MATH 243 EXAM 3 04/20/2016

Name: ______________________________

> Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the Sometimes/Always/Never questions. For all other questions, please circle your answers and justify your work for full credit. There are 10 questions for a total of 100 points.

Sometimes/Always/Never: Read the statement and decide whether the statement is sometimes true, always true, or never true. No work is necessary.

_______ 1. (5 points) Suppose that D is a bounded closed region in $\mathbb{R}^2$ and $f(x,y)$ is continuous on D. The integral $\iint_D f(x,y)\,dA$ is the area of the region D. A. Sometimes B. Always C. Never

_______ 2. (5 points) Let D_1 and D_2 be two solids bounded between the planes $z=0$ and $z=1$. For any $c \in [0,1]$, let $S_1(c)$ and $S_2(c)$ be the slices obtained by cutting the solids D_1 and D_2, respectively, with the plane $z=c$. If $S_1(c)$ and $S_2(c)$ have the same area for all $c \in [0,1]$, then D_1 and D_2 have the same volume.

A. Sometimes B. Always C. Never

_______ 3. (5 points) Let W be the top half of the unit sphere centered at the origin. The integral $\iiint_W (z^2 - z)\,dV$ is positive.

A. Sometimes B. Always C. Never

_______ 4. (5 points) The Jacobian of the transformation $x = u^2 + v^2$ and $y = -u^2 + v^2$ is given by $4u^2 + 4v^2$.

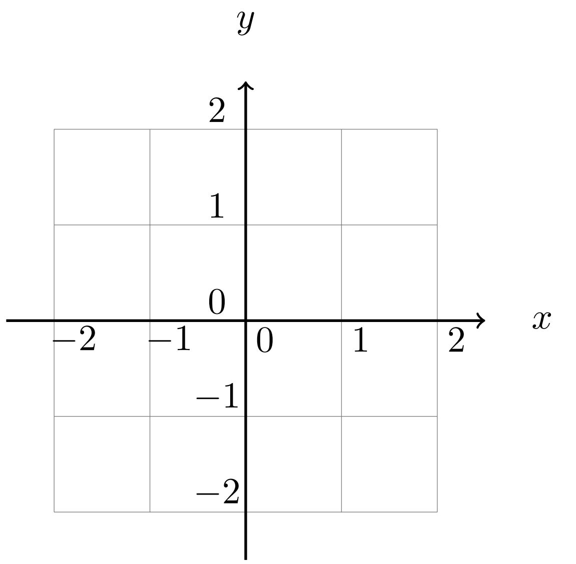
A. Sometimes B. Always C. Never

_______ 5. (5 points) The vector function $r(s,t) = \langle s^3, t^3, 1 - s^3 - t^3 \rangle$ is a parameterization of the plane $x + y + z = 1$.

A. Sometimes B. Always C. Never

Short Answer: Justify your answers for full credit.

6. (a) (7 points) Consider the integral $\int_0^2 \int_x^2 \sin(y^2)\, dydx$. Draw the region of integration on the graph below.

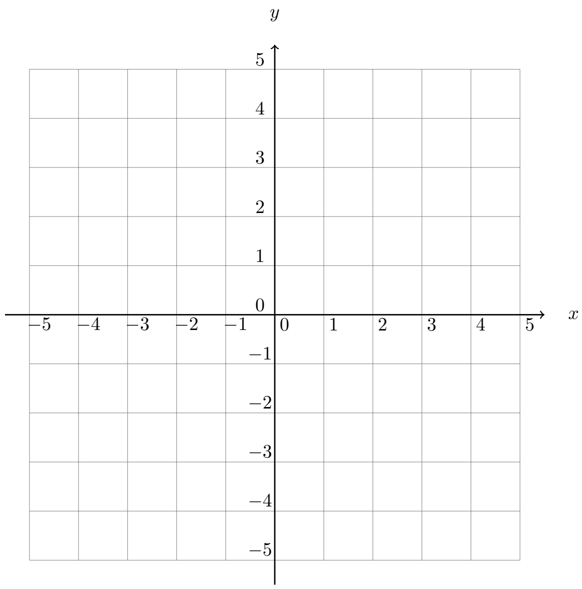


(b) (8 points) Setup, **but do not integrate**, the integral in part (a) using the form $\iint_R f(x,y)\, dxdy$.

7. (15 points) In this problem we wish to find the volume under the paraboloid $z = x^2 + y^2$ and above the annulus in the xy-plane $4 \leqslant x^2 + y^2 \leqslant 16$.

   (a) Sketch the region of integration on the graph provided;

   (b) Use polar coordinates to find the volume.



8. (15 points) Consider the iterated integral

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}\;dz\;dy\;dx.$$

(a) Describe the region of integration (what is the geometric object?).

(b) Convert this integral to a different coordinate system. Do not integrate.

9. (15 points) Using the chemical process known as Ipulledthisoutmyass, scientists are able to model the density of organisms in a circular petri dish. This density is given by the equation $P(x, y) = y^2 + x^2 + 1$, where x and y are the rectangular coordinates assuming the center of the dish is the origin. Estimate how many organisms are on the top half of a 4 inch diameter petri dish.

10. (15 points) Let D' be the region defined by the ellipse $\dfrac{x^2}{7} + \dfrac{y^2}{3} = 1$.

(a) What change of variables would you use to convert D' to the unit disk D?

(b) Find the area of the ellipse D'.