Math 390 Homework 7

Due Friday, March 25

Solutions should be written LaTeX or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

1. Determine the number of labeled spanning trees of the Petersen graph. (Hint: Use the Matrix Tree Theorem, Theorem 10.3 in Edition 4 and Theorem 3.5 in Edition 5. I recommend using a computer to compute the determinant.)

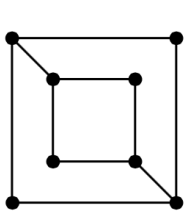
2. (a) Read the definition of the *chromatic number* (5th edition p. 101, 4th edition p. 81). The Four Color Theorem (Thm 5.5/Thm 17.5) says that every simple planar graph is 4-colorable. Give an example of a planar graph whose chromatic number is 4.

   (b) A graph is *outerplanar* if it has a planar drawing with all the vertices lying on the outside face. Use the Four Color Theorem to show that the chromatic number of an outerplanar graph is at most three. (Hint: Add a vertex to the outside face with edges to all the other vertices.)

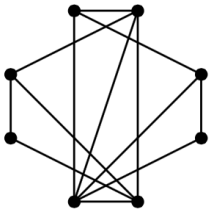
3. (a) Use Kuratowski's Theorem to show that $K_{2,3}$ and K_4 are not outerplanar.

   (b) Use Kuratowski's Theorem to show that a graph is outerplanar if and only if it does not contain a subgraph homeomorphic to $K_{2,3}$ or K_4.

   (c) For each of the following graphs, determine if the graph is outerplanar. If the graph is outerplanar, show a planar drawing with all the vertices lying on the outside face. If the graph is not outerplanar, show that it contains a subgraph homeomorphic to $K_{2,3}$ or K_4.



Graph 1



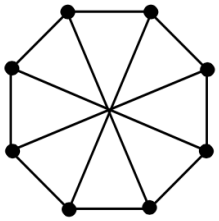
Graph 2

4. A graph has genus g if the graph can be drawn on a g-holed-torus without edge crossings but cannot be drawn on a $(g-1)$-holed-torus without edge crossings. The genus of a graph G is denoted $g(G)$.

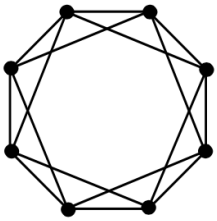
   (a) Determine the genus of $K_{3,5}$.

   (b) Show that $g(K_{r,s}) \geqslant \frac{1}{4}(r-2)(s-2)$. (*Hint:* Recall that for graphs drawn on a g-holed-torus, $n - m + f = 2 - 2g$.)

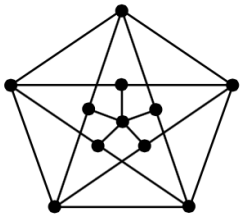
5. For each of the following graphs, determine the chromatic number of the graph, and show a coloring that uses the minimum number of colors. (You do not need to prove that your answer is a minimum.)



Graph 1



Graph 2



Graph 3