

STAT/MATH 495: Problem Set 03

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Question

For both `data1` and `data2` tibbles (a tibble is a data frame with some metadata attached):

- Find the splines model with the best out-of-sample predictive ability.
- Create a visualization arguing why you chose this particular model.
- Create a visualization of this model plotted over the given (x_i, y_i) points for $i = 1, \dots, n = 3000$.
- Give your estimate $\hat{\sigma}$ of σ where the noise component ϵ_i is distributed with mean 0 and standard deviation σ .

Data 1

```
head(data1)
```

```
## # A tibble: 6 x 3
##       ID         x         y
##   <int>     <dbl>     <dbl>
## 1     1 52.4585218 -4.694998
## 2     2 63.6341266 19.854723
## 3     3 33.1341217 10.878075
## 4     4  0.1977558 -5.649503
## 5     5 36.7559002 24.449026
## 6     6 66.2841361 -3.776968
```

```
data1$y
```

```
## [1] -4.69499756 19.85472273 10.87807540 -5.64950323 24.44902568
## [6] -3.77696791 -20.35697791 14.22626564 -5.27389357 -25.19274795
## [11]  7.60297037  6.57117208 -5.71872128  0.60943994 -13.66550609
## [16] -10.98673315  7.01732100 -36.28363142  0.21560802  7.20986091
## [21] -18.55970040 12.54958886 -14.23673480 13.53734030 -34.96185071
## [26] -15.78728789 10.84861255 30.80980682 14.84891560 24.09980639
## [31] -21.16745529 10.40804519 -9.95464987 -1.38555316  5.36595712
## [36] -10.22767555 21.97710456 37.31768098 -35.74687227 14.64836030
## [41]  0.92048422 47.41890312 -8.02591417 -7.67560276 -12.29413322
## [46] 27.08462007  9.49520448  2.26174413 -15.74204147 -21.78082261
## [51] -4.02714689 13.12363201  3.81876972 -14.45703359  9.92639754
```

##	[56]	16.31055284	2.65675571	-4.24623263	8.92319624	-8.40446406
##	[61]	21.52599091	3.62516856	19.86795272	0.75260395	26.86248654
##	[66]	0.23595169	22.10816379	-4.74817239	-6.20421627	-19.79023200
##	[71]	-8.21818109	-14.37465592	-3.77526366	-1.38414767	8.84852980
##	[76]	11.83209332	-25.65861535	-4.03442096	4.19124934	11.51744178
##	[81]	-12.87002075	-10.84525158	2.15514061	-38.57601892	-23.61465924
##	[86]	-19.75988942	14.11776726	-0.32593005	1.56661439	-7.25658637
##	[91]	24.62234383	11.26231325	-8.10579646	9.65004710	38.33791464
##	[96]	-1.91828878	-21.41827902	7.88610532	2.26887734	-2.79847293
##	[101]	4.95515886	6.38706237	9.98536217	-13.30669018	16.25597413
##	[106]	-8.73421176	-1.14232577	-21.67897141	-13.97075700	-21.54563588
##	[111]	1.88005578	31.14610286	3.88159123	8.88346125	42.09563756
##	[116]	16.75797363	12.19682198	-19.97010086	-3.87246219	-13.39212958
##	[121]	-6.04025737	5.78118854	-12.87548356	16.47077732	8.53474753
##	[126]	-4.53845825	-13.02936555	15.88267002	12.42375076	-7.84975188
##	[131]	-40.57785337	-19.87535353	-29.32645573	4.30697208	-32.12142612
##	[136]	-5.05748948	14.95036404	-24.75598302	15.82146811	11.19248591
##	[141]	8.46521177	8.48359411	-0.37169103	-21.81761207	-19.31224617
##	[146]	-31.04920982	6.97505141	20.42447836	16.27468560	-15.39677174
##	[151]	16.43736204	-6.24213305	-3.51512699	26.92796563	-2.52371640
##	[156]	-19.24454556	-26.29085931	0.05024197	19.56645478	7.95463466
##	[161]	14.47861447	11.02093545	4.79987215	5.31167078	-2.62883573
##	[166]	8.59301298	23.90593583	-22.03272433	7.68890082	7.59779250
##	[171]	-17.31820937	-11.71011505	0.76523783	-16.75734603	-14.44420526
##	[176]	-10.30081454	4.38915335	-10.68199983	21.02754763	-10.41889134
##	[181]	-22.02787422	-20.08948549	9.77410104	7.39424150	22.70954928
##	[186]	9.13652966	-15.33390491	7.73783938	-30.67242552	-2.83984210
##	[191]	8.69379796	-2.60727462	17.04779160	-31.74943529	-7.85803970
##	[196]	-6.22049305	-6.87925372	-18.40291464	-19.09388137	-21.44325980
##	[201]	-17.46917999	-11.07474219	-3.77852862	18.26707527	-8.36778823
##	[206]	-24.66237255	0.35860803	-21.25913489	-9.49761221	0.39697185
##	[211]	-24.06966428	-8.90776079	13.50118445	-22.98700352	-37.26989306
##	[216]	-6.82224418	-21.95309283	-23.46643444	-17.16701570	-27.13505057
##	[221]	12.86532312	3.84393508	-23.46447129	-8.47995431	-22.93869844
##	[226]	-19.41270940	16.66819470	3.50322235	24.48377816	10.05936595
##	[231]	9.67297426	-1.33499769	13.39758695	12.16338024	-0.14767126
##	[236]	-24.85160252	-5.28445122	9.74509153	-4.82711169	3.82849615
##	[241]	28.89976746	0.59376032	23.63336367	-7.79108416	-29.49027161
##	[246]	-21.48789914	-7.64965011	-13.21662024	-15.71963559	5.47733512
##	[251]	5.82938383	1.73592303	-3.17177775	26.39083006	-0.80987315
##	[256]	-13.21414138	-1.72141908	13.60219706	2.54496436	-22.06454274
##	[261]	16.83798005	1.23340168	7.80806245	-0.55096952	15.65054470
##	[266]	-24.82007084	-17.01758308	32.35873195	-16.66955085	-19.82712745
##	[271]	11.87679555	-21.83601090	14.07865440	-10.29346407	0.55791346
##	[276]	-21.37915638	-17.53275088	-21.61809752	-1.37402131	10.74511447
##	[281]	23.52588297	7.88919048	10.74016231	12.01315837	20.13989254
##	[286]	-30.43720591	-11.66544499	19.65465761	22.86768941	30.02484525
##	[291]	-10.89631334	8.90443057	-24.19103460	-6.69249238	-16.13369030
##	[296]	-1.60176074	-16.16286656	-0.47864219	-4.25927646	4.08749496
##	[301]	-21.98641351	9.33009116	-7.84549370	-19.26969741	-27.66743044
##	[306]	3.61551372	-17.90980505	14.45879275	-0.75804701	8.20973945
##	[311]	-13.22247299	-7.35417285	-9.42640408	-13.59119032	-43.37477303
##	[316]	-0.95956423	-5.92255921	4.86294843	16.81227987	7.03235069
##	[321]	-38.20684680	-27.97312885	-32.10800940	8.47332426	8.24678255

##	[326]	-21.96686595	-15.70348214	-22.98831415	2.77884264	1.80025087
##	[331]	12.45681940	11.25652027	3.14508014	24.67875474	-4.26472384
##	[336]	-23.51124133	-4.55138893	-17.84757252	-10.76880886	0.06389098
##	[341]	-29.85167191	15.77043552	-20.73979928	-2.26850525	4.66538968
##	[346]	-3.93892445	15.40468342	-6.26459863	41.22165823	28.59478059
##	[351]	-0.46066898	3.19327870	4.73013870	-4.66834612	7.50803524
##	[356]	-19.90747033	42.13768019	-22.65567879	-18.38883143	-7.30853513
##	[361]	6.11671992	20.48785876	3.50385184	9.35732517	-31.22683632
##	[366]	24.53003838	14.33980237	-10.99476037	25.32352855	-4.29340121
##	[371]	25.99213425	10.97425642	-17.25746581	-16.97898440	4.75253229
##	[376]	7.32874174	-11.37499751	9.28260564	3.20065103	14.98968979
##	[381]	-7.02861236	-4.54639133	4.70229781	18.62174410	26.11404448
##	[386]	6.00900056	-13.50265519	-9.71840470	-12.84763075	-11.42881013
##	[391]	3.58484242	-14.91639848	15.17315011	-18.53296782	-10.81242026
##	[396]	-2.06705156	-6.96558657	-11.14162488	9.72944938	30.67372695
##	[401]	0.05145666	-29.48322490	-23.14669973	-7.89289986	22.67830233
##	[406]	14.04782695	-3.95047430	2.65205815	0.05381763	-21.54435185
##	[411]	4.80478408	-8.88803799	-40.80057610	-8.32474043	-5.53840452
##	[416]	-10.41171774	-3.60390114	-15.40269289	-1.96610664	2.12724189
##	[421]	-35.17837646	0.53412566	-10.44311542	6.55399819	26.72903834
##	[426]	-6.66386677	15.30815591	-7.99919517	18.07685326	-12.69452656
##	[431]	6.06007877	-8.54497915	19.35472004	22.31054506	-6.61672531
##	[436]	3.47913083	-26.49405425	23.89256105	-34.32559153	-23.05438038
##	[441]	20.75862608	-11.44145281	11.58430094	26.25555125	-6.14340824
##	[446]	-1.57401370	8.22349968	11.39605873	8.27646585	-2.14191782
##	[451]	21.05591777	6.94698478	-3.64742139	26.92557732	-13.17334154
##	[456]	14.39211693	-17.04050109	9.11282702	-43.39267231	-27.16322145
##	[461]	7.66049229	23.25771084	-16.68287840	-14.98294065	-16.29530748
##	[466]	3.59924942	-7.49116087	-21.64702779	-29.30146569	14.30138024
##	[471]	-4.48356086	-18.43038403	-28.19056410	1.03159766	-0.46424490
##	[476]	-15.95166104	14.52152555	-30.52804424	-11.35858729	-8.43625725
##	[481]	-4.19597379	20.96750537	14.20963380	-17.10440946	6.84922058
##	[486]	-20.51339307	11.79175410	13.09395556	-12.78308559	0.92088896
##	[491]	-36.34460409	-17.47793304	-2.71043956	12.63007195	25.25936316
##	[496]	28.58352849	-11.91168867	-0.56471507	-10.05772530	3.37458533
##	[501]	-8.42558917	7.58698159	6.81832561	-8.61796988	-7.41711870
##	[506]	-9.31897165	0.95150341	-0.85670947	15.17534925	-6.31263721
##	[511]	-21.40224402	34.21177323	-3.80688483	-16.73571806	0.54892176
##	[516]	16.56801244	-2.25042320	32.18348593	-6.21229153	7.23276860
##	[521]	-3.15762420	-16.99781568	-13.36586000	-13.45318676	22.82394629
##	[526]	24.09152187	7.86236832	4.35974727	-6.50721576	33.07669679
##	[531]	-6.74616438	26.03949184	-11.40116591	5.08537613	3.47194224
##	[536]	-6.71795573	-15.60557784	11.73807676	8.16716947	-2.64089380
##	[541]	15.12374608	4.92672509	-10.96437790	8.12795783	-8.85026657
##	[546]	-6.69643466	-3.68824530	20.51965350	11.22756050	37.76931327
##	[551]	-5.34003818	28.70414732	14.48735158	-9.27947482	4.29776461
##	[556]	20.49875338	-24.90861261	-8.75233229	0.51994781	27.36449611
##	[561]	12.12094865	-25.87547015	-16.20725159	-5.14490448	-10.06009803
##	[566]	-8.59747455	-14.63368453	-9.83862007	13.76650669	-36.31690535
##	[571]	5.29540507	-4.18108435	-13.71117340	4.77271196	48.10255157
##	[576]	24.64200542	6.08834258	-17.36029537	-41.17016869	-20.37839763
##	[581]	-4.50713500	13.36576770	1.30385235	27.55045268	-8.89739460
##	[586]	-25.16632578	-5.77494090	-10.11284360	28.57541275	21.57533648
##	[591]	-26.65362892	-8.59510385	27.38715472	-18.26948575	-22.09707245

##	[596]	-13.55088408	-2.99035436	-22.02685150	15.33336103	-9.83705699
##	[601]	-8.13629109	24.50983204	-20.56924242	-20.96987231	31.34785831
##	[606]	34.22521880	10.06018613	-16.19040173	-7.76455370	-21.63428726
##	[611]	-29.86307822	-36.63328517	5.46009248	9.82436145	-1.63238638
##	[616]	-35.79719596	-16.25100298	-6.52257868	-28.28116504	9.13046491
##	[621]	-19.15989669	-18.04024359	17.99746685	-3.23419705	-6.52180996
##	[626]	4.70485442	13.78390467	-8.62253111	-18.56106430	16.38978191
##	[631]	-5.27343632	19.81830548	4.98549383	9.60182596	2.04151439
##	[636]	-21.45499584	-13.19204932	-18.64083833	21.16848276	3.79903220
##	[641]	9.10245448	-23.48561310	17.18363420	29.74747106	-18.24256724
##	[646]	-19.49343126	-38.66985763	-32.45437256	-16.91580342	-8.25866431
##	[651]	-4.95561875	-21.02347433	-2.47239029	12.05523158	31.03403821
##	[656]	9.53836950	-3.34770979	-6.03994818	1.19141909	21.72752543
##	[661]	-3.68005334	-19.50703799	-9.33954142	0.27205277	-13.18583813
##	[666]	-9.42824690	-20.80569724	29.52332253	18.24533773	6.89757015
##	[671]	19.89591709	20.56845746	-8.94282457	1.37761728	-13.93785846
##	[676]	9.78353720	-7.73365186	9.98933913	-9.59969345	-4.50672472
##	[681]	-6.85228371	10.29863733	-3.58529307	-1.79898666	-44.63107704
##	[686]	8.58766020	-22.85739868	-16.83900518	-7.66710019	-1.33804864
##	[691]	10.90068319	2.28167986	-2.48946139	5.51606942	-23.01918143
##	[696]	-12.91555502	-5.23659881	1.22028289	-0.33572054	23.85170585
##	[701]	13.83519790	37.18610840	-4.84500928	-1.04843930	-17.92045992
##	[706]	25.52915084	-12.02553066	-0.52858656	16.41577363	19.31707477
##	[711]	0.64518954	-11.00089935	8.14846328	-24.82010441	2.59203105
##	[716]	-4.49611909	-20.79921621	-5.77273444	-6.56028174	-13.80170004
##	[721]	-12.71150791	-27.41821184	5.50547719	-24.87629122	-29.03576463
##	[726]	20.01725229	-23.04432821	46.27991571	-4.36218339	10.79927605
##	[731]	20.19952992	-5.51623125	-14.18762182	7.78375755	19.55244735
##	[736]	7.55018508	-4.57615623	22.56253828	-17.22871317	7.99247591
##	[741]	-18.42129201	-10.72941114	9.54090675	19.24047824	3.78572099
##	[746]	-12.05269702	-27.99661573	-9.40958770	-21.17354679	6.16112168
##	[751]	21.19734307	3.65531960	-16.24276451	-7.15140742	-11.04335021
##	[756]	21.25620354	21.32074399	-22.37787069	-1.98792606	-17.54271074
##	[761]	24.87087697	21.97624705	-17.96583522	-10.55217856	-15.77991194
##	[766]	-24.09552072	-11.23597926	-6.77611319	-2.10373662	11.25458651
##	[771]	-34.42881170	12.36561864	7.33838061	7.78443484	13.16248937
##	[776]	-7.60918392	16.60354923	-16.27474883	-26.82810537	15.25955201
##	[781]	-13.10862618	-16.13026268	-3.94947052	-0.26831570	-23.21246455
##	[786]	21.28521291	3.98017621	-1.31678907	-3.58068331	-33.10358610
##	[791]	-7.98557715	-4.74012405	11.98148000	8.25930780	4.77401652
##	[796]	-17.25657068	-11.87229726	-12.56479182	-1.25591988	-18.74743050
##	[801]	-12.88121101	-14.30779863	-18.65358990	28.23490385	12.26413163
##	[806]	30.90588262	38.67431246	14.54912233	10.13676791	-5.89704657
##	[811]	-11.61632790	4.43263553	4.92520841	-11.51910207	-30.63656261
##	[816]	16.17750638	-7.17284953	24.96657125	9.08638218	-36.16684302
##	[821]	15.00954654	-0.05499015	-22.86401075	-31.56873606	5.37466147
##	[826]	11.51706478	-12.69160611	-19.71930163	41.30501523	8.58084999
##	[831]	-15.21617745	-4.40199731	-17.43167747	-4.87024187	-25.15629031
##	[836]	0.15238030	6.05501886	-41.87087898	-13.91412120	8.51681498
##	[841]	34.57589404	-6.21594477	-17.05539081	12.87449284	-26.36171433
##	[846]	16.71739370	-2.92716015	13.22904617	-17.93659748	-4.16824150
##	[851]	8.63110061	-9.10240098	25.05043301	-30.11853243	-36.95438477
##	[856]	2.72097721	-36.40540212	-14.95779446	13.09809142	2.32285315
##	[861]	10.97776134	10.14444163	-18.21685179	-8.57070591	-5.01694456

##	[866]	-33.00590267	-6.05377998	-18.18183291	4.96100007	-0.46355373
##	[871]	11.78053664	-0.60041011	10.34746735	9.06556801	12.75168325
##	[876]	15.00807314	-6.33031002	-19.48204235	13.36946200	4.64397372
##	[881]	-0.44047451	4.03744223	6.82173778	4.47610943	4.51135303
##	[886]	-24.00904368	2.12886615	-1.02856641	13.95338965	2.60608490
##	[891]	9.59631318	-15.80942730	31.77512521	12.76614497	8.67819082
##	[896]	16.28909037	-15.69831469	25.88712822	-3.33840279	21.87972944
##	[901]	-5.62219184	-13.57267595	-25.01732034	-11.99706374	0.95052231
##	[906]	12.47764410	-13.64401381	0.23660706	16.20322227	7.15011109
##	[911]	-10.40454698	-3.58636382	-13.23186178	-2.07700100	-3.22604285
##	[916]	-20.95435726	-3.57970814	-5.87106647	-10.60235117	23.52361800
##	[921]	19.77749927	-0.28017686	26.43861353	20.52054500	1.96183616
##	[926]	-38.77367065	8.68026418	-8.36531696	3.99846080	2.77338284
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##	[1916]	-1.18288240	8.29568941	-7.96733911	18.20933807	-12.45880955
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##	[1951]	-8.03749537	-22.38223333	-0.54739399	0.93413823	-20.69418681
##	[1956]	-13.59217183	19.99980651	23.34617824	-27.64286040	-10.25636240
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##	[1971]	0.84184260	7.86817548	-18.19209133	14.82454098	20.45171332
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##	[1981]	-1.48533706	-27.12281227	3.32674234	-3.20462029	-12.96906836
##	[1986]	-11.03401453	-17.73731481	27.29244321	8.85087611	-8.46525165
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##	[1996]	10.11347630	-10.50433143	0.16710541	3.78083640	20.78753483
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## [2871] -4.51965490 1.33234130 8.07295912 -9.00098312 -11.30305282
## [2876] -0.31034327 -9.62761653 2.78173561 18.98601882 16.65344420
## [2881] 18.47741123 23.55729286 -9.66365982 11.74353399 12.77110357
## [2886] -3.05807773 -38.84813656 14.64896694 11.04072185 -0.07897591
## [2891] -6.56823398 10.47228413 17.88288592 17.62554812 5.03506897
## [2896] -10.86439254 4.39968374 -2.24271504 -14.90332870 21.83524002
## [2901] 6.00522680 -9.57660799 20.36252563 -3.87693856 13.39425300
## [2906] 4.75822717 -25.21544149 1.23774453 -11.92640732 1.91569484
## [2911] -4.56889121 -2.58344440 0.22814642 24.33550303 -18.04045028
## [2916] -2.53948838 11.40947174 11.62545114 13.97298743 1.72454660
## [2921] 11.71992803 2.55954870 7.52800307 -17.58693947 7.35019005
## [2926] -18.25811670 3.39749092 -4.72283593 6.69014681 9.05472369
## [2931] -3.40439202 -24.47789097 0.28707653 -15.84953366 7.16601564
## [2936] -22.35170474 29.96946254 -10.11009026 -22.25771090 -9.65534267
## [2941] 6.54867563 8.84477468 22.39915542 -13.23902727 -3.25337925
## [2946] -10.09377872 -12.58343553 12.15541662 8.92119062 5.56003002
## [2951] 29.31637149 4.92593873 -18.33766106 -5.40991538 27.76386704
## [2956] -7.43231815 -0.75373856 16.33430755 -14.32067869 24.41312146
## [2961] -0.83707604 -3.17752902 -14.54720160 8.92501766 -14.56585875
## [2966] -9.49566670 11.36466041 4.78816825 5.79012868 -4.11297848
## [2971] 7.35673615 22.94551353 7.78437573 17.01484984 15.01331522
## [2976] 8.03114793 -18.81418389 0.54145745 11.34650693 -4.79585627
## [2981] -8.71066708 1.28778658 -11.55897157 -14.98742881 -10.12430027
## [2986] -19.36793178 14.37116876 -4.24196692 -25.50099267 -46.55547606
## [2991] 15.09740795 8.01189165 17.57361829 21.06827972 -19.69358067
## [2996] -2.07895076 7.50059230 2.50841802 13.98386935 -0.54748703
```

```
yhat1 <- (smooth.spline(data1$x,data1$y, df = 2))$y
RMSE <-sqrt(mean((yhat1-data1$y)^2))
```

```
## Warning in yhat1 - data1$y: longer object length is not a multiple of
## shorter object length
```

```
## Warning in yhat1 - data1$y: longer object length is not a multiple of
## shorter object length
```

```
RMSE
```

```
## [1] 16.51371
```

```
spline1 <- smooth.spline(data1$x, data1$y, df=26) %>%
  broom::augment() %>%
  ggplot(aes(x=x)) +
  geom_point(aes(y=y)) +
  geom_line(aes(y=.fitted), col="blue", size=1)
```

Data 2

```
head(data2)
```

```
## # A tibble: 6 x 3
##       ID          x          y
##   <int>    <dbl>    <dbl>
## 1     1 52.4585218 -4.127757
## 2     2 63.6341266 33.821437
## 3     3 33.1341217  8.944431
## 4     4  0.1977558 -7.617335
## 5     5 36.7559002 33.977201
## 6     6 66.2841361 -9.548001
```

```
smooth.spline(data2$x, data2$y, df=19) %>%
  broom::augment() %>%
  ggplot(aes(x=x)) +
  geom_point(aes(y=y)) +
  geom_line(aes(y=.fitted), col="blue", size=1)
```

