7 Problems: Linear Transformations

1. Show that the pair of conditions:

   (i) $L(u+v) = L(u) + L(v)$

   (ii) $L(cv) = cL(v)$

   is equivalent to the single condition:

   (iii) $L(ru+sv) = rL(u) + sL(v)\,.$

   Your answer should have two parts. Show that (i,ii)⇒(iii), and then show that (iii)⇒(i,ii).

2. Let P_n be the space of polynomials of degree n or less in the variable t. Suppose L is a linear transformation from $P_2 \to P_3$ such that $L(1) = 4, L(t) = t^3$, and $L(t^2) = t - 1$.

   - Find $L(1 + t + 2t^2)$.
   - Find $L(a + bt + ct^2)$.
   - Find all values a, b, c such that $L(a + bt + ct^2) = 1 + 3t + 2t^3$.

3. Show that integration is a linear transformation on the vector space of polynomials. What would a matrix for integration look like? Be sure to think about what to do with the constant of integration.