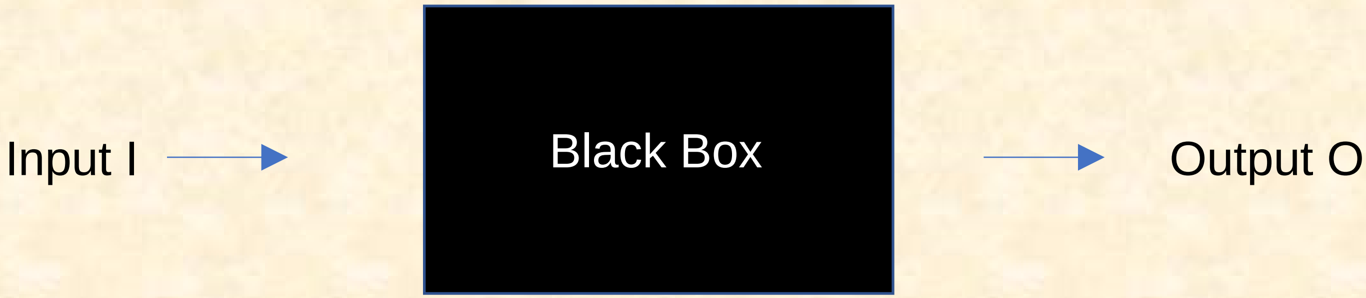
Linear systems



A*I -> A*O

If I1 -> O1 and I2 -> O2, then I1 + I2 -> O1 + O2

Together, A*I1 + B*I2 -> A*O1 + B*O2

Linear, time-invariant systems

X(t) → Y(t) – signals are functions of time. Like a tone.

If X(t + t1) -> Y(t + t1), for all t1, then time-invariant

Shifting X by time, produces same output but shifted by same time.
The system properties are not changing with time.

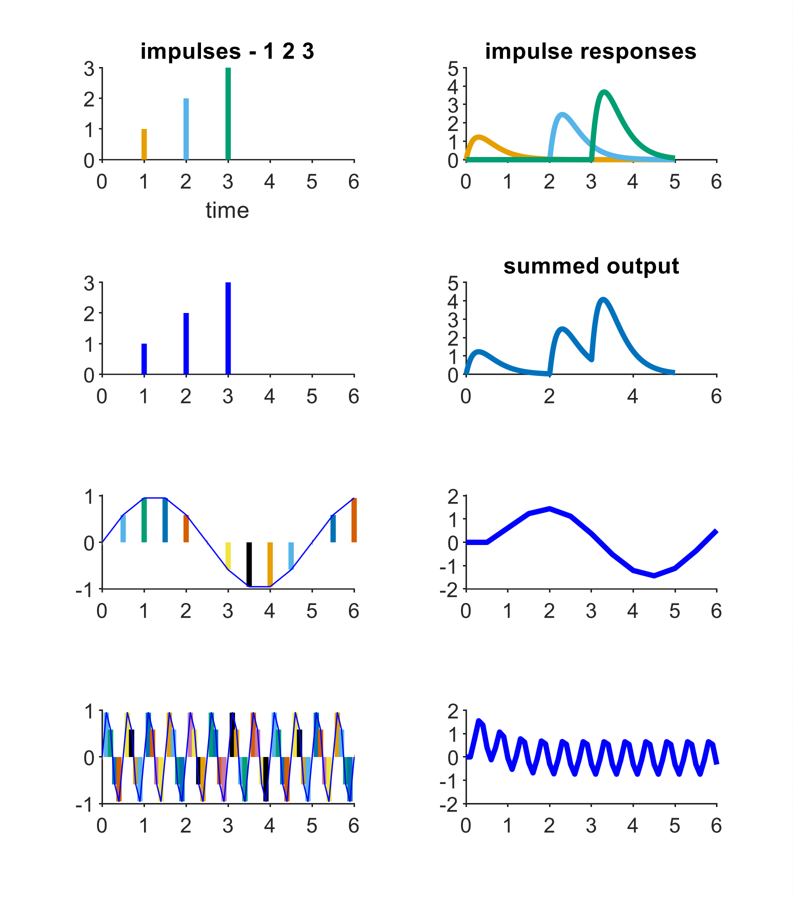
E.g.

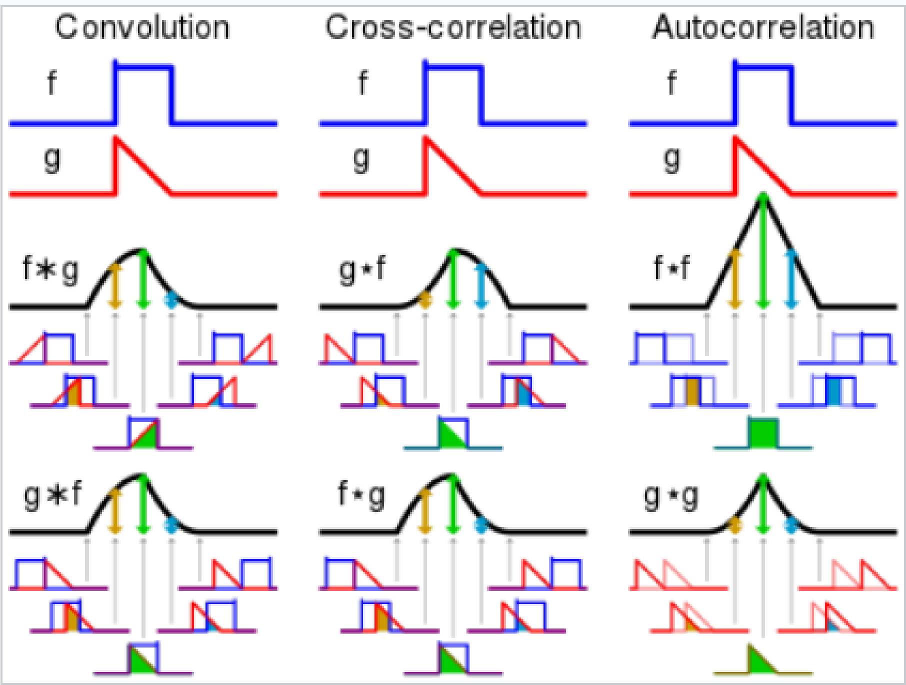
X(t) → A*X(t), say A=10

X(t+5) → 10*X(t+5) since A is a constant, and not a function of time

Linear, time-invariant (LTI) systems are very useful, even if wrong baseline models

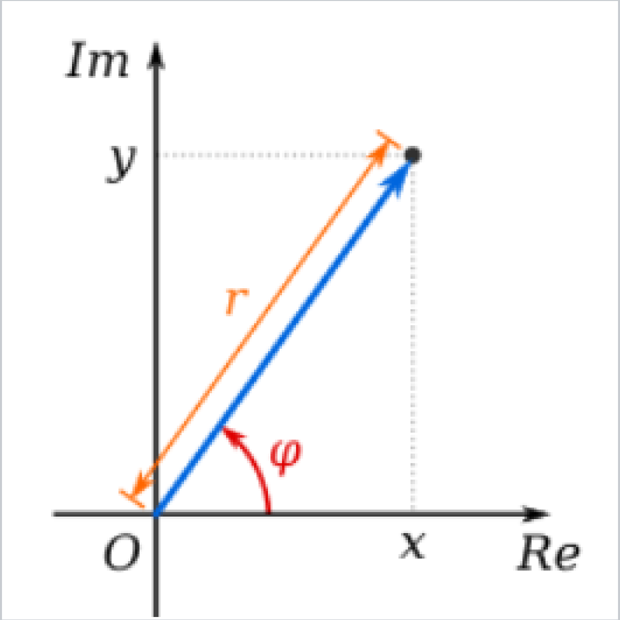
Convolution gives output





Visual comparison of convolution, cross-correlation, and autocorrelation. For the operations involving function f, and assuming the height of f is 1.0, the value of the result at 5 different points is indicated by the shaded area below each point. Also, the symmetry of f is the reason $g * f$ and $f \star g$ are identical in this example.

Complex numbers



Argument φ and modulus r locate a point in the complex plane.

Figure from Wikipedia article.

$x + iy$

$r*e^{i\varphi} = r * \cos\varphi + i * r * \sin\varphi$

$x = r * \cos\varphi\ ;\ y = r * \sin\varphi$

$r * e^{i\varphi}$ is a complex exponential

$r = \mathrm{sqrt}(x^2 + y^2)$ is the amplitude

φ is the phase

And $r * e^{i(\omega t+\varphi)}$ is a complex exponential function of time and is equal to $\cos(\omega t+\varphi) + \sin(\omega t+\varphi)$

Sinusoids go through with only scaling and phase-shifting

If you pass a complex exponential through an LTI system, you get another complex exponential, with only amplitude and phase being possibly different.

Here, complex exponentials include real numbers, when the phase is 0.

That is, exponentials are *eigenfunctions* for LTI systems.

Simple proof in Wikipedia.

Because real part of exponential is a cosine and imaginary part a sine, cosines and sines are also eigenfunctions of LTI systems. *Hence the lower graphs on left column figure in page 5.*

Basis functions

For a given class of target functions, a basis set is a minimal spanning set of basis functions: that is, by suitably scaling and adding these functions, you can get any target function.

It is minimal, because for this property to be true for this basis, all the basis functions are needed – you cannot delete any of them.

For a very large class of functions, the set of sinusoids and co-sinusoids are basis functions… aka the Fourier transform. Each target function can be characterized as the amplitude and phase of its basis functions, that when added produce the target function.

A sufficiently long pure-tone's frequency domain representation is a single sinusoid, for example. Shorter tones will need more frequencies to capture the onset and offset.

LTI systems – frequency domain

For a very large class of functions, the set of sinusoids and co-sinusoids are basis functions… aka the Fourier transform. Each target function can be characterized as the amplitude and phase of its basis functions.

If you add these basis functions, you will get the target function back – this is the inverse Fourier transform.

If you pass a complex exponential through an LTI system, you get another complex exponential, with only amplitude and phase being possibly different.

To get the output of a linear system, often it is easier, to break the input signal into its Fourier basis, scale and phase-shift each basis to capture LTI system's response, and then add the transformed basis functions to get the output.

Basis functions - example

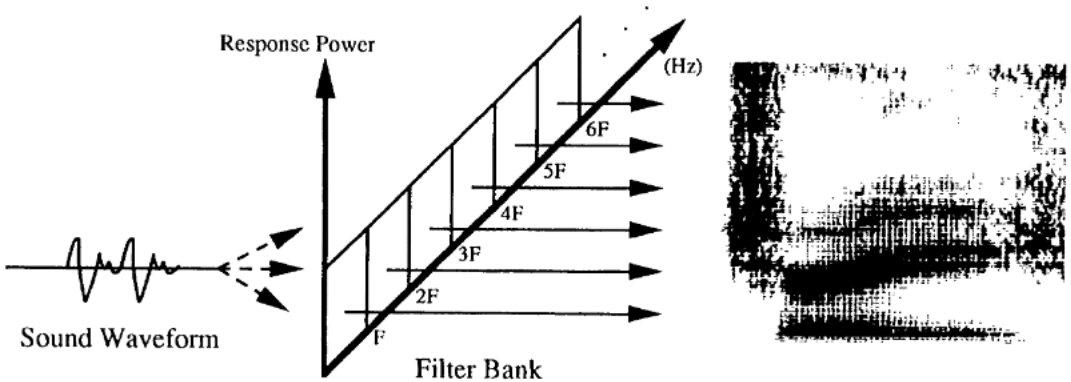
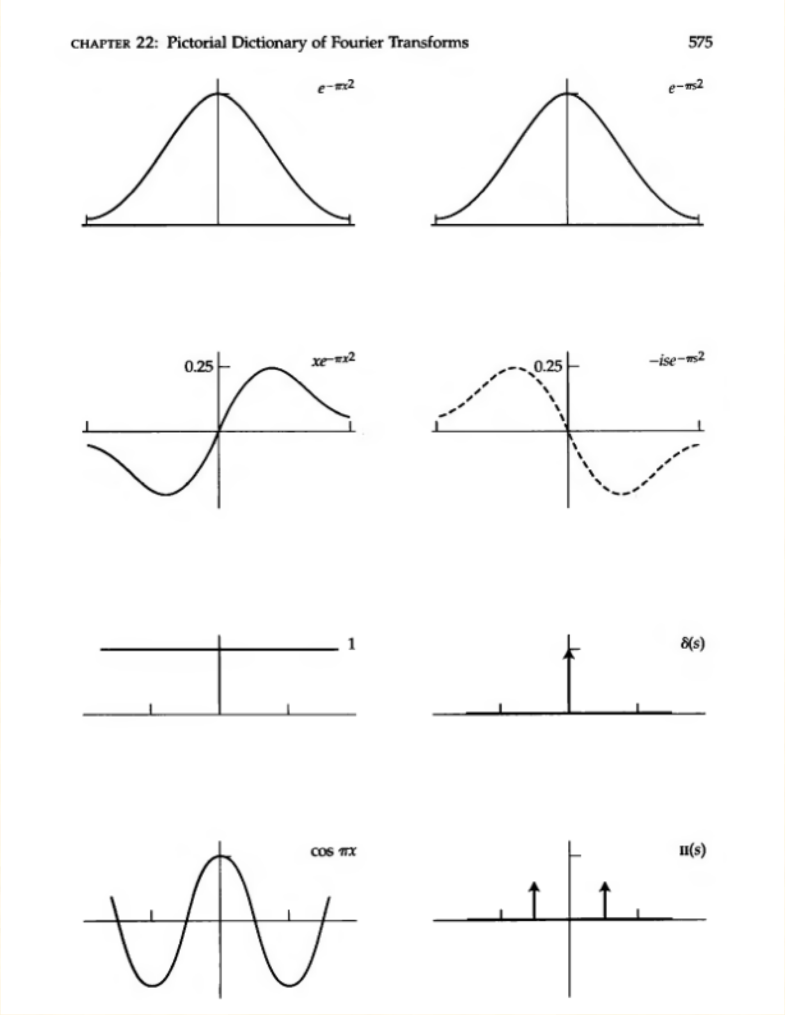
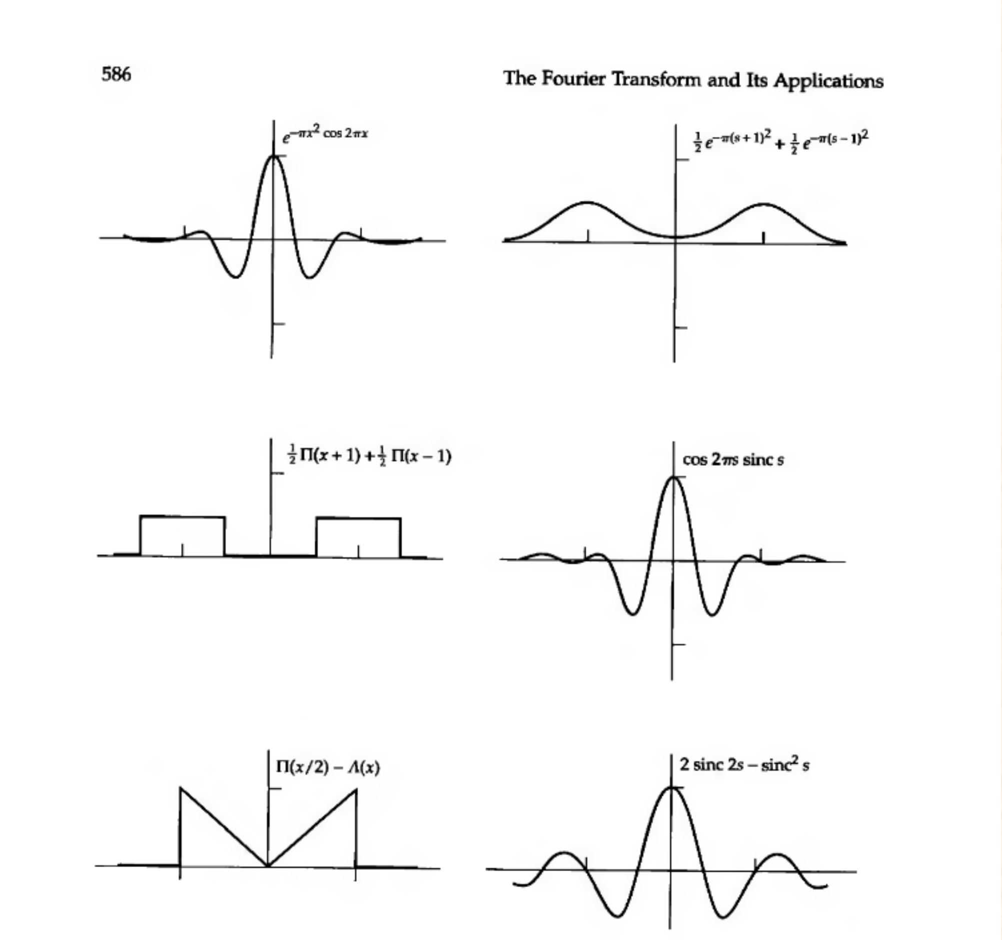


FIGURE 6.2. In creating a spectrogram, the sound waveform is separated into a number of channels equally spaced along a linear frequency axis, and the short-time power in each channel is reported as a function of time.

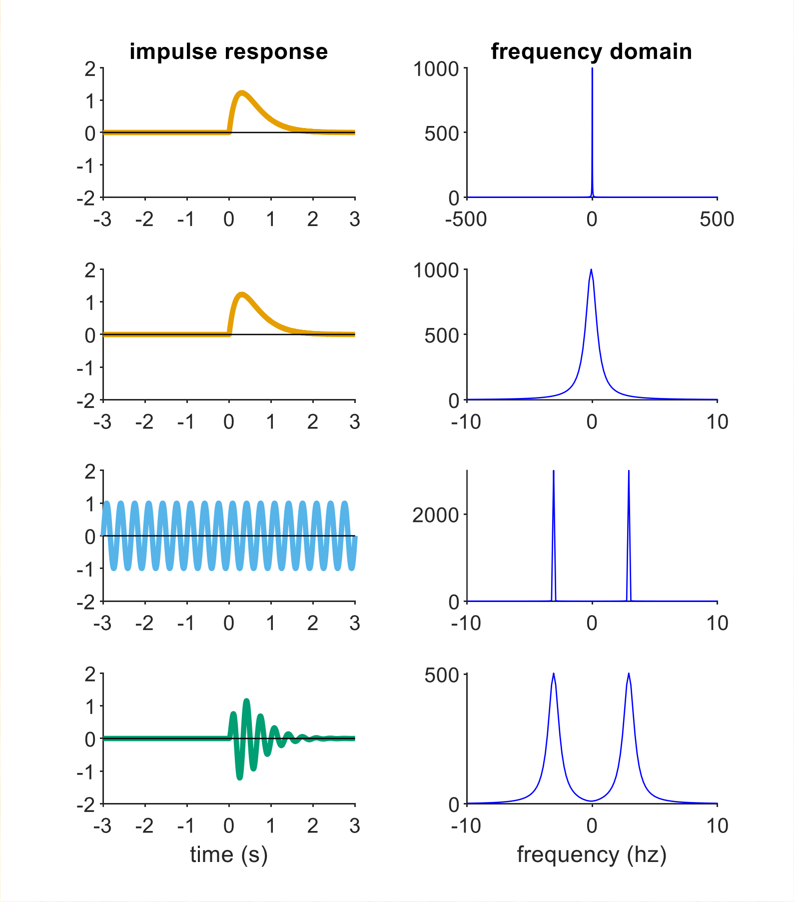
LTI systems – frequency domain



LTI systems – frequency domain



Auditory example: gammatone



Great book for intuitive understanding

Book for 2D as well, I think.