8 Problems: Matrices

1. Compute the following matrix products

$$\begin{pmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \\ 7 & 8 & 2 \end{pmatrix} \begin{pmatrix} -2 & \frac{4}{3} & -\frac{1}{3} \\ 2 & -\frac{5}{3} & \frac{2}{3} \\ -1 & 2 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} \frac{1}{2} \\ 3 \\ 4 \\ 5 \end{pmatrix},$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix}, \quad \begin{pmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \\ 7 & 8 & 2 \end{pmatrix} \begin{pmatrix} -2 & \frac{4}{3} & -\frac{1}{3} \\ 2 & -\frac{5}{3} & \frac{2}{3} \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \\ 7 & 8 & 2 \end{pmatrix},$$

$$\begin{pmatrix} 2 & 1 & 2 & 1 & 2 \\ 0 & 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 2 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 0 & 2 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix},$$

$$\begin{pmatrix} -2 & \frac{4}{3} & -\frac{1}{3} \\ 2 & -\frac{5}{3} & \frac{2}{3} \\ -1 & 2 & -1 \end{pmatrix} \begin{pmatrix} 4 & \frac{2}{3} & -\frac{2}{3} \\ 12 & -\frac{16}{3} & \frac{10}{3} \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 4 & 5 & 2 \\ 7 & 8 & 2 \end{pmatrix}.$$

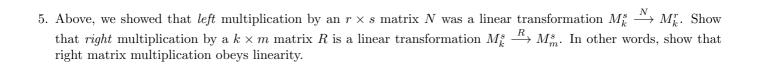
2. Let's prove the theorem $(MN)^T = N^T M^T$.

Note: the following is a common technique for proving matrix identities.

- (a) Let $M = (m_j^i)$ and let $N = (n_j^i)$. Write out a few of the entries of each matrix in the form given at the beginning of this chapter.
- (b) Multiply out MN and write out a few of its entries in the same form as in part a. In terms of the entries of M and the entries of N, what is the entry in row i and column j of MN?
- (c) Take the transpose $(MN)^T$ and write out a few of its entries in the same form as in part a. In terms of the entries of M and the entries of N, what is the entry in row i and column j of $(MN)^T$?
- (d) Take the transposes N^T and M^T and write out a few of their entries in the same form as in part a.
- (e) Multiply out $N^T M^T$ and write out a few of its entries in the same form as in part a. In terms of the entries of M and the entries of N, what is the entry in row i and column j of $N^T M^T$?
- (f) Show that the answers you got in parts c and e are the same.

3.	Let M be problem.)	e any $m \times n$ if What are the	matrix. Show t eir sizes?	hat M^TM and	$1 MM^T$ are sy	mmetric. (Hint	: use the result	of the previous

4. Let $x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$ and $y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$ be column vectors. Show that the dot product $x \cdot y = x^T \, \mathbbm{1} \, y$.



- 6. Explain what happens to a matrix when:
 - (a) You multiply it on the left by a diagonal matrix.
 - (b) You multiply it on the right by a diagonal matrix.

Give a few simple examples before you start explaining.