MATH 243 **EXAM 2** **02/29/2016**

Name: ____

Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the Sometimes/Always/Never questions. For all other questions, please circle your answers and justify your work for full credit. There are 12 questions for a total of 100 points.

Sometimes/Always/Never: Read the statement and decide whether the statement is sometimes true, always true, or never true. No work is necessary.

_______ 1. (5 points) If a function $f(x, y)$ is defined at (a, b) and the functions f_x and f_y are differentiable, then

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial f}{\partial x} \cdot \frac{\partial f}{\partial y}.$$

A. Sometimes B. Always C. Never

_______ 2. (5 points) The function $f(x, y) = xe^y - e^3x^2$ has an absolute maximum value on the largest possible domain. (Hint: Don't use optimization.)

A. Sometimes B. Always C. Never

_______ 3. (5 points) For a differentiable function $f(x, y)$, $\nabla f(a, b)$ gives the direction of the fastest increase of f at (a, b).

A. Sometimes B. Always C. Never

_______ 4. (5 points) The limit exists at $(0, 0)$ of a function s such that

$$\lim_{(x,x)\to(0,0)} s(x, x) = 4 \quad \text{and} \quad \lim_{(x,2x)\to(0,0)} s(x, 2x) = 7.$$

A. Sometimes B. Always C. Never

_______ 5. (5 points) Given the formula $\dfrac{1}{R} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$, we have that $\dfrac{\partial R}{\partial R_1} = 1$.

A. Sometimes B. Always C. Never

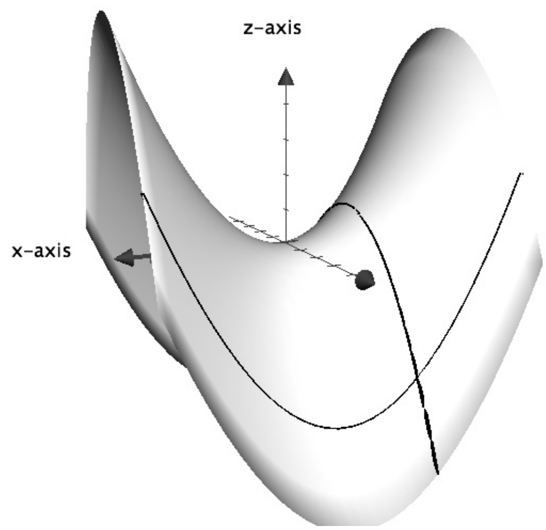

Figure 1: $f(x,y)$

Multiple Choice. Circle only one answer, no work is necessary.

_______ 6. (5 points) Given the graph above, let $(a, b, f(a, b))$ be the point of intersection of the two grid lines. Which of the following is true?

A. $f_x(a,b) < 0$ and $f_{xx}(a,b) > 0$

B. $f_y(a,b) < 0$ and $f_{yy}(a,b) > 0$

C. $f_x(a,b) > 0$ and $f_{yy}(a,b) < 0$

D. $f_x(a,b) > 0$ and $f_y(a,b) > 0$

E. $f_{xx}(a,b) > 0$ and $f_{yy}(a,b) > 0$

_______ 7. (5 points) To Find the directional derivative of $f(x, y)$ at $(1, 2)$ in the direction $\mathbf{v} = \langle -1, 2\rangle$, one would calculate:

A. $\langle -1, 2\rangle \cdot \langle f_x(1,2), f_y(1,2)\rangle$

B. $\frac{1}{\sqrt{5}}\langle 1, 2\rangle \cdot \langle f_x(-1,2), f_y(-1,2)\rangle$

C. $\frac{1}{\sqrt{5}}\langle -1, 2\rangle \cdot \langle f_x(1,2), f_y(1,2)\rangle$

D. $\langle -1, 2\rangle \cdot \langle 1, 2\rangle$

E. $\frac{1}{\sqrt{5}}\langle f_x(1,2), f_y(1,2)\rangle$

_______ 8. (5 points) Let $f(x, y)$ be a function such that $f_x(a, b) = 0$, $f_y(a, b) = 0$, $f_{xx}(a, b) = 3$, $f_{xy}(a, b) = 4$, and $f_{yy}(a, b) = 5$. Which of the following is known to be true?

A. $f(x,y)$ has a local maximum at (a,b).

B. $f(x,y)$ has a local minimum at (a,b).

C. $f(x,y)$ has an absolute maximum at (a,b).

D. $f(x,y)$ has an absolute minimum at (a,b).

E. $f(x,y)$ has a saddle point at (a,b).

Short Answer: Justify your answers for full credit.

9. (a) (3 points) What function would you optimize in order to find the shortest distance between the point $(1, 0, -2)$ and the plane $x + 2y + z = 4$?

   (b) (10 points) Let $f(x, y)$ be the name of the function you wrote down in part (a). Assume that the first order partials are as follows.

$$f_x = 4x + 4y - 14$$
$$f_y = 4x + 10y - 24$$

   Use this information to find and classify the critical point(s).

(c) (2 points) Use parts (a) and (b) to find the shortest distance between the point $(1, 0, -2)$ and the plane $x + 2y + z = 4$?

10. (a) (5 points) Show that the function $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$

(b) (5 points) If f is differentiable at $(1, 0)$, use the linearization of f at $(1, 0)$ to approximate $f(1.1, -0.1)$.

(c) (5 points) Why did we need to first determine if the function was differentiable at the point $(1, 0)$?

11. (15 points) There are several proposed formulas to approximate the surface area of the human body. One model uses the formula

$$A(h, w) = 0.0072h^{0.725}w^{0.425},$$

where A is the surface area in square meters, h is the height in centimeters, and w is the weight in kilograms.

Since a person's height h and weight w change over time, h and w are functions of time t. Let us think about what is happening to a child whose height is 60 centimeters and weight is 9 kilograms. Suppose, furthemore, that h is increasing at an instantaneous rate of 20 centimeters per year and w is increasing at an instantaneous rate of 5 kg per year.

Determine the instantaneous rate at which the child's surface area is changing.

12. (15 points) Follow the parts below to find the points on $x^2 + 4y^2 - z^2 = 4$ whose tangent plane is parallel to the plane $2x + 2y + z = 5$.

(a) Finding the tangent plane of points on the surface $x^2 + 4y^2 - z^2 = 4$ can be difficult as we would need to solve for z. However, life is easier if we look at the function

$$G(x, y, z) = x^2 + 4y^2 - z^2.$$

The reason we are looking at G is because we can view our surface as the level curve (or level surface)

$$G(x, y, z) = 4.$$

Find ∇G.

(b) Recall that the equation of a plane is

$$A(x - a) + B(y - b) + C(z - c) = 0.$$

It turns out that $\langle A, B, C\rangle = \nabla G(a, b, c)$. Why do you think this is true?

(c) Use parts (a) and (b) to find the points on the surface $x^2 + 4y^2 - z^2 = 4$ where the tangent plane is parallel to the plane $2x + 2y + z = 5$. (Hint: Parallel vectors are scalar multiples of each other.)