Subject : Control lec 1:- chapter 1.   Date: /   /

Z-Trans form:- $x(Z) = \sum_{n=0}^{\infty} x(nT)\, Z^{-n}$

where :- $n \Rightarrow 0, 1, 2, ---$ $T \rightarrow$ Sampling time.

for sequence:- $T_s = 1$ sec:-

$$\boxed{x(Z) = \sum_{n=0}^{\infty} x(n)\, Z^{-n}}$$

* Z-Transform for some elementary functions:-

① Impulse :- $\delta(n) \xrightarrow{Z.T} 1$

② Unit step:- $u(n) \xrightarrow{Z.T} \frac{Z}{Z-1}$

③ Ramp :- $t\,u(t) \xrightarrow{Z.T} \frac{ZT}{(Z-1)^2}$

④ Polynominal:- $a^n u(n) \xrightarrow{Z.T} \frac{Z}{Z-a}$

⑤ Exponential:- $e^{-at} u(t) \xrightarrow{Z.T} \frac{Z}{Z-e^{-aT}}$

⑥ :- $n a^n u(n) \xrightarrow{Z.T} \frac{aZ}{(Z-a)^2}$

⑦ Sin Soidal:- $\sin(wt) \xrightarrow{Z.T} \frac{Z \sin wT}{Z^2 - 2Z\cos wT + 1}$

⑧ ~~Cos~~ :- $\cos(wt) \xrightarrow{Z.T} \frac{Z^2 - Z\cos wT}{Z^2 - 2Z\cos wT + 1}$

* Inverse Z-Transform:-

① Partial fraction method:-

ex:- Find the inverse Z-transform of $x(z) = \dfrac{10z+5}{(z-1)(z+2)}$

Sol:-

$$x(z) = \frac{10z+5}{(z-1)(z+2)} = \frac{A}{(z-1)} + \frac{B}{(z+2)}$$

$$A = \lim_{z\to 1} \frac{10z+5}{z+2} = \frac{15}{3} = 5$$

$$B = \lim_{z\to -2} \frac{10z+5}{z-1} = \frac{-15}{-3} = 5$$

$$\therefore x(z) = \frac{5}{z-1} + \frac{5}{z+2} \qquad * \, z z^{-1}$$

$$x(z) = 5z^{-1}\frac{z}{z-1} + 5z^{-1}\frac{z}{z-(-2)}$$

$$\boxed{x(n) = 5\,u(n-1) + 5(-2)^{n-1}}$$

ex: Find the inverse z transform for $x(z) = \dfrac{1}{(z-1)^2(z+2)}$

Sol:-

$$x(z) = \frac{A}{(z-1)^2} + \frac{B}{z+2} + \frac{C}{z-1}$$

$$A = \lim_{z\to 1} \frac{1}{z+2} = \frac{1}{3}$$

$$B = \lim_{z\to -2} \frac{1}{(z-1)^2} = \frac{1}{9}$$

$$C = \lim_{z\to 1} \frac{d}{dz}\left(\frac{1}{z+2}\right) = \lim_{z\to 1} \frac{-1}{(z+2)^2} = \frac{-1}{9}$$

$$\therefore x(z) = \frac{1}{3}\frac{1}{(z-1)^2} + \frac{1}{9}\frac{1}{z+2} - \frac{1}{9}\frac{1}{z-1} \quad * z z^{-1}$$

$$x(z) = \frac{1}{3} z^{-1} \frac{z}{(z-1)^2} + \frac{1}{9} z^{-1} \frac{z}{z-(-2)} - \frac{1}{9} z^{-1} \frac{z}{z-1}$$

$$x(n) = \frac{1}{3}(n-1)(1)^{n-1} u(n-1) + \frac{1}{9}(-2)^{n-1} u(n-1) - \frac{1}{9} u(n-1).$$

② Direct long Division: *approximated and used for non-closed form*

ex: find $x(n)$ for $n = 0, 1, 2, 3, 4,$ when $x(t)$ is given by:-

$$x(z) = \frac{z^2 + 2z}{z^3 + z^2 + z + 1} * \frac{z^{-3}}{z^{-3}}$$

نضرب البسط والمقام × أعلى أس في المقام بس بإشارة مخالفة:

$$x(z) = \frac{z^{-1} + 2z^{-2}}{1 + z^{-1} + z^{-2} + z^{-3}} \begin{matrix} \leftarrow \text{dividend} \\ \leftarrow \text{divisor} \end{matrix}$$

Steps:-

1. Divide first term of dividend by first term of divisor and place the result in quotient.

2. Multiply first term on quotient by divisor.

3. flip signs of result and add it with dividend

4. Repeat this process, each time use the new dividend until you get $z^{-(n)}$ where $x(n)$ is the last required sample

5. Compare coff. of quotient with general equation.

Subject : .... Date: / /

$$x(z)=\sum_{n=0}^{\infty} x(n)\, z^{-n}$$

$$x(z)=x(0)+x(1)z^{-1}+x(2)z^{-2}+x(3)z^{-3}+ \cdots$$

ex: Find $x(n)$ for $n=0,1,2,3,4$ when $x(t)$ is given by:

$$x(t)=\frac{z^{2}+2z}{z^{3}+z^{2}+z+1} * \frac{z^{-3}}{z^{-3}}$$

sol.

$$x(t)=\frac{z^{-1}+2z^{-2}}{1+z^{-1}+z^{-2}+z^{-3}}$$

$$\begin{array}{r|l}
 & z^{-1}+z^{-2}-2z^{-3}+z^{-5} \\
\hline
1+z^{-1}+z^{-2}+z^{-3} & z^{-1}+2z^{-2} \\
 & -(z^{-1}+z^{-2}+z^{-3}+z^{-4}) \\
\hline
 & z^{-2}-z^{-3}-z^{-4} \\
 & -(z^{-2}+z^{-3}+z^{-4}+z^{-5}) \\
\hline
 & -2z^{-3}-2z^{-4}-z^{-5} \\
 & -(-2z^{-3}-2z^{-4}-2z^{-5}-2z^{-6}) \\
\hline
 & z^{-5}+2z^{-6}
\end{array}$$

Quotient $= z^{-1}+z^{-2}-2z^{-3} \rightarrow$ (I) $\rightarrow$ the largest negative power of "z" should be $\leq$ maximum number of n.

General equation: $x(z)=x(0)+x(1)z^{-1}+x(2)z^{-2}+x(3)z^{-3}+x(4)z^{-4} \rightarrow$ (II)

Compare (I), (II) we get:

$x(0)=0$, $x(1)=1$, $x(2)=1$, $x(3)=-2$, $x(4)=0$