Numerical aPerture

$$N_A = \sqrt{n_1^2 - n_2^2}$$

$$N_A = n_1\sqrt{2\Delta}$$

n_1 : refractive index of core

n_2 : refractive index of cladding

$n = \frac{c}{v}$ → velocity of light (3×10^8 m/sec)

↓ velocity in the medium

$$\Delta = \frac{n_1 - n_2}{n_1}$$

The (V) number
The normalized frequency

$$V = \frac{\pi d}{\lambda} N_A$$

$$V = \frac{2\pi a}{\lambda} N_A$$

d: core diameter

$(d = 2a,\ a = d/2)$

a: core radius

note

For Single Mode optical fiber (SM)

$$V = 2.405$$

The No. of Modes (M)

$$M|_{SI} = \frac{V^2}{2}$$

$$M|_{GI} = \left(\frac{\alpha}{\alpha+2}\right) M|_{SI}$$

Assume $(\alpha = 2)$

$$M|_{GI} = \left(\frac{\alpha}{\alpha+2}\right) a^2 K^2 n_1^2 \Delta$$

$K = \frac{2\pi}{\lambda}$ → wave no. (rad/m)

The Fractional Power wasted in cladding

$$\frac{P_{cladding}}{P_{Total}} = \frac{4}{3\sqrt{M}}$$

(dcore) / cladding diameter (μm)

core diameter (μm) (50/125) SI

$n_1 = 1.48$

$n_2 = 1.465$

$\lambda = 1320\ nm$

a) $N_A = \sqrt{n_1^2 - n_2^2} = \boxed{0.21}$

b) $\theta_{max} = \sin^{-1}(N_A) = \boxed{12.13^\circ}$

c) $M|_{SI} = \frac{V^2}{2} = \frac{(25)^2}{2} = \boxed{312.5}$

d) $\frac{P_{cladding}}{P_{total}} = \frac{4}{3\sqrt{M}} = 0.075 = \boxed{7.5\%}$

e) $d_{core} = ?$ in Case of (SI) fiber

Support (SM)

$V = 2.405$

$V = \frac{\pi d}{\lambda} N_A$

$V = \frac{\pi \times 50 \times 10^{-6}}{1320 \times 10^{-9}} \times 0.21$

$V = 25$

2.405 (SM)

$d = \frac{V\lambda}{\pi N_A} = \frac{2.405 \times 1320 \times 10^{-9}}{\pi \times 0.21} = \boxed{4.812\ \mu m}$

No. 6

$a = 25\ \mu m$

$n_1 = 1.48$

$\Delta = 0.01$

$\lambda = 0.84\ \mu m$

a) $N_A = n_1\sqrt{2\Delta} = \boxed{0.21}$

b) $V = \frac{2\pi a}{\lambda} \cdot N_A = \boxed{39.27}$

c) $M|_{SI} = \frac{V^2}{2} = \boxed{771}$

d) $\frac{P_{cladding}}{P_{total}} = \frac{4}{3\sqrt{M}} = 0.048 = \boxed{4.8\%}$

No. 7

SM $\longrightarrow$ $V = 2.405$

$d = 10\ \mu m$

$\lambda = 1.3\ \mu m$

$n_1 = 1.55$

a) $n_2 = 1.5468$

b) $\Delta = 0.206\%$

c) $\theta_{max} = 5.712^\circ$

$V = \frac{\pi d}{\lambda} \cdot N_A \longrightarrow N_A = \frac{V\lambda}{\pi d} = 0.09352$

$N_A = \sqrt{n_1^2 - n_2^2}$

$N_A^2 = n_1^2 - n_2^2$

$n_2 = \sqrt{n_1^2 - N_A^2} = 1.5468$

$\Delta = \frac{n_1 - n_2}{n_1} = 0.206\%$

$\theta_{max} = \sin^{-1}(N_A) = 5.712^\circ$

No. 8

$n_1 = 1.55$
$n_2 = 1.51$
$d = 50\,\mu m$
$\lambda = 0.8\,\mu m$

a) $N_A = \sqrt{n_1^2 - n_2^2} = \boxed{0.35}$

b) $\theta_{max} = \sin^{-1}(N_A) = \boxed{20.48^\circ}$

c) $V = \frac{\pi d}{\lambda} N_A = \boxed{68.72}$

d) $M|_{SI} = \frac{V^2}{2} = \boxed{2361}$

$M|_{GI} = \frac{\alpha}{\alpha+2} M|_{SI} = \boxed{1180}$

No. 9

$\lambda = 1300\,nm$

GI → graded index ($\alpha = 1$)

$d = 50\,\mu m$
$n_1 = 1.48$
$n_2 = 1.46$

a) $M|_{SI} \propto \frac{P_{cladding}}{P_T}$

b) $M|_{GI} \propto \frac{P_{cladding}}{P_T}$

$N_A = \sqrt{n_1^2 - n_2^2} \simeq 0.24$

$V = \frac{\pi d}{\lambda} \cdot N_A = \frac{\pi \times 50 \times 10^{-6}}{1300 \times 10^{-9}} \times 0.24 \simeq 29$

$M|_{SI} = \frac{V^2}{2} = \boxed{420}$

$\frac{P_{cladding}}{P_T} = \frac{4}{\sqrt{M}} = \boxed{6.5\%}$

$M|_{GI} = \frac{\alpha}{\alpha+2} M|_{SI} = \boxed{140}$

$\frac{P_{cladding}}{P_t} = \frac{4}{\sqrt{M}} = \boxed{11.3\%}$