HOMEWORK #3

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1. From the book

1.1. 1.8 Page 28: 6, 8, 14.

6)

(a) $\bigcup_{i \in \mathbb{N}}$ [0,i+1] =$[0, \infty)$

(b) $\bigcap_{i \in \mathbb{N}}$ [0,i+1] = $[0, 2]$

8)

(a) $\bigcup_{\alpha \in \mathbb{R}}$ $\{\alpha\}$ x [0,1] = $\{(x, y) \mid x \in \mathbb{R}$ and $y \in [0, 1]\}$

(b) $\bigcap_{\alpha \in \mathbb{R}}$ $\{\alpha\}$ x [0,1] = ø the empty set is the only solution.

14) If J $\neq$ ø and J $\subseteq$ I, does it follow that $\bigcap_{\alpha \in I} A_\alpha \subseteq \bigcap_{\alpha \in I} A_\alpha$? Explain.

Let J and I be two sets such that J $\neq$ ø and J $\subseteq$ I. So I $\neq$ ø. let x $\in \bigcap_{\alpha \in I} A_\alpha$, then x $\in A_\alpha$ for all $\alpha \in$ I. Let $\alpha \in$ J be an arbitrary element and $\alpha \in J \subseteq I$, thus $\alpha \in$ I and then x $\in A_\alpha$. So x $\in A_\alpha$ for all $\alpha \in$ J. So we can conclude that for each x $\in \bigcap_{\alpha \in I} A_\alpha$, then x $\in \bigcap_{\alpha \in J} A_\alpha$. So this is true when J $\neq$ ø and J $\subseteq$ I.

1.2. 2.1 Page 37: evens.

(2) Every even integer is a real number.
Is both a Statement, and is true.

(4) Sets $\mathbb{Z}$ and $\mathbb{N}$
This is not a sentence, there is no conclusion, thus it is not a statement.

(6) Sets $\mathbb{Z}$ and $\mathbb{N}$ are finite.
This sentence is a True Statement.

(8) $\mathbb{N} \notin \mathbb{P}(\mathbb{N})$
This is a Statement, however it is False.

(10) ($\mathbb{R}$ x $\mathbb{N}$) $\cap$ ($\mathbb{N}$ x $\mathbb{R}$) = $\mathbb{N}$ x $\mathbb{N}$
This is a Statement, and it is True. The intersection only contains pairs of natural numbers.

(12) If the integer x is a multiple of 7, then it is divisible by 7.
This is a Statement and it is True, By definition of how being a multiple of something works.

(14) Call me Ishmael.
Not a Statement, I might do it though...

1.3. **2.2 Page 41: 6,8,12.**

6) There is a quiz scheduled for Wednesday or Friday.
P: There will be a quiz on Wednesday. Q: There will be a quiz on Friday. Either could be true depending on the intention of the english sentence.
P $\vee$ Q could be either p or q , or both.
(P $\vee$ Q) $\wedge \sim$ (P $\wedge$ Q) could be P or Q and not both.

8) At least one of the numbers x and y equals 0.
P: the number x is 0.
Q: the number y is 0. P $\vee$ Q So either X or y is equal to 0 or both are equal to 0.

12) Happy families are all alike, but each unhappy family is unhappy in its own way
P: Happy families are all alike
Q:each unhappy family is unhappy in its own way.
P $\wedge$ Q

2(A)

p	q	$p \vee q$	$p \wedge q$	$((\sim p) \wedge (\sim q)\}$	$((\sim p) \vee (\sim q)\}$	$\sim (p \vee q)$	$\sim (p \wedge q)$
T	T	T	T	F	F	F	F
T	F	T	F	F	T	F	T
F	T	T	F	F	T	F	T
F	F	F	F	T	T	T	T

p, q, if p or q, if p and q, not p and not q, not p or not q, not p or q, not p and q
Following the logic of the first and second column, the subsequent columns hold logically true.

2(B) (P $\vee$ Q) $\wedge$ ($\sim$ (P $\wedge$ Q)). This statement translates to: if P or Q, and if not P and Q. kind of like you can have this or that, and not this and that. Like a mother telling her kid they can choose one or the other toy, but decidely not both.

2(C) P $\rightarrow$ (Q $\wedge$ ($\sim$ Q)) and ($\sim$ P)

P	Q	$(\sim Q)$	$(\sim P)$	$P \rightarrow (Q \wedge (\sim Q))$
T	T	F	F	F
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

The reason that this holds true is because P $\rightarrow$ (Q $\wedge$($\sim$ Q)) is vacuously true when P is false (ie. $\sim$P).