MATH 243 EXAM 4 05/18/2016

Name: ______________________________

Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the Sometimes/Always/Never questions. For all other questions, please circle your answers and justify your work for full credit. There are 10 questions for a total of 100 points.

Sometimes/Always/Never: Read the statement and decide whether the statement is sometimes true, always true, or never true. No work is necessary.

_______ 1. (5 points) The vector field $\mathbf{F} = x\mathbf{i} + (x - y)\mathbf{j}$ is conservative.

A. Sometimes B. Always C. Never

_______ 2. (5 points) Let C be a closed curve. The integral $\oint_C \mathbf{F} \cdot d\mathbf{s} = 0$.

A. Sometimes B. Always C. Never

_______ 3. (5 points) $grad(div\ \mathbf{F})$ is a vector field.

A. Sometimes B. Always C. Never

_______ 4. (5 points) The path $\mathbf{x}(t) = \langle 2\cos t, 4\sin t, t\rangle$ is the flow line of the vector field $\mathbf{F}(x, y, z) = -\frac{y}{2}\mathbf{i} + 2x\mathbf{j} + z\mathbf{k}$.

A. Sometimes B. Always C. Never

_______ 5. (5 points) If the curve C is the level set at height c of a function $f(x, y)$, then $\int_C f(x, y)ds$ is c times the length of C.

A. Sometimes B. Always C. Never

Short Answer: Justify your answers for full credit.

6. (15 points) Calculate the area under the surface $z = x^2y - x$ above the line segment from $(3, 2)$ to $(5, 7)$. (HINT: Parameterize the "height funciton".)

7. (15 points) Calculate the integral $\int \mathbf{F} \cdot d\mathbf{s}$ for the vector field $\mathbf{F}(x,y) = \langle x+y, x-2y \rangle$ along the path $\mathbf{x}(t) = \langle 3t^2, 2t+3 \rangle$ for $1 \leqslant t \leqslant 2$.

8. (a) (10 points) Show that the vector field $\mathbf{F}(x, y) = \langle 2xy^2 + 4x + y, 2x^2y + x + 2y + 2 \rangle$ is a gradient field by finding the potential.

(b) (5 points) Using the potential function, calculate the integral $\int \mathbf{F} \cdot d\mathbf{s}$ for the vector field $\mathbf{F}(x, y) = \langle 2xy^2 + 4x + y, 2x^2y + x + 2y + 2 \rangle$ long the path $\mathbf{x}(t) = \langle t^2, t^3 \rangle$ for $1 \leqslant t \leqslant 2$.

9. (15 points) Give a vector field $\mathbf{F}(x, y)$ and a curve $\mathbf{x}(t)$ from $a \leqslant t \leqslant b$, and assuming that $\nabla f = \mathbf{F}$, $\mathbf{x}(a) = A$, and $\mathbf{x}(b) = B$, explain why

$$\int \mathbf{F} \cdot d\mathbf{s} = f(B) - f(A).$$

(Hint: I don't want to see explanations along the lines of "it's a theorem". Draw a picture.)

10. (15 points) Evaluate $\int_C x^2y\ dx - xy\ dy$, where C is the curve with equation $y^2 = x^3$, from $(1, -1)$ to $(1, 1)$. (HINT: Parameterize with respect to y.)