Math 243 **Weekly Assignment 6** **DUE 11/18/2016**

Name: _____

Answer the questions on the worksheet and not on a separate sheet of paper. Please circle your answers and justify your work for full credit.

1. (5 points) Determine the absolute maximum and absolute minimum of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 0$, and $y = 9 - x$.

2. (5 points) Let $f(x, y) = 4x^2e^y - 2x^4 - e^{4y}$.
   (a) Find all of the critical points of f, and show that f has relative maximum values at each of these critical points.
   (b) Show that f has no absolute maximum value and no absolute minimum value.
   (c) Why is this example surprising? (Think of the analogy of having two mountain peaks without some sort of valley in between).

3. (5 points) A rectangular box with length x, width y, and height z is being built. The box is positioned so that one corner is stationed at the origin and the box lies in the first octant where x, y, and z are all positive. There is an added constraint on how the box is constructed: it must fit underneath the plane with equation $x + 2y + 3z = 6$. In fact, we will assume that the corner of the box "opposite" the origin must actually lie on this plane. Find the maximum volume of the box.

4. (5 points) Consider the box with a sloped top that is given by the following description: the base is the rectangle $R = [0, 4] \times [0, 3]$, while the top is given by the plane $z = p(x, y) = 20 - 2x - 3y$.
   (a) Estimate the value of $\iint_R p(x, y)\, dA$ by using a double Riemann sum with four subintervals in the x direction and three subintervals in the y direction, and choosing (x_i^*, y_j^*) to be the point that is the midpoint of each subrectangle.
   (b) What important quantity does your double Riemann sum in (a) estimate?
   (c) Suppose it can be determined that $\iint_R p(x, y)\, dA = 138$. What is the exact average value of p over R?
   (d) If you wanted to build a rectangular box (with the same base) that has the same volume as the box with the sloped top described here, how tall would the rectangular box have to be?