A REPORT ON THE MODELING OF A TWO TANK FLUID SYSTEM

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ABSTRACT

This paper will cover the approach and techniques used to model the two tank fluid system and how the dynamic height of each tank was determined. The orifices were modeled 3 different ways and it was found that the model chosen impacted the results appreciably.

INTRODUCTION

This system consisted of two 8 cm diameter cylinders stacked on top of each other with an orifice drilled into the bottom of each cylinder. One circumstance of note is that both cylinders were emptying into the open air, and the resulting stream drained into the tank below. A waste tank was at the bottom of the system with a bilge pump that returned the water to top tank.

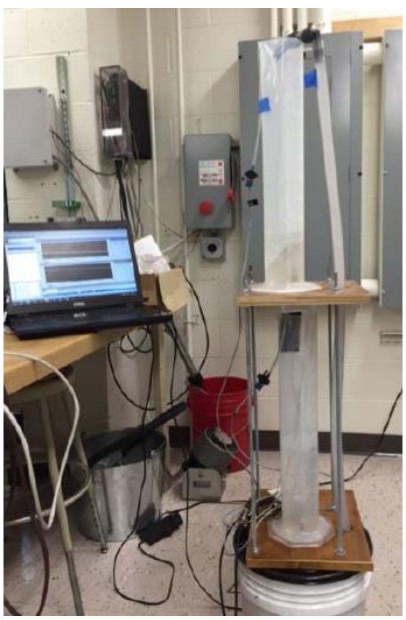

FIGURE 1. TWO TANK FLUID SYSTEM

DETERMINING THE INPUT FLOW

In order to properly model the system, the relationship between voltage and flow rate created by the bilge pump must first be determined. This was accomplished by holding the pump at a constant voltage and recording the change in height in the top tank, which is plugged so that no water could drain out. This was repeated with 4 additional voltage values. It was discovered that a constant voltage of 6.9V must be given to the pump in order to negate head losses. Henceforth, and voltage values given as an input to the system will be the amount of volts required above the 6.9 value. Using the 5 height data sets and their respective voltages, the static gain required to convert voltage to flowrate could be determined.

By creating a control volume around the top tank:

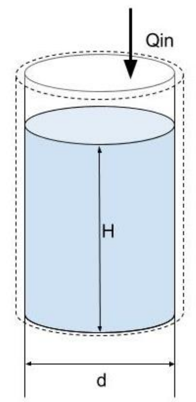

FIGURE 2. CV FOR VOLTAGE GAIN CALCS

There is only one input to the control volume. Using the continuity equation:

$$\frac{d}{dt}M = \dot{m}_{in} - \dot{m}_{out} \qquad (1)$$

Simplifying for constant density and cross sectional area:

$$A\dot{H} = \dot{q}_{in} - \dot{q}_{out} \qquad (2)$$

Where A is the cross sectional area of the cylinder

Setting the input flow to the Voltage times a constant:

$$A\dot{H} = Z * V_{cmd} \qquad (3)$$

Using this equation, we can use the slopes from the pump calibration data to determine the gain Z. Note that equation 3 is of the form y = mx + b. By subtracting 6.9 from the voltage we effectively set the b term to zero. Plotting each height slope against its respective voltage divided by the area yields a linear trend, with the slope of the trend line being the static gain Z. For this experiment, it was determined that the static gain, Z, was 3.4575e-5.

ESTIMATION OF THE MODELING PAREMETERS

The first modeling approach for the orifice was assuming laminar flow. This means that the flow out of the orifice in the bottom of the cylinder is defined as:

$$\dot{q} = \frac{1}{R}\Delta P \qquad (4)$$

Where R is the resistance parameter of the orifice and ΔP represents the pressure drop across the orifice. The control volume for the initial value system:

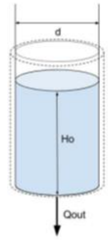

FIGURE 3. CV FOR RESISTANCE CALCS

Simplifying pressure:

$$\dot{q} = \frac{1}{R}(P + P_{atm} - P_{atm}) = \frac{1}{R}(P) \qquad (5)$$

Where the pressure in the tank can be defined using density and height of the fluid:

$$\dot{q} = \frac{1}{R}(\rho g H) \qquad (6)$$

Simplifying for constant cross sectional area:

$$\dot{H} = \frac{\rho g}{RA}(H) \qquad (7)$$

The tanks were filled to an initial height and allowed to drain out of their orifice. This initial value problem can be solved using the time constant method:

$$\dot{H} = \frac{1}{\tau}(H) \qquad (8)$$

Where τ is the time constant of the system or the time it takes for the tank to drain by 63%. Using the top and bottom tank draining data, the time constants for each tank could be experimentally determined. τ for the top tank was found to be 11.2 seconds and τ for the bottom tank was found to be 15.3 seconds. Since equations 7 and 8 are of the same form, the resistance parameters can be found using the time constant and known values.

$$R = \frac{\tau \rho g}{A} \qquad (9)$$

The second modeling method for the orifice was using Bernoulli's equation for flow:

$$\dot{q} = Cd * A_0\sqrt{2\Delta P/\rho} \qquad (10)$$

Where Cd is the discharge coefficient of the orifice and A_0 is the area of the orifice profile.

Since the tanks are emptying into open air, the equation can be further simplified by the same method as equations 5 and 6:

$$\dot{q} = Cd * A_0\sqrt{2gH} \qquad (11)$$

For a cylinder with a constant cross sectional area:

$$A\dot{H} = Cd * A_0\sqrt{2gH} \qquad (12)$$

Using the same control volume depicted in figure 3, the model can be simulated for a range of Cd values. Cd is typically in the range of 0.4 to 1.2. Plotting the results of each Cd value simulation against the drain data, once can visually determine which Cd value yields the best results.

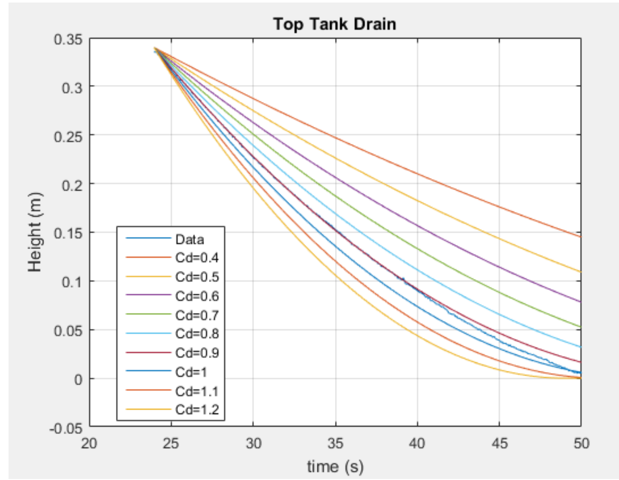

FIGURE 4. CD APRROXIMATIONS – TOP TANK

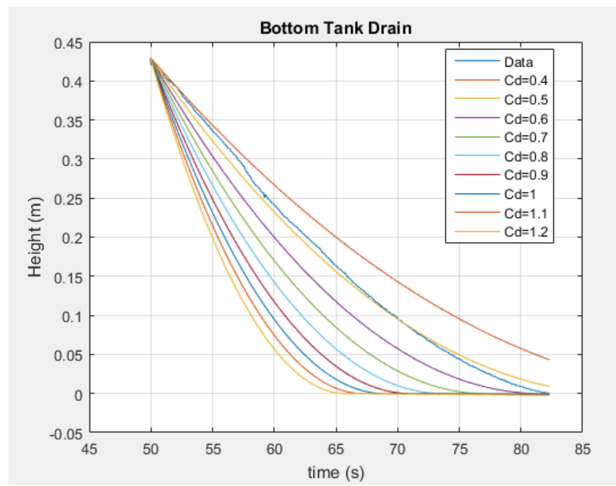

FIGURE 5. CD APRROXIMATIONS – BOTTOM TANK

It can be seen that for the top tank, a Cd value of 0.9 and a value of 0.5 for the bottom tank yield the best results to the tank drain data.

The third and final model was an empirical model for flow:

$$\dot{q} = k(\Delta P)^m \qquad (13)$$

Where k and m are empirically determined coefficients. Equation 13 can be simplified using the same method as equations 6 and 7 since the cylinder has a constant cross sectional area, the water has constant density and the tank drains into open air.

$$A\dot{H} = k(\rho g H)^m \qquad (14)$$

The coefficients of this system can be determined using the same control volume depicted in figure 3 as well as the draining data for the tanks filled to an initial height. Using matlab's fminsearch function, k and m can be approximated for each system. For the top and bottom tanks, k and m wear similar. k was found to be 2.23e-06 and m was found to 0.4773. $\dot{H}$

MODELING OF THE SYSTEM

For the first model, the control volume for the combined system is:

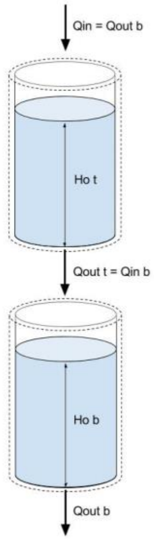

FIGURE 6. CONTROL VOLUMES FOR TANK SYSTEM

This system can be defined by

$$\begin{bmatrix} H_t \\ H_b \end{bmatrix}^{\cdot} = \begin{bmatrix} -\rho g/R_t A & 0 \\ \rho g/R_t A & \rho g/R_b A \end{bmatrix} \begin{bmatrix} H_t \\ H_b \end{bmatrix} + \begin{bmatrix} Z/A \\ 0 \end{bmatrix} V_{cmd} \qquad (15)$$

This system can be solved using ode45 with initial conditions a time span and the input Voltage. The resulting simulation is:

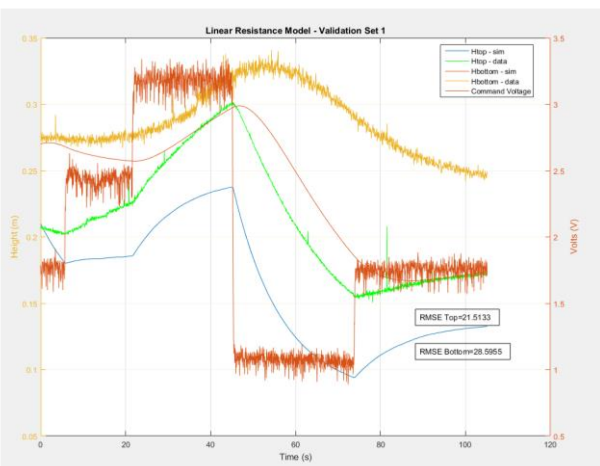

FIGURE 7. SIMULATION – 1ST MODEL

Using the flow defined by Bernoulli relationship, with the same control volumes as the first combined tank model, the system can be defined using equation 2 as:

$$\dot{H}_t = {}^{Z*Vcmd}/_{A} - Cd_t * {}^{A_{0t}}/_{A}\sqrt{2gH_t}$$

$$\dot{H}_b = Cd_t * {}^{A_{0t}}/_{A}\sqrt{2gH_t} - Cd_b * {}^{A_{0b}}/_{A}\sqrt{2gH_b} \qquad (16)$$

This system can be solved using ode45 with initial conditions a time span and the input Voltage. The resulting simulation is:

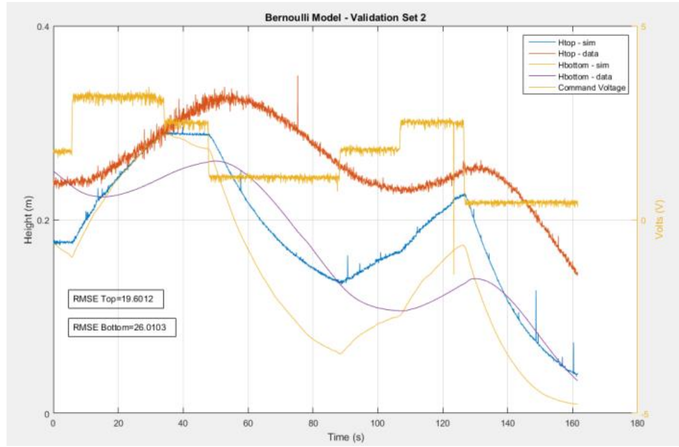

FIGURE 8. SIMULATION – 2ND MODEL

The Third empirical model can be modeled using the same control volume

$$\dot{H}_t = {}^{Z*Vcmd}/_{A} - {}^{k}/_{A}(\rho g H_t)^m$$
$$\dot{H}_b = {}^{k}/_{A}(\rho g H)^m - {}^{k}/_{A}(\rho g H_b)^m \qquad (17)$$

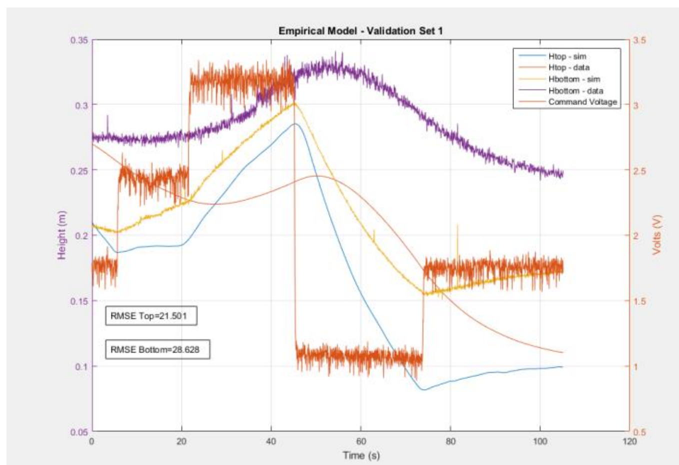

FIGURE 8. SIMULATION – 3RD MODEL

CONCLUSIONS

Overall the modeling produced results that we close, but did not exactly match the data measured. This could be attributed to the assumptions made about the pump converting to a flow rate linearly or assumption made about the orifices. The orifices could be more like a nozzle than an actual flat plate orifice. Overall, the 2nd Bernoulli model produced the best results of all 3 models, as indicated by the decrease in RMSE

REFERENCES

[1] ASME, 2003. ASME Manual MS-4, An ASME Paper, latest ed. The American Society of Mechanical Engineers, New York. See also URL http://www.asme.org/pubs/MS4.html