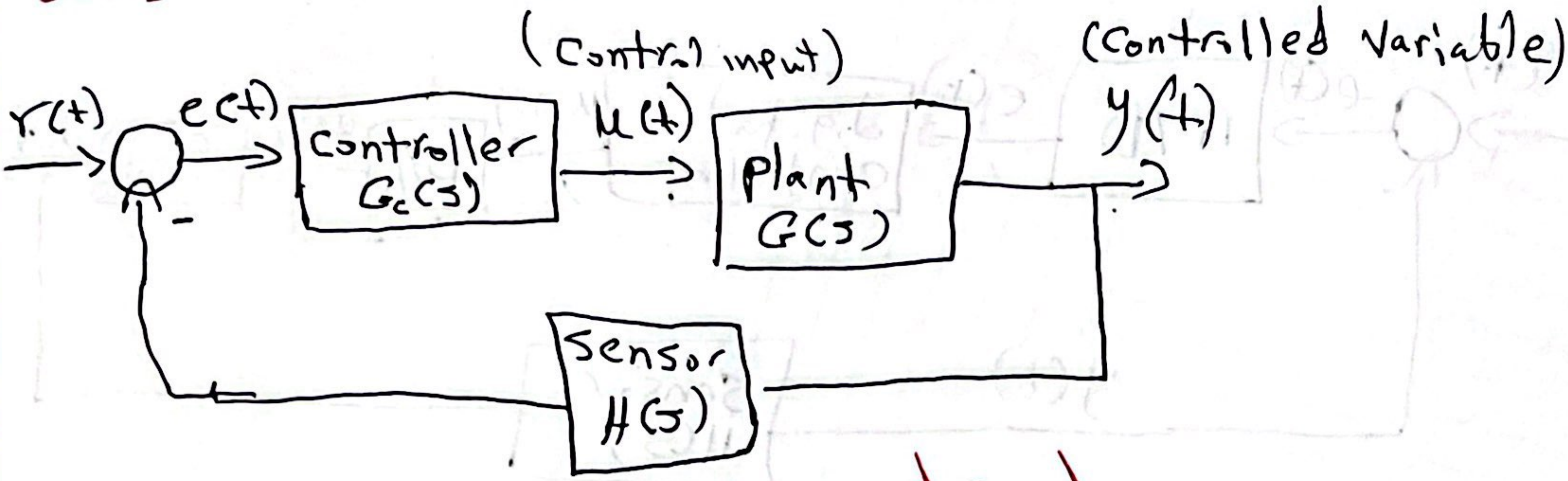
sec1

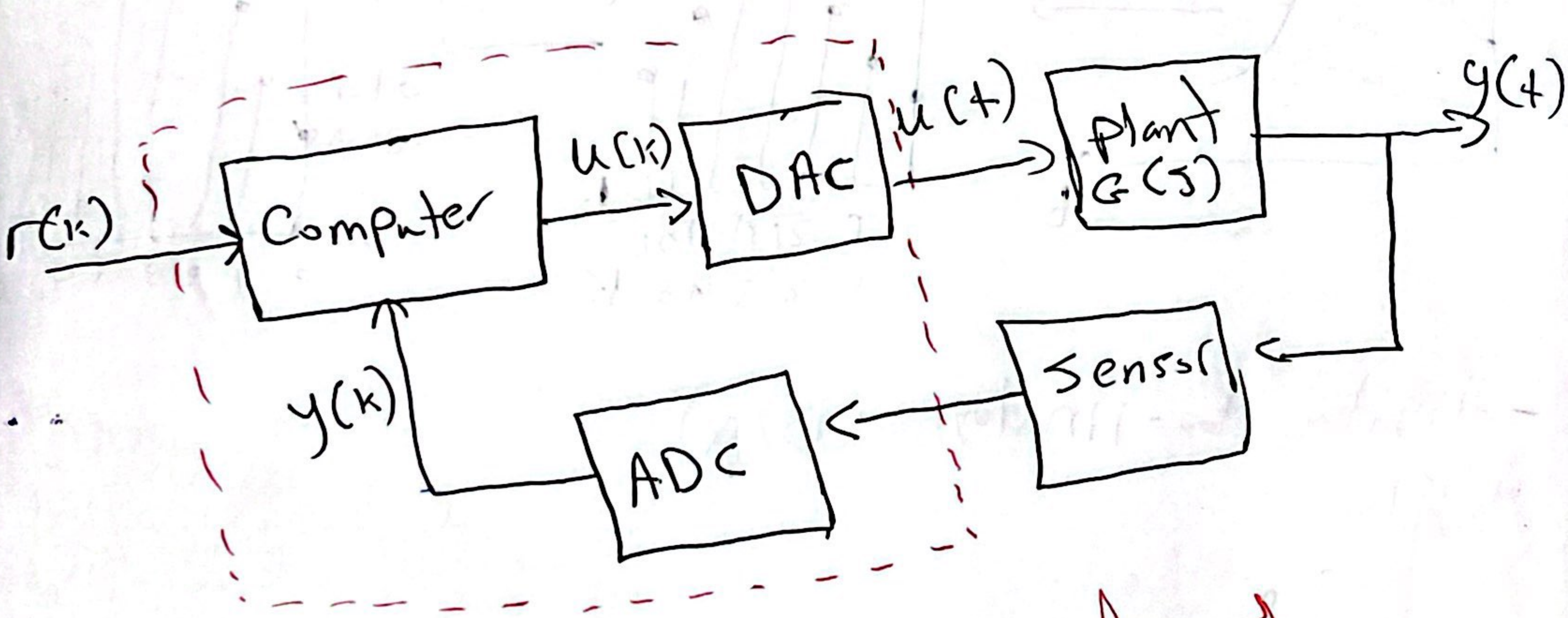
digital Control



(Analog control system)

* why digital control?

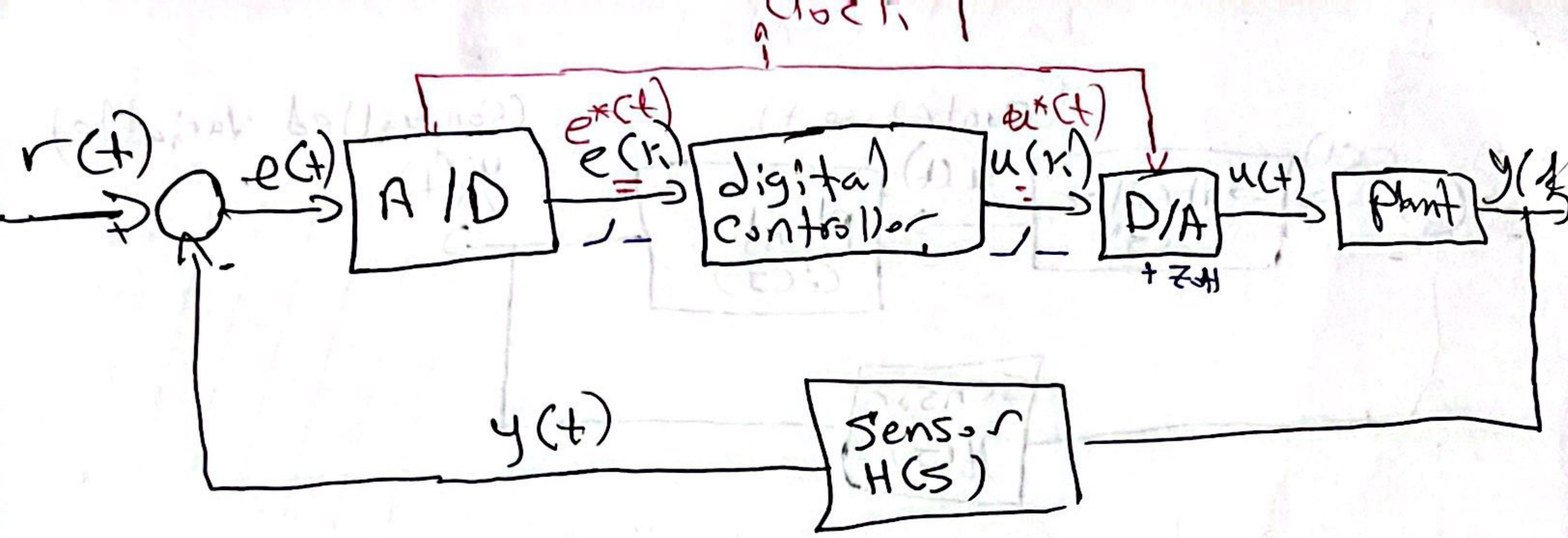
1- Accuracy
2- implementation errors
3- flexibility
4- speed
5- Cost



(digital control system)

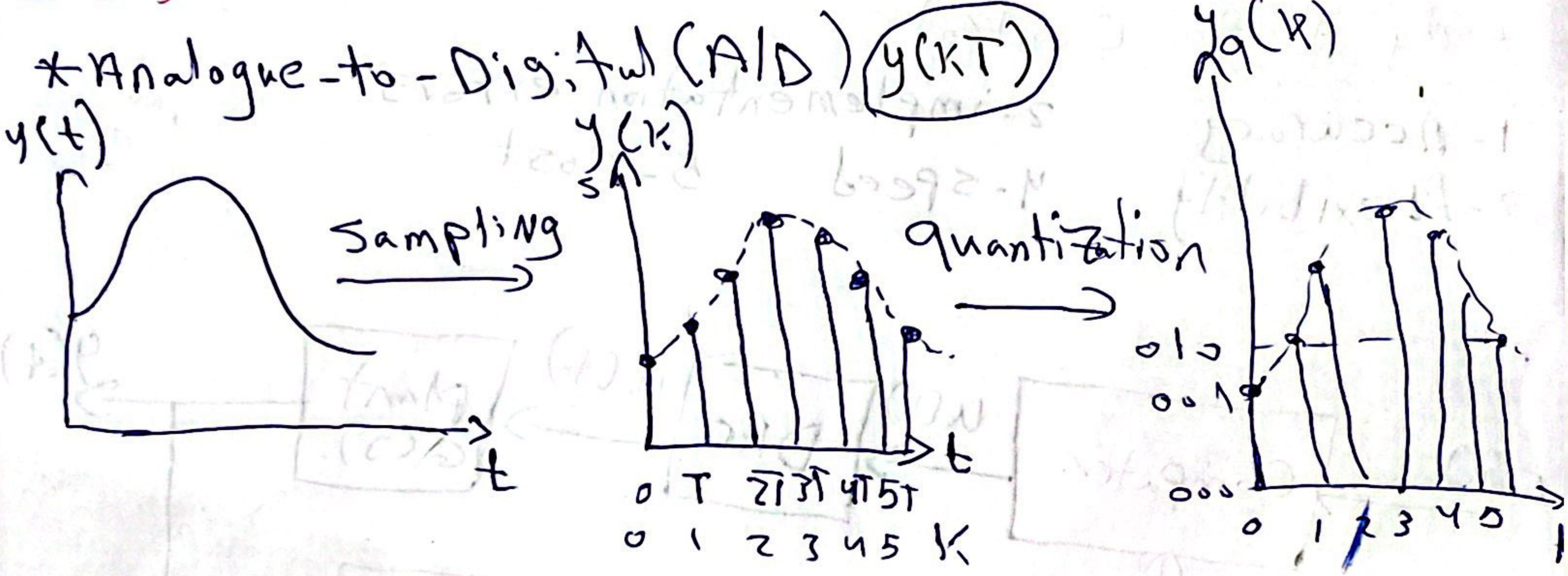
ADC: change Analog signal to digital signal

DAC: change digital signal to analog signal

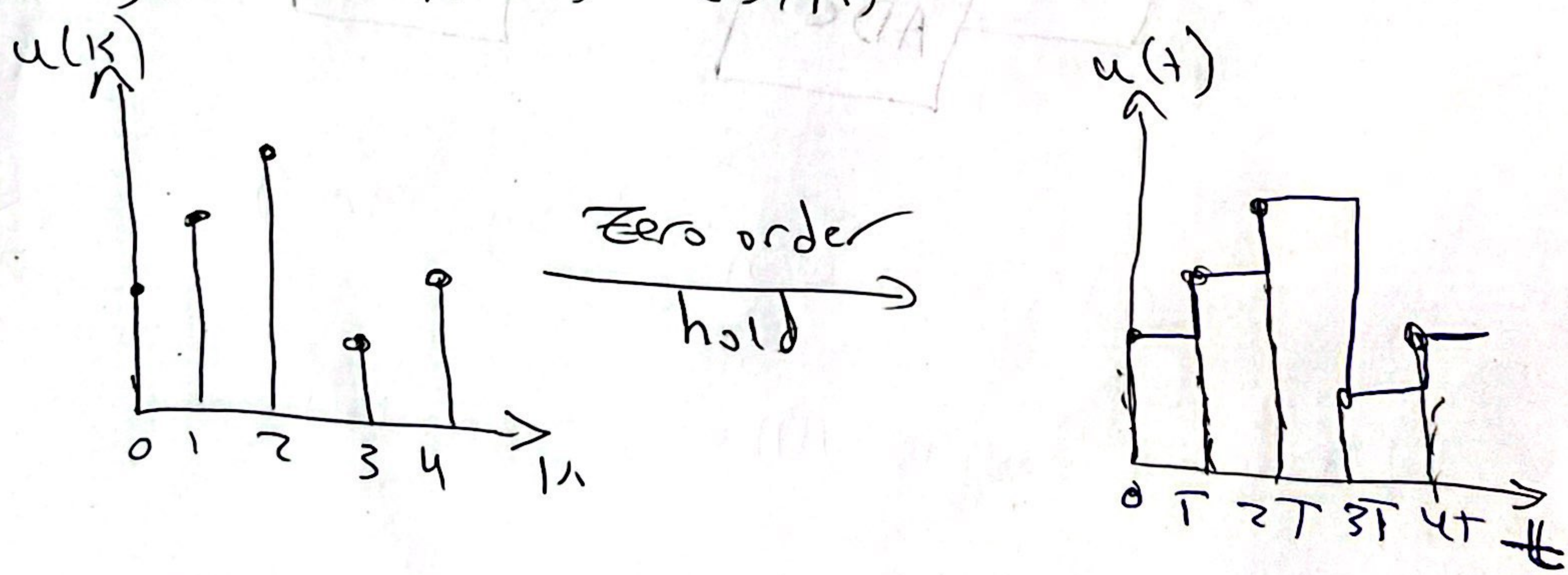


- Signal Conversion

* Analogue-to-Digital (A/D) y(KT)



- digital-to-Analogue (D/A)



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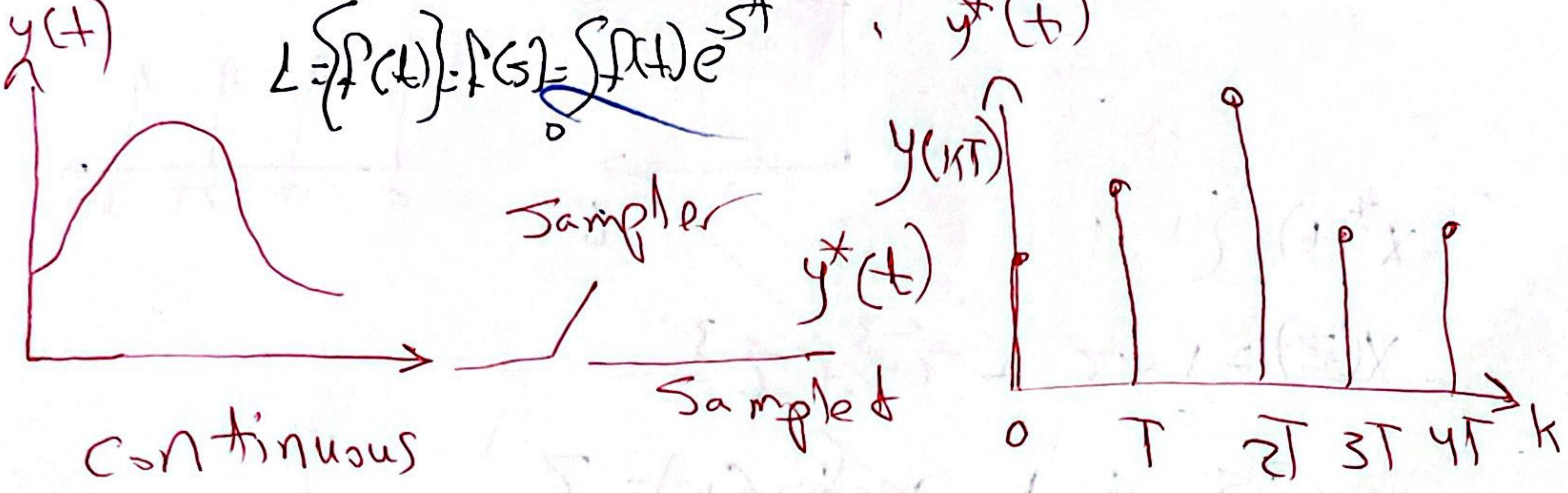
$$L\{f(t)\} = F(s) = \int_0^\infty f(t)\, e^{-st}$$



$$y^*(t) = y(t) \cdot \delta_T(t) = y(t) \cdot \sum_{K=0}^{\infty} \delta(t - KT)$$

$K = 0 - \infty$

$$y^*(t) = \sum_{K=0}^{\infty} y(KT)\, \delta(t - KT)$$

$$L[y^*(t)] = \sum_{K=0}^{\infty} y(KT)\, e^{-KST}$$

$$\boxed{z = e^{ST}}$$

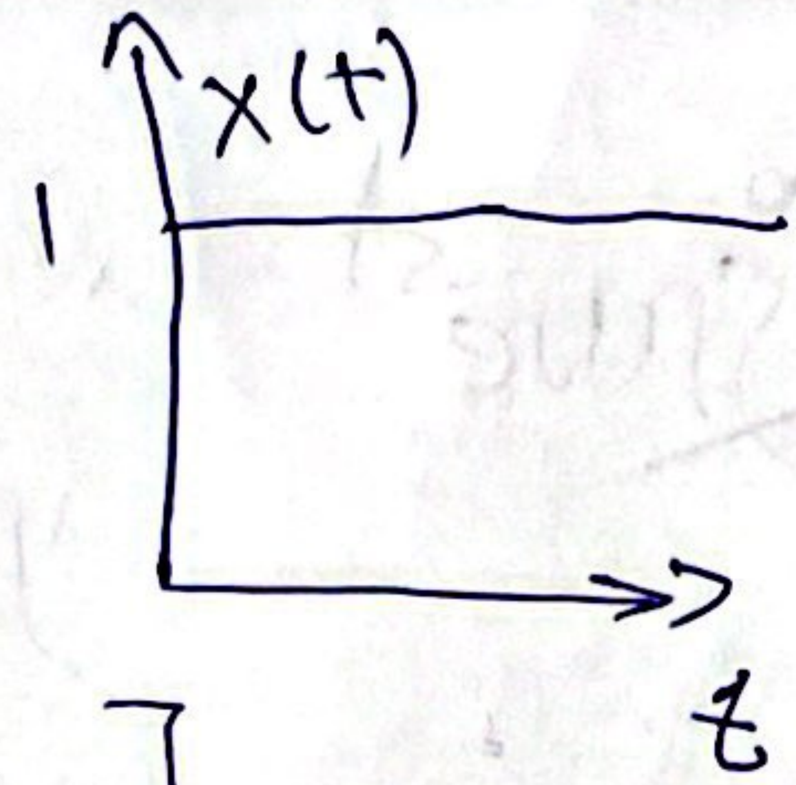
$$Z\{y(t)\} = Z\{y^*(t)\} = \sum_{K=0}^{\infty} y(KT)\, z^{-K}$$

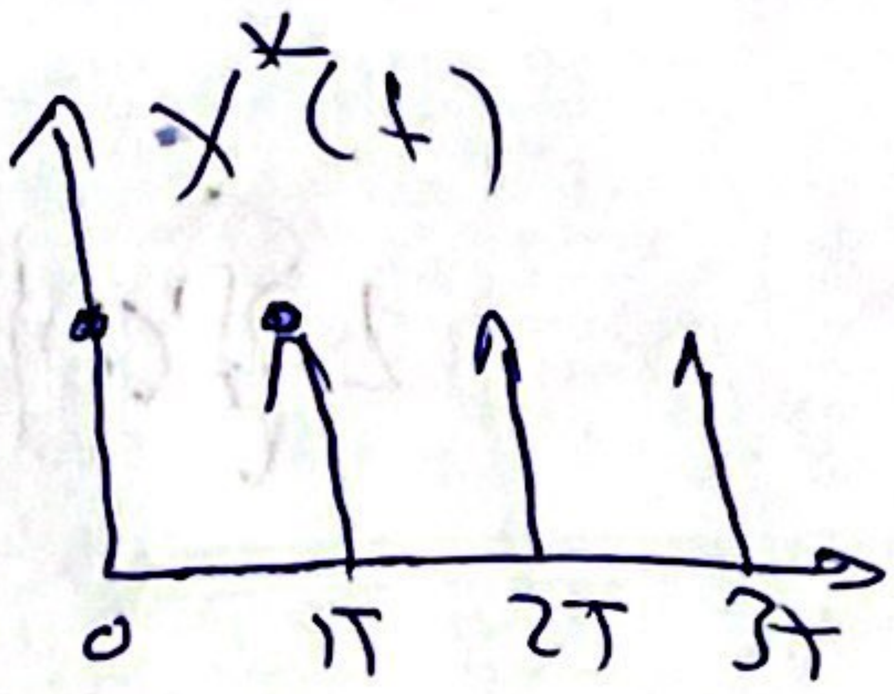
$$= y(0) z^0 + y(1) z^{-1} + y(2) z^{-2} + y(3) z^{-3} + \cdots$$

$$y^*(t) = \{1, 3, 2, 0, 4, 0, 0, \}$$

$$y(z) = 1 + 3z^{-1} + 2z^{-2} + 4z^{-4}$$ #

- step function





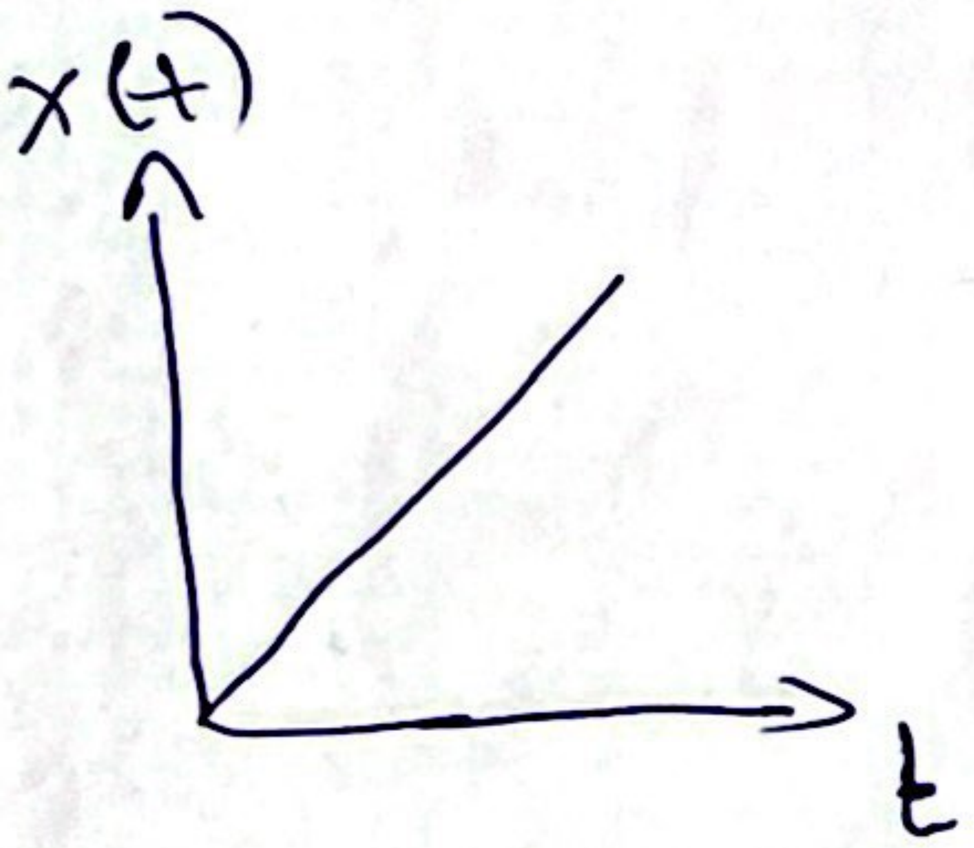
$x^*(t) = \{1, 1, 1, \ldots\}$

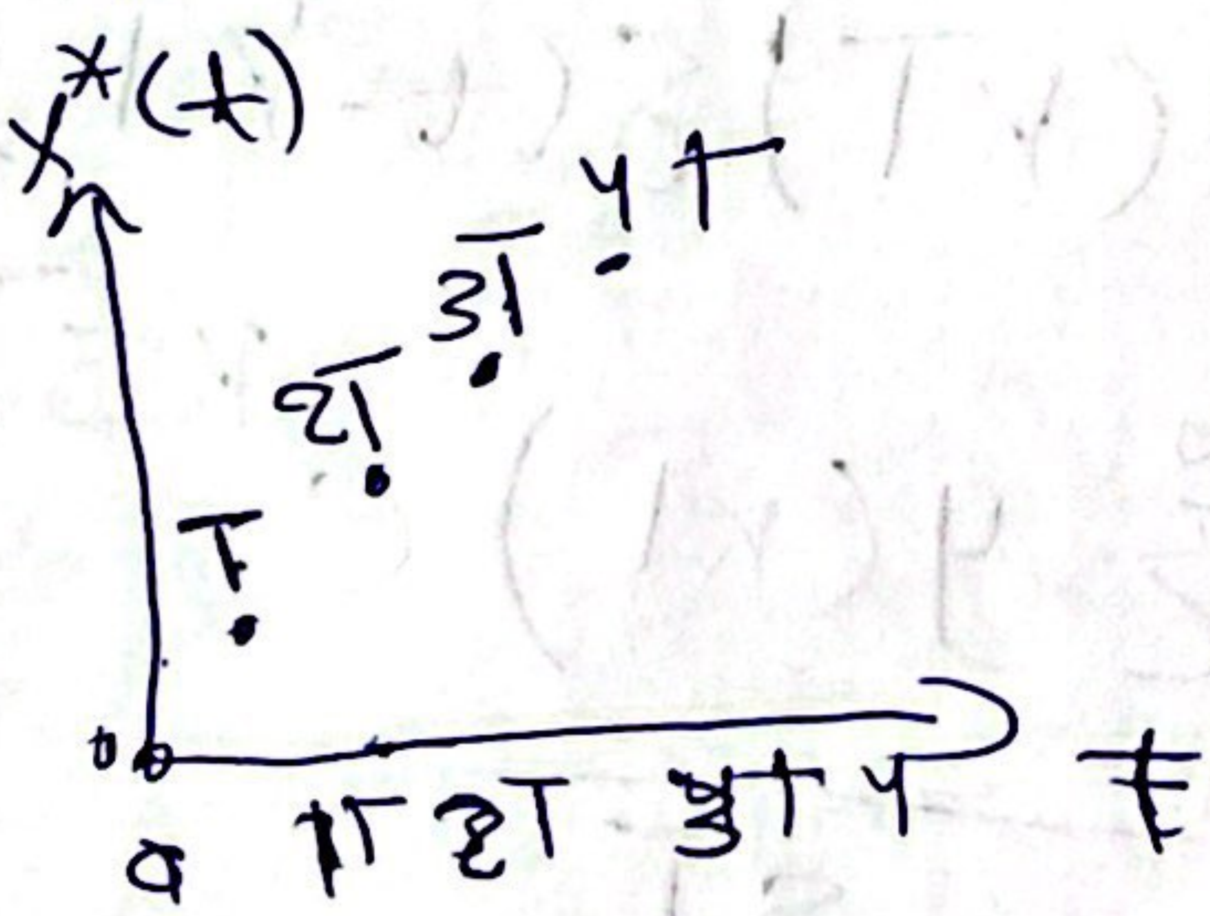
$X(z) = 1 + z^{-1} + z^{-2} + z^{-3} \ldots$

GS $a = 1$ ratio $(r) = z^{-1}$

$$X(z) = \frac{a}{1-r} = \frac{1}{1-\frac{1}{z}} * \frac{z}{z} = \frac{z}{z-1} \quad \#$$

* ramp function $f(t) = t$





$$X(z) = \sum_{k=0}^{\infty} x(kT) z^{-k} = 0 + Tz^{-1} + 2Tz^{-2} + 3Tz^{-3} \ldots$$

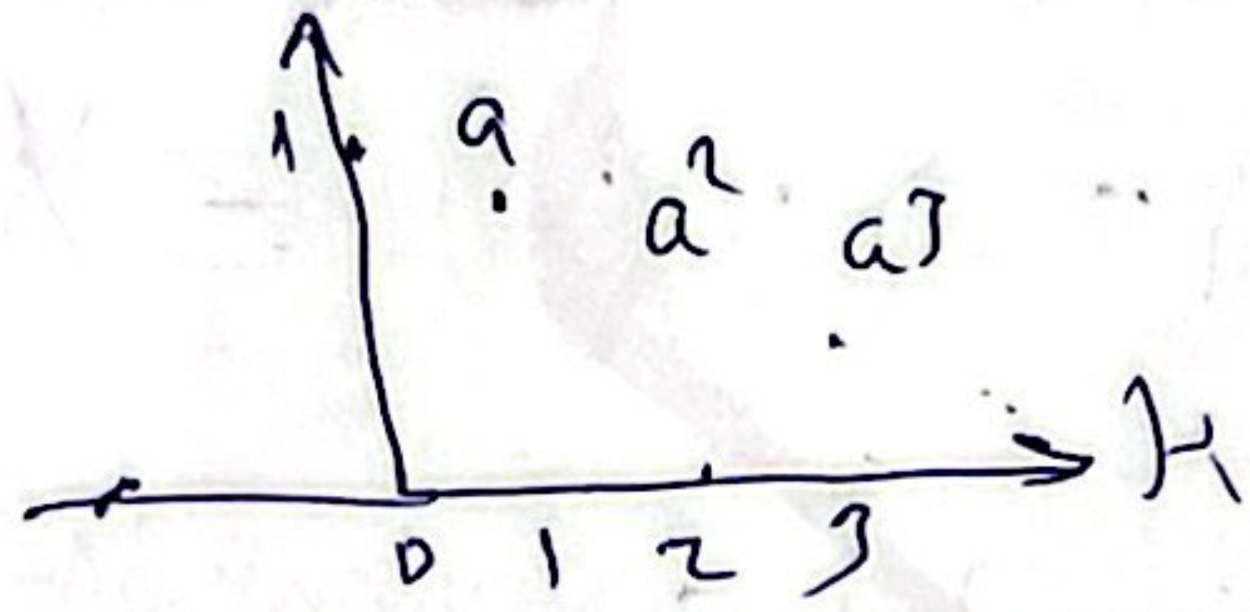
$$= T[z^{-1} + 2z^{-2} + 3z^{-3} + \ldots]$$

$$S = ar + 2ar^2 + 3ar^3 + 4ar^4 = \frac{ar}{(1-r)^2}$$

$a = 1$, $r = z^{-1}$

$$X(z) = \frac{Tz^{-1}}{(1-z^{-1})^2} * \frac{z^2}{z^2} = \frac{Tz}{(z-1)^2} \quad \#$$

$$u(k) = \begin{cases} a^k & k \geq 0 \\ 0 & k < 0 \end{cases}$$



$$\sum_{0}^{\infty} u(k) z^{-k} = 1 + a z^{-1} + a^2 z^{-2} + \cdots + a^k z^{-k}$$

$$U(z) = \frac{1}{1 - a/z} = \frac{z}{z - a} \quad \#$$

Time domain	LaPlace domain	z-domain $(t = kT)$
$\delta(t)$	1	1
$1(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
e^{-at}	$\frac{1}{s+a}$	$\frac{z}{z - e^{-aT}}$
t	$\frac{1}{s^2}$	$T \frac{z}{(z-1)^2}$
$\sin wt$	$\frac{w}{s^2 + w^2}$	$\frac{z \sin wT}{z^2 - 2z \cos wT + 1}$
$\cos wt$	$\frac{s}{s^2 + w^2}$	$\frac{z(z - \cos wT)}{z^2 - 2z \cos wT + 1}$

- properties of the z-transform

① Linearity $Z\{\alpha X_1(k) + B X_2(k)\} = \alpha X_1(z) + B X_2(z)$

② Time delay $x(t) = x(t - kT) \Rightarrow X(z) = z^{-kT} X(z)$

③ Time Advance $x(t) = x(t + kT) \Rightarrow X(z) = z^k X(z)$

$- z^{k-1} x(1) - z\, x(k-1)$

ex:
$$x(t) = x(t+1) \Longrightarrow X(z) = z\,X(z) - z\,x(0)$$

④ multiplication by exponentional

$$\mathcal{Z}\{a^{-k}x(k)\} = X(az) \Longrightarrow X(z)\big|_{az} = X(az)$$

⑤ Complex Translation theorem

$$\mathcal{Z}\left[e^{-at}f(t)\right] = F(z_1)\Big|_{z_1 = ze^{aT}} = F(ze^{aT}).$$

ex_2: $$\mathcal{Z}(e^{-at}\cos(wt)) = \mathcal{Z}(\cos(wt))\Big|_{z_1 = ze^{aT}}$$

$$= \frac{z_1^2 - z_1\cos(wT)}{z_1^2 - 2z_1\cos(wT) + 1}\Bigg|_{z_1 = ze^{aT}} = \frac{z^2e^{2aT} - ze^{aT}\cos(wT)}{z^2e^{2aT} - 2ze^{aT}\cos(wT) + 1}$$

ex_3: $f(t) = 1 - 0.8\,e^{-\frac{t}{2}}\cos(2.5t)$

$$\mathcal{Z}\{\cos(wt)\} = \frac{z^2 - z\cos(wT)}{z^2 - 2z\cos(wT) + 1}$$

$$F(z) = \frac{z}{z-1} - 0.8\,\mathcal{Z}[\cos(2.5t)]\Big|_{z_1 = ze^{T/2}}$$

$$F(z) = \frac{z}{z-1} - 0.8\,\frac{z_1^2 - z_1\cos(2.5T)}{z_1^2 - 2z_1\cos(2.5T) + 1}\Bigg|_{z_1 = ze^{T/2}}$$

$$= \frac{z}{z-1} - 0.8\,\frac{z^2e^{T} - ze^{T/2}\cos(2.5T)}{z^2e^{T} - 2ze^{T/2}\cos(2.5T) + 1}$$

ex5: $f(t) = t\, e^{-2t}$

$$Z(t) = Z(TK) = T\,Z(K) = T\,\frac{z}{(z-1)^2}$$

$$f(z) = Z(t)\Big|_{z_1 = ze^{2T}} = T\,\frac{z_1}{(z_1-1)^2}\Big|_{z_1 = ze^{2T}}$$

$$f(z) = \frac{Tze^{2T}}{(ze^{2T}-1)^2}$$

⑥ intial and final Values

in → $f(0) = \lim_{t \to 0} f(t) = \lim_{K \to 0} f(KT) = \lim_{z \to \infty} f(z)$

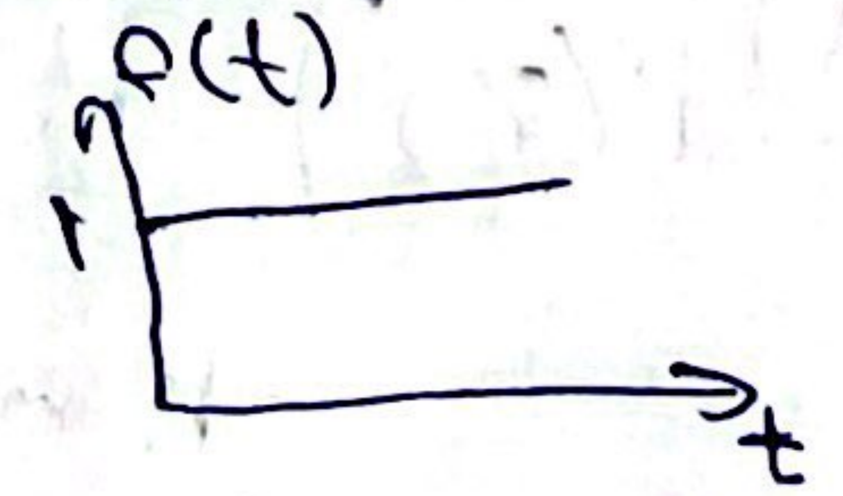
final $f(\infty) = \lim_{t \to \infty} f(t) = \lim_{K \to \infty} f(KT) = \lim_{z \to 1} \left[\frac{z-1}{z} f(z)\right]$

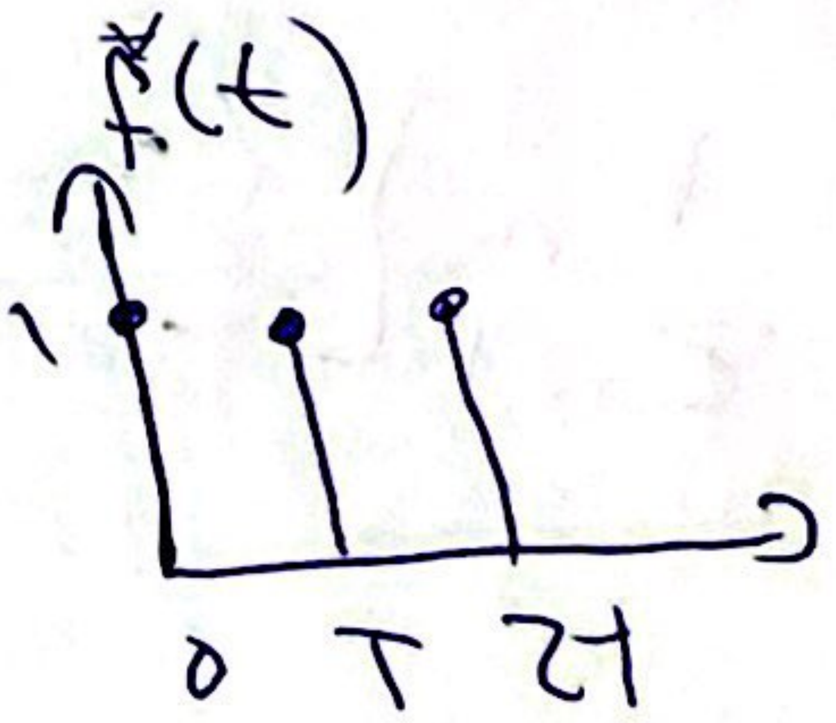
$$= \lim_{z \to 1} \left[(1 - z^{-1}) f(z)\right]$$

ex6: determine the intial and final Values for discrete unit step

$f(t) = u(t) = 1$

$= 1(t) \quad (t \geq 0)$





$$f(z) = \frac{z}{z-1}$$

intial value: $f(0) = \lim_{t \to 0} f(t) = \lim_{K \to 0} f(KT) = 1$

$$f(0) = \lim_{z \to \infty} f(z) = \lim_{z \to \infty} \frac{z}{z-1} = \lim_{z \to \infty} \frac{1}{1 - \frac{1}{z}} = \frac{1}{1 - \frac{1}{\infty}} = 1 =$$

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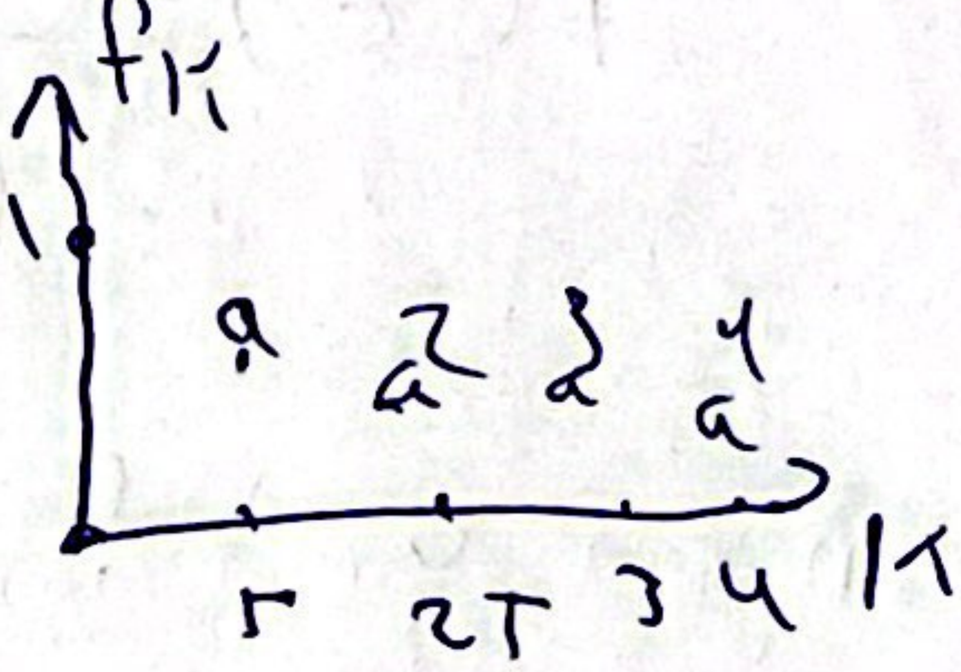
final value: $f(\infty) = \lim_{t\to\infty} f(t) = \lim_{k\to\infty} f(kT) = 1$

$$f(\infty) = \lim_{z\to 1}\left[\left(\frac{z-1}{z}\right)F(z)\right] = \lim_{z\to 1}\left(\frac{\cancel{z-1}}{\cancel{z}}\cdot\frac{\cancel{z}}{\cancel{z-1}}\right) = 1$$

Ex7: Determine the intial and final values for discrete exponential

$$f(k) = \begin{cases} a^k & k \geq 0 \\ 0 & k < 0 \end{cases}$$



$$F(z) = \frac{z}{z-a}$$

intial value: $f(0) = \lim_{t\to 0} f(t) = \lim_{k\to 0} f(kT) = a^0 = 1$

$$f(0) = \lim_{z\to\infty} F(z) = \lim_{z\to\infty}\left(\frac{z}{z-a}\right) = \lim_{z\to\infty}\frac{1}{1-\frac{a}{z}} = 1$$

final value: $f(\infty) = \lim_{t\to\infty} f(t) = \lim_{k\to\infty} f(kT) = a^\infty = 0$

$$f(\infty) = \lim_{z\to 1}\left[\frac{(z-1)}{z}F(z)\right] = \lim_{z\to 1}\left(\frac{z-1}{z}\right)\frac{z}{z-a}$$

$$= \lim_{z\to 1}\left[\frac{z-1}{z-a}\right] = 0$$

· inverse of the Z-transform

$Z^{-1}\{X(z)\} = \{x_k\}$ → causal sequence

ex: direc inverse Z-Transform (from table)

① $f(z) = \frac{5z}{z-1} \Rightarrow f(k) = 5 * 1^k = 5 * 1(k) = 5 \quad (k \geq 0)$

② $G(z) = \frac{5z}{z+1} \Rightarrow g(k) = 5 * (-1)^k \quad (k \geq 0)$

③ $H(z) = \frac{5z}{z-5} \Rightarrow h(k) = 5 * 5^k = 5^{k+1} \quad (k \geq 0)$

④ $c(z) = \frac{2z}{z+5} \Rightarrow c(k) = 2 * (-5)^k \mid k \geq 0)$

⑤ $f(z) = \frac{1.2z^{-1}}{1-2z^{-1}+z^{-2}} = \frac{1.2z}{z^2-2z+1} = \frac{1.2z}{(z-1)^2}$

$\{f(k)\} = \{1.2k\} \quad k \geq 0$

⑥ $G(z) = \frac{0.5z^{-1}}{1-z^{-1}+0.25z^{-2}} = \frac{0.5z}{z^2-z+0.25} = \frac{0.5z}{(z-0.5)^2}$

$\{g(k)\} = \{0.5k(0.5)^{k-1}\} \quad k \geq 1$

$\{g(k)\} = \{k(0.5)^k\} \quad k \geq 1$