
4. DFT.
5. FFT.
6. Digital Filter Design (FIR & IIR).

---

*** References**


1. J. Proakis, "Digital Signal Processing: Principles, Algorithms & Applications".
2. Vinay K. Ingle, "Digital Signal Processing using MATLAB"


---

*** Grading System:**

* **Intro. to DSP:**

[1] what is the diff. bet. analog & digital signals?

[2] Examples of A&D Systems.

[3] what we need to remember from signals course?

[4] why we need Transformation?

[5] **Examples of Transformers:**

① **L.T**

$$L[f(t)] = \int_0^\infty f(t)\, e^{-st}\, dt = F(s)$$

$\boxed{\sigma = 0}$

② **F.T**

$$F[x(t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = X(j\omega)$$

③ **Z.T**

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

what is the diff. bet. L.T, F.T & Z.T?

④ DCT
⑤ DST
⑥ W.T
⑦ DFT
⑧ FFT

| Cont. | Discrete |
|---|---|
| $x(t)$ | $x[n]$ |
| $t$ | $n$ |
| $T$ | $N$ |
| $f$ | $F = \frac{f}{f_s}$ |
| $f = \frac{1}{T}$ | $F = \frac{K}{N}$ |
| $\omega = 2\pi f$ | $\Omega = 2\pi F$ |
| $\int$ | $\Sigma$ |

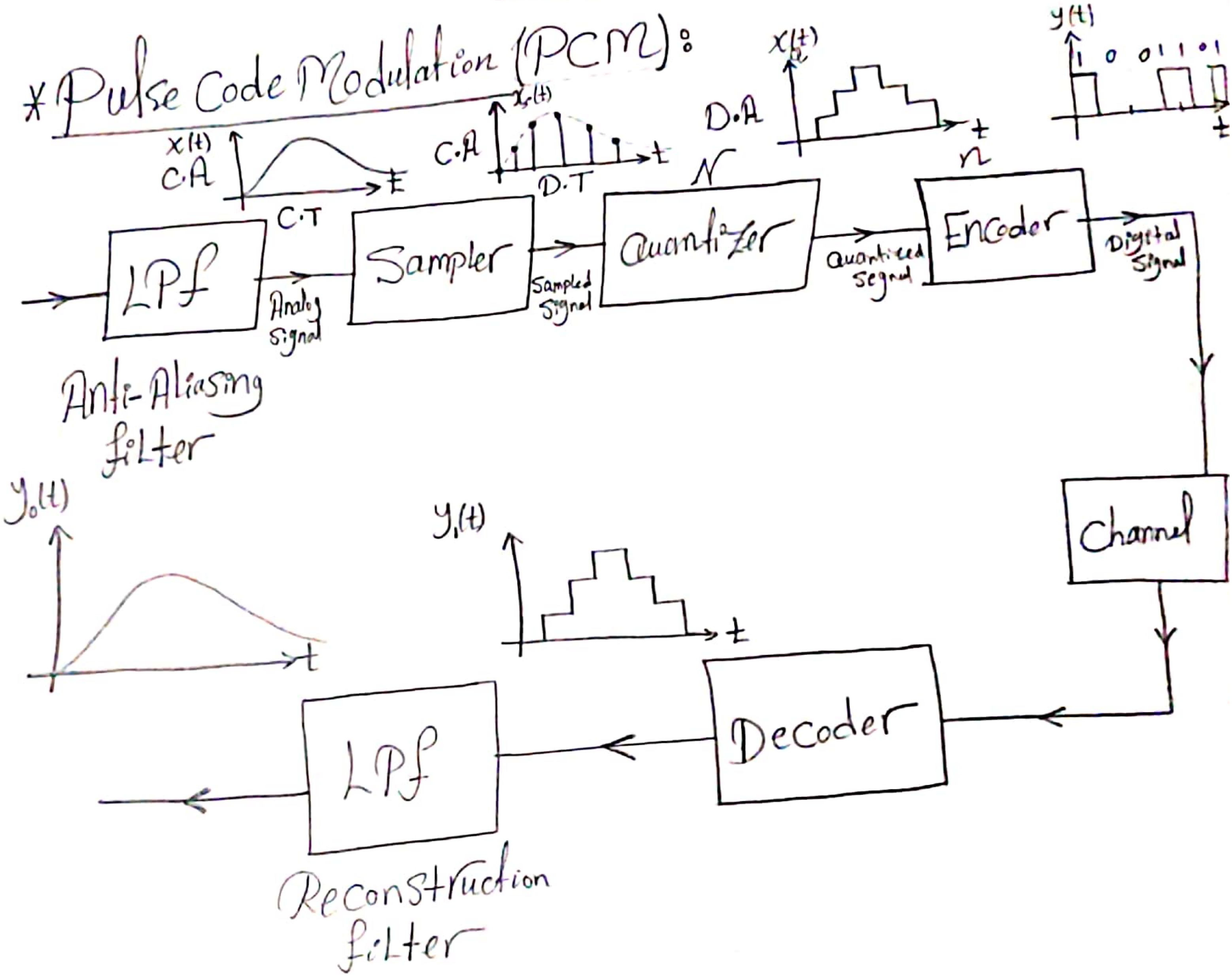
$\frac{d}{dt}$ diff.

* **System Representation :**

1. Impulse response. $h(n)$
2. Transfer fn. $H(z)$
3. Diff. eqn.
4. freq. response. $H(e^{j\omega})$

# DSP

## * Pulse Code Modulation (PCM):




### * Note

$f_s \geq 2B$

$f_s$ → Sampling rate

$B$ → B.W of analog signal

$N = 2^n$

$n$ → No. of bits.

$N$ → No. of Quantization levels

$\boxed{N \uparrow \rightarrow \text{Quantization error} \downarrow \rightarrow n \uparrow \rightarrow \underline{BW} \uparrow}$

# * System classification:

## 1 Static & Dynamic

* Static
- O/P depends only on the present i/P.
- No memory.

Ex: $y(n) = 4x(n) + (x(n))^3$

* Dynamic
- O/P depends on the Past &/ future i/P.
- has memory

Ex: $y(n) = 4x(n) - x(n-1)$

## 2 Linear & Non-Linear

Satisfy Superposition principle. $\Rightarrow$ linear

If $x_1(n) \longrightarrow y_1(n)$

$x_2(n) \longrightarrow y_2(n)$

$\therefore \alpha x_1(n) + \beta x_2(n) \longrightarrow \alpha y_1(n) + \beta y_2(n)$

Not Satisfy S.P.P $\Rightarrow$ N.L

## 3 Time Variant & Time-invariant

T.I.V

$x_1(n) \longrightarrow y_1(n)$

$x_1(n-k) \longrightarrow y_1(n-k)$

fixed system.

T.V

Not fixed.

## 4 Causal & Non-Causal

**Causal**

Response depends only on the present &/ past value of the i/p

$$h(n) = 0 \text{ for } n < 0$$

**Non-Causal**

$$h(n) \neq \text{ for } n < 0$$

## 5 Stable & Non-stable

**Stable**

BIBO

$$y(n) = n.x(n)$$

$$\sum_{n=-\infty}^{\infty} |h(n)| < 0$$

**Unstable**

$$y(n) = n.x(n)$$

$$y(n) \to \infty \text{ as } n \to \infty$$

# DSP

## Z-Transform

$$x(n) \xrightarrow{Z.T} X(Z)$$

Time Domain (T.D) — Z-Domain (Z.D)

**L.T :** $$X(s) = \int_0^{\infty} x(t)\, e^{-st}\, dt$$

$s = \sigma + j\omega$

$\sigma$ → AMP.

$j\omega$ → EXPonential (oscillation)

Fourier Transform is a special case of LaPlace Transform. $\boxed{s = j\omega}$

L.T : is used for System analysis

F.T : " " " Signal "

**F.T :**

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$x(t) \xrightarrow{F.T} X(f)$$

TD — F.D

$$\boxed{X(Z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}}$$

$$\boxed{Z = e^{j2\pi F}}$$

$$\boxed{Z = e^{ST}}$$

L.T for C.T System.

Z.T : " D.T " .

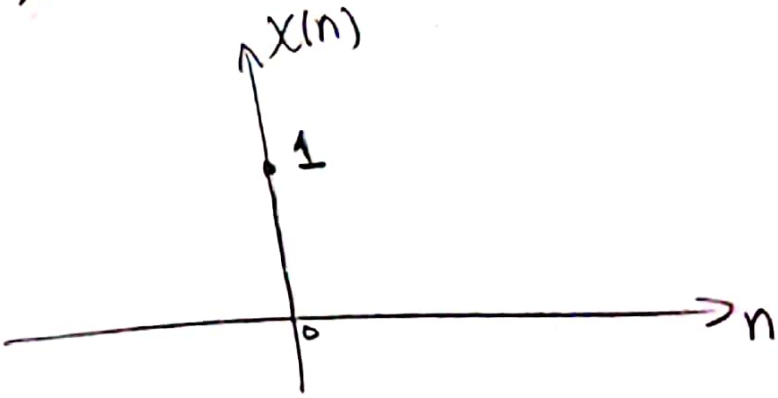
Design, Analysis, Representation

---

**EX1:** $x(n) = \delta(n)$ : impulse fn.

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$$

$$X(Z) = 1 \cdot Z^{0} = \boxed{1}$$




$$\boxed{\delta(n) \xrightarrow{Z.T} 1}$$

**Roc**

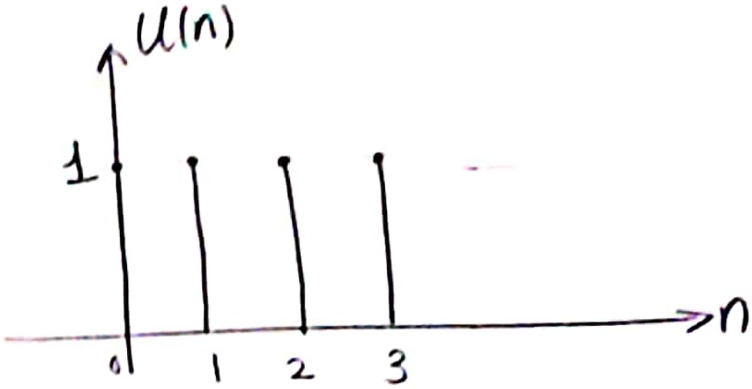
ايه ال Cond. اللي يخلي $\sum$ بتاعي
لقيمة finite وليس $\infty$

Roc: all Z-domain.

لا يوجد شرط على $Z$

Ex2: $x(n) = u(n)$

$X(z) = (1)z^0 + (1)z^{-1} + (1)z^{-2} + \cdots$




$$X(z) = 1 + \frac{1}{z} + \left(\frac{1}{z}\right)^2 + \left(\frac{1}{z}\right)^3 + \cdots$$

$$S = \frac{a}{1-r} = \frac{1}{1 - 1/z}$$

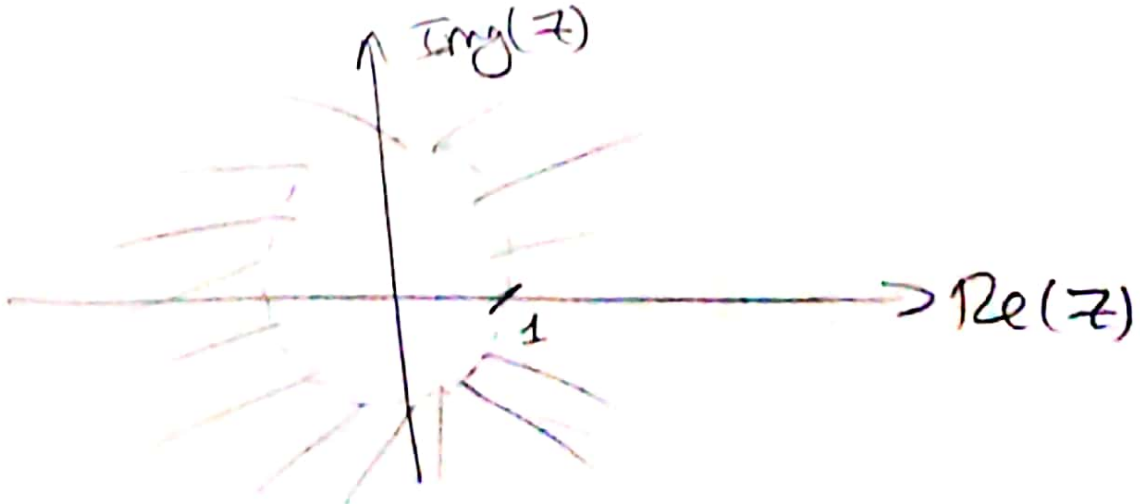
$$X(z) = \frac{z}{z-1}$$

$\boxed{|r| < 1}$

$\left|\frac{1}{z}\right| < 1$

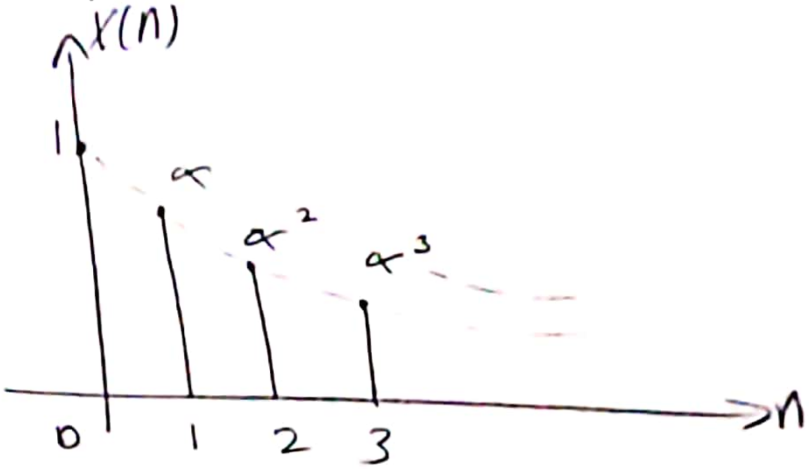
$$\boxed{u(n) \xrightarrow{z.T} \frac{z}{z-1}}$$

Roc: $|z| > 1$




Roc

<u>EX3:</u> $x(n) = \alpha^{n} u(n)$




$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= 1 + \alpha^{1} z^{-1} + \alpha^{2} z^{-2} + \cdots$$

$$= 1 + \frac{\alpha}{z} + \left(\frac{\alpha}{z}\right)^{2} + \left(\frac{\alpha}{z}\right)^{3} + \cdots$$

$r = \dfrac{\alpha}{z}$ $\qquad |r| < 1$

$\left|\dfrac{\alpha}{z}\right| < 1$ $\qquad \boxed{|z| > |\alpha|}$

$$X(z) = \frac{1}{1 - \frac{\alpha}{z}}$$

$\boxed{X(z) = \dfrac{z}{z-\alpha}}$ $\qquad$ R.O.C $\boxed{|z| > |\alpha|}$

$$\boxed{\alpha^{n} u(n) \xrightarrow{Z.T} \frac{z}{z-\alpha}}$$

# Common Signals

| $x(n)$ | $X(z)$ | R.o.c |
|---|---|---|
| $\delta(n)$ | $1$ | all $z$-domain |
| $u(n)$ | $\frac{z}{z-1}$ | $\lvert z\rvert > 1$ |
| $\alpha^n u(n)$ | $\frac{z}{z-\alpha}$ | $\lvert z\rvert > \lvert\alpha\rvert$ |
| $(-\alpha)^n u(n)$ | $\frac{z}{z+\alpha}$ | $\lvert z\rvert > \lvert\alpha\rvert$ |
| $n(\alpha)^n u(n)$ | $\frac{\alpha z}{(z-\alpha)^2}$ | $\lvert z\rvert > \lvert\alpha\rvert$ |
| $\cos(\Omega n)u(n)$ | $\frac{z^2 - z\cos\Omega}{z^2 - 2z\cos\Omega + 1}$ | $\lvert z\rvert > 1$ |
| $\sin(\Omega n)\,u(n)$ | $\frac{z\sin(\Omega)}{z^2 - 2z\cos\Omega + 1}$ | $\lvert z\rvert > 1$ |

# Properties of Z-Transform.

| $x(n)$ | $X(z)$ |
|---|---|
| $x(n-N)$ | $z^{-N} X(z)$ |
| $x(-n)$ | $X(1/z)$ |
| $\alpha^n x(n)$ | $X(z/\alpha)$ |
| $n\,x(n)$ | $-z \frac{d}{dz} X(z)$ |
| $\cos(\Omega n)\ x(n)$ | $\frac{1}{2}\{X(ze^{-j\Omega})+X(ze^{j\Omega})\}$ |
| $\sin(\Omega n)\ x(n)$ | $\frac{1}{2j}\{X(ze^{-j\Omega})-X(ze^{j\Omega})\}$ |
| $x(n) * h(n)$ | $X(z) \cdot H(z)$ |

* obtain Z-transform of: & R.o.c

**1** $x_1(n) = 2^{n+2}\, u(n)$

$= 2^2\, 2^n\, u(n)$

$$\boxed{X_2(z) = 4\,\frac{z}{z-2}}$$

$$\boxed{\text{R.o.c}: |z| > |2|}$$

**2** $x_2(n) = (n-1)\, 2^{n+2}\, u(n)$

$= n\, 2^{n+2}\, u(n) - 2^{n+2}\, u(n)$

$= 2^2\, n 2^n u(n) - 2^2\, 2^n u(n)$

$$\boxed{X_2(z) = 4\,\frac{2z}{(z-2)^2} - 4\,\frac{z}{z-2}}$$

$$\boxed{\text{R.o.c}: |z| > |2|}$$

**3** $x_3(n) = \{1, 2, 3, \underset{\uparrow}{2}, 1\}$

$\quad\;\; -3 \;\; -2 \;\; -1 \;\; 0 \;\; 1$

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$X(z) = z^{-(-3)} + 2z^{-(-2)} + 3z^{-(-1)} + 2z^{-0} + z^{-1}$

$$\boxed{X(z) = z^3 + 2z^2 + 3z + 2 + \frac{1}{z}}$$

$$\boxed{\text{Roc: all } z\text{-Plane except } z = 0}$$

$\boxed{4}$ $x_4(n) = \cos(0.25\pi n - 0.25\pi)\, u(n)$ ⑧

$= \left[\cos(0.25\pi n)\cos(0.25\pi) + \sin(0.25\pi n)\sin(0.25\pi)\right] u(n)$

$$\cos(0.25\pi n)\,u(n) \xrightarrow{Z.T} \frac{z^2 - z\cos(\frac{\pi}{4})}{z^2 - 2z\cos(\pi/4) + 1}$$

$$= \frac{z^2 - 1/\sqrt{2}\, z}{z^2 - \sqrt{2}\, z + 1}$$

$$\sin(0.25\pi n)\,u(n) \xrightarrow{Z.T} \frac{z\sin(\frac{\pi}{4})}{z^2 - 2z\cos(\pi/4) + 1}$$

$$= \frac{1/\sqrt{2}\, z}{z^2 - \sqrt{2}\, z + 1}$$

$$X_4(z) = \frac{1}{\sqrt{2}} \frac{z^2 - 1/\sqrt{2}\, z}{z^2 - \sqrt{2}\, z + 1} + \frac{1}{\sqrt{2}} \frac{1/\sqrt{2}\, z}{z^2 - \sqrt{2}\, z + 1}$$

R.o.c $|z| > 1$

---

$$X_4(z) = \frac{1}{\sqrt{2}} \left( \frac{z^2 + 1}{z^2 - \sqrt{2}\, z + 1} \right)$$

(a)

$\boxed{5}$ $x_5(n) = (0.5)^n u(n-2)$

$= (0.5)^{n+2-2} u(n-2)$

$= (0.5)^{+2} (0.5)^{n-2} u(n-2)$

$$X_5(z) = (0.5)^2 \frac{z}{z-0.5} z^{-2}$$

$$= \frac{1}{4} \frac{z^{-1}}{z-0.5}$$

R.o.c : $|z| > |\tfrac{1}{2}|$

$\boxed{10}$ $x_{10}(n) = e^{-0.1n} \cos(0.25\pi n)$

$$X_{10}(z) = \frac{(z/e^{-0.1})^2 - (z/e^{-0.1}) \cos(0.25\pi)}{(z/e^{-0.1})^2 - 2(z/e^{-0.1}) \cos(0.25\pi) + 1}$$

$\boxed{11}$ $x_{11}(n) = 3\delta(n) - 2\delta(n-1)$

$$X_{11}(z) = 3 - 2z^{-1}$$

$\boxed{13}$ $x_{13}(t) = t e^{-2t}$

$$L.T \quad X_{13}(s) = \frac{1}{(s+2)^2}$$

$$x_{13}(n) = n e^{-2n}$$

$$x_{13}(n) = (e^{-2})^n \, n \, u(n)$$

$$X_{13}(z) = \frac{e^{-2} z}{(z - e^{-2})^2}$$

$$R.o.c: \; |z| > |e^{-2}|$$

---

$\boxed{14}$ $x_{14}(t) = t^3$

$$x_{14}(n) = n^3 = n \; n \; n \, u(n)$$

$$-z \frac{d}{dz} \frac{z}{z-1} = -z \frac{-1}{(z-1)^2}$$

$$= \frac{z}{(z-1)^2}$$

$$-z \frac{d}{dz} \frac{z}{(z-1)^2} = -z \frac{(z-1)^2 - 2(z-1)z}{(z-1)^3} = -z \frac{(-z-1)}{(z-1)^3}$$

$$-z \frac{d}{dz} \frac{z(z+1)}{(z-1)^3} = -z \frac{(2z+1)(z-1)^3 - 3(z-1)^2 z(z+1)}{(z-1)^{\cancel{6}\,4}} = \frac{z(z^2+4z+\dots}{(z-1)^4}$$

$$R.o.c: \; |z| > |1|$$

# Inverse Z-Transform

$$x(n) = Z^{-1}\left[X(Z)\right] = \frac{1}{2\pi j}\oint_C X(Z)\, Z^{n-1}\, dZ \Rightarrow \text{Generally}$$

## * Methods of I.Z.T:

[1] Direct Division method (Long Division)

[2] Power series "

[3] Partial fraction " ✓✓

[4] Inversion Integral method.

$$A_k = \frac{1}{(k-1)!}\left[\frac{d^{(k-1)}}{ds^{(k-1)}}\left\{(S+a)^n F(S)\right\}\right]\Bigg|_{S=-a}$$

# DSP

## Sheet II

### I Obtain Z-Transform & R.O.C:

① $x(n) = 2^{n+2}\, u(n)$

$= 2^n\, 2^2\, u(n) = 4 \dfrac{Z}{Z-2}$

R.O.C : $|Z| > 2$

② $x(n) = (n-1)\left(2^{n+2}\right) u(n)$

$= n\, 2^{n+2} u(n) - 2^{n+2}\, u(n)$

$= 4\, n 2^n u(n) - 4\, 2^n\, u(n)$

$= 4 \dfrac{2Z}{(Z-2)^2} - 4 \dfrac{Z}{Z-2}$

R.O.C : $|Z| > 2$

③ $x(n) = \left\{ \underset{-3}{1}, \underset{-2}{2}, \underset{-1}{3}, \underset{\underset{0}{\uparrow}}{2}, \underset{1}{1} \right\}$

$X(Z) = \sum\limits_{n=-\infty}^{\infty} x(n)\, Z^{-n}$

$= Z^{-(-3)} + 2Z^{-(-2)} + 3Z^{-(-1)} + 2Z^{0} + Z^{-1}$

$$X(Z) = Z^3 + 2Z^2 + 3Z + 2 + \frac{1}{Z}$$

R.o.c: all Z-Plane except $Z=0$

④ $x(n) = \cos(0.25\pi n - 0.25\pi)\, u(n)$

$$= \left[\cos\left(\tfrac{\pi}{4}n\right)\cos(\pi/4) + \sin\left(\tfrac{\pi}{4}n\right)\sin(\pi/4)\right] u(n)$$

$$X(Z) = \frac{1}{\sqrt{2}} \frac{Z^2 - Z\cos(\pi/4)}{Z^2 - 2Z\cos(\pi/4) + 1} + \frac{1}{\sqrt{2}} \frac{Z\sin(\pi/4)}{Z^2 - 2Z\cos(\pi/4) + 1}$$

$$= \frac{1}{\sqrt{2}} \left[\frac{Z^2 - Z\frac{1}{\sqrt{2}}}{Z^2 - \sqrt{2}Z + 1} + \frac{Z \cdot \frac{1}{\sqrt{2}}}{Z^2 - \sqrt{2}Z + 1}\right]$$

$$= \frac{1}{\sqrt{2}} \left[\frac{Z^2}{Z^2 - \sqrt{2}Z + 1}\right] \quad \text{Roc: } |Z| > 1$$

⑤ $x(n) = (0.5)^n\, u(n-2)$

$$= \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^{n-2} u(n-2)$$

$$X(Z) = \frac{1}{4} Z^{-2} \frac{Z}{Z - 1/2}$$

$$= \frac{1}{4} \frac{Z^{-1}}{Z - 1/2}$$

R.o.c: $|Z| > 0.5$

③

$$x(n) = \left(\frac{1}{3}\right)^n \left[u(n) - u(n-4)\right]$$

$$= \left(\frac{1}{3}\right)^n u(n) - \left(\frac{1}{3}\right)^4 \left(\frac{1}{3}\right)^{n-4} u(n-4)$$

$$X(z) = \frac{z}{z - 1/3} - \frac{1}{81} z^{-4} \frac{z}{z - 1/3}$$

$$= \frac{1}{z - 1/3} - \frac{1}{81} \frac{z^{-3}}{z - 1/3}$$

ROC: $|z| > 1/3$

⑦ $x(n) = n\left(\frac{1}{2}\right)^n u(n)$

$$X(z) = \frac{0.5 z}{(z - 1/2)^2} \quad \text{ROC}: |z| > \frac{1}{2}$$

⑧ $x(n) = n(0.5)^n \cos(0.25\pi n)\, u(n)$

$$X(z) = -z \frac{d}{dz} \frac{(z/0.5)^2 - z/0.5 * \frac{1}{\sqrt{2}}}{(z/0.5)^2 - 2z/0.5 * \frac{1}{\sqrt{2}} + 1}$$

$$= -z \frac{d}{dz} \frac{4z^2 - 2z * \frac{1}{\sqrt{2}}}{4z^2 - 4z * \frac{1}{\sqrt{2}} + 1}$$

$$= -z \frac{d}{dz} \frac{z^2 - \frac{1}{2\sqrt{2}} z}{z^2 - \frac{1}{\sqrt{2}} z + 1/4}$$

$$X(z) = -z \frac{\left(2z - \frac{1}{2\sqrt{2}}\right)\left(z^2 - \frac{1}{\sqrt{2}}z + 1/4\right) - \left(2z - \frac{1}{\sqrt{2}}\right)\left(z^2 - \frac{1}{2\sqrt{2}}z\right)}{\left(z^2 - \frac{1}{\sqrt{2}}z + 1/4\right)^2}$$

$$\text{R.O.C: } |z| > 1$$

(9) $x(n) = (2)^{-0.3n}\, n\, u(n)$

$$X(z) = \frac{2^{-0.3}\, z}{\left(z - 2^{-0.3}\right)^2}$$

$$\text{R.O.C: } |z| > 2^{-0.3}$$

(10) $x(n) = \left(e^{-0.1}\right)^n \cos\left(\pi/4\, n\right) u(n)$

$$X(z) = \frac{\left(z/e^{-0.1}\right)^2 - \left(z/e^{-0.1}\right)\cos(\pi/4)}{\left(z/e^{-0.1}\right)^2 - 2\left(z/e^{-0.1}\right)\cos(\pi/4) + 1}$$

$$= \frac{\left(e^{0.1}\right)^2 z^2 - \frac{e^{0.1}}{\sqrt{2}}\, z}{\left(e^{0.1}\right)^2 z^2 - \frac{1}{\sqrt{2}} e^{0.1}\, z + 1}$$

$$\text{R.O.C: } |z| > 1$$

$x(n) = 3\delta(n) - 2\delta(n-1)$

$X(z) = 3(1) - 2z^{-1}$

(12) $x(n) = 5^n n^2 u(n)$

$$= -z \frac{d}{dz} \frac{5z}{(z-5)^2}$$

$$= -z \frac{5(z-5)^2 - 2\cancel{(z-5)} \cdot 5z}{(z-5)^3}$$

$$= -z \frac{5z - 25 - 10z}{(z-5)^3}$$

$$= -z \frac{-5z-25}{(z-5)^3} = \frac{5z^2+25z}{(z-5)^3}$$

R.o.c : $|z| > 5$

(13) $x(t) = t e^{-2t}$

$$\boxed{\begin{array}{l} t = nT \\ \text{Let } T = 1 \text{ sec} \end{array}}$$

$x(n) = n e^{-2n} u(n)$

$$X(z) = \frac{e^{-2} z}{(z - e^{-2})^2}$$

R.o.c : $|z| > e^{-2}$

$x(t) = n^3 u(n) \qquad t = nT$

$$n u(n) \xrightarrow{Z.T} -z \frac{d}{dz} \frac{z}{z-1} \quad \text{Let } T = 1 \text{ sec}$$

$$\longrightarrow -z \frac{(\cancel{z}-1) - \cancel{z}}{(z-1)^2} = \frac{z}{(z-1)^2}$$

$$n^2 u(n) \xrightarrow{Z.T} -z \frac{d}{dz} \frac{z}{(z-1)^2}$$

$$\longrightarrow -z \frac{(z-1)^2 - 2\cancel{(z-1)}\, z}{(z-1)^3}$$

$$\longrightarrow \frac{-z\left[z-1-2z\right]}{(z-1)^3}$$

$$\longrightarrow \frac{z^2+z}{(z-1)^3}$$

$$n^3 u(n) \xrightarrow{Z.T} -z \frac{d}{dz} \frac{z^2+z}{(z-1)^3}$$

$$\longrightarrow -z \frac{(2z+1)(z-1)^3 - 3\cancel{(z-1)^2}(z^2+z)}{(z-1)^4}$$

ROC $|z| > 1$

obtain I Z T

(15) $X(z) = \frac{z+1}{z+2}$

$$\frac{X(z)}{z} = \frac{z+1}{z(z+2)} = \frac{A}{z} + \frac{B}{z+2}$$

$$A = \lim_{z \to 0} \left( \frac{X(z)}{z} z \right) = \boxed{\frac{1}{2}}$$

$$B = \boxed{\frac{1}{2}}$$

$$X(z) = \frac{1}{2} + \frac{z}{2(z+2)}$$

$$x(n) = \frac{1}{2} \delta(n) + \frac{1}{2} (-2)^n u(n)$$

(16) $$\frac{X(z)}{z} = \frac{z+4}{z(z^2-3z+2)} = \frac{A}{z} + \frac{B}{(z-1)} + \frac{C}{z-2}$$

$A = 2$, $B = -5$ & $C = 3$

$$x(n) = 2\delta(n) - 5u(n) + 3(2)^n u(n)$$

⑧

$$\frac{X(Z)}{Z} = \frac{Z^2+2Z+3}{Z(Z+1)(Z+2)^2}$$

$$= \frac{A}{Z} + \frac{B}{Z+1} + \frac{C}{(Z+2)^2} + \frac{D}{Z+2}$$

$$A = \lim_{Z\to 0}\left(\frac{X(Z)}{Z}\,Z\right) = \boxed{\frac{3}{4}}$$

$$B = \frac{1-2+3}{(-1)(1)^2} = \boxed{-2}$$

$$C = \frac{\cancel{4}-\cancel{4}+3}{(-2)(-1)} = \boxed{\frac{3}{2}}$$

$$D = \lim_{Z\to -2}\left(\frac{d}{dZ}\,\frac{Z^2+2Z+3}{Z(Z+1)}\right)$$

$$= \lim_{Z\to -2}\left[\frac{(2Z+2)(Z^2+Z)-(2Z+1)(Z^2+2Z+3)}{Z^2(Z+1)^2}\right]$$

$$= \frac{\overset{-4}{(-2)(2)} - \overset{-9}{(-3)(3)}}{(4)} = \boxed{\frac{5}{4}}$$

$$\boxed{x(n) = \frac{3}{4}\delta(n) - 2(-1)^n u(n) + \frac{1.5}{-2}\, n(-2)^n u(n) + \frac{5}{4}(-2)^n u(n)}$$

(9)

$X(z) = 3z^{2} + 2z + 3 + 5z^{-1} + 6z^{-2}$

$x(n) = 3\delta(n+2) + 2\delta(n+1) + 3\delta(n) + 5\delta(n-1) + 6\delta(n-2)$

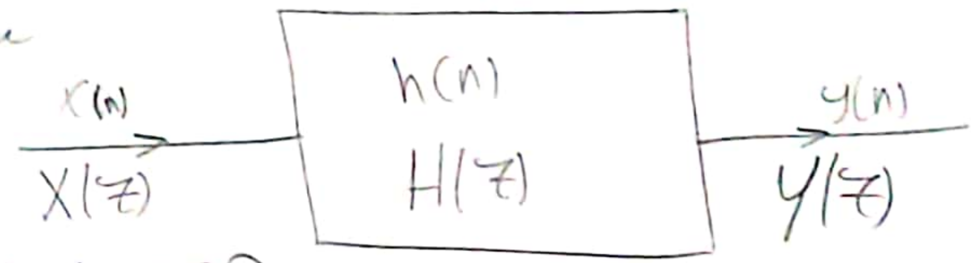
---

$\times$ for $y(n) = x(n) - x(n-2) - 1.3\,y(n-1) - 0.36\,y(n-2)$

find $H(z)$

$h(n)$: impulse response

$H(z)$ : T.f




$y(n) = x(n) * h(n)$ T.D

$Y(z) = X(z) \cdot H(z)$ Z.D

$$H(z) = \frac{Y(z)}{X(z)}$$

$Y(z) = X(z) - X(z)\,z^{-2} - 1.3\,Y(z)\,z^{-1} - 0.36\,Y(z)\,z^{-2}$

$\left[1 + 1.3z^{-1} + 0.36z^{-2}\right] Y(z) = \left[1 - z^{-2}\right] X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-2}}{1 + 1.3z^{-1} + 0.36z^{-2}}$$

$$= \frac{z^{2} - 1}{z^{2} + 1.3z + 0.36}$$

Scanned with CamScanner

Given $H(z) = \frac{z^2 - 0.5z + 0.36}{z^2}$

find Difference eqn

$$\frac{Y(z)}{X(z)} = \frac{1 - 0.5z^{-1} + 0.36z^{-2}}{1}$$

$$Y(z) = \left[1 - 0.5z^{-1} + 0.36z^{-2}\right] X(z)$$

$$y(n) = x(n) - 0.5\,x(n-1) + 0.36\,x(n-2)$$

---

* Given a T.f :

$$H(z) = \frac{z+1}{z-0.5}$$

1- $h(n) = ?$

$$\frac{H(z)}{z} = \frac{z+1}{z(z-0.5)} = \frac{A}{z} + \frac{B}{z-0.5}$$

$A = -2$ & $B = 3$

$$\boxed{h(n) = -2\delta(n) + 3\left(\tfrac{1}{2}\right)^n u(n)}$$

$\therefore S(n) = ?$

$$S(z) = H(z) \cdot U(z)$$

$$= \frac{z}{z-1} \cdot \frac{z+1}{z-0.5}$$

$$\frac{S(z)}{z} = \frac{A}{z-1} + \frac{B}{z-0.5}$$

$A = 4$ & $B = -3$.

$$\boxed{S(n) = 4\,u(n) - 3\left(\tfrac{1}{2}\right)^n u(n)}$$

3- if input $x(n) = (0.25)^n u(n)$

$y(n) = ?$

$y(n) = x(n) * h(n)$

$$Y(z) = X(z) \cdot H(z)$$

$$Y(z) = \frac{z}{z-0.25} \cdot \frac{z+1}{z-0.5}$$

$y(n) =$

Given $h(n) = (0.5)^n u(n) - (0.25)^n u(n)$

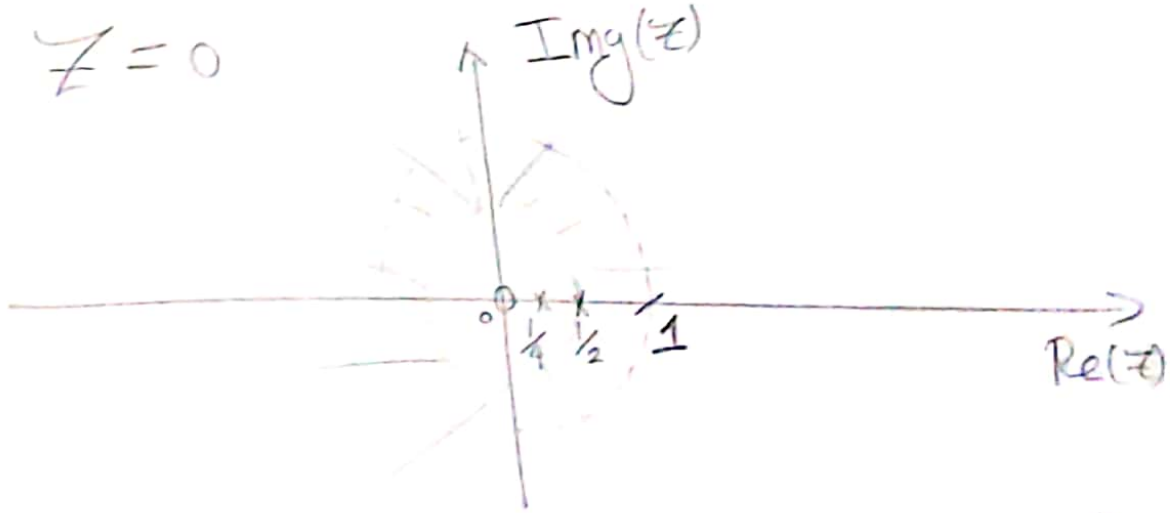
1) $H(Z) = \dfrac{Z}{Z - \frac{1}{2}} - \dfrac{Z}{Z - \frac{1}{4}} = \dfrac{\cancel{Z^2} - \frac{1}{2}Z - \cancel{Z^2} + \frac{1}{2}Z}{(Z-\frac{1}{2})(Z-\frac{1}{4})}$

$= \dfrac{\frac{1}{4} Z}{(Z-\frac{1}{2})(Z-\frac{1}{4})}$

R.o.c : $|Z| > \dfrac{1}{2}$

2) Poles. $Z = \dfrac{1}{2}$ & $Z = \dfrac{1}{4}$

Zeros. $Z = 0$




Causal system since R.o.c exterior of a circle with finite radius $r = 0.5$

$h(n) = 0 \quad n < 0$

Stable system since Poles lie inside the unit circle

$\sum^{\infty} |h(n)| < \infty$

$$h(n) = \{3, \underset{\uparrow}{5}, 4, 1\} \quad \& \quad x(n) = \{\underset{\uparrow}{7}, 2, 3\}$$

$$\begin{array}{cccc} 3 & 5 & 4 & 1 \\ -1 & 0 & 1 & 2 \end{array} \qquad \begin{array}{ccc} 7 & 2 & 3 \\ 0 & 1 & 2 \end{array}$$

$\boxed{1}$ $y(n) = x(n) * h(n)$

$$\begin{array}{rcccccc}
x(n): & 7 & 2 & 3 & & & \\
h(n): & 3 & 5 & 4 & 1 & & \\
\hline
 & 21 & 35 & 28 & 7 & & \\
 & & 6 & 10 & 8 & 2 & \\
 & & & 9 & 15 & 12 & 3 \\
\hline
\end{array}$$

$$\{21, \underset{\uparrow}{41}, 47, 30, 14, 3\}$$

$$\begin{array}{cccccc} -1 & 0 & 1 & 2 & 3 & 4 \end{array}$$

---

$\boxed{2}$ $Y(z) = X(z) \cdot H(z)$

$$= [3z + 5 + 4z^{-1} + z^{-2}] \cdot [7 + 2z^{-1} + 3z^{-2}]$$

$$= 21z + 35 + 28z^{-1} + 7z^{-2} + 6 + 10z^{-1} + 8z^{-2}$$

$$+ 2z^{-3} + 9z^{-1} + 15z^{-2} + 12z^{-3} + 3z^{-4}$$

$$= 21z + 41 + 47z^{-1} + 30z^{-2} + 14z^{-3} + 3z^{-4}$$

$$y(n) = 21\delta(n+1) + 41 + 47\delta(n-1) + 30\delta(n-2) + 14\delta(n-3)$$

$$+ 3\delta(n-4)$$

## * Timing shift property:

Let $y(n) = x(n-k)$

$$Y(Z) = \sum_{n=-\infty}^{\infty} y(n) Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (x(n-k)) Z^{-n}$$

Let $m = n-k$

$$Y(Z) = \sum_{m=-\infty}^{\infty} x(m) Z^{-(m+k)} = \sum_{m=-\infty}^{\infty} x(m) Z^{-m} Z^{-k}$$

$$= Z^{-k} \left[ \sum_{m=-\infty}^{\infty} x(m) Z^{-m} \right] = \boxed{Z^{-k} X(Z)}$$

Scanned with CamScanner