

# Compiler Design

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# Lexical Analysis

## Lecture 4

Compilers *Principles, Techniques, & Tools*

Second Edition



# How to convert NFA with epsilon to DFA

In this method, we first convert Non-deterministic finite automata (NFA) with  $\epsilon$  to NFA without  $\epsilon$ . Then, NFA without  $\epsilon$  can be converted to its equivalent Deterministic Finite Automata (DFA).

## □ Method for conversion

The method for converting the NFA with  $\epsilon$  to DFA is explained below –

**Step 1** – Consider  $M = \{Q, \Sigma, \delta, q_0, F\}$  is NFA with  $\epsilon$ . We have to convert this NFA with  $\epsilon$  to equivalent DFA denoted by

- $M_0 = (Q_0, \Sigma, \delta_0, q_0, F_0)$
- Then obtain,  $\epsilon\text{-closure}(q_0) = \{p_1, p_2, p_3, \dots, p_n\}$
- then  $[p_1, p_2, p_3, \dots, p_n]$  becomes a start state of DFA
- now  $[p_1, p_2, p_3, \dots, p_n] \in Q_0$

**Step 2** – We will obtain  $\delta$  transition on  $[p_1, p_2, p_3, \dots, p_n]$  for each input.

- $\delta_0([p_1, p_2, p_3, \dots, p_n], a) = \epsilon\text{-closure}(\delta(p_1, a) \cup \delta(p_2, a) \cup \dots \cup \delta(p_n, a))$
- $= \cup_{i=1 \text{ to } n} \epsilon\text{-closure}(\delta(p_i, a))$
- Where  $a$  is input  $\in \Sigma$

**Step 3** – The state obtained  $[p_1, p_2, p_3, \dots, p_n] \in Q_0$ .

- The states containing final state in  $p_i$  is a final state in DFA

The steps involved in the conversion of NFA to DFA are,

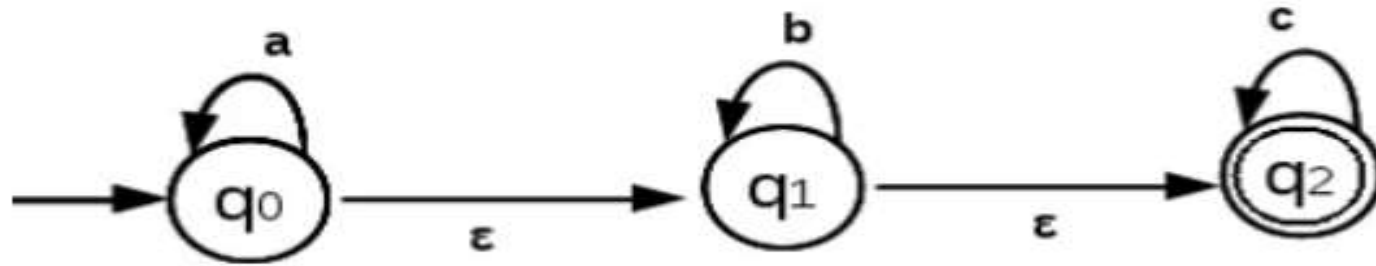
1. Transform the NFA with Epsilon transitions to NFA without epsilon transitions.
2. convert the resulting NFA to DFA.

These steps are explained in detail as follows:

### **1. Transform the NFA with $\epsilon$ transitions to NFA without $\epsilon$ transitions.**

#### **Example 1:**

Consider the following NFA with epsilon transitions:



The above NFA has states,  $q_0, q_1, q_2$ . The start state is  $q_0$ . The final state is  $q_2$ .

Step a: Find the Epsilon closure of all states.

$$ECLOSE(q_0) = \{q_0, q_1, q_2\}$$

$$ECLOSE(q_1) = \{q_1, q_2\}$$

$$ECLOSE(q_2) = \{q_2\}$$

The states of the new NFA will be  $\{q_0, q_1, q_2\}, \{q_1, q_2\}, \{q_2\}$ .

The transition table for the new NFA is,

Current state	Input	Symbol		
	a	b	c	
$\longrightarrow * \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$	
$* \{q_1, q_2\}$	$\phi$	$\{q_1, q_2\}$	$\{q_2\}$	
$* \{q_2\}$	$\phi$	$\phi$	$\{q_2\}$	

Let us say,

$\{q_0, q_1, q_2\}$  as  $q_x$

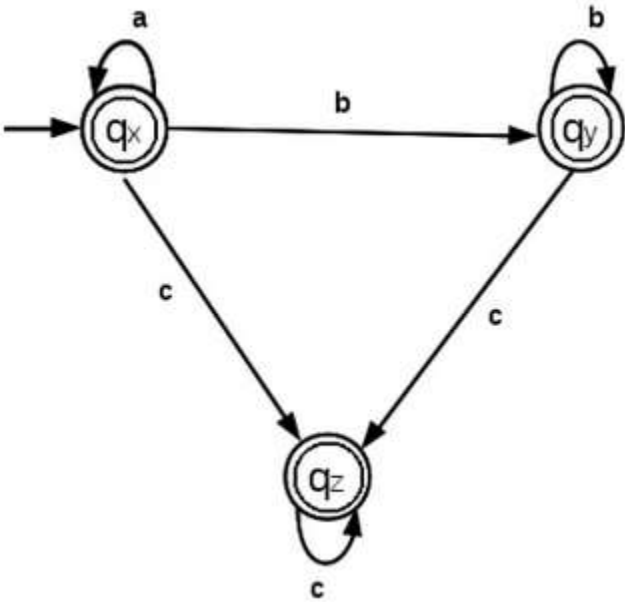
$\{q_1, q_2\}$  as  $q_y$

$\{q_2\}$  as  $q_z$

Then the transition table will become,

Current state	Input	Symbol		
	a	b	c	
$\longrightarrow *q_x$	$q_x$	$q_y$	$q_z$	
$*q_y$	$\phi$	$q_y$	$q_z$	
$*q_z$	$\phi$	$\phi$	$q_z$	

NFA to DFA ConversionNFA to DFA Conversion



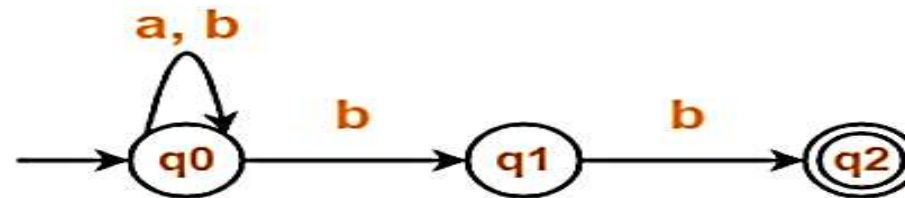
The transition diagram for the new NFA is,

# Subset Construction Algorithm

- Converting NFA to DFA

**Problem-01:**

Convert the following Non-Deterministic Finite Automata (NFA) to Deterministic Finite Automata (DFA)-



Transition table for the given Non-Deterministic Finite Automata (NFA) is-

State / Alphabet	a	b
→q0	q0	q0, q1
q1	—	*q2
*q2	—	—

# Converting NFA to DFA

**Step-02** Add transitions of start state  $q_0$  to the transition table  $T'$ .

State / Alphabet	a	b
$\rightarrow q_0$	$q_0$	$\{q_0, q_1\}$

## **Step-03:**

New state present in state  $Q'$  is  $\{q_0, q_1\}$ .

Add transitions for set of states  $\{q_0, q_1\}$  to the transition table  $T'$

State / Alphabet	a	b
$\rightarrow q_0$	$q_0$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$q_0$	$\{q_0, q_1, q_2\}$

# Converting NFA to DFA

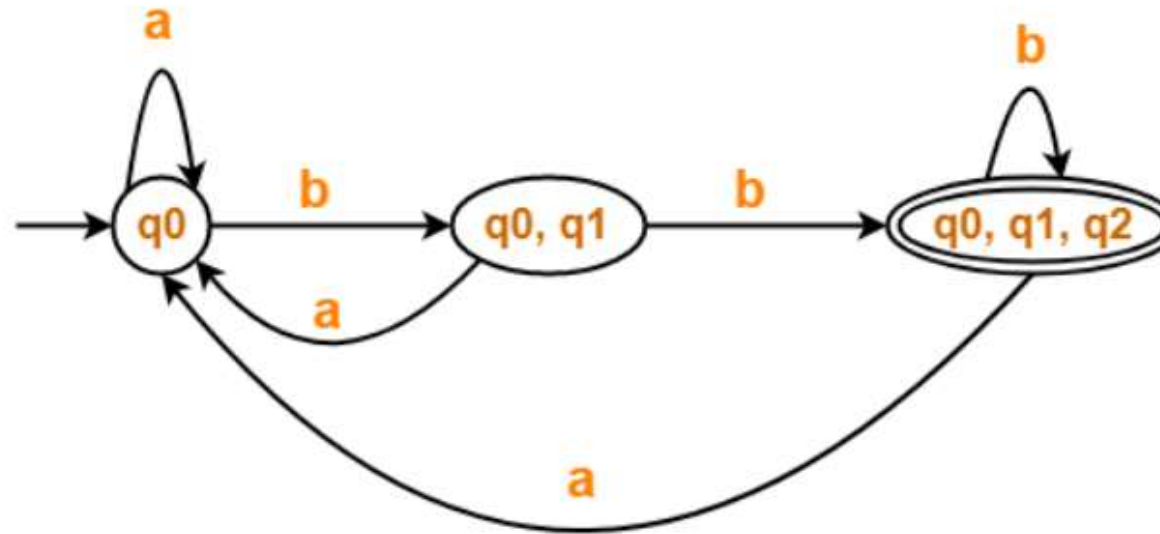
State / Alphabet	a	b
$\rightarrow q_0$	$q_0$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$q_0$	$\{q_0, q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$q_0$	$\{q_0, q_1, q_2$

State / Alphabet	a	b
$\rightarrow q_0$	$q_0$	$\{q_0, q_1\}$
$\{q_0, q_1\}$	$q_0$	$^*\{q_0, q_1, q_2\}$
$^*\{q_0, q_1, q_2\}$	$q_0$	$^*\{q_0, q_1, q_2$



# Converting NFA to DFA

Now, Deterministic Finite Automata (DFA) may be drawn as-

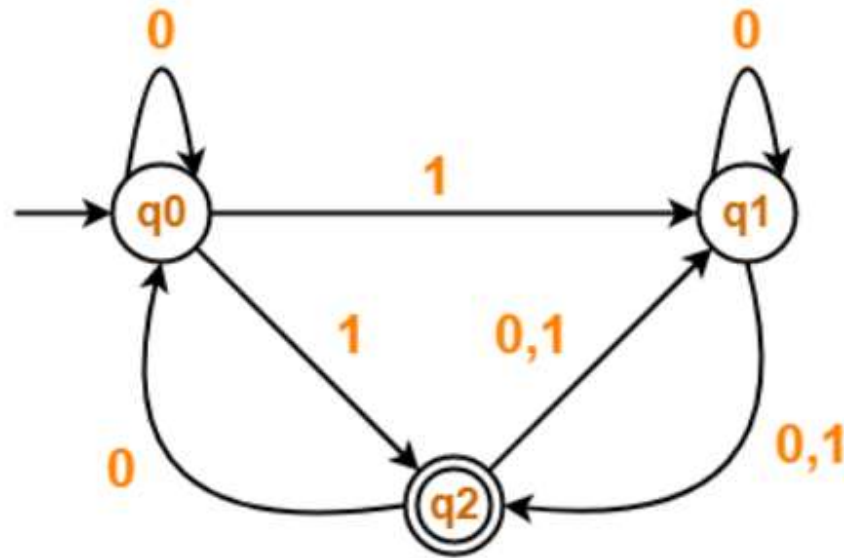


**Deterministic Finite Automata (DFA)**

# Converting NFA to DFA

## Problem-02:

Convert the following Non-Deterministic Finite Automata (NFA) to Deterministic Finite Automata (DFA)-



# Converting NFA to DFA

- Transition table for the given Non-Deterministic Finite Automata (NFA) is

State / Alphabet	0	1
→q0	q0	q1, *q2
q1	q1, *q2	*q2
*q2	q0, q1	q1

## Step-02:

Add transitions of start state q0 to the transition table T'.

State / Alphabet	0	1
→q0	q0	{q1, q2}

# Converting NFA to DFA

- **Step-03:**
- New state present in state Q' is {q1, q2}.
- Add transitions for set of states {q1, q2} to the transition table T'.

State / Alphabet	0	1
→q0	q0	{q1, q2}
{q1, q2}	{q0, q1, q2}	{q1, q2}

## **Step-04:**

New state present in state Q' is {q0, q1, q2}.

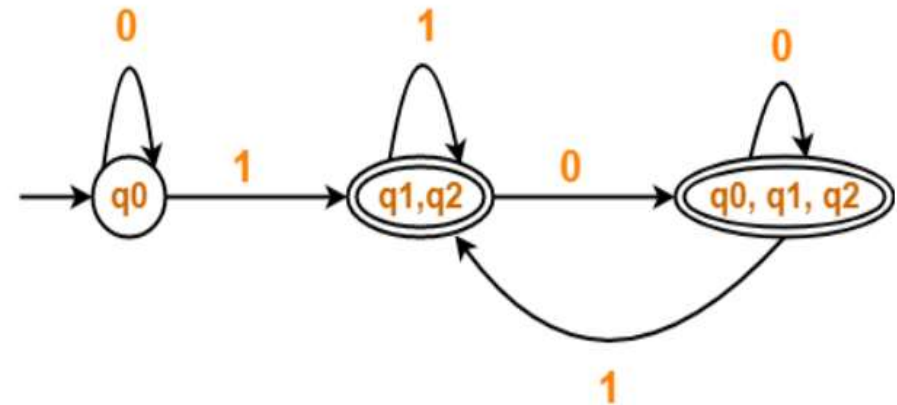
Add transitions for set of states {q0, q1, q2} to the transition table

State / Alphabet	0	1
→q0	q0	{q1, q2}
{q1, q2}	{q0, q1, q2}	{q1, q2}
{q0, q1, q2}	{q0, q1, q2}	{q1, q2}

# Converting NFA to DFA

- **Step-05:**
- Since no new states are left to be added in the transition table  $T'$ , so we stop.
- States containing  $q_2$  as its component are treated as final states of the DFA.
- Finally, Transition table for Deterministic Finite Automata (DFA) is-

State / Alphabet	0	1
$\rightarrow q_0$	$q_0$	$\{q_1, q_2\}$
$\{q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$



**Deterministic Finite Automata (DFA)**

# Minimization of DFA

Minimize the given DFA-

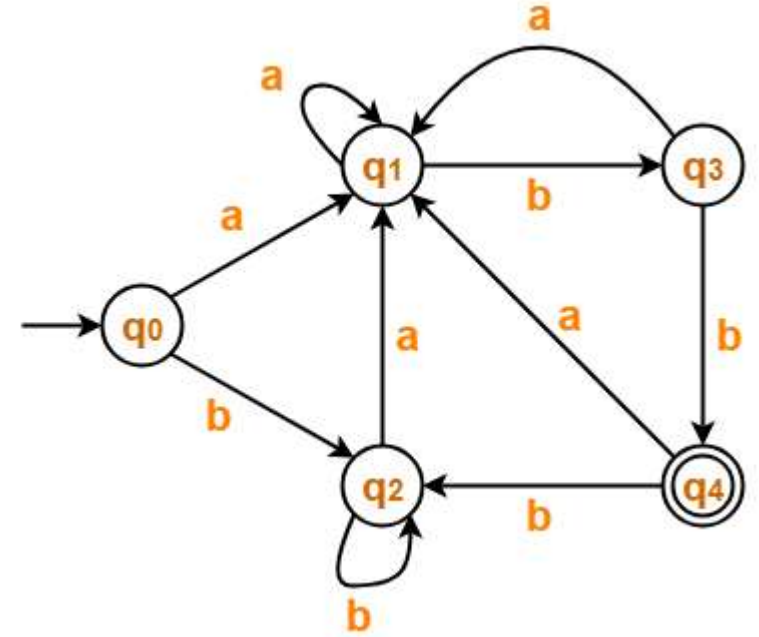
## Solution-

### Step-01:

The given DFA contains no dead states and inaccessible states.

Step-02: Draw a state transition table

	a	b
→q0	q1	q2
q1	q1	q3
q2	q1	q2
q3	q1	*q4
*q4	q1	q2



### Step-03:

Now using Equivalence Theorem, we have-

$$P_0 = \{ q_0, q_1, q_2, q_3 \} \{ q_4 \}$$

$$P_1 = \{ q_0, q_1, q_2 \} \{ q_3 \} \{ q_4 \}$$

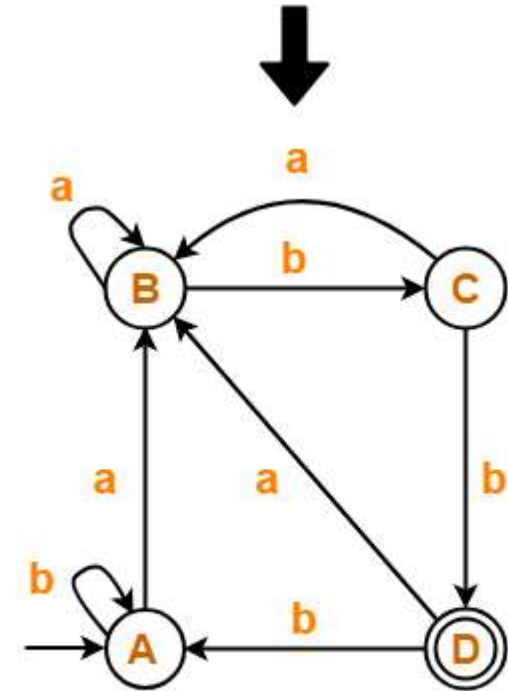
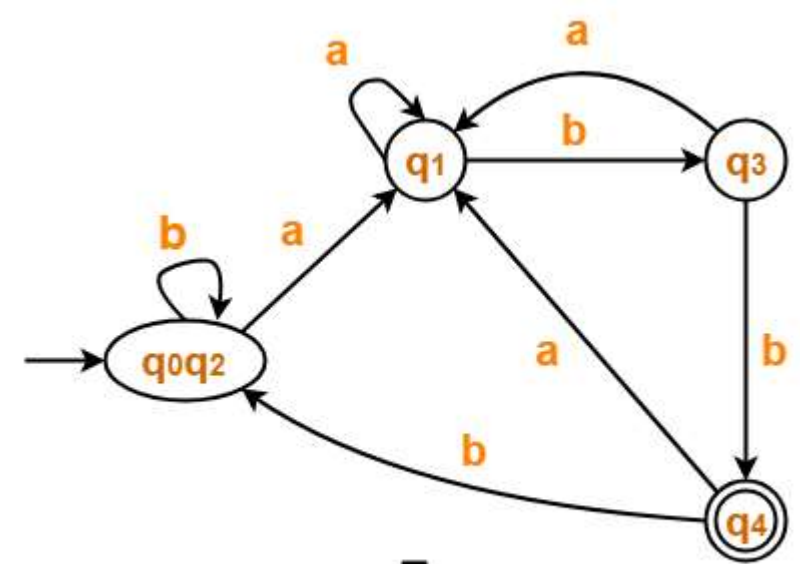
$$P_2 = \{ q_0, q_2 \} \{ q_1 \} \{ q_3 \} \{ q_4 \}$$

$$P_3 = \{ q_0, q_2 \} \{ q_1 \} \{ q_3 \} \{ q_4 \}$$

Since  $P_3 = P_2$ , so we stop.

From  $P_3$ , we infer that states  $q_0$  and  $q_2$  are equivalent and can be merged together.

So, Our minimal DFA is-



**Minimal DFA**

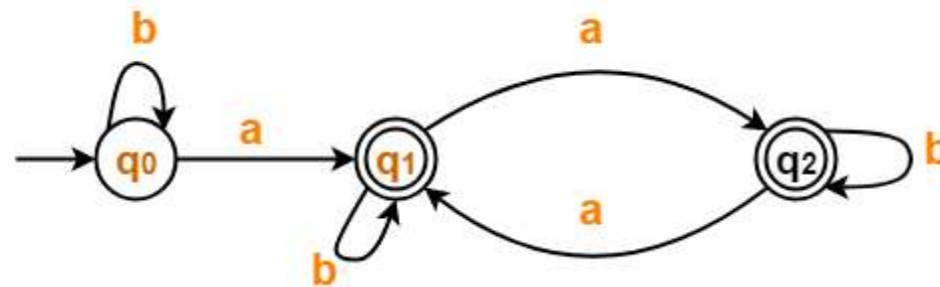
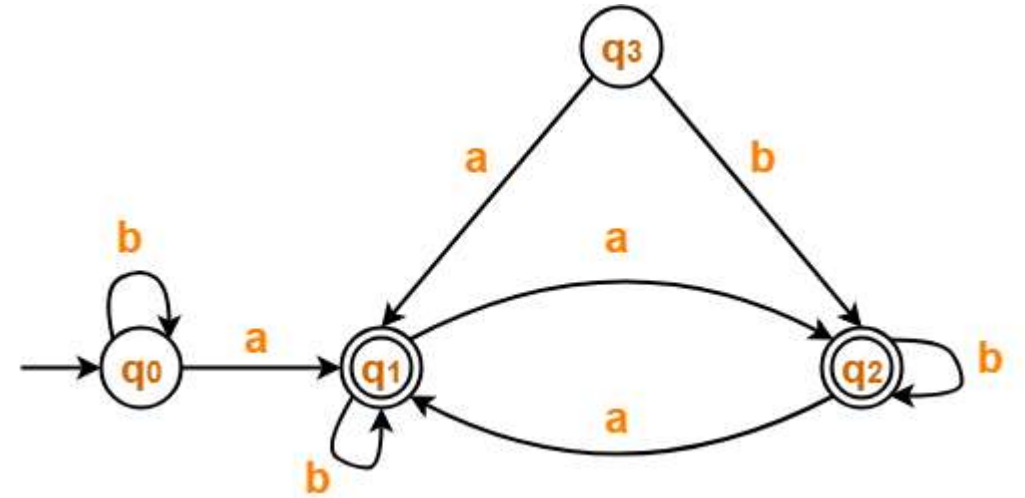
EX: 2 Minimize the given DFA-

## Solution-

### Step-01:

- State  $q_3$  is inaccessible from the initial state.
- So, we eliminate it and its associated edges from the DFA.

The resulting DFA is-





## Step-02:

Draw a state transition table-

	a	b
→q0	*q1	q0
*q1	*q2	*q1
*q2	*q1	*q2

## Step-03:

Now using Equivalence Theorem, we have-

$$P_0 = \{ q_0 \} \{ q_1, q_2 \}$$

$$P_1 = \{ q_0 \} \{ q_1, q_2 \}$$

Since  $P_1 = P_0$ , so we stop.

From  $P_1$ , we infer that states  $q_1$  and  $q_2$  are equivalent and can be merged together. So, Our minimal DFA is-

