

48. (a) $y = \ln(-x)$

(b) $y = \ln|x|$

49–52 Solve each equation for x .

49. (a) $e^{7-4x} = 6$

(b) $\ln(3x - 10) = 2$

50. (a) $\ln(x^2 - 1) = 3$

(b) $e^{2x} - 3e^x + 2 = 0$

51. (a) $2^{x-5} = 3$

(b) $\ln x + \ln(x - 1) = 1$

52. (a) $\ln(\ln x) = 1$

(b) $e^{ax} = Ce^{bx}$, where $a \neq b$

53–54 Solve each inequality for x .

53. (a) $e^x < 10$

(b) $\ln x > -1$

54. (a) $2 < \ln x < 9$

(b) $e^{2-3x} > 4$

55–56 Find (a) the domain of f and (b) f^{-1} and its domain.

55. $f(x) = \sqrt{3 - e^{2x}}$

56. $f(x) = \ln(2 + \ln x)$

- CAS** 57. Graph the function $f(x) = \sqrt{x^3 + x^2 + x + 1}$ and explain why it is one-to-one. Then use a computer algebra system to find an explicit expression for $f^{-1}(x)$. (Your CAS will produce three possible expressions. Explain why two of them are irrelevant in this context.)

- CAS** 58. (a) If $g(x) = x^6 + x^4$, $x \geq 0$, use a computer algebra system to find an expression for $g^{-1}(x)$.
 (b) Use the expression in part (a) to graph $y = g(x)$, $y = x$, and $y = g^{-1}(x)$ on the same screen.

59. If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is $n = f(t) = 100 \cdot 2^{t/3}$. (See Exercise 29 in Section 1.5.)
 (a) Find the inverse of this function and explain its meaning.
 (b) When will the population reach 50,000?

60. When a camera flash goes off, the batteries immediately begin to recharge the flash's capacitor, which stores electric charge given by

$$Q(t) = Q_0(1 - e^{-t/a})$$

(The maximum charge capacity is Q_0 and t is measured in seconds.)

- (a) Find the inverse of this function and explain its meaning.
 (b) How long does it take to recharge the capacitor to 90% of capacity if $a = 2$?

61. Starting with the graph of $y = \ln x$, find the equation of the graph that results from
 (a) shifting 3 units upward
 (b) shifting 3 units to the left
 (c) reflecting about the x -axis
 (d) reflecting about the y -axis
 (e) reflecting about the line $y = x$
 (f) reflecting about the x -axis and then about the line $y = x$
 (g) reflecting about the y -axis and then about the line $y = x$
 (h) shifting 3 units to the left and then reflecting about the line $y = x$

62. (a) If we shift a curve to the left, what happens to its reflection about the line $y = x$? In view of this geometric principle, find an expression for the inverse of $g(x) = f(x + c)$, where f is a one-to-one function.
 (b) Find an expression for the inverse of $h(x) = f(cx)$, where $c \neq 0$.

1.7 Parametric Curves

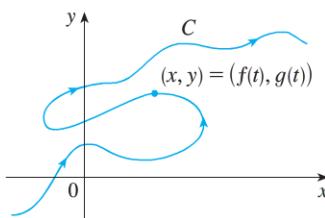


FIGURE 1

Imagine that a particle moves along the curve C shown in Figure 1. It is impossible to describe C by an equation of the form $y = f(x)$ because C fails the Vertical Line Test. But the x - and y -coordinates of the particle are functions of time and so we can write $x = f(t)$ and $y = g(t)$. Such a pair of equations is often a convenient way of describing a curve and gives rise to the following definition.

Suppose that x and y are both given as functions of a third variable t (called a **parameter**) by the equations

$$x = f(t) \quad y = g(t)$$

(called **parametric equations**). Each value of t determines a point (x, y) , which we can plot in a coordinate plane. As t varies, the point $(x, y) = (f(t), g(t))$ varies and traces out a curve C , which we call a **parametric curve**. The parameter t does not necessarily represent time and, in fact, we could use a letter other than t for the parameter. But in many applications of parametric curves, t does denote time and therefore we can interpret $(x, y) = (f(t), g(t))$ as the position of a particle at time t .

EXAMPLE 1 Graphing a parametric curve Sketch and identify the curve defined by the parametric equations

$$x = t^2 - 2t \quad y = t + 1$$

SOLUTION Each value of t gives a point on the curve, as shown in the table. For instance, if $t = 0$, then $x = 0$, $y = 1$ and so the corresponding point is $(0, 1)$. In Figure 2 we plot the points (x, y) determined by several values of the parameter t and we join them to produce a curve.

t	x	y
-2	8	-1
-1	3	0
0	0	1
1	-1	2
2	0	3
3	3	4
4	8	5

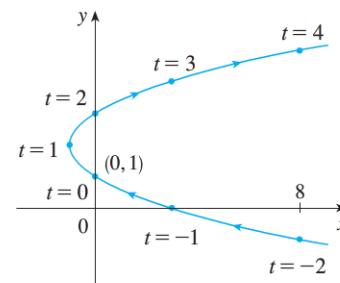


FIGURE 2

A particle whose position is given by the parametric equations moves along the curve in the direction of the arrows as t increases. Notice that the consecutive points marked on the curve appear at equal time intervals but not at equal distances. That is because the particle slows down and then speeds up as t increases.

It appears from Figure 2 that the curve traced out by the particle may be a parabola. This can be confirmed by eliminating the parameter t as follows. We obtain $t = y - 1$ from the second equation and substitute into the first equation. This gives

$$x = t^2 - 2t = (y - 1)^2 - 2(y - 1) = y^2 - 4y + 3$$

and so the curve represented by the given parametric equations is the parabola $x = y^2 - 4y + 3$.

This equation in x and y describes where the particle has been, but it doesn't tell us when the particle was at a particular point. The parametric equations have an advantage—they tell us when the particle was at a point. They also indicate the direction of the motion.

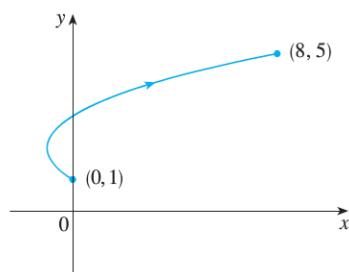


FIGURE 3

No restriction was placed on the parameter t in Example 1, so we assumed that t could be any real number. But sometimes we restrict t to lie in a finite interval. For instance, the parametric curve

$$x = t^2 - 2t \quad y = t + 1 \quad 0 \leq t \leq 4$$

shown in Figure 3 is the part of the parabola in Example 1 that starts at the point $(0, 1)$ and ends at the point $(8, 5)$. The arrowhead indicates the direction in which the curve is traced as t increases from 0 to 4.

In general, the curve with parametric equations

$$x = f(t) \quad y = g(t) \quad a \leq t \leq b$$

has **initial point** $(f(a), g(a))$ and **terminal point** $(f(b), g(b))$.

V EXAMPLE 2 Identifying a parametric curve What curve is represented by the following parametric equations?

$$x = \cos t \quad y = \sin t \quad 0 \leq t \leq 2\pi$$

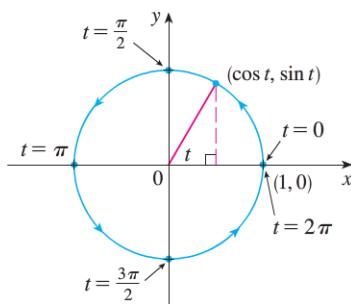


FIGURE 4

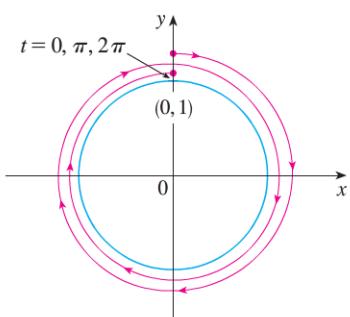


FIGURE 5

SOLUTION If we plot points, it appears that the curve is a circle. We can confirm this impression by eliminating t . Observe that

$$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$$

Thus the point (x, y) moves on the unit circle $x^2 + y^2 = 1$. Notice that in this example the parameter t can be interpreted as the angle (in radians) shown in Figure 4. As t increases from 0 to 2π , the point $(x, y) = (\cos t, \sin t)$ moves once around the circle in the counterclockwise direction starting from the point $(1, 0)$.

EXAMPLE 3 What curve is represented by the given parametric equations?

$$x = \sin 2t \quad y = \cos 2t \quad 0 \leq t \leq 2\pi$$

SOLUTION Again we have

$$x^2 + y^2 = \sin^2 2t + \cos^2 2t = 1$$

so the parametric equations again represent the unit circle $x^2 + y^2 = 1$. But as t increases from 0 to 2π , the point $(x, y) = (\sin 2t, \cos 2t)$ starts at $(0, 1)$ and moves *twice* around the circle in the clockwise direction as indicated in Figure 5.

Examples 2 and 3 show that different sets of parametric equations can represent the same curve. Thus we distinguish between a *curve*, which is a set of points, and a *parametric curve*, in which the points are traced in a particular way.

EXAMPLE 4 Find parametric equations for the circle with center (h, k) and radius r .

SOLUTION If we take the equations of the unit circle in Example 2 and multiply the expressions for x and y by r , we get $x = r \cos t$, $y = r \sin t$. You can verify that these equations represent a circle with radius r and center the origin traced counterclockwise. We now shift h units in the x -direction and k units in the y -direction and obtain parametric equations of the circle (Figure 6) with center (h, k) and radius r :

$$x = h + r \cos t \quad y = k + r \sin t \quad 0 \leq t \leq 2\pi$$

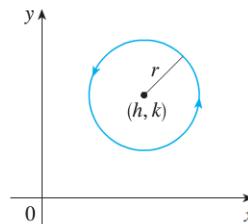


FIGURE 6

$$x = h + r \cos t, y = k + r \sin t$$

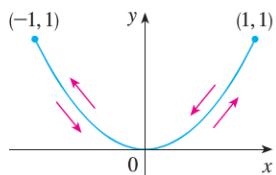


FIGURE 7

EXAMPLE 5 Sketch the curve with parametric equations $x = \sin t$, $y = \sin^2 t$.

SOLUTION Observe that $y = (\sin t)^2 = x^2$ and so the point (x, y) moves on the parabola $y = x^2$. But note also that, since $-1 \leq \sin t \leq 1$, we have $-1 \leq x \leq 1$, so the parametric equations represent only the part of the parabola for which $-1 \leq x \leq 1$. Since $\sin t$ is periodic, the point $(x, y) = (\sin t, \sin^2 t)$ moves back and forth infinitely often along the parabola from $(-1, 1)$ to $(1, 1)$. (See Figure 7.)

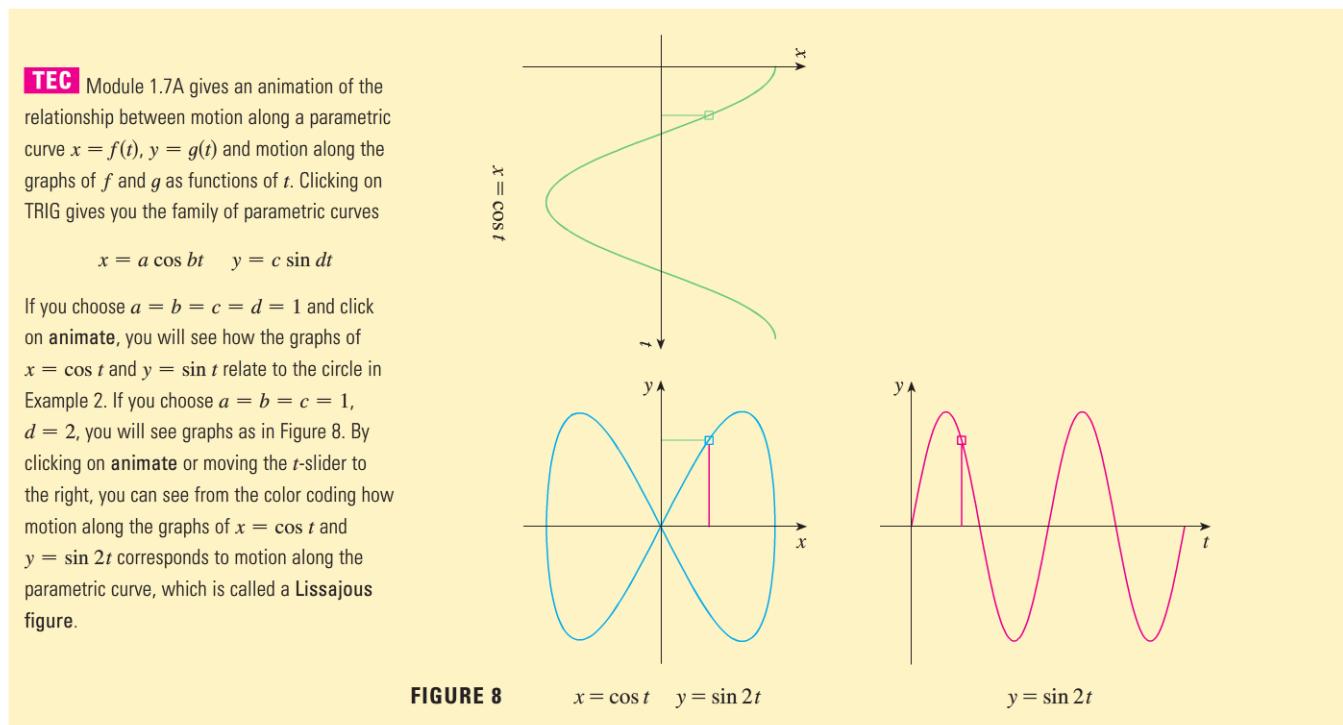


FIGURE 8

Graphing Devices

Most graphing calculators and computer graphing programs can be used to graph curves defined by parametric equations. In fact, it's instructive to watch a parametric curve being drawn by a graphing calculator because the points are plotted in order as the corresponding parameter values increase.

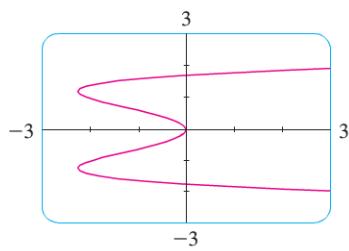


FIGURE 9

EXAMPLE 6 Graphing x as a function of y

Use a graphing device to graph the curve $x = y^4 - 3y^2$.

SOLUTION If we let the parameter be $t = y$, then we have the equations

$$x = t^4 - 3t^2 \quad y = t$$

Using these parametric equations to graph the curve, we obtain Figure 9. It would be possible to solve the given equation ($x = y^4 - 3y^2$) for y as four functions of x and graph them individually, but the parametric equations provide a much easier method.

In general, if we need to graph an equation of the form $x = g(y)$, we can use the parametric equations

$$x = g(t) \quad y = t$$

Notice also that curves with equations $y = f(x)$ (the ones we are most familiar with—graphs of functions) can also be regarded as curves with parametric equations

$$x = t \quad y = f(t)$$

Graphing devices are particularly useful when sketching complicated curves. For instance, the curves shown in Figures 10, 11, and 12 would be virtually impossible to produce by hand.

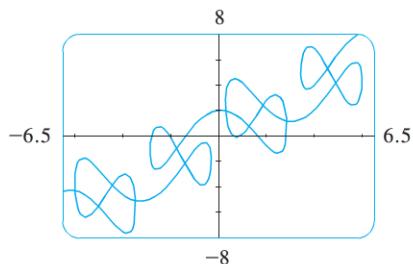


FIGURE 10
 $x = t + 2 \sin 2t$
 $y = t + 2 \cos 5t$

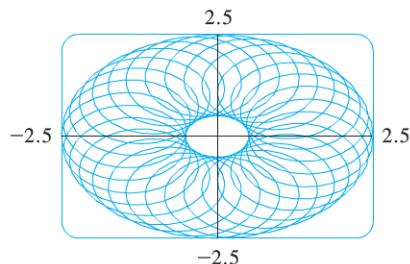


FIGURE 11
 $x = 1.5 \cos t - \cos 30t$
 $y = 1.5 \sin t - \sin 30t$

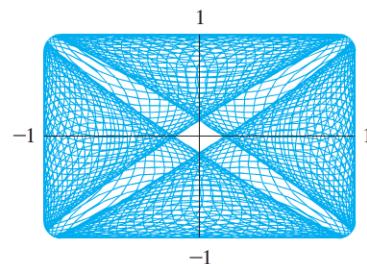


FIGURE 12
 $x = \sin(t + \cos 100t)$
 $y = \cos(t + \sin 100t)$

One of the most important uses of parametric curves is in computer-aided design (CAD). In the Laboratory Project after Section 3.4 we will investigate special parametric curves, called **Bézier curves**, that are used extensively in manufacturing, especially in the automotive industry. These curves are also employed in specifying the shapes of letters and other symbols in laser printers.

The Cycloid

TEC An animation in Module 1.7B shows how the cycloid is formed as the circle moves.

EXAMPLE 7 Deriving parametric equations for a cycloid The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a **cycloid** (see Figure 13). If the circle has radius r and rolls along the x -axis and if one position of P is the origin, find parametric equations for the cycloid.

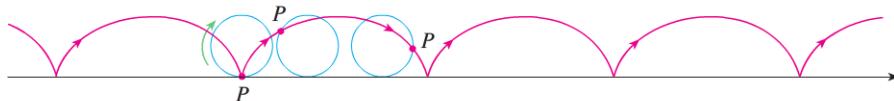


FIGURE 13

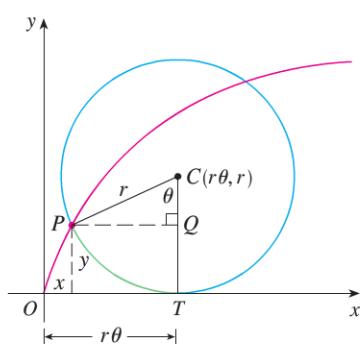


FIGURE 14

SOLUTION We choose as parameter the angle of rotation θ of the circle ($\theta = 0$ when P is at the origin). Suppose the circle has rotated through θ radians. Because the circle has been in contact with the line, we see from Figure 14 that the distance it has rolled from the origin is

$$|OT| = \text{arc } PT = r\theta$$

Therefore the center of the circle is $C(r\theta, r)$. Let the coordinates of P be (x, y) . Then from Figure 14 we see that

$$x = |OT| - |PQ| = r\theta - r \sin \theta = r(\theta - \sin \theta)$$

$$y = |TC| - |QC| = r - r \cos \theta = r(1 - \cos \theta)$$

Therefore parametric equations of the cycloid are

1

$$x = r(\theta - \sin \theta) \quad y = r(1 - \cos \theta) \quad \theta \in \mathbb{R}$$

One arch of the cycloid comes from one rotation of the circle and so is described by $0 \leq \theta \leq 2\pi$. Although Equations 1 were derived from Figure 14, which illustrates the case where $0 < \theta < \pi/2$, it can be seen that these equations are still valid for other values of θ (see Exercise 37).

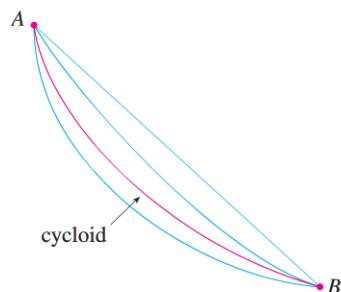


FIGURE 15

Although it is possible to eliminate the parameter θ from Equations 1, the resulting Cartesian equation in x and y is very complicated and not as convenient to work with as the parametric equations.

One of the first people to study the cycloid was Galileo, who proposed that bridges be built in the shape of cycloids and who tried to find the area under one arch of a cycloid. Later this curve arose in connection with the **brachistochrone problem**: Find the curve along which a particle will slide in the shortest time (under the influence of gravity) from a point A to a lower point B not directly beneath A . The Swiss mathematician John Bernoulli, who posed this problem in 1696, showed that among all possible curves that join A to B , as in Figure 15, the particle will take the least time sliding from A to B if the curve is part of an inverted arch of a cycloid.

The Dutch physicist Huygens had already shown that the cycloid is also the solution to the **tautochrone problem**; that is, no matter where a particle P is placed on an inverted cycloid, it takes the same time to slide to the bottom (see Figure 16). Huygens proposed that pendulum clocks (which he invented) should swing in cycloidal arcs because then the pendulum would take the same time to make a complete oscillation whether it swings through a wide or a small arc.

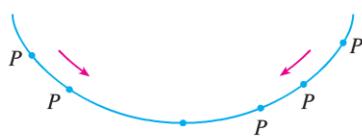


FIGURE 16

1.7 Exercises

- 1–4** Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.

1. $x = t^2 + t, \quad y = t^2 - t, \quad -2 \leq t \leq 2$
2. $x = t^2, \quad y = t^3 - 4t, \quad -3 \leq t \leq 3$
3. $x = \cos^2 t, \quad y = 1 - \sin t, \quad 0 \leq t \leq \pi/2$
4. $x = e^{-t} + t, \quad y = e^t - t, \quad -2 \leq t \leq 2$

5–8

- (a) Sketch the curve by using the parametric equations to plot points. Indicate with an arrow the direction in which the curve is traced as t increases.
(b) Eliminate the parameter to find a Cartesian equation of the curve.

5. $x = 3t - 5, \quad y = 2t + 1$
6. $x = 1 + 3t, \quad y = 2 - t^2$
7. $x = \sqrt{t}, \quad y = 1 - t$
8. $x = t^2, \quad y = t^3$

9–16

- (a) Eliminate the parameter to find a Cartesian equation of the curve.
(b) Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.
9. $x = \sin \frac{1}{2}\theta, \quad y = \cos \frac{1}{2}\theta, \quad -\pi \leq \theta \leq \pi$

10. $x = \frac{1}{2} \cos \theta, \quad y = 2 \sin \theta, \quad 0 \leq \theta \leq \pi$
11. $x = \sin t, \quad y = \csc t, \quad 0 < t < \pi/2$
12. $x = \tan^2 \theta, \quad y = \sec \theta, \quad -\pi/2 < \theta < \pi/2$
13. $x = e^{2t}, \quad y = t + 1$
14. $x = e^t - 1, \quad y = e^{2t}$
15. $x = \sin \theta, \quad y = \cos 2\theta$
16. $x = \ln t, \quad y = \sqrt{t}, \quad t \geq 1$

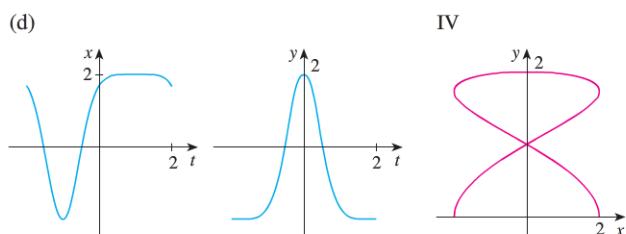
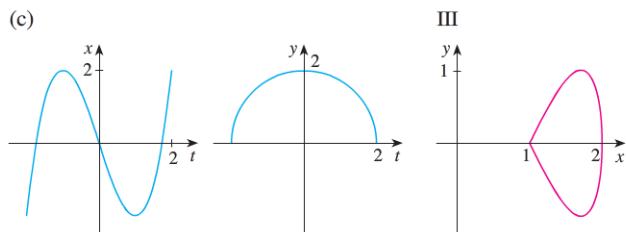
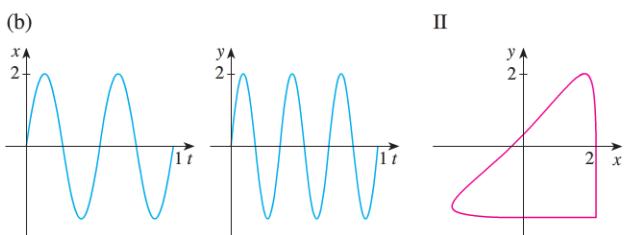
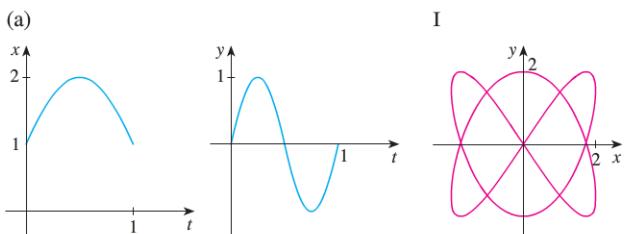
- 17–20** Describe the motion of a particle with position (x, y) as t varies in the given interval.

17. $x = 3 + 2 \cos t, \quad y = 1 + 2 \sin t, \quad \pi/2 \leq t \leq 3\pi/2$
18. $x = 2 \sin t, \quad y = 4 + \cos t, \quad 0 \leq t \leq 3\pi/2$
19. $x = 5 \sin t, \quad y = 2 \cos t, \quad -\pi \leq t \leq 5\pi$
20. $x = \sin t, \quad y = \cos^2 t, \quad -2\pi \leq t \leq 2\pi$

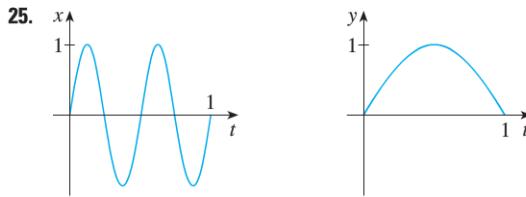
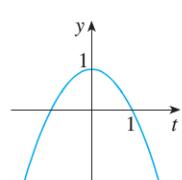
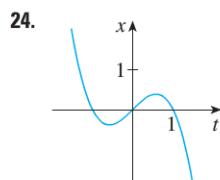
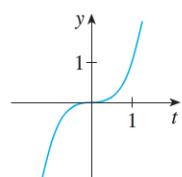
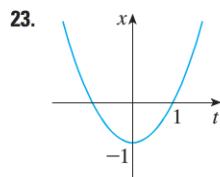
21. Suppose a curve is given by the parametric equations $x = f(t)$, $y = g(t)$, where the range of f is $[1, 4]$ and the range of g is $[2, 3]$. What can you say about the curve?
22. Match the graphs of the parametric equations $x = f(t)$ and $y = g(t)$ in (a)–(d) with the parametric curves labeled I–IV. Give reasons for your choices.

Graphing calculator or computer with graphing software required

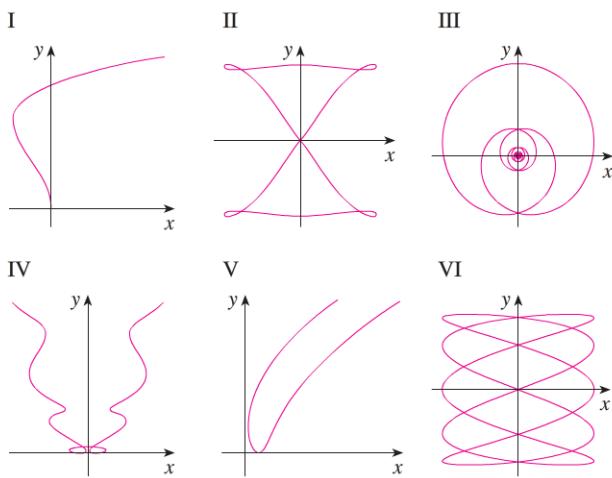
1. Homework Hints available in TEC



- 23–25** Use the graphs of $x = f(t)$ and $y = g(t)$ to sketch the parametric curve $x = f(t)$, $y = g(t)$. Indicate with arrows the direction in which the curve is traced as t increases.



- 26.** Match the parametric equations with the graphs labeled I–VI. Give reasons for your choices. (Do not use a graphing device.)
- $x = t^4 - t + 1$, $y = t^2$
 - $x = t^2 - 2t$, $y = \sqrt{t}$
 - $x = \sin 2t$, $y = \sin(t + \sin 2t)$
 - $x = \cos 5t$, $y = \sin 2t$
 - $x = t + \sin 4t$, $y = t^2 + \cos 3t$
 - $x = \frac{\sin 2t}{4 + t^2}$, $y = \frac{\cos 2t}{4 + t^2}$



- 27.** Graph the curve $x = y - 2 \sin \pi y$.

- 28.** Graph the curves $y = x^3 - 4x$ and $x = y^3 - 4y$ and find their points of intersection correct to one decimal place.

- 29.** (a) Show that the parametric equations

$$x = x_1 + (x_2 - x_1)t \quad y = y_1 + (y_2 - y_1)t$$

where $0 \leq t \leq 1$, describe the line segment that joins the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

- (b) Find parametric equations to represent the line segment from $(-2, 7)$ to $(3, -1)$.

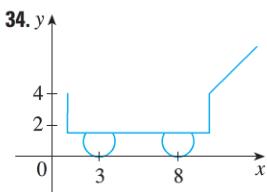
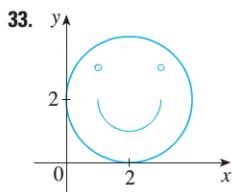
- 30.** Use a graphing device and the result of Exercise 29(a) to draw the triangle with vertices $A(1, 1)$, $B(4, 2)$, and $C(1, 5)$.

- 31.** Find parametric equations for the path of a particle that moves along the circle $x^2 + (y - 1)^2 = 4$ in the manner described.

- Once around clockwise, starting at $(2, 1)$
- Three times around counterclockwise, starting at $(2, 1)$
- Halfway around counterclockwise, starting at $(0, 3)$

32. (a) Find parametric equations for the ellipse $x^2/a^2 + y^2/b^2 = 1$. [Hint: Modify the equations of the circle in Example 2.]
 (b) Use these parametric equations to graph the ellipse when $a = 3$ and $b = 1, 2, 4$, and 8 .
 (c) How does the shape of the ellipse change as b varies?

- 33–34 Use a graphing calculator or computer to reproduce the picture.



- 35–36 Compare the curves represented by the parametric equations. How do they differ?

35. (a) $x = t^3$, $y = t^2$ (b) $x = t^6$, $y = t^4$
 (c) $x = e^{-3t}$, $y = e^{-2t}$
36. (a) $x = t$, $y = t^{-2}$ (b) $x = \cos t$, $y = \sec^2 t$
 (c) $x = e^t$, $y = e^{-2t}$

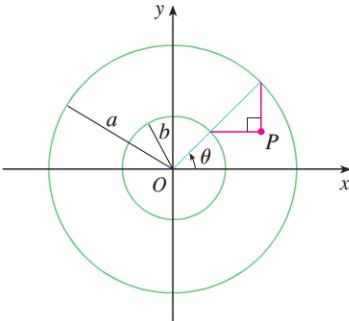
37. Derive Equations 1 for the case $\pi/2 < \theta < \pi$.

38. Let P be a point at a distance d from the center of a circle of radius r . The curve traced out by P as the circle rolls along a straight line is called a **trochoid**. (Think of the motion of a point on a spoke of a bicycle wheel.) The cycloid is the special case of a trochoid with $d = r$. Using the same parameter θ as for the cycloid and, assuming the line is the x -axis and $\theta = 0$ when P is at one of its lowest points, show that parametric equations for the trochoid are

$$x = r\theta - d \sin \theta \quad y = r - d \cos \theta$$

Sketch the trochoid for the cases $d < r$ and $d > r$.

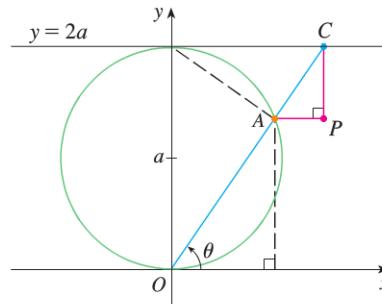
39. If a and b are fixed numbers, find parametric equations for the curve that consists of all possible positions of the point P in the figure, using the angle θ as the parameter. Then eliminate the parameter and identify the curve.



40. A curve, called a **witch of Maria Agnesi**, consists of all possible positions of the point P in the figure. Show that parametric equations for this curve can be written as

$$x = 2a \cot \theta \quad y = 2a \sin^2 \theta$$

Sketch the curve.



41. Suppose that the position of one particle at time t is given by

$$x_1 = 3 \sin t \quad y_1 = 2 \cos t \quad 0 \leq t \leq 2\pi$$

and the position of a second particle is given by

$$x_2 = -3 + \cos t \quad y_2 = 1 + \sin t \quad 0 \leq t \leq 2\pi$$

- (a) Graph the paths of both particles. How many points of intersection are there?
 (b) Are any of these points of intersection *collision points*? In other words, are the particles ever at the same place at the same time? If so, find the collision points.
 (c) Describe what happens if the path of the second particle is given by

$$x_2 = 3 + \cos t \quad y_2 = 1 + \sin t \quad 0 \leq t \leq 2\pi$$

42. If a projectile is fired with an initial velocity of v_0 meters per second at an angle α above the horizontal and air resistance is assumed to be negligible, then its position after t seconds is given by the parametric equations

$$x = (v_0 \cos \alpha)t \quad y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2$$

where g is the acceleration due to gravity (9.8 m/s^2).

- (a) If a gun is fired with $\alpha = 30^\circ$ and $v_0 = 500 \text{ m/s}$, when will the bullet hit the ground? How far from the gun will it hit the ground? What is the maximum height reached by the bullet?
 (b) Use a graphing device to check your answers to part (a). Then graph the path of the projectile for several other values of the angle α to see where it hits the ground. Summarize your findings.
 (c) Show that the path is parabolic by eliminating the parameter.

43. Investigate the family of curves defined by the parametric equations $x = t^2$, $y = t^3 - ct$. How does the shape change as c increases? Illustrate by graphing several members of the family.

44. The **swallowtail catastrophe curves** are defined by the parametric equations $x = 2ct - 4t^3$, $y = -ct^2 + 3t^4$.

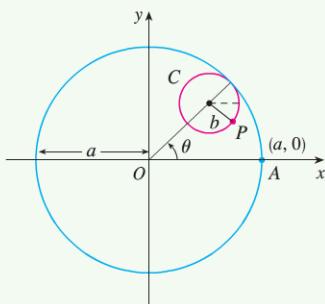
Graph several of these curves. What features do the curves have in common? How do they change when c increases?

-  45. The curves with equations $x = a \sin nt$, $y = b \cos t$ are called **Lissajous figures**. Investigate how these curves vary when a , b , and n vary. (Take n to be a positive integer.)

-  46. Investigate the family of curves defined by the parametric equations $x = \cos t$, $y = \sin t - \sin ct$, where $c > 0$. Start by letting c be a positive integer and see what happens to the shape as c increases. Then explore some of the possibilities that occur when c is a fraction.

LABORATORY PROJECT

Running Circles Around Circles



TEC Look at Module 1.7B to see how hypocycloids and epicycloids are formed by the motion of rolling circles.

In this project we investigate families of curves, called *hypocycloids* and *epicycloids*, that are generated by the motion of a point on a circle that rolls inside or outside another circle.

1. A **hypocycloid** is a curve traced out by a fixed point P on a circle C of radius b as C rolls on the inside of a circle with center O and radius a . Show that if the initial position of P is $(a, 0)$ and the parameter θ is chosen as in the figure, then parametric equations of the hypocycloid are

$$x = (a - b) \cos \theta + b \cos\left(\frac{a - b}{b} \theta\right) \quad y = (a - b) \sin \theta - b \sin\left(\frac{a - b}{b} \theta\right)$$

2. Use a graphing device (or the interactive graphic in TEC Module 1.7B) to draw the graphs of hypocycloids with a a positive integer and $b = 1$. How does the value of a affect the graph? Show that if we take $a = 4$, then the parametric equations of the hypocycloid reduce to

$$x = 4 \cos^3 \theta \quad y = 4 \sin^3 \theta$$

This curve is called a **hypocycloid of four cusps**, or an **astroid**.

3. Now try $b = 1$ and $a = n/d$, a fraction where n and d have no common factor. First let $n = 1$ and try to determine graphically the effect of the denominator d on the shape of the graph. Then let n vary while keeping d constant. What happens when $n = d + 1$?
4. What happens if $b = 1$ and a is irrational? Experiment with an irrational number like $\sqrt{2}$ or $e - 2$. Take larger and larger values for θ and speculate on what would happen if we were to graph the hypocycloid for all real values of θ .
5. If the circle C rolls on the *outside* of the fixed circle, the curve traced out by P is called an **epicycloid**. Find parametric equations for the epicycloid.
6. Investigate the possible shapes for epicycloids. Use methods similar to Problems 2–4.

 Graphing calculator or computer with graphing software required

1 Review

Concept Check

1. (a) What is a function? What are its domain and range?
(b) What is the graph of a function?
(c) How can you tell whether a given curve is the graph of a function?
2. Discuss four ways of representing a function. Illustrate your discussion with examples.
3. (a) What is an even function? How can you tell if a function is even by looking at its graph?
(b) What is an odd function? How can you tell if a function is odd by looking at its graph?
4. What is an increasing function?
5. What is a mathematical model?
6. Give an example of each type of function.

(a) Linear function	(b) Power function
(c) Exponential function	(d) Quadratic function
(e) Polynomial of degree 5	(f) Rational function
7. Sketch by hand, on the same axes, the graphs of the following functions.

(a) $f(x) = x$	(b) $g(x) = x^2$
(c) $h(x) = x^3$	(d) $j(x) = x^4$
8. Draw, by hand, a rough sketch of the graph of each function.

(a) $y = \sin x$	(b) $y = \tan x$
(c) $y = e^x$	(d) $y = \ln x$
(e) $y = 1/x$	(f) $y = x $
(g) $y = \sqrt{x}$	
9. Suppose that f has domain A and g has domain B .
 - (a) What is the domain of $f + g$?
 - (b) What is the domain of fg ?
 - (c) What is the domain of f/g ?
10. How is the composite function $f \circ g$ defined? What is its domain?
11. Suppose the graph of f is given. Write an equation for each of the graphs that are obtained from the graph of f as follows.
 - (a) Shift 2 units upward.
 - (b) Shift 2 units downward.
 - (c) Shift 2 units to the right.
 - (d) Shift 2 units to the left.
 - (e) Reflect about the x -axis.
 - (f) Reflect about the y -axis.
 - (g) Stretch vertically by a factor of 2.
 - (h) Shrink vertically by a factor of 2.
 - (i) Stretch horizontally by a factor of 2.
 - (j) Shrink horizontally by a factor of 2.
12. (a) What is a one-to-one function? How can you tell if a function is one-to-one by looking at its graph?
(b) If f is a one-to-one function, how is its inverse function f^{-1} defined? How do you obtain the graph of f^{-1} from the graph of f ?
13. (a) What is a parametric curve?
(b) How do you sketch a parametric curve?
(c) Why might a parametric curve be more useful than a curve of the form $y = f(x)$?

True-False Quiz

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. If f is a function, then $f(s + t) = f(s) + f(t)$.
2. If $f(s) = f(t)$, then $s = t$.
3. If f is a function, then $f(3x) = 3f(x)$.
4. If $x_1 < x_2$ and f is a decreasing function, then $f(x_1) > f(x_2)$.
5. A vertical line intersects the graph of a function at most once.
6. If f and g are functions, then $f \circ g = g \circ f$.
7. If f is one-to-one, then $f^{-1}(x) = \frac{1}{f(x)}$.
8. You can always divide by e^x .
9. If $0 < a < b$, then $\ln a < \ln b$.
10. If $x > 0$, then $(\ln x)^6 = 6 \ln x$.
11. If $x > 0$ and $a > 1$, then $\frac{\ln x}{\ln a} = \ln \frac{x}{a}$.
12. The parametric equations $x = t^2$, $y = t^4$ have the same graph as $x = t^3$, $y = t^6$.