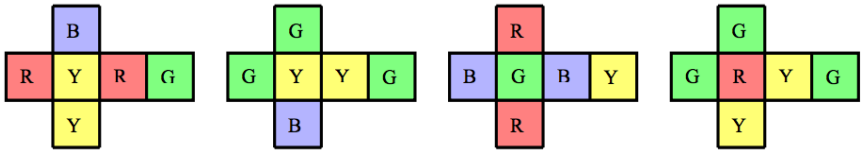
Math 390 Homework 1

Due Wednesday, February 3

Solutions should be written LaTeX or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

Many graph theory proofs use induction, so it will be helpful for you to review induction from your discrete/proofs course. Problems 2 and 3 below are not graph theory problems, but are intended to help you review proofs using induction.

1. Consider four cubes whose faces are colored red, blue, green, and yellow as in the following diagram:



   (Note: These cubes are colored differently than the ones from class.) Find a way to stack all four cubes so that all four colors appear on each side of the resulting stack.

2. Use induction to prove that the following formula holds for all $n \in \mathbb{Z}_{n>1}$:

$$2 + 4 + 8 + 16 + 32 + \ldots + 2^n = 2^{n+1} - 2.$$

3. A store offers gift card in the amounts of \$15 and \$25. What amounts can be made using gift cards of these two types? Decide on an answer and then prove your answer using induction.