9 Problems: Properties of Matrices

1. Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 3 & -1 & 4 \end{pmatrix}$. Find AA^T and A^TA. What can you say about matrices MM^T and M^TM in general? Explain.

2. Compute $\exp(A)$ for the following matrices:

- $A = \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}$
- $A = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix}$
- $A = \begin{pmatrix} 0 & \lambda \\ 0 & 0 \end{pmatrix}$

Hint

3. Suppose $ad - bc \neq 0$, and let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

   (a) Find a matrix M^{-1} such that $MM^{-1} = I$.

   (b) Explain why your result explains what you found in a previous homework exercise.

   (c) Compute $M^{-1}M$.

4. Let $M = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$. Divide M into named blocks, and then multiply blocks to compute M^2.