MATH 243 **EXAM 1** **02/24/2016**

Name: ____________________

> Answer the questions on the exam and not on a separate sheet of paper. No work is necessary for the Sometimes/Always/Never questions. For all other questions, please circle your answers and justify your work for full credit. There are 13 questions for a total of 100 points.

Sometimes/Always/Never: Read the statement and decide whether the statement is sometimes true, always true, or never true. No work is necessary.

_______ 1. (5 points) The two sets of parametric equations

$$\begin{aligned} x &= 3t-1 \\ y &= -t+2 \\ z &= 2t+5 \end{aligned} \quad \text{and} \quad \begin{aligned} x &= -6t-7 \\ y &= 2t+2 \\ z &= -4t+1 \end{aligned}$$

both represent the same line.

A. Sometimes B. Always C. Never

_______ 2. (5 points) If $\mathbf{a}, \mathbf{u} \in \mathbb{R}^n$ and $||\mathbf{u}|| = 1$, then $\text{proj}_{\mathbf{u}}\mathbf{a} = (\mathbf{a}\cdot\mathbf{u})\mathbf{u}$.

A. Sometimes B. Always C. Never

_______ 3. (5 points) $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{c} - (\mathbf{a}\times\mathbf{c})\cdot\mathbf{b}$ is a scalar.

A. Sometimes B. Always C. Never

_______ 4. (5 points) Let a be a constant. The following planes are parallel:

$$2x + 3y - z = 4;$$
$$-10x + 3ay - az = 4.$$

A. Sometimes B. Always C. Never

_______ 5. (5 points) The parametric equation $\mathbf{r}(t) = \langle 3t^5, 3t^5, -t^5\rangle$ is the equation of a line in $\mathbb{R}^3$.

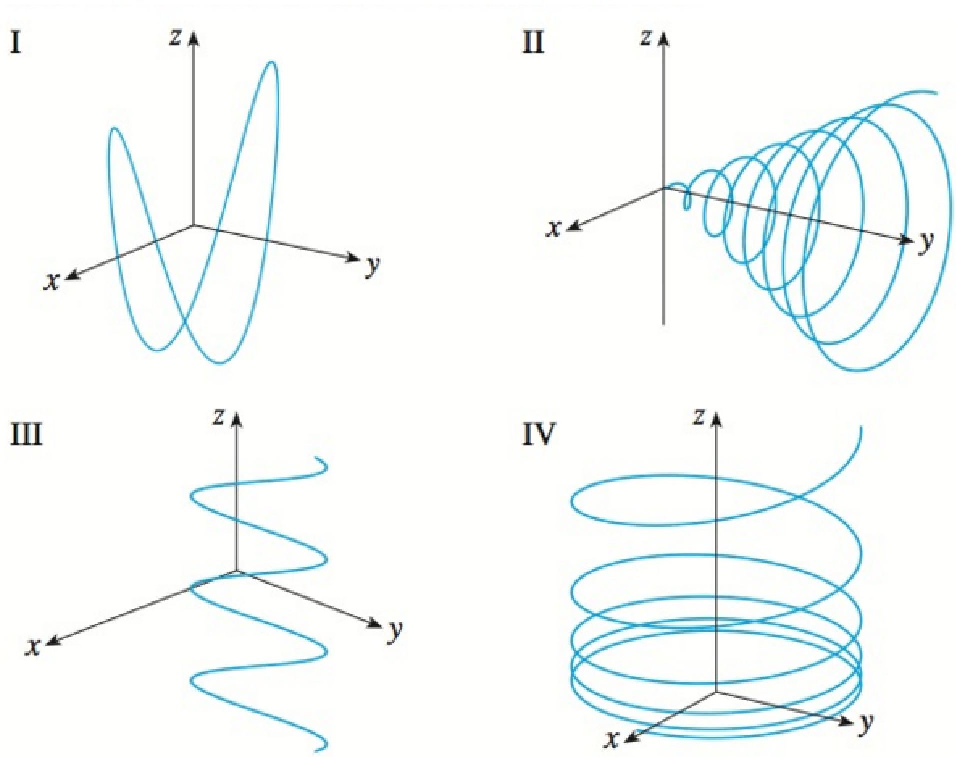
A. Sometimes B. Always C. Never

_______ 6. (5 points) Let $\mathbf{r}(t)$ be a vector equation in $\mathbb{R}^3$. If $|\mathbf{r}(t)| = 1$ for all t, then $\mathbf{r}'$ is orthogonal to $\mathbf{r}$.

A. Sometimes B. Always C. Never

Matching. For each question match **exactly one** item one group with **exactly one** item from the other group.

7. (5 points) Match the parametric equations with the graphs.



______ (a) $x = t\cos t,\ y = t,\ z = t\sin t,\ t \geqslant 0$

______ (b) $x = \cos t,\ y = \sin t,\ z = \cos 2t$

______ (c) $x = \cos 8t,\ y = \sin 8t,\ z = e^{0.8t},\ t \geqslant 0$

______ (d) $x = \cos^2 t,\ y = \sin^2 t,\ z = t$

Short Answer: Show your work for full credit.

8. (10 points) Give a parametric equation for the line through $(1, 4, 5)$ and $(2, 4, -1)$.

9. (10 points) Give an equation for the plane that passes through the three points $(0, 2, 1)$, $(7, -1, 5)$, and $(-1, 3, 0)$.

10. (15 points) Find an equation for the tangent plane to the surface given by $2xz+yz-xy+10=0$ at $(1,-5,5)$. (Hint: Don't be afraid to fix a variable and parameterize like we did in the homework.)

11. (10 points) Find $f'(2)$ where $f(t) = \mathbf{u}(t) \cdot \mathbf{v}(t)$, $\mathbf{u}(2) = \langle 1, 2, -1 \rangle$, $\mathbf{u}'(2) = \langle 3, 0, 4 \rangle$, and $\mathbf{v}(t) = \langle t, t^2, t^3 \rangle$.

12. (5 points) Show that the curve with parametric equations

$$\begin{aligned} x &= t \cos t; \\ y &= t \sin t; \\ z &= t; \end{aligned}$$

lies on the cone $z^2 = x^2 + y^2$. (Hint: If your solution needs more than the space provided, then you are doing it wrong.)

13. (15 points) Suppose you start at the point $(0, 0, 3)$ and move 5 units along the curve

$$\begin{aligned} x &= 3\sin t; \\ y &= 4t; \\ z &= 3\cos t; \end{aligned}$$

in the positive direction. Where are you now?