Math 42 HW 4

stewartben02

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Collaborators: Evan Coons, Jessie Baker, and Ronan Nayak

1

I was very interested in chapter 6 how much of certain graphs can be erased without the loss of any data. The bar chart in the beginning of the chapter seemed like a relatively simple chart without much excess ink, but the addition of the white grid allowed the author to erase the tick marks–a technique I hadn't seen before. I also do not think I have ever seen the technique of erasing the frame where no data points lie in a scatter plot by taking the minimum and maximum x as well as the minimum and maximum y.

2

a

$$x(n) = \frac{x(n-1)}{1+x(n-1)}$$

$$x^\star = \frac{x^\star}{1+x^\star}$$

$$(1+x^\star)x^\star = x^\star$$

$$1+x^\star = 1$$

$$x^\star = 0$$

Stability test:

$$f'(x) = \frac{1}{(x+1)^2}$$

$$f'(0) = 1$$

The stability test is inconclusive

b

$$x(n) = x(n-1)e^{rx(n-1)}$$
$$x^\star = x^\star e^{rx^\star}$$
$$1 = e^{rx^\star}$$
$$0 = rx^\star$$
$$x^\star = 0$$

Stability test:

$$f'(x) = e^{rx} + re^{rx}x$$
$$f'(0) = 1$$

The stability test is inconclusive.

c

$$x(n) = x(n-1)^2 - 6$$
$$x^\star = x^{\star 2} - 6$$
$$x^{\star 2} - x^\star - 6 = 0$$
$$(x^\star - 3)(x^\star + 2) = 0$$
$$x^\star = -2, 3$$

Stability test:

$$f'(x) = 2x$$

$x^\star = -2$:

$$f'(-2) = -4$$
$$|-4| > 1$$

The fixed point $x^\star = -2$ is unstable.
$x^\star = 3$:

$$f'(3) = 6$$
$$|6| > 1$$

The fixed point $x^\star = 3$ is unstable.

d

$$x(n) = x(n-1)^2 + 0.7x(n-1) + 0.02$$
$$x^\star = x^{\star 2} + 0.7x^\star + 0.02$$
$$x^{\star 2} - 0.3x^\star + 0.02 = 0$$
$$x = \frac{0.3 \pm \sqrt{0.3^2 - 4(1)(0.02)}}{2}$$
$$x = 0.1, 0.2$$

Stability test:

$$f'(x) = 2x + 0.7$$

$x^\star = 0.1$**:**

$$f'(x) = 0.9$$
$$|0.9| < 1$$

The fixed point $x^\star = 0.1$ is stable.
$x^\star = 0.2$**:**

$$f'(x) = 1.1$$
$$|1.1| > 1$$

The fixed point $x^\star = 0.2$ is unstable.

3

Stable at $x = 1$, $|f'(1)| > 1$
Unstable at $x = 3$, $|f'(3)| > 1$
We will define a function f as $f(x) = ax^2 + bx + c$ with a, b, and c being constants.

$$f(x) = ax^2 + bx + c$$
$$f(1) = a + b + c = 1$$
$$f(3) = 9a + 3b + c = 3$$

Now to solve for b and c in terms of a:

$$f(3) - f(1) : 8a + 2b = 2$$
$$4a + b = 1$$
$$b = 1 - 4a$$
$$a + (1 - 4a) + c = 1$$
$$-3a + c + 1 = 1$$
$$c = 3a$$

Plug back into original function and take derivatives:

$$f(x) = ax^2 + (1 - 4a)x + 3a$$

$$f'(x) = 2ax - 4a + 1$$
$$f'(1) = 1 - 2a$$
$$f'(3) = 1 + 2a$$

Find a given the constraints:

$$-1 < 1 - 2a < 1$$

$$1 + 2a < -1 or 1 + 2a > 1$$

We can define a as $a = 0.1$ Find the function:

$$f(x) = 0.1x^2 + 0.6x + 0.3$$

and finally:

$$x(n + 1) = 0.1x(n)^2 + 0.6x(n) + 0.3$$

4

$$N(t + 1) = 1.2N(t) - 0.02N(t)^2 - 0.04N(t)P(t)$$
$$P(t + 1) = 0.9P(t) - 0.01P(t)^2 + 0.02N(t)P(t)$$

a

$$N = 1.2N - 0.02N^2 - 0.04NP$$
$$0 = N(0.2 - 0.02N - 0.04P)$$

$$P = 0.9P - 0.01P^2 + 0.02NP$$
$$0 = P(-0.1 - 0.01P + 0.02N)$$

<u>if $N = 0$:</u>

$$0 = P(-0.1 - 0.01P)$$
$$P = 0 \rightarrow (N, P) = (0, 0)$$

and

$$-0.1 - 0.01P = 0$$
$$P = -10 \rightarrow (N, P) = (0, -10)$$

<u>if $P = 0$:</u>

$$0 = N(0.2 - 0.02N)$$
$$N = 0 \rightarrow (N, P) = (0, 0)$$

and

$$0.2 - 0.02N = 0$$

$$N = 10 \rightarrow (N, P) = (10, 0)$$

<u>if $0.2 - 0.02N - 0.04P = 0$:</u>

$$0.02N = 0.2 - 0.04P$$

$$N = 10 - 2P$$

$$-0.1 - 0.01P + 0.02(10 - 2P) = 0$$

$$-0.1 - 0.01P + 0.2 - 0.04P = 0$$

$$0.1 - 0.05P = 0$$

$$P = 2$$

$$N = 10 - 2(2)$$

$$N = 6 \rightarrow (N, P) = (6, 2)$$

The fixed points are: $(0, 0), (0, -10), (10, 0), (6, 2)$ Now for the Jacobian:

$$J = \begin{bmatrix} \frac{df_1}{dN} & \frac{df_1}{dP} \\ \frac{df_2}{dN} & \frac{df_2}{dP} \end{bmatrix}$$

$$J = \begin{bmatrix} 1.2 - 0.04N - 0.04P & -0.04P \\ 0.02P & 0.9 - 0.02P + 0.02N \end{bmatrix}$$

[16]:
```
def x(N, P):
    J = np.matrix([[1.2-0.04*N-0.04*P, -0.04*P],
                   [0.02*P, 0.9-0.02*P+0.02*N]])
    eigenvalues = np.linalg.eig(J)[0]
    if any(abs(eigenvalues) > 1):
        return "Unstable"
    return "Stable"
```

[17]:
```
x(0,0)
```

[17]: 'Unstable'

[18]:
```
x(0, -10)
```

[18]: 'Unstable'

[19]:
```
x(10, 0)
```

[19]: 'Unstable'

[20]:
```
x(6, 2)
```

[20]: 'Stable'

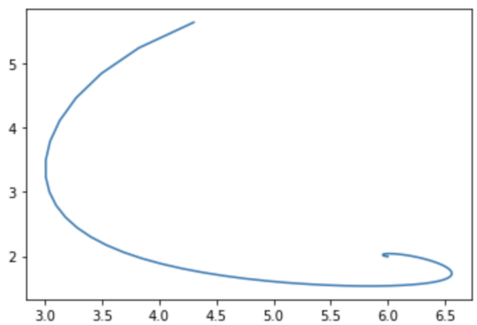
b

```
def model(state):
    n, p= state
    return(1.2*n-0.02*n**2-0.04*n*p, 0.9*p-0.01*p**2+0.02*n*p)
```

```
initial = (5, 6)
x = []
y=[]
for i in range(100):
    initial = model(initial)
    x.append(initial[0])
    y.append(initial[1])
```

```
plt.plot(x, y)
```

```
[<matplotlib.lines.Line2D at 0x7fee1afb05b0>]
```



This does agree with my results from earlier because it approaches the stable point (6, 2).

5

a

```
[84]: def P(p, d):
          return ((1 - p**(d-1))/(1-p**d)) * p
```

```
[97]: def G(p, d):
          return ((p**d) * (1 - p)) / (1 - p**d)
```

```
[98]: P2 = P(0.7857, 7)
```

```
[99]: G2 = G(0.7857, 7)
```

```
[100]: P3 = P(0.6758 , 8)
```

```
[101]: G3 = G(0.6758, 8)
```

```
[102]: P4 = P(0.7425 , 6)
```

```
[136]: G4 = G(0.7425 , 6)
```

b

```
[167]: A = np.diagflat([0.6747, G2, G3, G4, 0.8091 , 0.8091], -1)
       np.fill_diagonal(A, [0, P2, P3, P4, 0, 0, 0.8091])
       A[0,] = [0, 0, 0, 0, 127, 4, 80]
```

```
[177]: A
```

```
[177]: array([[0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
                1.27000000e+02, 4.00000000e+00, 8.00000000e+01],
               [6.74700000e-01, 7.37106646e-01, 0.00000000e+00, 0.00000000e+00,
                0.00000000e+00, 0.00000000e+00, 0.00000000e+00],
               [0.00000000e+00, 4.85933545e-02, 6.61053937e-01, 0.00000000e+00,
                0.00000000e+00, 0.00000000e+00, 0.00000000e+00],
               [0.00000000e+00, 0.00000000e+00, 1.47460633e-02, 6.90667193e-01,
                0.00000000e+00, 0.00000000e+00, 0.00000000e+00],
               [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 5.18328073e-02,
                0.00000000e+00, 0.00000000e+00, 0.00000000e+00],
               [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
                8.09100000e-01, 0.00000000e+00, 0.00000000e+00],
               [0.00000000e+00, 0.00000000e+00, 0.00000000e+00, 0.00000000e+00,
                0.00000000e+00, 8.09100000e-01, 8.09100000e-01]])
```

c

```
[185]: np.matmul(np.linalg.matrix_power(A, 10), initial)

[185]: matrix([[5.75109503e+02],
               [2.29784980e+03],
               [3.72237032e+02],
               [1.52970547e+01],
               [7.30169881e-01],
               [5.24392846e-01],
               [5.33052305e+00]])
```

The population seems to be slightly declining.

d

```
[172]: abs(np.linalg.eig(A)[0])

[172]: array([0.14889702, 0.14889702, 0.26625279, 0.37093228, 0.77612369,
              0.77612369, 0.94528704])

[186]: b = np.linalg.eig(A)[1][:,6]

[187]: b

[187]: array([2.90965719e-01+0.j, 9.43002191e-01+0.j, 1.61218517e-01+0.j,
              9.33681507e-03+0.j, 5.11964424e-04+0.j, 4.38205959e-04+0.j,
              2.60342273e-03+0.j])

[188]: (1 / sum(b)) * b

[188]: array([2.06640513e-01+0.j, 6.69709328e-01+0.j, 1.14495539e-01+0.j,
              6.63089886e-03+0.j, 3.63591255e-04+0.j, 3.11208841e-04+0.j,
              1.84892093e-03+0.j])
```

From the eigenvalues, we can conclude that the overall population of the loggerheads will decline by 0.055 or 5.5% each year. From the eigenvectors, we get the distribution of turtles in each age range in the long run. We see that age ranges 1-3 will have the most turtles while 4-7 will have very few. This makes sense because the survival percentages are not very high, so for every age group it becomes less and less likely that turtles from the prior age group will survive until reaching the age group above. This is because raising a decimal to a high power will quickly make the proportion decline.

e

```
[189]: B = A = np.diagflat([0.6747, G2, G3, G4, 0.8091 , 0.8091], -1)
       np.fill_diagonal(B, [0, P2, P3, P4, 0, 0, 0.8091])
       B[0,] = [0, 0, 0, 0, 150, 20, 100]
```

```
[190]: abs(np.linalg.eig(B)[0])
```

```
[190]: array([0.15124253, 0.15124253, 0.31371928, 0.31371928, 0.78061002,
              0.78061002, 0.95667464])
```

```
[196]: c = np.linalg.eig(B)[1][:,6]
```

```
[197]: (1 / sum(c)) * c
```

```
[197]: array([2.16647416e-01+0.j, 6.65725488e-01+0.j, 1.09430206e-01+0.j,
              6.06623892e-03+0.j, 3.28669936e-04+0.j, 2.77969995e-04+0.j,
              1.52401198e-03+0.j])
```

The overall growth does not change very much as the population is still declining by around 5% each year. The distribution of sea turtles by age also does not change much, with only slightly more in the first age range. This suggests that protecting eggs does not significantly increase population.

f

```
[207]: C = np.diagflat([0.6747, G2 + 0.05, G3+0.05, G4+0.05, 0.8091 , 0.8091], -1)
       np.fill_diagonal(C, [0, P2+0.05, P3+0.05, P4+0.05, 0, 0, 0.8091])
       C[0,] = [0, 0, 0, 0, 127, 4, 80]
```

```
[211]: abs(np.linalg.eig(C)[0])
```

```
[211]: array([0.32436831, 0.32436831, 1.17712194, 0.91051886, 0.91051886,
              0.46099968, 0.46099968])
```

```
[213]: c = np.linalg.eig(C)[1][:,2]
```

```
[215]: (1 / sum(c)) * c
```

```
[215]: array([0.31593618+0.j, 0.54654815+0.j, 0.11561835+0.j, 0.01715145+0.j,
              0.00148377+0.j, 0.00101988+0.j, 0.00224221+0.j])
```

When increasing the percentage of surviving and staying within the age class as well as surviving and entering the new age class by 0.05 for groups 2, 3, and 4, I found that the population would increase by 17.7% each year. In the long term, the population would grow exponentially and the distribution of age groups would be much more evenly distributed with most in the first, second, and third age groups.

g

If I were in a policy-making position in 1987 regarding the measures to protect the loggerhead sea turtles, I would explain how protecting the eggs of the sea turtles will not significantly increase the population of the turtles, and the overall population will still decline. I would suggest that protecting the turtles in the younger age groups be the best course of action, as ever so slightly increasing their survival rates results in an increase in the overall population. I would make the use of the Turtle Excluder Device (TED) mandatory, as it would stop the accidental killing of turtles when fishing for shrimp and fish. Because this is the main source of death in juvenile turtles in the region the loggerheads reside, this would most likely increase the survival rates of juveniles to rates where the population would increase.

6

a

After discussing my findings with Jessie Baker, Ronan Nayak, and Evan Coons, we all came to the conclusion that the most effective strategy to protect the loggerhead turtles and increase their population would be to protect the turtles in the younger life stages. We found that just protecting the eggs does not make increase the overall population by very much while protecting the turtles in the first stages of life has significant effects on the population. Most notably, we found that after protecting more eggs, the population still declines, while after protecting more eggs, the population increases.

b

A generalization of the project is how to use discrete time models to model different ideas for protecting the loggerhead turtle population. By running different test scenarios, we are able to find the best approach to increasing the number of loggerheads based on factors such as probability of survival and reproduction rates.