

Math 351 Homework 1

Due Friday, September 9 at 5pm

Solutions should be written L^AT_EX or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

Many proofs use induction, so it will be helpful for you to review induction from your discrete/proofs course. Problem 1 below is intended to help you review proofs using induction.

1. Use induction to prove that the following formula holds for all $n \in \mathbb{Z}_{n>1}$:

$$2 + 4 + 8 + 16 + 32 + \dots + 2^n = 2^{n+1} - 2.$$

2. Show that the set $\mathbb{Z}[\sqrt{-5}] = \{a + b\sqrt{-5} \mid a, b \in \mathbb{Z}\}$ is closed under addition and multiplication.
3. Compute the greatest common divisor $\gcd(455, 1235)$ by hand.
4. Let a, b, c, n be integers. Prove that
 - (a) if $a \mid n$ and $b \mid n$ with $\gcd(a, b) = 1$, then $ab \mid n$.
 - (b) if $a \mid bc$ and $\gcd(a, b) = 1$, then $a \mid c$.
5. In each of the following, apply the division algorithm to find q and r such that $a = bq + r$ and $0 \leq r < |b|$:

$$a = 300, b = 17, \quad a = 729, b = 31, \quad a = 300, b = -17, \quad a = 389, b = 4.$$

6. Prove that a positive integer n is prime if and only if n is not divisible by any prime p with $1 < p \leq \sqrt{n}$.