Math 253 **Exam 1** **03/25/2015**

Name: ____

> Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the True/False questions. For all other questions, please circle your answers and justify your work for full credit. There are 12 questions for a total of 100 points.

True or False: Please circle either true or false. No work is necessary.

_______ 1. (5 points) The set $\{\pi + t \mid t \in \mathbb{Z}\}$ is a vector space.

A. True B. False

_______ 2. (5 points) For any real number t, the matrix $\begin{pmatrix} 1 & -\frac{2}{5} \\ 0 & t \end{pmatrix}$ is invertible.

A. True B. False

_______ 3. (5 points) If L is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$, then $L\begin{pmatrix} 1 \\ 1 \end{pmatrix} = L\begin{pmatrix} 1 \\ 0 \end{pmatrix} + L\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

A. True B. False

_______ 4. (5 points) The following is a two dimensional hyperplane in $\mathbb{R}^4$:

$$\left\{ \begin{pmatrix} 1 \\ \frac{3}{5} \\ -3 \\ 0 \end{pmatrix} + \begin{pmatrix} 5 \\ \frac{1}{5} \\ 0 \\ 1 \end{pmatrix} t \begin{pmatrix} 0 \\ \frac{3}{2} \\ -1 \\ \frac{6}{5} \end{pmatrix} s \;\middle|\; t, s \in \mathbb{R} \right\}.$$

A. True B. False

_______ 5. (5 points) The following is an LU factorization for the matrix $\begin{pmatrix} 2 & -1 \\ 3 & 0 \end{pmatrix}$:

$$\begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 7 & 3 \\ 0 & -2 \end{pmatrix}.$$

A. True B. False

Short Answer. Make sure and justify your answer for full credit.

6. (10 points) Assume that L is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$. If $L\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}a\\b\end{pmatrix}$ and $L\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}c\\d\end{pmatrix}$, then use the definition of a linear transformation to find the matrix that represents L.

7. Consider the system of equations

$$\begin{aligned} x - y + 2z + u &= 1 \\ 2x + 2z + u &= 1 \\ x - y + 4z + u &= 2. \end{aligned}$$

(a) (2 points) Write an augmented matrix for this system.

(b) (5 points) Use elementary row operations to find its reduced row echelon form.

(c) (3 points) Write the solution set for the system in the form $S = \{X_0 + \sum_i \mu_i Y_i \mid \mu_i \in \mathbb{R}\}$.

8. Which of the following is a vector space over $\mathbb{R}$? Explain why or why not. For those that are not vector spaces, modify one part of the definition to make it into a vector space. (You don't need to fully verify that your new set is a vector space. You just need to explain what part you changed.)

   (a) (5 points) $V = \{(a, b) \in \mathbb{R}^2 \mid a, b \geqslant 0\}$

   (b) (5 points) $V = \left\{ \begin{pmatrix} a & b \\ c & 1 \end{pmatrix} \middle| a, b, c \in \mathbb{R} \right\}$

9. (a) (2 points) Find the matrix A such that $\begin{pmatrix} 1 & 0 \\ 1.5 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 0 & -1.5 \end{pmatrix}$ is an LU decomposition for.

(b) (3 points) Is the matrix A from Part (a) invertible? Symmetric? (Don't forget to justify your answer.)

(c) (5 points) Given the matrix in Part (a), use the LU decomposition to find a solution set to the homogeneous equation $Ax = 0$. Write your answer in vector form.

10. Let $M = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -3 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$.

(a) (5 points) Determine whether or not M is invertible. If so, find the inverse.

(b) (5 points) If possible, use the inverse to find the solution to $Mx = b$.

11. Let $M = \begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$ and $X = \begin{pmatrix} x \\ y \end{pmatrix}$.

(a) (2 points) Find the dot product between MX and X.

(b) (3 points) Compute the lengths of MX and X.

(c) (5 points) Find the angle between MX and X.

12. (15 points) Let $L(x, y, z) = (3x, x - y, 2x + y + z)$. Determine whether or not L is a linear operator. If so, is it invertible?