Laser Spectroscopy Prelab

Björn Sumner
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Prelab Questions

1

$s = \frac{1}{2}$, always.
$5s^1$: $s \implies \ell = 0$ and since $j = |\ell - s|, \cdots, \ell + s = \frac{1}{2}$.
Furthermore, $2s + 1 = 2$, so this gives us

$$^{2s+1}L_j \to 5^2S_{1/2}$$

$5p^1$: $p \implies \ell = 1$ and $j = |1 - \frac{1}{2}|, 1 + \frac{1}{2} = \frac{1}{2}, \frac{3}{2}$

$$^{2s+1}L_j \to 5^2P_{1/2} \text{ and } 5^2P_{3/2}$$

2

The Hamiltonian associated with spin orbit coupling is given by equation (5):

$$H_{SO} = \zeta(r)\frac{\hbar^2}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right]$$

For the higher energy $5^2P_{3/2}$, $j = \frac{3}{2}, \ell = 1, s = \frac{1}{2}$, and we get an associated energy:

$$\begin{aligned} H_{SO} &= \zeta(r)\frac{\hbar^2}{2}\left[\frac{3}{2}(\frac{5}{2}) - 1(2) - \frac{1}{2}(\frac{3}{2})\right] \\ &= \zeta(r)\frac{\hbar^2}{2}\left[\frac{15}{4} - \frac{8}{4} - \frac{3}{4}\right] \\ &= \zeta(r)\frac{\hbar^2}{2} \end{aligned}$$

For the lower energy $5^2P_{3/2}$, $j = \frac{1}{2}, \ell = 1, s = \frac{1}{2}$, and we get an associated energy:

$$\begin{aligned} H_{SO} &= \zeta(r)\frac{\hbar^2}{2}\left[\frac{1}{2}(\frac{3}{2}) - 1(2) - \frac{1}{2}(\frac{3}{2})\right] \\ &= \zeta(r)\frac{\hbar^2}{2}\left[-2\right] \\ &= -\zeta(r)\hbar^2 \end{aligned}$$

To get the energy splitting, we take the difference between the two:

$$\begin{aligned} \Delta E &= \zeta(r)\frac{\hbar^2}{2} + \zeta(r)\hbar^2 \\ &= \zeta(r)\frac{3\hbar^2}{2} \end{aligned}$$

3

For ^{87}Rb, $I = \frac{3}{2}$, and $F = J + I$, with F taking on values $F = |J - I|, |J - I| + 1, \cdots, J + I$

(1)

$5^2S_{1/2} \implies j = \frac{1}{2}$
So

$$\begin{aligned} F &= |\frac{1}{2} - \frac{3}{2}|, \frac{1}{2} + \frac{3}{2} \\ &= 1,\ 2 \end{aligned}$$

(2)

$5^2P_{1/2} \implies j = \frac{1}{2}$
So

$$\begin{aligned} F &= |\frac{1}{2} - \frac{3}{2}|, \frac{1}{2} + \frac{3}{2} \\ &= 1,\ 2 \end{aligned}$$

(3)

$5^2P_{3/2} \implies j = \frac{3}{2}$
So

$$\begin{aligned} F &= |\frac{3}{2} - \frac{3}{2}|,\ |\frac{3}{2} - \frac{3}{2}| + 1,\ |\frac{3}{2} - \frac{3}{2}| + 2,\ \frac{3}{2} + \frac{3}{2} \\ &= 0,\ 1,\ 2,\ .3 \end{aligned}$$

4

The presence of crossover resonances means that there are frequencies between the 'standard' transition frequencies of ^{87}Rb that can absorb light. I expect to observe 12 spectral lines in the transition from $5^2P_{3/2}$ to $5^2S_{1/2}$.

5

For the $5^2S_{1/2}$ state of ^{87}Rb, we have $I = \frac{3}{2}$, $J = \frac{1}{2}$, $F = 1$ or 2. However, when we plug this into equation 10:

$$B\frac{\frac{3}{4}C(C+1) - I(I+1)J(J+1)}{2I(2I-1)J(2J-1)}$$

We notice that the denominator results in zero, due to the value of J: $2\frac{1}{2} - 1 = 0$. Since this term is a term in the total hyperfine interaction, we cannot have it diverge, or else we would have a divergent hyperfine transition! The only way to prevent this is if we force $B = 0$, regardless of our choice in $F = 1$ or 2.