Math 243 **Weekly Assignment 7** **DUE 05/04/2016**

Name: ______________________________

Answer the questions on the worksheet and not on a separate sheet of paper. Please circle your answers and show your work for full credit.

Fill In the Blank. *Read Section 15.1 in Strang's book to answer these questions.*

1. A vector field assigns a ____________ to each point $\{x, y\}$ or $\{x, y, z\}$. In two dimensions $\mathbf{F}(x, y) =$ ____________$\mathbf{i}$ + ____________$\mathbf{j}$. An example is the position field $\mathbf{R} =$ ____________. Its magnitude is $|\mathbf{R}| =$ ____________ and its direction is ____________. IT is the gradient field for $f =$ ____________. The level curves are ____________, and the are ____________ to the vectors $\mathbf{R}$.

2. Reversing this picture, the spin field is $\mathbf{S} =$ ____________. Its magnitude is $|\mathbf{S}| =$ ____________ and its direction is ____________. It is not a gradient field, because no function has $\partial f/\partial x =$ ____________ and $\partial f/\partial y =$ ____________. $\mathbf{S}$ is the velocity field for flow going ____________. The streamlines or ____________ lines or integral ____________ are ____________.

3. A gravity field from the origin is proportional to $\mathbf{F} =$ ____________ which has $|\mathbf{F}| =$ ____________. This is Newton's ____________ square law. It is a gradient field, with potential $f =$ ____________. The equipotential curves $f(x, y) = c$ are ____________. They are ____________ to the field lines which are ____________. This illustrates that the ____________ of a function $f(x, y)$ is ____________ to its level curves.

True or False. *No work is necessary.*

4. The constant field $\mathbf{i} + 2\mathbf{k}$ is a gradient field

5. *grad* f is a scalar field.

6. *curl* $\mathbf{F}$ is a vector field.

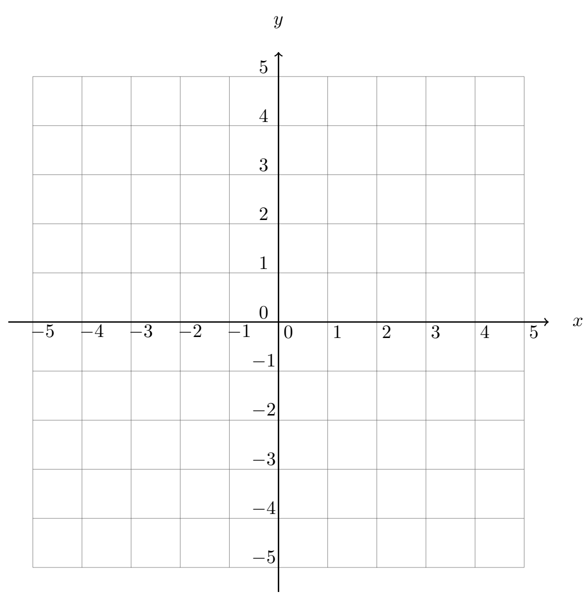
7. $\nabla \times (\nabla f) = \mathbf{0}$ for all function $f : \mathbb{R}^3 \to \mathbb{R}$.

8. if $\mathbf{F}$ and $\mathbf{G}$ are gradient fields, then $\mathbf{F} \times \mathbf{G}$ is incompressible.

Short Answer.

9. Write down the the vector field $\mathbf{F} = M\mathbf{i} + N\mathbf{j}$ that points radially away from origin with magnitude 5.

10. Sketch the vector field $\mathbf{F} = x^2\mathbf{i} + x\mathbf{j}$ on $\mathbb{R}^2$.



11. Calculate the flow line $\mathbf{x}(t)$ of the vector field $\mathbf{F}(x, y, z) = 2\mathbf{i} + 3y\mathbf{j} + z^3\mathbf{k}$ that passes through $\mathbf{x}(0) = (3, 5, 7)$.

12. Show the vector field $\mathbf{F} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ is a gradient field and describe the equipotential surfaces of $\mathbf{F}$ in words.

13. The **circulation** of a vector field on a closed curve is given by $\int_C \mathbf{F} \cdot d\mathbf{s}$. Find the circulation of $\mathbf{F} = (x^2 - y)\mathbf{i} + (xy + x)\mathbf{j}$ along the circle $x^2 + y^2 = 16$, oriented counterclockwise.

14. Let C be a level curve of the function $f(x, y)$. Show that $\int_C \nabla f \cdot d\mathbf{s} = 0$.

15. Let $\mathbf{B}$ be a uniform (constant) vector field and $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$. If we let

$$\mathbf{A}(\mathbf{r}) = -\frac{1}{2}(\mathbf{r} \times \mathbf{B}),$$

show that $\nabla \cdot \mathbf{A} = 0$ and $\nabla \times \mathbf{A} = \mathbf{B}$. (You can thank Dr. Matt Wright for this problem.)