HOMEWORK #4

BY: BEN CRANE

Due 02/16

1. From the book

1.1. **2.3 Page 44: even.**

(2) For a function to be continuous, it is sufficient that it is differentiable.
P: A function f is differentiable
Q: the function f is continuous.
If P then Q. If A function f is differentiable, then the function f is continuous.

(4) A function is rational if it is a polynomial.
P: a function is a polynomial
Q: the function is rational.
If P then Q. If a function is a polynomial, then the function is rational.

(6) Whenever a surface has only one side, it is non-orientable.
P: A surface has only one side
Q: The surface is non-orientable
If P then Q. If a surface has only one side, then the surface is non-orientable.

(8) A geometric series with ratio r converges if $|r| < 1$.
P: $|r| < 1$
Q: the geometric series with ratio r converges
If $|r| < 1$, then the geometric series with ratio r converges.

(10) The discriminant is negative only if the quadratic equation has no real solutions.
P: the discriminant is negative
Q: the quadratic equation has no real solution
IF P then Q. If the discriminant is negative then the quadratic equation has no real solution.

(12) People will generally accept facts as truth only if the facts agree with what they already believe. (Andy Rooney)
P: the facts agree wi what they already believe. Q: the people will generally accept facts as truth. if P then Q. If the facts agree with what they already believe then the people will generally accept the facts as truth.

,

1.2. **2.5 Page 48: evens.**

(2) (Q $\vee$ R) $\Leftrightarrow$ (R $\wedge$ Q)

Q	R	(Q $\vee$ R)	(R $\wedge$ Q)	(Q $\vee$ R) $\Leftrightarrow$ (R $\wedge$ Q))
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

(4) ($\sim$P $\vee$ Q) $\vee$($\sim$P)

P	Q	(P $\vee$ Q)	$\sim$(P $\vee$$Q$)	$\sim P$	$\sim$(P $\vee$ Q) $\vee$($\sim$ P))
T	T	T	F	F	F
T	F	T	F	F	F
F	T	T	F	T	T
F	F	F	T	T	T

(6) (P $\wedge$($\sim$P)$\wedge$ Q

P	Q	$\sim$P	(P $\wedge$($\sim P$)	(P $\wedge$($\sim P$))$\wedge$Q
T	T	F	F	F
T	F	F	F	F
F	T	T	F	F
F	F	T	F	F

(8) $P \vee (Q \wedge \sim R))$

P	Q	R	$\sim$R	$Q \wedge \sim R$	$P \vee (Q \wedge \sim R))$
T	T	T	F	F	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	T	T
F	F	T	F	F	F
F	F	F	T	F	F

(10) Suppose the statement $((P \wedge Q) \vee R) \Rightarrow (R \vee S)$ is false. Find the truth values of P,Q,R and S. (This can be done without a truth table.)

$(P \wedge Q) \vee R$	$R \vee$ S	$(\sim (P \wedge Q) \vee R) \Rightarrow (R \vee S)$
T	F	F

R	S	$R \vee S$
F	F	F

$P$$\wedge$Q	R	$(P \wedge Q) \vee R$
T	F	T

P	Q	$P \wedge Q$
F	F	F

So P is True, Q is True, R is False, and S is False

2. WORKSHEET

(1)

P	Q	$\sim (P \Rightarrow Q)$	$\sim (P \Leftrightarrow Q)$	$(\sim (P \vee Q) \Rightarrow (\sim P) \wedge Q$	$(P \Rightarrow Q) \vee (P \Rightarrow (\sim Q)$	$(P \Rightarrow Q) \wedge (P \wedge (\sim Q)$
T	T	F	F	F	T	F
T	F	T	T	F	T	F
F	T	F	T	F	T	F
F	F	F	F	F	T	F

(2) Write a logical statement for the sentence P or Q but not both. Justify your answer using a truth table. $(P \vee Q) \wedge \sim (P \wedge Q)$This is the same as $\sim (P \Leftrightarrow Q)$

P	Q	$(\sim Q)$	$(\sim P)$	$\sim (P \Leftrightarrow Q)$
T	T	F	F	F
T	F	T	F	T
F	T	F	T	T
F	F	T	T	F

(3) Write a tautology and a contradiction in one variable P using $\wedge, \vee, \Rightarrow, and \Leftrightarrow$
P ∨ ∼P ⊤
P ⇒ P ⊤
P $\Rightarrow \sim P \perp$

(4) Write a logical statement equivalent to P ⇒Q that uses only $\vee, \wedge$ and $\sim$. Use your conclusion to do the same exercise for P ⇔ Q. Conclude that the logical connectors ⇒ and ⇔ are redundant. Are > and ⊥ redundant?
P$\Rightarrow Q = \sim P \vee Q$
While the logical connectors might seem redundant $\Rightarrow and \Leftrightarrow$, there actually does remain some distinction between the two. But that is more of a set theory distinction.

(5) Write a truth table for the following statement $(Q \Rightarrow (\sim R) \wedge (\sim P)) \wedge (P \vee R)$.

P	Q	R	$(\sim R)$	$(\sim P)$	$(Q \Rightarrow (\sim R) \wedge (\sim P))$	$(P \vee$ R)	$(Q \Rightarrow (\sim R) \wedge (\sim P)) \wedge (P \vee R)$.
T	T	F	T	F	F	T	F
T	F	T	F	F	F	T	F
F	T	F	T	T	T	F	F
F	F	T	F	T	F	T	F

(6) i. What is the number of possible logical statements in two variables P and Q up to logical equivalence (i.e., how many possible truth tables are there)? Can each possibility in terms of a basic logical statement? Justify your answer.

2^{2^2} or 16 truth tables which is the same as 2^{2^n} where n = 2. This makes sense for all of the logical ways of arranging the possible different micro-states without repeating.

ii. How many rows are there in a truth table for a statement in n variables? Justify your answer.

For n variables there would be 2^n rows, if there are three variables then $2^3 = 8$ rows would be needed. This is because there is a truth and a false for each option.

iii. What is the number of possible logical statements in n variables up to logical equivalence? Justify your answer.

There are 2^{2^n} boolean functions of n variables.

3. PROOFS

(a)

Lemma 3.1. *Let x and y be integers. If x and y are odd, then x*y is odd.*

Proof. Given that x and y are integers, x is odd and y is odd implies that x*y is also an odd number. x is an odd number if x = 2n+1 and x is an integer, so n is also an integer. Since y is an odd number implies that y = 2m+1 and y is an integer which implies that m is also an integer. So x*y = (2n+1)(2m+1) = 4n*m+2n+2m+1 = 2(2n*m+n+m)+1 = 2k+1. where k = 2n*m+n+m. Since n and m are integers, therefore k is also an integer. This all implies that x*y is an odd number; supposing x is an odd number and y is an odd number. □

(b)

Lemma 3.2. *Let x be an integer. If x is odd, then x^3 is odd.*

Proof. Since x is an odd number then x^2=x*x is also an odd number, as proven by Lemma 3.1, then $(x^2) * x = x^3$ must be an odd number, still following the logic of Lemma 3.1. □