22 Problems: Gram-Schmidt and Orthogonal Complements

1. Find the QR factorization of

$$M = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 2 & 0 \\ -1 & -2 & 2 \end{pmatrix}.$$

2. Suppose u and v are linearly independent. Show that u and $v^{\perp}$ are also linearly independent. Explain why $\{u, v^{\perp}\}$ are a basis for $\text{span}\{u, v\}$.

3. Repeat the previous problem, but with three independent vectors u, v, w, and $v^{\perp}$ and $w^{\perp}$ as defined in the lecture.

4. Given any three vectors u, v, w, when do $v^{\perp}$ or $w^{\perp}$ vanish?

5. For U a subspace of W, use the subspace theorem to check that $U^{\perp}$ is a subspace of W.