Math 351 Homework 4

Due Friday, October 7 at 5pm

Solutions should be written LaTeX or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

(1) Find an $x \in \mathbf{Z}$ such that $x \equiv -4 \pmod{17}$ and $x \equiv 3 \pmod{23}$.

(2) Find rules for divisibility of an integer by 5, 9, and 11, and prove each of these rules using arithmetic modulo a suitable n.

(3) Find an integer x such that $37x \equiv 1 \pmod{101}$.

(4) Let $n = \varphi(20!) = 416084687585280000$. Compute the prime factorization of n using the multiplicative property of φ.

(5) Show that if p is a positive integer such that both p and $p^2 + 2$ are prime, then $p = 3$.

(6) Seven competitive math students try to share a huge hoard of stolen math books equally between themselves. Unfortunately, six books are left over, and in the fight over them, one math student is expelled. The remaining six math students, still unable to share the math books equally since two are left over, again fight, and another is expelled. When the remaining five share the books, one book is left over, and it is only after yet another math student is expelled that an equal sharing is possible. What is the minimum number of books that allows this to happen?