15 Problems: Subspaces and Spanning Sets

1. (Subspace Theorem) Suppose that V is a vector space and that $U \subset V$ is a subset of V. Show that

$$\mu u_1 + \nu u_2 \in U \text{ for all } u_1, u_2 \in U, \mu, \nu \in \mathbb{R}$$

implies that U is a subspace of V. (In other words, check all the vector space requirements for U.)

2. Let $P_3^{\mathbb{R}}$ be the vector space of polynomials of degree 3 or less in the variable x. Check whether

$$x - x^3 \in \operatorname{span}\{x^2, 2x + x^2, x + x^3\}$$

3. Let U and W be subspaces of V. Are:

   (a) $U \cup W$

   (b) $U \cap W$

   also subspaces? Explain why or why not. Draw examples in $\mathbb{R}^3$.

Hint