

# Math 390 Homework 10

Due Friday, May 13

Solutions should be written L<sup>A</sup>T<sub>E</sub>X or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

1. Suppose that there exists a polynomial-time algorithm to determine whether a graph is 4-colorable. Show that there exists a polynomial time algorithm to determine whether a graph is 3-colorable. (This shows that if 3-colorability is NP-complete, then 4-colorability is NP-complete also.)
2. Prove that  $k^4 - 4k^3 + 3k^2$  is not a chromatic polynomial.
3. There are six graphs with chromatic polynomial  $k^6 - 5k^5 + 10k^4 - 10k^3 + 5k^2 - k$ . Find all six graphs.
4. Let  $T$  be a tree. Prove that  $T$  has a complete matching if and only if for every vertex  $v$ , exactly one component of  $T - v$  has an odd number of vertices. (Hint: For the backwards direction, use induction on the number of vertices of  $T$ . Find a leaf with a neighbor of degree 2 and delete them both.)