Math 390 Homework 9

Due Friday, May 6

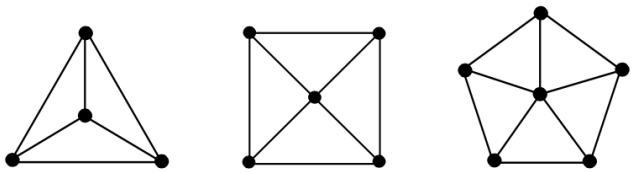
Solutions should be written LaTeX or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

1. (a) Let $G = (V_1, V_2)$ be a cubic bipartite graph. Show that there is a complete matching from V_1 to V_2 as well as a complete matching from V_2 to V_1. (I.e. there is a one-to-one correspondence from V_1 onto V_2.)

   (b) Show Tutte's 5-flow conjecture is true for cubic bipartite graphs.

2. Let G be a simple graph that is not a null graph. Prove that the sum of the coefficients of $P_G(k)$ is 0. (Hint: When a function is a polynomial, how can one obtain the sum of the coefficients?)

3. The wheel graph W_n is the n-vertex graph consisting of a cycle with $n-1$ vertices and an additional vertex that is adjacent to all of the vertices in the cycle. The wheel graphs W_4, W_5, and W_6 are shown below:



W_4 W_5 W_6

   Determine the chromatic polynomial of W_n for $n \geqslant 4$. Prove your answer.

4. An **infinite graph** is a graph G where both the vertex set $V(G)$ and the edge set $E(G)$ are infinite. (In Edition 4 of the textbook, see Section 16 for more information about infinite graphs. In Edition 5 of the textbook, see pages 24, 38, and 44.)

   (a) Find an Eulerian trail in the infinite square lattice. (The infinite square lattice is Figure 16.1 in Edition 4 and Figure 1.45 in Edition 5 of the textbook.)

   (b) Give an example of a connected infinite graph in which every vertex has even degree, but the graph is not Eulerian.