16 Problems: Linear Independence

1. Let B^n be the space of $n \times 1$ bit-valued matrices (*i.e.*, column vectors) over the field $\mathbb{Z}_2 := \mathbb{Z}/2\mathbb{Z}$. Remember that this means that the coefficients in any linear combination can be only 0 or 1, with rules for adding and multiplying coefficients given here.

   (a) How many different vectors are there in B^n?

   (b) Find a collection S of vectors that span B^3 and are linearly independent. In other words, find a basis of B^3.

   (c) Write each other vector in B^3 as a linear combination of the vectors in the set S that you chose.

   (d) Would it be possible to span B^3 with only two vectors?

`Hint for Problem 1`

2. Let e_i be the vector in $\mathbb{R}^n$ with a 1 in the ith position and 0's in every other position. Let v be an arbitrary vector in $\mathbb{R}^n$.

   (a) Show that the collection $\{e_1, \ldots, e_n\}$ is linearly independent.

   (b) Demonstrate that $v = \sum_{i=1}^{n}(v \cdot e_i)e_i$.

   (c) The span$\{e_1, \ldots, e_n\}$ is the same as what vector space?