Math122 Review Problems for Midterm - Summer 2011
(Sec 8.1 - 8.7, 8.9, H.1, H.2, 9.1 - 9.7)
Exam 2 (Midterm), Friday, Julu 1, 8:00 - 10:00 am.

Review the **Concept Check** *problems: Page 631/1 - 11, Page 690/1 - 20*

PART 1: True-False Problems

Ch.8. Page 632 True-False Quiz Problems 1 – 18.

Ch.9. Page 691 True-False Quiz Problems 1 – 16.

Additional True-False Problems.

1. If the series $\sum a_n$ converges, then the sequence $\{a_n\}$ converges.
2. If $\lim\limits_{n\to\infty} |a_n| = 0$, then $\sum a_n$ converges.
3. If $\sum a_n$ converges, then $\sum |a_n|$ converges.
4. If $\{a_n\}$ converges, then $\{|a_n|\}$ converges.
5. If $\{|a_n|\}$ converges, then $\{a_n\}$ converges.
6. If $\{a_n\}$ converges but $\{b_n\}$ diverges, then $\{a_n + b_n\}$ diverges.
7. If $\{a_n\}$ converges but $\{b_n\}$ diverges, then $\{a_n b_n\}$ diverges.
8. If $0 \le a_n \le b_n$ for $n \ge 1$ and $\{b_n\}$ converges, then $\{a_n\}$ converges.
9. If $1 \le a_n \le b_n$ for $n \ge 1$ and $\lim\limits_{n\to\infty} b_n = 1$, then $\lim\limits_{n\to\infty} a_n$ converges.
10. If $\{a_n\}$ is bounded then $\{a_n\}$ is convergent.
11. If $0 \le a_n \le b_n$ and $\sum b_n$ converges, then $\sum a_n$ converges.
12. If $\sum a_n$ converges, then $\lim\limits_{n\to\infty} e^{a_n} = 1$.
13. If $\sum a_n$ is absolutely convergent, then $\sum a_n^3$ converges.
14. If $f(x) = 1 - 3(x-1)^2 + 5(x-1)^3 + \cdots$ converges for all x, then $f''(1) = -3$.
15. If $\sum\limits_{n=0}^{\infty} c_n(x-1)^n$ is the Taylor series of $f(x) = \ln x$ at $a = 1$, then $c_2 = -1$.
16. The points $\left(1, \dfrac{4\pi}{3}\right)$ and $\left(-1, \dfrac{\pi}{3}\right)$ represent the same point in the polar coordinate system.
17. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $\mathbf{u} \cdot \mathbf{v}$ is a vector.

18. For any vectors $\mathbf{u}$ and $\mathbf{v}$ in V_3, $\mathbf{u} \times \mathbf{v}$ is a vector.

19. For any vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in V_3, $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$ is a vector.

20. If $|\mathbf{u}|^2 + |\mathbf{v}|^2 = |\mathbf{u} + \mathbf{v}|^2$, then $\mathbf{u} \cdot \mathbf{v} = 0$.

21. For any $\mathbf{u}$ in V_3, $\mathbf{u} \cdot \mathbf{u} = 0$.

22. For any $\mathbf{u}$ in V_3, $\mathbf{u} \times \mathbf{u} = \mathbf{0}$.

23. If two lines are perpendicular to a third line, then they are parallel.

24. If two lines are parallel to a third line, then they are parallel.

25. If two planes are perpendicular to a line, then they are parallel.

26. If two planes are parallel to a line, then they are parallel.

PART II. Multiple-Choice Problems

1. Exactly one of the following sequences diverges. Which is it?

   (A) $\left\{\dfrac{\sqrt{n^4+1}}{n^2}\right\}$ (B) $\left\{\dfrac{n^2}{3^n}\right\}$ (C) $\left\{\dfrac{2^n}{n!}\right\}$ (D) $\left\{\dfrac{n}{(\ln n)^2}\right\}$ (E) $\left\{\cos\dfrac{1}{\sqrt{n}}\right\}$

2. Exactly one of the following sequences diverges. Which is it?

   (A) $\left\{\sin\dfrac{1}{\sqrt{n}}\right\}$ (B) $\left\{\cos\dfrac{1}{\sqrt{n}}\right\}$ (C) $\left\{n\sin\dfrac{1}{n}\right\}$ (D) $\left\{e^{\frac{1}{\sqrt{n}}}\right\}$ (E) $\left\{\dfrac{(-1)^n\sqrt{n+1}}{\sqrt{n}}\right\}$

3. Exactly one of the following series diverges. Which is it?

   (A) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(-2)^n}$ (B) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4}$ (C) $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^3}$ (D) $\displaystyle\sum_{n=1}^{\infty}\frac{n^3+n^2}{n^4+1}$ (E) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{\ln n}$

4. Exactly one of the following series diverges. Which is it?

   (A) $\displaystyle\sum_{n=1}^{\infty}\frac{\sin n}{n^2}$ (B) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ (C) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n\ln n}{n}$ (D) $\displaystyle\sum_{n=2}^{\infty}\sin\frac{1}{n\ln n}$ (E) $\displaystyle\sum_{n=1}^{\infty}\frac{n^5}{n!}$

5. The series $\displaystyle\sum_{n=1}^{\infty}(-r)^n$ for $0 < r < 1$ converges to

   (A) $\dfrac{1}{1-r}$ (B) $\dfrac{1}{1+r}$ (C) $\dfrac{r}{1-r}$ (D) $\dfrac{r}{1+r}$ (E) None of the above is true.

6. The sum of the geometric series $\sum_{n=1}^{\infty} (-\pi)^{n-1} 2^{-2n}$ is

(A) $\dfrac{1}{4-\pi}$ (B) $\dfrac{1}{4+\pi}$ (C) $-\dfrac{\pi}{4+\pi}$ (D) $\dfrac{1}{2+\pi}$ (E) $\dfrac{\pi}{4+\pi}$

7. The series $\sum_{n=1}^{\infty} \dfrac{1}{n(n+1)}$ converges to

(A) $\dfrac{1}{2}$ (B) 1 (C) $\dfrac{1}{3}$ (D) ∞ (E) None of the above is true.

8. The sum of the series $\sum_{n=1}^{\infty} \left(\dfrac{1}{\sqrt{n}} - \dfrac{1}{\sqrt{n+2}} \right)$ is

(A) $1 + \dfrac{1}{\sqrt{3}}$ (B) $1 - \dfrac{1}{\sqrt{3}}$ (C) $1 + \dfrac{1}{\sqrt{2}}$ (D) $1 - \dfrac{1}{\sqrt{2}}$ (E) ∞

9. The sum of the series $\sum_{n=0}^{\infty} \dfrac{1}{n!}$ is

(A) $\sqrt{2}$ (B) π (C) e (D) $\ln 2$ (E) ∞

10. The sum of the series $\sum_{n=0}^{\infty} (-1)^n \dfrac{\pi^{2n+1}}{(2n+1)!6^{2n+1}}$ is

(A) 0 (B) 1/2 (C) $\pi/6$ (D) $\sqrt{3}/2$ (E) ∞

11. The radius of convergence of the series $\sum_{n=1}^{\infty} \dfrac{2 \ln n}{3n+1} x^n$ is

(A) $\dfrac{1}{3}$ (B) $\dfrac{2}{3}$ (C) 1 (D) 0 (E) ∞

12. The interval of convergence of the series $\sum_{n=1}^{\infty} \dfrac{3^n}{n}(x-1)^n$ is

(A) [2/3, 4/3] (B) (2/3, 4/3) (C) (2/3, 4/3]
(D) [2/3, 4/3) (E) None of the above is true.

13. The Maclaurin series for $\cos(x^2)$ is

(A) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$ (B) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$ (C) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+2}}{(2n+1)!}$
(D) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$ (E) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n)!}$

14. The Maclaurin series for e^{-2x} is

(A) $\sum_{n=0}^{\infty}(-1)^n\frac{2^n x^n}{n}$ (B) $\sum_{n=0}^{\infty}\frac{2^n x^n}{n!}$ (C) $\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{(2n)!}$
(D) $\sum_{n=0}^{\infty}(-1)^n\frac{2^n x^n}{n!}$ (E) $\sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$

15. The 2nd degree Taylor polynomial of the function $f(x) = \ln x$ at $a = 1$ is

(A) $x-1-\frac{1}{2}(x-1)^2$ (B) $x-1-(x-1)^2$ (C) $x-x^2$ (D) $x-\frac{1}{2}x^2$ (E) $1-x$

16. The 3rd degree Taylor polynomial of the function $f(x) = \sin x$ at $a = 0$ is

(A) x (B) $x - \frac{x^3}{3!}$ (C) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ (D) $1 - \frac{x^2}{2!}$ (E) $1 + x - x^3$

17. The polar equation for the curve of the Cartesian equation $x^2 + xy = 1$ is

(A) $\sin^2\theta + \sin\theta\cos\theta = 0$ (B) $r(\sin^2\theta + \sin\theta\cos\theta) = 1$ (C) $r^2(\sin^2\theta + \sin\theta\cos\theta) = 1$
(D) $r(\cos^2\theta + \sin\theta\cos\theta) = 1$ (E) $r^2(\cos^2\theta + \sin\theta\cos\theta) = 1$

18. The Certesian equation of the polar equation $r = \frac{1}{\cos\theta - \sin\theta}$ is

(A) $y = x$ (B) $y-x = 1$ (C) $x-y = 1$ (D) $x^2+y^2 = \frac{1}{x-y}$ (E) $x^2+y^2 = \frac{1}{y-x}$

19. The slope of the line tangent to the polar curve $r = \sin\theta$ at $\theta = \frac{\pi}{6}$ is

(A) $\sqrt{3}$ (B) $\frac{\sqrt{3}}{3}$ (C) $-\sqrt{3}$ (D) $-\frac{\sqrt{3}}{3}$ (E) ∞

20. The slope of the line tangent to the polar curve $r = \theta$ at $\theta = \pi$ is

(A) 0 (B) -1 (C) 1 (D) $-\pi$ (E) π

PART III. Essay Problems

1. Determine whether the sequence converges. Find the limit if it is convergent.

   (a) $a_n = \dfrac{n^2-n}{2n^2-3}$ (b) $a_n = \dfrac{\ln(2n+1)}{n}$ (c) $a_n = \dfrac{2^{2n}}{e^{n+2}}$

   (d) $a_n = \dfrac{n\cos(n)}{e^n}$ (e) $a_n = \ln(n^2+1) - \ln(2n^2-n)$ (f) $a_n = (1+2/n)^{2n}$

2. Let the sequence $\{a_n\}$ satisfy

$$a_1 = 10, \quad a_{n+1} = \sqrt{6+a_n}, \qquad n \geq 1.$$

   (a) (optional) Show that $a_n > 0$ and $a_n \geq a_{n+1}$ for $n \geq 1$. (Hence $\{a_n\}$ has a limit since it is decreasing and bounded below.)

   (b) Find the limit.

3. Determine whether the series is convergent, or divergent, or absolutely convergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^n n}{2^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n \cos(n\pi)}{e^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(\ln n)^2}{n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2+e^{-n}}{2n^2+1}$ (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n!}{1\cdot 3\cdot 5\cdots(2n+1)}$ (f) $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)2^n}{n^3(-3)^n}$

4. Use the partial sum s_5 to estimate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$, and estimate the error in using s_5 as an approximation to the sum of the series.

5. Use the partial sum s_5 to estimate the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$, and estimate the error in using s_5 as an approximation to the sum of the series.

6. Find the radius of convergence and interval of convergence of the power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(1-x)^n}{2n+1}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{n3^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^2 3^n}$ (d) $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}(x-3)^n}{n!}$

7. Find the Taylor series of the function at $a = 0$.

   (a) $\dfrac{1}{1-x^2}$ (b) $\dfrac{1}{(1-x)^2}$ (c) $\ln(1+x)$ (d) $\sin x^2$ (e) $x^2 e^{-x}$.

8. Find the Maclaurin series of e^{-x^2} and approximate $\displaystyle\int_0^{0.1} e^{-x^2}\,dx$ correct to within an error of 10^{-5}.

9. Find the Taylor polynomial $T_3(x)$ of the function $e^{\sin x}$ at $a = 0$.

10. Let $T_3(x)$ be the degree 3 Taylor polynomial of e^{x^2} at $a = 0$. Using the Taylor inequality to find a bound for
$$|R_3(x)| = \left|e^{x^2} - T_3(x)\right| \qquad \text{for } x \in [0,\ 0.1].$$

11. Find the Cartesian coordinates of the point given in polar coordinates.
$$\text{(a)}\ \left(2, \frac{\pi}{6}\right) \quad \text{(b)}\ \left(4, \frac{3\pi}{4}\right) \quad \text{(c)}\ \left(0, \frac{\pi}{5}\right) \quad \text{(d)}\ \left(5, -\frac{\pi}{2}\right) \quad \text{(e)}\ \left(3, -\frac{\pi}{3}\right)$$

12. Find the polar coordinates (r, θ) with $r \geq 0$ and $0 \leq \theta < 2\pi$ of the point given in the Certesian coordinates.
$$\text{(a)}\ (1,0) \quad \text{(b)}\ \left(3, \sqrt{3}\right) \quad \text{(c)}\ (-2,2) \quad \text{(d)}\ \left(-1, \sqrt{3}\right) \quad \text{(e)}\ (0,-2)$$

13. Find the slope of the line tangent to the polar curve at the point specified by the value of θ.
$$\text{(a)}\ r = \sin\theta + \cos\theta,\quad \theta = \frac{\pi}{6} \qquad \text{(b)}\ r = 1 + \theta^2,\quad \theta = \frac{\pi}{2} \qquad \text{(c)}\ r = 4\cos 3\theta,\quad \theta = \frac{\pi}{6}$$

14. Find the points on the given curve where the tangent line is horizontal or vertical.
$$\text{(a)}\ r = 1 + \cos\theta, \qquad \text{(b)}\ r^2 = \cos 2\theta$$

15. Find the area of the region that is bounded by the given curve and lies in the specified sector.
$$\text{(a)}\ r = 1 - \cos\theta,\quad 0 \leq \theta \leq \frac{\pi}{2} \qquad \text{(b)}\ r = 3 - \theta,\quad 0 \leq \theta \leq 3$$

16. Find the area of the region enclosed by one loop of the curve.
$$\text{(a)}\ r = 2\sin\theta \qquad \text{(b)}\ r^2 = \cos 2\theta$$

17. Find the area of the region that lies inside $r = \sqrt{2}\cos\theta$ and outside $r = 1$.

18. Find the exact length of the polar curve.
$$\text{(a)}\ r = \theta^2,\quad 0 \leq \theta \leq \pi \qquad \text{(b)}\ r = 2\cos\theta,\quad 0 \leq \theta \leq \frac{\pi}{2}$$