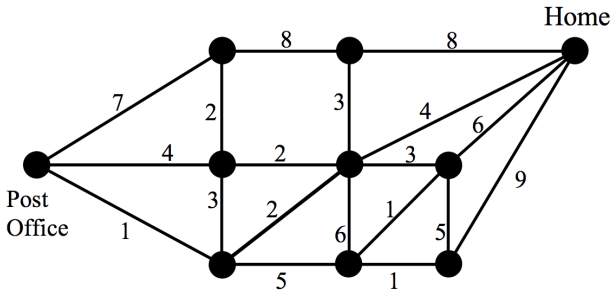
Math 390 Homework 4

Due Wednesday, February 24

Solutions should be written LaTeX or Markdown and converted to a PDF. You are encouraged to work with others on the assignment, but you should write up your own solutions independently. This means no copy pasting. You should reference all of your sources, including your collaborators.

1. Every morning a postman takes the bus to the post office. From there, he chooses a route to reach home as quickly as possible (**NOT** ending at the post office). His route must include all of the streets in the map below. The map gives the number of minutes required to walk each block. Find the optimal route for the postman to travel (give the route and the total time that the route takes), and explain why the route is optimal.



2. (Exercise 3.8/1.51) The **complement** of a simple graph G is a simple graph $\overline{G}$ with vertex set $V(G)$ where two vertices in $\overline{G}$ are adjacent if and only if they are not adjacent in G. A simple graph that is isomorphic to its complement is called **self-complementary**.
   (a) Prove that, if G is self-complementary, then G has $4k$ or $4k+1$ vertices, where k is an integer.
   (b) Find all self-complementary graphs with 4 and 5 vertices.
   (c) Find a self-complementary graph with 8 vertices.
3. First look up the definition of connectivity in your book. Now construct a 3-regular simple graph G with connectivity $\kappa(G) = 1$.
4. (Exercise 7.7/2.34)
   (a) Let G be a simple graph with n vertices and $\binom{n-1}{2} + 2$ edges. Use Ore's Theorem to prove that G is Hamiltonian.
   (b) Find a simple non-Hamiltonian graph with n vertices and $\binom{n-1}{2} + 1$ edges.
5. (Exercise 10.3/3.17)
   (a) Find the number of labeled trees on n vertices in which vertex 1 is a leaf.
   (b) Prove that if n is large, then the probability that a given vertex of a tree with n vertices is a leaf is approximately e^{-1}.