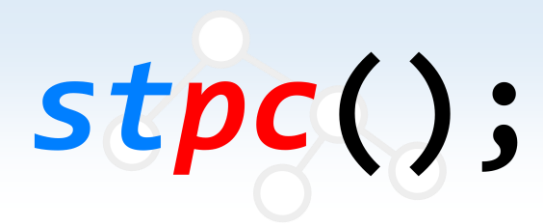# B – Decode II

Chin Ka Wang `{rina__owo}`

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Background

Problem Idea by `rina__owo`

Preparation by `rina__owo,pepper1208`

Problem Restatement

There is total of Q queries. Each query provides an integer x.

Define $rev(x)$ as the reverse order of integer x.

Determine whether $x + rev(x)$ is a palindrome.

Subtasks Constraints

	Score	Constraints
1	7	$x \leq 100$
2	3	$x \leq 10^{14}$
3	8	Every digit in $x < 5$
4	8	Every digit in $x > 5$
5	74	No additional constraints

Subtask 1 (7%): N ≤ 100

- Notice that the constraint is very small, we can brute force for the answer.

- Literally reverse the whole input number using an array or STL function, then add *x* to it.

- Literally check if the result is a palindrome using basic for loop.

- Expected score: **7** (Culminative score: **7**)

- Time complexity: O(Q * |N|)

Subtask 2 (3%): N ≤ 10^{14}

- Note that the maximum integer stored in **int** is approximately 10^9.
- Use **long long** instead.
- Expected score: **3** (Culminative score: **10**)
- Time complexity: O(Q * |N|)

Subtask 3 (8%): Every digit in x < 5

- Observe that for every digit in *x* < 5, no carry occurs.
- It is not hard to observe that *x* + *rev*(*x*) is **symmetric** in case of digits, so it must be a palindrome.
- We just need to output Yes in this subtask.
- Expected score: **8** (Culminative score: **18**)
- Time complexity: O(1)

Subtask 4 (8%): Every digit in x > 5

- Observe that if every digit in x > 5, carry must occur in each corresponding digit.

- Note that a carry will add one to the digit left to the current one.

- It is not hard to observe that this must break the symmetry of *x* + *rev*(*x*).

- We just need to output No in this subtask.

- Expected score: **8** (Culminative score: **26**)

- Time complexity: O(1)

Subtask 5 (74%): No additional constraints

- Note that a carry will add one to the digit left to the current one. Also, the result must be a palindrome if there are no carries occur.

- However, is it possible that the result be a palindrome even carries occur?

Subtask 5 (74%): No additional constraints

- Assume that there appear carries when calculating *x* + *rev*(*x*).

- **Observation 1**: For the result to be a palindrome, the first digit and the last digit of it must be the same.
  - Proof: Trivial, from the definition of a palindrome.

- **Observation 2**: If carry do not occur on the first digit in the addition, the result must not be a palindrome.
  - The proof is left as exercise. ☺

Subtask 5 (74%): No additional constraints

- By observation 1 and 2, it is easily to deduce that the sum of the first digit and the last digit of x must be 11 for $x + rev(x)$ to be a palindrome.

- However, the carry on the last digit will break the symmetry of the middle part of the result.

- After extending the idea, we can conclude that each addition of digits in $x + rev(x)$ have to be 11 or 0, symmetrically.

- Using a simple for loop can achieve the construction above.

- Expected score: **74** (Culminative score: **100 AC!**)

- Time complexity: O(N)

Alternative Method: APA

- Implement an array with sufficient size to store an integer digit-by-digit.
  - Referring to the constraint, the size of the array should exceed 3000.
- Then, implement *addition*, *reverse* and *palindrome checking* on array.
  - Addition: add the integers digit-by-digit, while maintaining carries.
  - Reverse: swap corresponding digits.
  - Palindrome checking: check if the integer is equal to the reversed integer
- Expected score: **100 AC!**
- Time complexity: O(N)

Takeaways

- The subtasks may give you inspirations for the full solution.

- Arbitrary-Precision Arithmetic is a simple and strong algorithm when processing large number calculations.

- Reasoning and observations are always important.