Z-Transform for common Signals

$x(n)$	$X(z)$	RoC
$\delta(n)$	1	all z-domain
$u(n)$	$\frac{z}{z-1}$	$\|z\| > 1$
$a^n u(n)$	$\frac{z}{z-a}$	$\|z\| > a$
$(-a)^n u(n)$	$\frac{z}{z+a}$	$\|z\| > \|a\|$
$n a^n u(n)$	$\frac{az}{(z-a)^2}$	$\|z\| > a$
$\cos(\omega n)\, u(n)$	$\frac{z^2 - z\cos\omega}{z^2 - 2z\cos\omega + 1}$	$\|z\| > 1$
$\sin(\omega n)\, u(n)$	$\frac{z \sin\omega}{z^2 - 2z\cos\omega + 1}$	$\|z\| > 1$

Properties of (Z-Transform)

$x(n) \longrightarrow X(z)$

$x(n-N) \longrightarrow z^{-N} X(z)$

$x(-n) \longrightarrow X\left(\frac{1}{z}\right)$

$a^n x(n) \longrightarrow X\left(\frac{z}{a}\right)$

$n\, x(n) \longrightarrow -z \frac{d}{dz} X(z)$

$x(n) * h(n) \longrightarrow X(z) \cdot H(z)$

$\cos(\omega n) x(n) \longrightarrow \frac{1}{2}\left\{ X(z e^{-j\omega}) + X(z e^{j\omega}) \right\}$

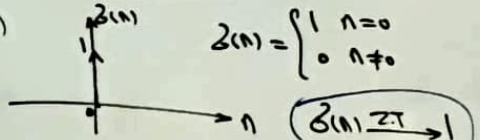
$\sin(\omega n) x(n) \longrightarrow \frac{1}{2j}\left\{ X(z e^{-j\omega}) - X(z e^{j\omega}) \right\}$

Z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

unit impulse ($\delta(n)$)

$\delta(n) = u(n) - u(n-1)$



$$\delta(n) = \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}$$

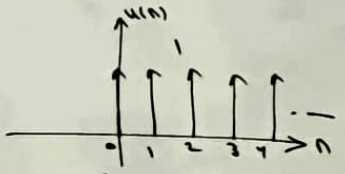
$\delta(n) \xrightarrow{Z.T} 1$

$X(z) = 1 \cdot z^{0} = \boxed{1}$ RoC : all z-domain

R.O.C → region of convergence

$X(z)$: Finite value not ∞

unit step fn $u(n)$



$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$u(n) \xrightarrow{Z.T} \frac{z}{z-1}$

RoC : $|z| > 1$

$$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

$$X(z) = 1 z^{0} + 1 z^{-1} + 1 z^{-2} + 1 z^{-3} + \cdots + 1 z^{-n}$$

$$= 1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \cdots + \frac{1}{z^n}$$

$$= \frac{1 - \frac{1}{z^{\infty}}}{1 - \frac{1}{z}} = \frac{1}{1-1} = \frac{1}{}$$

Sheet 1

No. 1

$X_1(n) = 2^{n+2}\, u(n)$

$X_1(n) = 2^2 \cdot 2^n\, u(n) = 4 \cdot 2^n u(n)$

$X(z) = 4\left(\frac{z}{z-2}\right)$

Roc : $|z| > 2$

No. 2

$X_2(n) = (n-1)\, 2^{n+2}\, u(n)$

$X_2(n) = n \cdot 2^2 \cdot 2^n\, u(n) - 2^2 \cdot 2^n\, u(n)$

$X_2(n) = 4\left[n 2^n u(n)\right] - 4\left[2^n u(n)\right]$

$X_2(z) = 4\left(\frac{2z}{(z-2)^2}\right) - 4\left(\frac{z}{z-2}\right)$

Roc : $|z| > 2$

No. 3

$X_3(n) = \{1, 2, 3, 2, 1\}$ (3 at $n = 0$; $n = -2, -1, 0, 1, 2$)

$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$

$X(z) = 1z^2 + 2z^1 + 3z^0 + 2z^{-1} + 1z^{-2}$

$X(z) = \left(z^2 + 2z + 3 + \frac{2}{z} + \frac{1}{z^2}\right)$

Roc :

all (z-Plane) except $z = 0$

No. 5

$$x_5(n) = (0.5)^n u(n-2)$$
$$= (0.5)^2 (0.5)^{n-2} u(n-2)$$

$$X(z) = \frac{1}{4} \frac{z}{z-0.5} \cdot z^{-2}$$
$$X(z) = \frac{1}{4}\left(\frac{z^{-1}}{z-0.5}\right)$$

ROC: $|z| > 0.5$

No. 13

$$x_{13}(t) = t e^{-2t}$$
$$x_{13}(n) = n e^{-2n} u(n)$$
$$x_{13}(n) = (e^{-2})^n \cdot n u(n)$$
$$X_{13}(z) = \frac{e^{-2} z}{(z - e^{-2})^2}$$

ROC $|z| > e^{-2}$

No. 10

$$x_{10}(n) = e^{-0.1n} \cos(0.25\pi n)$$
$$= (e^{-0.1})^n \cos(0.25\pi n)$$

$$X(z) = \frac{\left(\frac{z}{e^{0.1}}\right)^2 - \left(\frac{z}{e^{0.1}}\right)\cos 0.25\pi}{\left(\frac{z}{e^{0.1}}\right)^2 - 2\left(\frac{z}{e^{0.1}}\right)\cos 0.25\pi + 1}$$

ROC: $|z| > 1$

No. 11

$$x_{11}(n) = 3\delta(n) - 2\delta(n-1)$$
$$X(z) = 3 - 2z^{-1}$$

$$X(n) = \cos(0.25\pi n - 0.25\pi)\, u(n)$$

Note

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$X(n) = \left[\cos(0.25\pi n)\cos(0.25\pi) + \sin(0.25\pi n)\sin(0.25\pi)\right] u(n)$$

($\cos(0.25\pi) \to \frac{1}{\sqrt{2}}$, $\sin(0.25\pi) \to \frac{1}{\sqrt{2}}$)

$$X(n) = \frac{1}{\sqrt{2}}\cos(0.25\pi n)\, u(n) + \frac{1}{\sqrt{2}}\sin(0.25\pi n)\, u(n)$$

$$X(z) = \frac{1}{\sqrt{2}}\left(\frac{z^2 - z\cos 0.25\pi}{z^2 - 2z\cos 0.25\pi + 1}\right) + \frac{1}{\sqrt{2}}\left(\frac{z\sin 0.25\pi}{z^2 - 2z\cos 0.25\pi + 1}\right)$$

$$= \frac{1}{\sqrt{2}}\left(\frac{z^2 - \frac{1}{\sqrt{2}}z}{z^2 - \frac{2}{\sqrt{2}}z + 1}\right) + \frac{1}{\sqrt{2}}\left(\frac{\frac{1}{\sqrt{2}}z}{z^2 - \frac{2}{\sqrt{2}}z + 1}\right)$$

Right

$$x(t) = t^3$$

$$x(n) = n^3 u(n)$$

$$= n \cdot n \cdot n \, u(n)$$

$(z e^{j\omega})$

$- X(z e^{j\omega})$