Math122 Review Problems for Midterm - Summer 2011 (Sec 8.1 - 8.7, 8.9, H.1, H.2, 9.1 - 9.7)

Exam 2 (Midterm), Friday, Julu 1, 8:00 - 10:00 am.

Review the **Concept Check** *problems: Page 631/1 - 11, Page 690/1 - 20*

PART 1: True-False Problems

Ch.8. Page 632 True-False Quiz Problems 1 – 18.

Ch.9. Page 691 True-False Quiz Problems 1 – 16.

Additional True-False Problems.

1. If the series $\sum a_n$ converges, then the sequence $\{a_n\}$ converges.
2. If $\lim\limits_{n\to\infty} |a_n| = 0$, then $\sum a_n$ converges.
3. If $\sum a_n$ converges, then $\sum |a_n|$ converges.
4. If $\{a_n\}$ converges, then $\{|a_n|\}$ converges.
5. If $\{|a_n|\}$ converges, then $\{a_n\}$ converges.
6. If $\{a_n\}$ converges but $\{b_n\}$ diverges, then $\{a_n + b_n\}$ diverges.
7. If $\{a_n\}$ converges but $\{b_n\}$ diverges, then $\{a_n b_n\}$ diverges.
8. If $0 \le a_n \le b_n$ for $n \ge 1$ and $\{b_n\}$ converges, then $\{a_n\}$ converges.
9. If $1 \le a_n \le b_n$ for $n \ge 1$ and $\lim\limits_{n\to\infty} b_n = 1$, then $\lim\limits_{n\to\infty} a_n$ converges.
10. If $\{a_n\}$ is bounded then $\{a_n\}$ is convergent.
11. If $0 \le a_n \le b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.
12. If $\sum a_n$ converges, then $\lim\limits_{n\to\infty} e^{a_n} = 1$.
13. If $\sum a_n$ is absolutely convergent, then $\sum a_n^3$ converges.
14. If $f(x) = 1 - 3(x-1)^2 + 5(x-1)^3 + \cdots$ converges for all x, then $f''(1) = -3$.
15. If $\sum\limits_{n=0}^{\infty} c_n(x-1)^n$ is the Taylor series of $f(x) = \ln x$ at $a = 1$, then $c_2 = -1$.
16. The points $\left(1, \dfrac{4\pi}{3}\right)$ and $\left(-1, \dfrac{\pi}{3}\right)$ represent the same point in the polar coordinate system.
17. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $\mathbf{u} \cdot \mathbf{v}$ is a vector.

18. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $\mathbf{u} \times \mathbf{v}$ is a vector.

19. For any vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in V_3, $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ is a vector.

20. If $|\mathbf{u}|^2 + |\mathbf{v}|^2 = |\mathbf{u} + \mathbf{v}|^2$, then $\mathbf{u} \cdot \mathbf{v} = 0$.

21. For any $\mathbf{u}$ in V_3, $\mathbf{u} \cdot \mathbf{u} = 0$.

22. For any $\mathbf{u}$ in V_3, $\mathbf{u} \times \mathbf{u} = \mathbf{0}$.

23. If two lines are perpendicular to a third line, then they are parallel.

24. If two lines are parallel to a third line, then they are parallel.

25. If two planes are perpendicular to a line, then they are parallel.

26. If two planes are parallel to a line, then they are parallel.

PART II. Multiple-Choice Problems

1. Exactly one of the following sequences diverges. Which is it?

(A) $\left\{\dfrac{\sqrt{n^4+1}}{n^2}\right\}$ (B) $\left\{\dfrac{n^2}{3^n}\right\}$ (C) $\left\{\dfrac{2^n}{n!}\right\}$ (D) $\left\{\dfrac{n}{(\ln n)^2}\right\}$ (E) $\left\{\cos\dfrac{1}{\sqrt{n}}\right\}$

2. Exactly one of the following sequences diverges. Which is it?

(A) $\left\{\sin\dfrac{1}{\sqrt{n}}\right\}$ (B) $\left\{\cos\dfrac{1}{\sqrt{n}}\right\}$ (C) $\left\{n\sin\dfrac{1}{n}\right\}$ (D) $\left\{e^{\frac{1}{\sqrt{n}}}\right\}$ (E) $\left\{\dfrac{(-1)^n\sqrt{n+1}}{\sqrt{n}}\right\}$

3. Exactly one of the following series diverges. Which is it?

(A) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(-2)^n}$ (B) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4}$ (C) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^3}$ (D) $\displaystyle\sum_{n=1}^{\infty}\frac{n^3+n^2}{n^4+1}$ (E) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}$

4. Exactly one of the following series diverges. Which is it?

(A) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$ (B) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ (C) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\ln n}{n}$ (D) $\displaystyle\sum_{n=2}^{\infty}\sin\frac{1}{n\ln n}$ (E) $\displaystyle\sum_{n=1}^{\infty}\frac{n^5}{n!}$

5. The series $\displaystyle\sum_{n=1}^{\infty}(-r)^n$ for $0 < r < 1$ converges to

(A) $\dfrac{1}{1-r}$ (B) $\dfrac{1}{1+r}$ (C) $\dfrac{r}{1-r}$ (D) $\dfrac{r}{1+r}$ (E) None of the above is true.

6. The sum of the geometric series $\sum_{n=1}^{\infty} (-\pi)^{n-1} 2^{-2n}$ is

(A) $\frac{1}{4-\pi}$ (B) $\frac{1}{4+\pi}$ (C) $-\frac{\pi}{4+\pi}$ (D) $\frac{1}{2+\pi}$ (E) $\frac{\pi}{4+\pi}$

7. The series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges to

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{1}{3}$ (D) ∞ (E) None of the above is true.

8. The sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+2}} \right)$ is

(A) $1 + \frac{1}{\sqrt{3}}$ (B) $1 - \frac{1}{\sqrt{3}}$ (C) $1 + \frac{1}{\sqrt{2}}$ (D) $1 - \frac{1}{\sqrt{2}}$ (E) ∞

9. The sum of the series $\sum_{n=0}^{\infty} \frac{1}{n!}$ is

(A) $\sqrt{2}$ (B) π (C) e (D) $\ln 2$ (E) ∞

10. The sum of the series $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)! 6^{2n+1}}$ is

(A) 0 (B) $1/2$ (C) $\pi/6$ (D) $\sqrt{3}/2$ (E) ∞

11. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{2 \ln n}{3n+1} x^n$ is

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) 1 (D) 0 (E) ∞

12. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{3^n}{n} (x-1)^n$ is

(A) $[2/3,\ 4/3]$ (B) $(2/3,\ 4/3)$ (C) $(2/3,\ 4/3]$
(D) $[2/3,\ 4/3)$ (E) None of the above is true.

13. The Maclaurin series for $\cos(x^2)$ is

(A) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$ (B) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$ (C) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+2}}{(2n+1)!}$
(D) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$ (E) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n)!}$

14. The Maclaurin series for e^{-2x} is

(A) $\sum_{n=0}^{\infty}(-1)^n\frac{2^n x^n}{n}$ (B) $\sum_{n=0}^{\infty}\frac{2^n x^n}{n!}$ (C) $\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{(2n)!}$
(D) $\sum_{n=0}^{\infty}(-1)^n\frac{2^n x^n}{n!}$ (E) $\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$

15. The 2nd degree Taylor polynomial of the function $f(x) = \ln x$ at $a = 1$ is

(A) $x-1-\frac{1}{2}(x-1)^2$ (B) $x-1-(x-1)^2$ (C) $x-x^2$ (D) $x-\frac{1}{2}x^2$ (E) $1-x$

16. The 3rd degree Taylor polynomial of the function $f(x) = \sin x$ at $a = 0$ is

(A) x (B) $x - \frac{x^3}{3!}$ (C) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ (D) $1 - \frac{x^2}{2!}$ (E) $1 + x - x^3$

17. The polar equation for the curve of the Cartesian equation $x^2 + xy = 1$ is

(A) $\sin^2\theta + \sin\theta\cos\theta = 0$ (B) $r(\sin^2\theta + \sin\theta\cos\theta) = 1$ (C) $r^2(\sin^2\theta + \sin\theta\cos\theta) = 1$
(D) $r(\cos^2\theta + \sin\theta\cos\theta) = 1$ (E) $r^2(\cos^2\theta + \sin\theta\cos\theta) = 1$

18. The Certesian equation of the polar equation $r = \frac{1}{\cos\theta - \sin\theta}$ is

(A) $y = x$ (B) $y-x = 1$ (C) $x-y = 1$ (D) $x^2+y^2 = \frac{1}{x-y}$ (E) $x^2+y^2 = \frac{1}{y-x}$

19. The slope of the line tangent to the polar curve $r = \sin\theta$ at $\theta = \frac{\pi}{6}$ is

(A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{3}$ (C) $-\sqrt{3}$ (D) $-\frac{\sqrt{3}}{3}$ (E) ∞

20. The slope of the line tangent to the polar curve $r = \theta$ at $\theta = \pi$ is

(A) 0 (B) -1 (C) 1 (D) $-\pi$ (E) π

21. The area of the region bounded by the polar curve $r = \sqrt{\cos\theta}$ and the rays $\theta = \pi/6$, $\theta = \pi/2$ is

(A) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\cos\theta}\, d\theta$ (B) $\displaystyle\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{\cos\theta}\, d\theta$ (C) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos\theta\, d\theta$
(D) $\displaystyle\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos\theta\, d\theta$ (E) $\displaystyle\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos\theta)^2\, d\theta$

22. The exact length of the polar curve $r = 1 + \cos\theta$ with $\pi/6 \le \theta \le \pi/2$ is

(A) $\displaystyle\sqrt{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{1+\cos\theta}\, d\theta$ (B) $\displaystyle\frac{\sqrt{2}}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{1+\cos\theta}\, d\theta$ (C) $\displaystyle 2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1+\cos\theta)\, d\theta$
(D) $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1+\cos\theta)\, d\theta$ (E) $\displaystyle 4\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (1+\cos\theta)^2\, d\theta$

23. If $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$, and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\pi/3$, then $|\mathbf{a} - \mathbf{b}| =$

(A) $\sqrt{5}$ (B) 5 (C) $\sqrt{3}$ (D) $\sqrt{5 - 2\sqrt{3}}$ (E) 3

24. If $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$, and $|\mathbf{a} \times \mathbf{b}| = \sqrt{3}$, then the angle between $\mathbf{a}$ and $\mathbf{b}$ is

(A) either 0 or π (B) either $\pi/6$ or $5\pi/6$ (C) either $\pi/3$ or $2\pi/3$
(D) either $\pi/4$ or $3\pi/4$ (E) $\pi/2$

25. The scalar projection $\text{comp}_{\mathbf{v}}\mathbf{u}$ of $\mathbf{u} = \langle 1, -1, \sqrt{2}\rangle$ onto $\mathbf{v} = \langle 0, -3, 4\rangle$ is

(A) $3 + 4\sqrt{2}$ (B) $\dfrac{3+4\sqrt{2}}{2}$ (C) $\dfrac{3+4\sqrt{2}}{4}$ (D) $\dfrac{3+4\sqrt{2}}{24}$ (E) $\dfrac{3+4\sqrt{2}}{5}$

26. The vector projection $\text{proj}_{\mathbf{u}}\mathbf{v}$ of $\mathbf{v} = \langle 0, -3, 4\rangle$ onto $\mathbf{u} = \langle 1, -1, \sqrt{2}\rangle$ is

(A) $\dfrac{3+4\sqrt{2}}{2}$ (B) $\dfrac{3+4\sqrt{2}}{2}\langle 1, -1, \sqrt{2}\rangle$ (C) $\dfrac{3+4\sqrt{2}}{4}\langle 1, -1, \sqrt{2}\rangle$
(D) $\dfrac{3+4\sqrt{2}}{5}\langle 1, -1, \sqrt{2}\rangle$ (E) $\dfrac{3+4\sqrt{2}}{25}\langle 0, -3, 4\rangle$

27. The area of the triangle with vertices at the points $P(1, 2, 3)$, $Q(-1, 0, 1)$ and $R(1, 1, 0)$ is

(A) $\sqrt{14}$ (B) $2\sqrt{14}$ (C) $3\sqrt{14}$ (D) $4\sqrt{14}$ (E) $5\sqrt{14}$

28. The volume of the parallelepiped determined by the vectors $\mathbf{a} = \langle 1, 2, 3\rangle$, $\mathbf{b} = \langle -1, 0, 1\rangle$, and $\mathbf{c} = \langle 1, 1, 0\rangle$ is

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

29. The distance from the point $P(1,2,3)$ to the line through the points $Q(-1,0,1)$ and $R(1,1,0)$ is

(A) $\frac{\sqrt{42}}{6}$ (B) $\frac{\sqrt{42}}{3}$ (C) $\frac{2\sqrt{42}}{3}$ (D) $\frac{2\sqrt{21}}{3}$ (E) $\sqrt{14}$

30. The distance from the point $P(1,2,3)$ to the line $\mathbf{r}(t) = \langle 0,1,1\rangle + t\langle -2,1,2\rangle$ is

(A) 5 (B) $\sqrt{5}$ (C) $\sqrt{15}$ (D) 0 (E) 1

31. The distance from the point $P(1,2,3)$ to the plane through the points $Q(1,1,3)$, $R(4,1,0)$, and $S(-1,-1,3)$ is

(A) 1 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{5}{\sqrt{3}}$ (E) $\frac{1}{3}$

32. The distance from the point $P(1,2,3)$ to the plane $x - y + z = 3$ is

(A) 1 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\frac{5}{\sqrt{3}}$ (E) $\frac{1}{3}$

33. If θ is the angle between the planes $x - y + z = 3$ and $2x - y - z = 1$, then $\cos\theta$ is

(A) $\sqrt{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{3\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{3}$ (E) $\frac{\sqrt{2}}{6}$

34. The distance between the planes $x - 3y + 2z = 3$ and $2x - 6y + 4z = 3$ is

(A) $\frac{1}{2\sqrt{14}}$ (B) $\frac{1}{\sqrt{14}}$ (C) $\frac{3}{2\sqrt{14}}$ (D) $\frac{2}{\sqrt{14}}$ (E) $\frac{5}{2\sqrt{14}}$

35. Exactly one of the following vectors is parallel to the line described by $x = 1 + 2t$, $y = -3t$, $z = 3 - t$. Which is it?

(A) $\langle 1,0,3\rangle$ (B) $\langle -1,0,-3\rangle$ (C) $\mathbf{i}-3\mathbf{j}+3\mathbf{k}$ (D) $-2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ (E) $\langle 2,0,-1\rangle$

36. Exactly one of the following vectors is normal to the plane described by $4x-6y = 5-2z$. Which is it?

(A) $\langle 2,-3,1\rangle$ (B) $4\mathbf{i}-6\mathbf{j}-2\mathbf{k}$ (C) $2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ (D) $4\mathbf{i}+6\mathbf{j}+2\mathbf{k}$ (E) $\langle 4,-6,-5\rangle$

37. The domain of the function $f(x,y) = \dfrac{\ln(x-y^2)}{\sqrt{1-x}}$ is

(A) $\{(x,y) \mid x \geq y^2\}$ (B) $\{(x,y) \mid y^2 < x < 1\}$ (C) $\{(x,y) \mid x > y^2,\ x \geq 1\}$
(D) $\{(x,y) \mid y^2 < x \leq 1\}$ (E) $\{(x,y) \mid y^2 \leq x < 1\}$

FIGURE 1

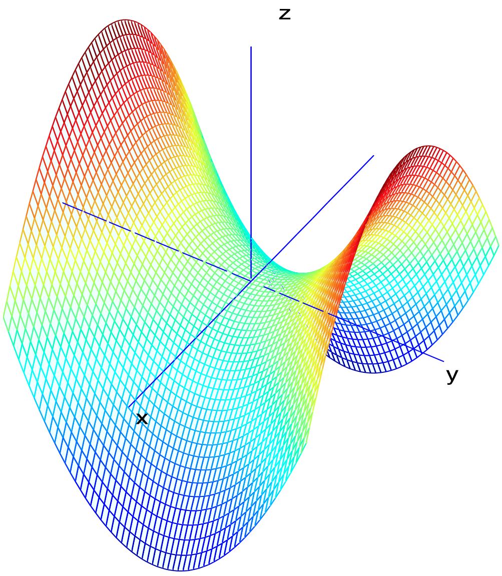
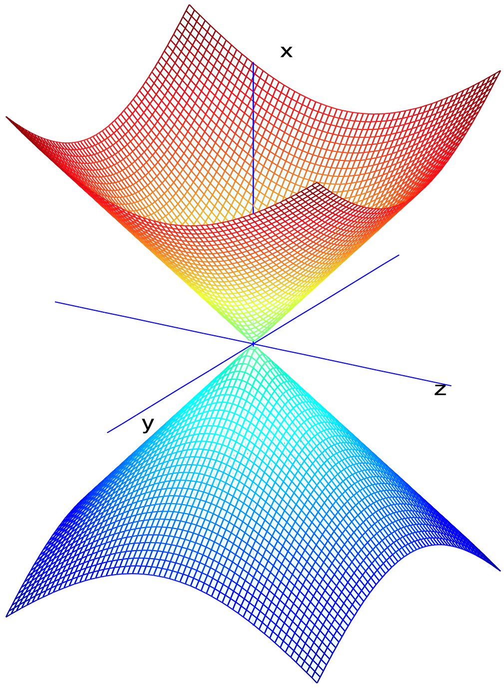


FIGURE 2



38. Exactly one of the following quadratic equations has the graph as shown in Figure 1. Which is it?

(A) $z = y^2 - x^2$ (B) $z = x^2 - y^2$ (C) $x = y^2 - z^2$
(D) $y = z^2 - x^2$ (E) $x = z^2 - y^2$

39. Exactly one of the following quadratic equations has the graph as shown in Figure 2. Which is it?

(A) $z^2 = x^2 + y^2 + 1$ (B) $x^2 = y^2 + z^2$ (C) $y^2 = z^2 + x^2$
(D) $x^2 = y^2 + z^2 - 1$ (E) $z^2 = x^2 + y^2$

40. Exactly one of the following equations in cylindrical coordinates completely describes the cone $x^2 + y^2 - z^2 = 0$. Which is it?

(A) $r = z$ (B) $r = -z$ (C) $r = z^2$ (D) $r^2 = z^2$ (E) $r^2 \cos 2\theta = z^2$

41. Exactly one of the following equations in spherical coordinates completely describes the ellipsoid $2x^2 + 5y^2 + 2z^2 = 1$. Which is it?

(A) $\rho^2(2 + 3\sin^2\phi \sin^2\theta) = 1$ (B) $\rho^2(2 + 3\cos^2\phi \sin^2\theta) = 1$ (C) $\rho^2(2 + 3\sin^2\phi \cos^2\theta) = 1$
(D) $\rho^2(2 + 3\cos^2\phi \cos^2\theta) = 1$ (E) $\rho^2(2 + 3\cos^2\phi) = 1$

PART III. Essay Problems

1. Determine whether the sequence converges. Find the limit if it is convergent.

(a) $a_n = \dfrac{n^2 - n}{2n^2 - 3}$ (b) $a_n = \dfrac{\ln(2n+1)}{n}$ (c) $a_n = \dfrac{2^{2n}}{e^{n+2}}$

(d) $a_n = \dfrac{n\cos(n)}{e^n}$ (e) $a_n = \ln(n^2+1) - \ln(2n^2 - n)$ (f) $a_n = (1 + 2/n)^{2n}$

2. Let the sequence $\{a_n\}$ satisfy

$$a_1 = 10, \quad a_{n+1} = \sqrt{6 + a_n}, \qquad n \geq 1.$$

(a) (optional) Show that $a_n > 0$ and $a_n \geq a_{n+1}$ for $n \geq 1$. (Hence $\{a_n\}$ has a limit since it is decreasing and bounded below.)

(b) Find the limit.

3. Determine whether the series is convergent, or divergent, or absolutely convergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^n n}{2^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n \cos(n\pi)}{e^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (\ln n)^2}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + e^{-n}}{2n^2 + 1}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n!}{1 \cdot 3 \cdot 5 \cdots (2n+1)}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)2^n}{n^3(-3)^n}$

4. Use the partial sum s_5 to estimate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$, and estimate the error in using s_5 as an approximation to the sum of the series.

5. Use the partial sum s_5 to estimate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$, and estimate the error in using s_5 as an approximation to the sum of the series.

6. Find the radius of convergence and interval of convergence of the power series.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(1-x)^n}{2n+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n3^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2 3^n}$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}(x-3)^n}{n!}$

7. Find the Taylor series of the function at $a = 0$.

(a) $\dfrac{1}{1-x^2}$ (b) $\dfrac{1}{(1-x)^2}$ (c) $\ln(1+x)$ (d) $\sin x^2$ (e) $x^2 e^{-x}$.

8. Find the Maclaurin series of e^{-x^2} and approximate $\displaystyle\int_0^{0.1} e^{-x^2}\,dx$ correct to within an error of 10^{-5}.

9. Find the Taylor polynomial $T_3(x)$ of the function $e^{\sin x}$ at $a = 0$.

10. Let $T_3(x)$ be the degree 3 Taylor polynomial of e^{x^2} at $a = 0$. Using the Taylor inequality to find a bound for

$$|R_3(x)| = \left|e^{x^2} - T_3(x)\right| \qquad \text{for } x \in [0,\ 0.1].$$

11. Find the Cartesian coordinates of the point given in polar coordinates.

$$\text{(a)}\ \left(2, \frac{\pi}{6}\right) \quad \text{(b)}\ \left(4, \frac{3\pi}{4}\right) \quad \text{(c)}\ \left(0, \frac{\pi}{5}\right) \quad \text{(d)}\ \left(5, -\frac{\pi}{2}\right) \quad \text{(e)}\ \left(3, -\frac{\pi}{3}\right)$$

12. Find the polar coordinates (r, θ) with $r \geq 0$ and $0 \leq \theta < 2\pi$ of the point given in the Certesian coordinates.

$$\text{(a)}\ (1,0) \quad \text{(b)}\ \left(3, \sqrt{3}\right) \quad \text{(c)}\ (-2,2) \quad \text{(d)}\ \left(-1, \sqrt{3}\right) \quad \text{(e)}\ (0,-2)$$

13. Find the slope of the line tangent to the polar curve at the point specified by the value of θ.

$$\text{(a)}\ r = \sin\theta + \cos\theta, \quad \theta = \frac{\pi}{6} \qquad \text{(b)}\ r = 1+\theta^2, \quad \theta = \frac{\pi}{2} \qquad \text{(c)}\ r = 4\cos 3\theta, \quad \theta = \frac{\pi}{6}$$

14. Find the points on the given curve where the tangent line is horizontal or vertical.

$$\text{(a)}\ r = 1 + \cos\theta, \qquad \text{(b)}\ r^2 = \cos 2\theta$$

15. Find the area of the region that is bounded by the given curve and lies in the specified sector.

$$\text{(a)}\ r = 1 - \cos\theta, \quad 0 \leq \theta \leq \frac{\pi}{2} \qquad \text{(b)}\ r = 3 - \theta, \quad 0 \leq \theta \leq 3$$

16. Find the area of the region enclosed by one loop of the curve.

$$\text{(a)}\ r = 2\sin\theta \qquad \text{(b)}\ r^2 = \cos 2\theta$$

17. Find the area of the region that lies inside $r = \sqrt{2}\cos\theta$ and outside $r = 1$.

18. Find the exact length of the polar curve.

$$\text{(a)}\ r = \theta^2, \quad 0 \leq \theta \leq \pi \qquad \text{(b)}\ r = 2\cos\theta, \quad 0 \leq \theta \leq \frac{\pi}{2}$$

19. Let $\mathbf{a}$ and $\mathbf{b}$ be vectors in V_2. Suppose $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$, and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\theta = \pi/4$. Find

    (a) $\mathbf{a} \cdot \mathbf{b}$ (b) $|\mathbf{a} + 2\mathbf{b}|$ (c) $|3\mathbf{a} - 2\mathbf{b}|$ (d) $|(2\mathbf{a}) \times \mathbf{b}|$
    (e) $\text{comp}_{\mathbf{a}}\mathbf{b}$ (f) $\text{comp}_{\mathbf{b}}\mathbf{a}$
    (g) The area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$.

20. Suppose that $\mathbf{a} = \langle 1, -1, 2\rangle$, $\mathbf{b} = -\mathbf{i} + 3\mathbf{k}$, and $\mathbf{c} = \langle -2, 3, 1\rangle$. Find

    (a) $\mathbf{a} \cdot (\mathbf{b} - 2\mathbf{c})$, (b) $(2\mathbf{a} - \mathbf{c}) \times \mathbf{b}$, (c) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$, (d) $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$
    (e) The angle between $\mathbf{a}$ and $\mathbf{b}$
    (f) The area of the parallelogram determined by $\mathbf{a}$ and $\mathbf{b}$
    (g) The volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$.

21. Given the points $P(1, 3, -1)$, $Q(2, -1, 1)$, $R(1, 1, 1)$, and $S(-2, 1, -3)$, find

    (a) The area of the triangle with vertices P, Q, R
    (b) The length of the line segment PQ
    (c) The angle between the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$
    (d) Parametric equations for the line that passes through the points Q and R
    (e) The distance from the point P to the line that passes through Q and R
    (f) A scalar equation for the plane through the points Q, R, and S
    (g) The distance from the point P to the plane through the points Q, R, and S
    (h) The volume of the parallelepiped with adjacent edges PQ, PR, and PS
    (i) The distance between the line through the points P, Q and the line through the points R and S
    (j) Symmetric equations for the line through the point P and normal to the plane through the points Q, R, and S
    (k) The angle between the plane through the points P, Q, R and the plane through the points P, Q, S.

22. Find the value(s) of t such that the vectors $\langle t, 2t - 1, 3\rangle$ and $\langle t + 1, -1, -1\rangle$ are orthogonal.

23. Find the work done by a force $\mathbf{F} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ that moves a particle from the point $P(1, 1, 1)$ to the point $Q(4, -1, 2)$.

24. Find the work done by a force of 10N applied at an angle of $\pi/6$ to the moving direction in moving an object 3m.

25. Find the line (its vector equation, parametric equations, and symmetric equations) that passes through the point $P(1, 2, 3)$ and is perpendicular to the plane $2x - y + 3z = 9$.

26. Find the line (its vector equation, parametric equations, and symmetric equations) that passes through the points $P(1, 2, 3)$ and $Q(3, 2, 1)$.

27. Find the line (its vector equation, parametric equations, and symmetric equations) that passes through the point $P(1, 1, 1)$ and is perpendicular to the vectors $\langle 1, 1, 0\rangle$ and $\langle 1, 0, 1\rangle$.

28. Find the line (its vector equation, parametric equations, and symmetric equations) of the intersection of the planes $z = x + y - 1$ and $y = x + z + 1$.

29. Find a scalar equation of the plane through the point $P(1, 0, 2)$ with normal vector $\mathbf{n} = \langle 2, -1, 3\rangle$.

30. Find a scalar equation of the plane that passes through the point $P(1, 2, 3)$ and is perpendicular to the line $2(x - 1) = \dfrac{y}{2} = -\dfrac{z-5}{4}$.

31. Find a scalar equation of the plane that contains the line $2(x - 1) = \dfrac{y}{2} = -\dfrac{z-5}{4}$ and is parallel to the vector $\langle 1, 1, 1\rangle$.

32. Find a scalar equation of the plane that passes through the point $P(1, 1, 1)$ and is parallel to the vectors $\langle 1, 1, 0\rangle$ and $\langle 1, 0, 1\rangle$.

33. Show that the lines

$$L_1 : \{x = 1 + t, y = -2 - t, z = 3t\} \text{ and } L_2 : \{x = 2 - 3t, y = 1 - t, z = 3 + 2t\}$$

are skew. Find the plane that contains the line L_1 and is parallel to the line L_2. Determine the distance between L_1 and L_2.

34. Find (a) the angle, (b) the distance between the two planes $x+3y = z+2$ and $z = x+y-1$.

35. Let $f(x, y) = \dfrac{\sqrt{xy - 1}}{x^2 - 4}$.

    (a) Find the domain of $f(x, y)$. (b) Evaluate $f(5, 2)$.

36. Let $f(x, y) = \sqrt{4 - y^2}\,\ln(y^2 - x)$.

    (a) Find the domain of $f(x, y)$. (b) Evaluate $f(0, -1)$.

37. The cylindrical coordinates of a point are $(2\sqrt{3},\ \pi/3,\ 2)$. Find the rectangular and spherical coordinates of the point.

38. The rectangular coordinates of a point are $(2,\ 2,\ -1)$. Find the cylindrical and spherical coordinates of the point.

39. The spherical coordinates of a point are $(8, \pi/4, \pi/6)$. Find the rectangular and cylindrical coordinates of the point.

40. Write the equation in cylindrical coordinates and in spherical coordinates.

(a) $x^2 + y^2 + z^2 = 4$ (b) $x^2 + z^2 = 4$.