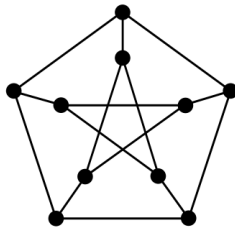


Math 390 Homework 2

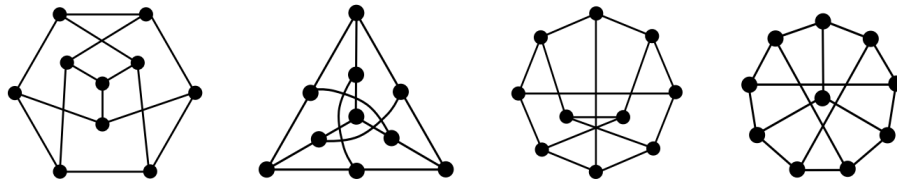
Due Wednesday, February 10

Solutions should be written \LaTeX or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

1. The following graph is called the **Petersen graph**:

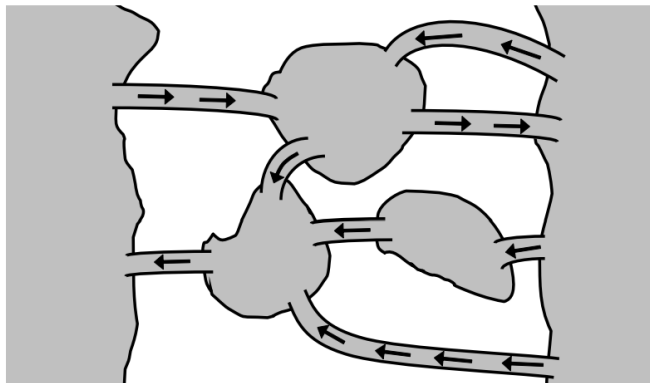


Prove (by labeling the vertices) that the graphs below are all isomorphic to the Petersen graph.



2. The **degree sequence** of a graph is the list of vertex degrees including repetitions. For each of the following sequences, find a simple graph (no loops or double edges) with the given degree sequence or prove that no such graph exists.
 - (a) $(1, 2, 2, 3, 4, 5)$
 - (b) $(1, 1, 2, 2, 3, 3)$
 - (c) $(2, 3, 3, 4, 5, 5)$
 - (d) $(3, 3, 5, 5, 5, 5)$
3. Suppose that the edges of the graph K_{17} are colored with three colors (red, blue, and green). Show that the coloring must contain a red triangle, a blue triangle, or a green triangle.
4. A **saturated hydrocarbon** is a molecule formed from hydrogen and carbon atoms, where each carbon atom is bonded to four other atoms, each hydrogen atom is bonded to one other atom, and no chain of carbon atoms can form a cycle. Consider a saturated hydrocarbon with k carbon atoms. Prove by induction on k that the saturated hydrocarbon has exactly $2k + 2$ hydrogen atoms.

5. (a) The following picture shows three islands with land on the left and the right. The islands and the land are connected by several one-way bridges. Cars can only travel on the bridges in the indicated direction. Is it possible to start on the land on the left, drive across each bridge exactly once, and end back on the left side? Prove your answer. (Hint: Look at Theorem 23.1 in the 4th Edition or Theorem 2.13 in the 5th Edition of the textbook.)



- (b) Now, consider the following picture with six islands and land on the left and the right. Some of these bridges are one-way, allowing cars to only travel in the indicated direction. The bridges without an indicated direction are two-way bridges, allowing cars to travel over the bridge in either direction. Is it possible to start on the land on the left, drive across each bridge exactly once, and end back on the left side? Prove your answer.

