13 Problems: Elementary Matrices and Determinants II

1. Let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $N = \begin{pmatrix} x & y \\ z & w \end{pmatrix}$. Compute the following:

   (a) $\det M$.

   (b) $\det N$.

   (c) $\det(MN)$.

   (d) $\det M \det N$.

   (e) $\det(M^{-1})$ assuming $ad - bc \neq 0$.

   (f) $\det(M^T)$

   (g) $\det(M + N) - (\det M + \det N)$. Is the determinant a linear transformation from square matrices to real numbers? Explain.

2. Suppose $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is invertible. Write M as a product of elementary row matrices times $\mathrm{RREF}(M)$.

3. Find the inverses of each of the elementary matrices, $E^i_j, R^i(\lambda), S^i_j(\lambda)$. Make sure to show that the elementary matrix times its inverse is actually the identity.