Math 362 **Midterm Review** **Spring 2017**

Definitions: You should know the following definitions as well as an example of each.

1. random variable
2. probability distribution
3. probability density function
4. population
5. sample
6. random sample
7. sample mean, sample median, sample mode
8. sample variance, sample standard deviation, sample range
9. statistic
10. sampling distribution
11. central limit theorem
12. sampling distribution of S^2 (χ^2, don't worry about the density function)
13. t-distribution (don't worry about the density function)
14. F-distribution (don't worry about the density function)
15. binomial distribution
16. statistical inference
17. estimation
18. tests of hypotheses
19. point estimate
20. unbiased estimator
21. confidence interval
22. prediction interval
23. tolerance limits
24. Bernoulli process
25. statistical hypothesis
26. null hypothesis
27. alternative hypothesis
28. type I error
29. type II error

General Questions

30. A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20?

31. Suppose a sample is taken from a population with the data set consisting of the following observations: 0.32 0.53 0.28 0.37 0.47 0.43 0.36 0.42 0.38 0.43. What is the sample mean, sample mode, sample median?

32. A comparison of coffee prices at 4 randomly selected grocery stores in San Diego showed increases from the previous month of 12, 15, 17, and 20 cents for a 1-pound bag. Find the variance of this random sample of price increase.

33. Find the variance of the data 3, 4, 5, 6, 6, and 7, representing the number of trout caught by a random sample of 6 fishermen on June 19, 1996, at Lake Muskoka.

34. What is the difference between μ, σ, $\overline{X}$, and S^2?

35. Why does $\mu_{\overline{X}} = \mu$ and $\sigma^2_{\overline{X}} = \frac{\sigma^2}{n}$?

36. What does the central limit theorem say? Why is it important?

37. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed, with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a random sample of 16 bulbs will have an average life of less than 775 hours.

38. Traveling between two campuses of a university in a city via shuttle bus takes, on average, 28 minutes with a standard deviation of 5 minutes. In a given week, a bus transported passengers 40 times. What is the probability that the average transport time was more than 30 minutes? Assume the mean time is measured to the nearest minute.

39. Two independent experiments are run in which two different types of paint are compared. Eighteen specimens are painted using type A, and the drying time, in hours, is recorded for each. The same is done with type B. The population standard deviations are both known to be 1.0.

    Assuming that the mean drying time is equal for the two types of paint, find $P(\overline{X}_A - \overline{X}_B > 1.0)$, where $\overline{x}_A$ and $\overline{X}_B$ are average drying times for samples of size $n_A = n_B = 18$.

40. Given the discrete uniform population

$$f(x) = \begin{cases} \frac{1}{3}, & x = 2, 4, 6, \\ 0, & \text{elsewhere}, \end{cases}$$

    find the probability that a random sample of size 54, selected with replacement, will yield a sample mean greater than 4.1 but less than 4.4. Assume the means are measured to the nearest tenth.

41. A manufacturer of car batteries guarantees that the batteries will last, on average, 3 years with a standard deviation of 1 year. If five of these batteries have lifetimes of 1.9, 2.4, 3.0, 3.5, and 4.2 years, should the manufacturer still be convinced that the batteries have a standard deviation of 1 year? Assume that the battery lifetime follows a normal distribution.

42. Find k such that $P(k < T < -1.761) = 0.045$ for a random sample of size 15 selected from a normal distribution and $T = \frac{\overline{X}-\mu}{S/\sqrt{n}}$.

43. What is the t-distribution used for?

44. A chemical engineer claims that the population mean yield of a certain batch process is 500 grams per milliliter of raw material. To check this claim he samples 25 batches each month. If the computed t-value falls between $-t_{0.05}$ and $t_{0.05}$, he is satisfied with this claim. What conclusion should he draw from a sample that has a mean $\overline{x} = 518$ grams per milliliter and a sample standard deviation $s = 40$ grams? Assume the distribution of yields to be approximately normal.

45. How do you know which unbiased estimator is better?

46. Show how to find the confidence interval on μ with σ^2 known.

47. The average zinc concentration recovered from a sample of measurements taken in 36 different locations in a river is found to be 2.6 grams per milliliter. Find the 95% and 99% confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3 gram per milliliter.

48. If $\overline{x}$ is used as an estimator of μ, we can be $100(1-\alpha)\%$ confident that the error does not exceed a certain bound. What is this bound?

49. In Question 47, how large a sample is required if we want to be 95% confident that our estimate of μ is off by less than 0.05?

50. The contents of seven similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 liters. Find a 95% confidence interval for the mean contents of all such containers, assuming an approximately normal distribution.

51. Due to the decrease in interest rates, the First Citizens Bank received a lot of mortgage applications. A recent sample of 50 mortgage loans resulted in an average loan amount of \$257,300. Assume a population standard deviation of \$25,000. For the next customer who fills out a mortgage application, find a 95% prediction interval for the loan amount.

52. A meat inspector has randomly selected 30 packs of 95% lean beef. The sample resulted in a mean of 96.2% with a sample standard deviation of 0.8%. Find a 99% prediction interval for the leanness of a new pack. Assume normality.

53. Consider Question 52. With the information given, find a tolerance interval that gives two-sided 95% bounds on 90% of the distribution of packages of 95% lean beef. Assume the data came from an approximately normal distribution.

54. What is the difference between Confidence Intervals, Prediction Intervals, and Tolerance Intervals?

55. Explain hypothesis testing.

56. A manufacturer of a certain brand of rice cereal claims that the average saturated fat content does not exceed 1.5 grams per serving. State the null and alternative hypotheses to be used in testing this claim and determine where the critical region is located.

57. A real estate agent claims that 60% of all private residences being built today are 3-bedroom homes. To test this claim, a large sample of new residences is inspected; the proportion of these homes with 3 bedrooms is recorded and used as the test statistic. State the null and alternative hypotheses to be used in this test and determine the location of the critical region.

58. Suppose that an allergist wishes to test the hypothesis that at least 30% of the public is allergic to some cheese products. Explain how the allergist could commit (a) a type I error; (b) a type II error.

59. A fabric manufacturer believes that the proportion of orders for raw material arriving late is p = 0.6. If a random sample of 10 orders shows that 3 or fewer arrived late, the hypothesis that p = 0.6 should be rejected in favor of the alternative p ¡ 0.6. Use the binomial distribution.

    1. Find the probability of committing a type I error if the true proportion is p = 0.6.
    2. Find the probability of committing a type II error for the alternatives p = 0.3, p = 0.4, and p = 0.

60. A sociologist is concerned about the effectiveness of a training course designed to get more drivers to use seat belts in automobiles.

    1. What hypothesis is she testing if she commits a type I error by erroneously concluding that the training course is ineffective?
    2. What hypothesis is she testing if she commits a type II error by erroneously concluding that the training course is effective?