25 Problems: Least Squares

1. Let $L : U \to V$ be a linear transformation. Suppose $v \in L(U)$ and you have found a vector u_{ps} that obeys $L(u_{\text{ps}}) = v$.

   Explain why you need to compute $\ker L$ to describe the solution space of the linear system $L(u) = v$.

Hint for Problem 1

2. Suppose that M is an $m \times n$ matrix with trivial kernel. Show that for any vectors u and v in $\mathbb{R}^m$:

   - $u^T M^T M v = v^T M^T M u$.
   - $v^T M^T M v \geq 0$. In case you are concerned (you don't need to be) and for future reference, the notation $v \geq 0$ means each entry $v^i \geq 0$.
   - If $v^T M^T M v = 0$, then $v = 0$.

   (Hint: Think about the dot product in $\mathbb{R}^n$.)

Hint for Problem 2