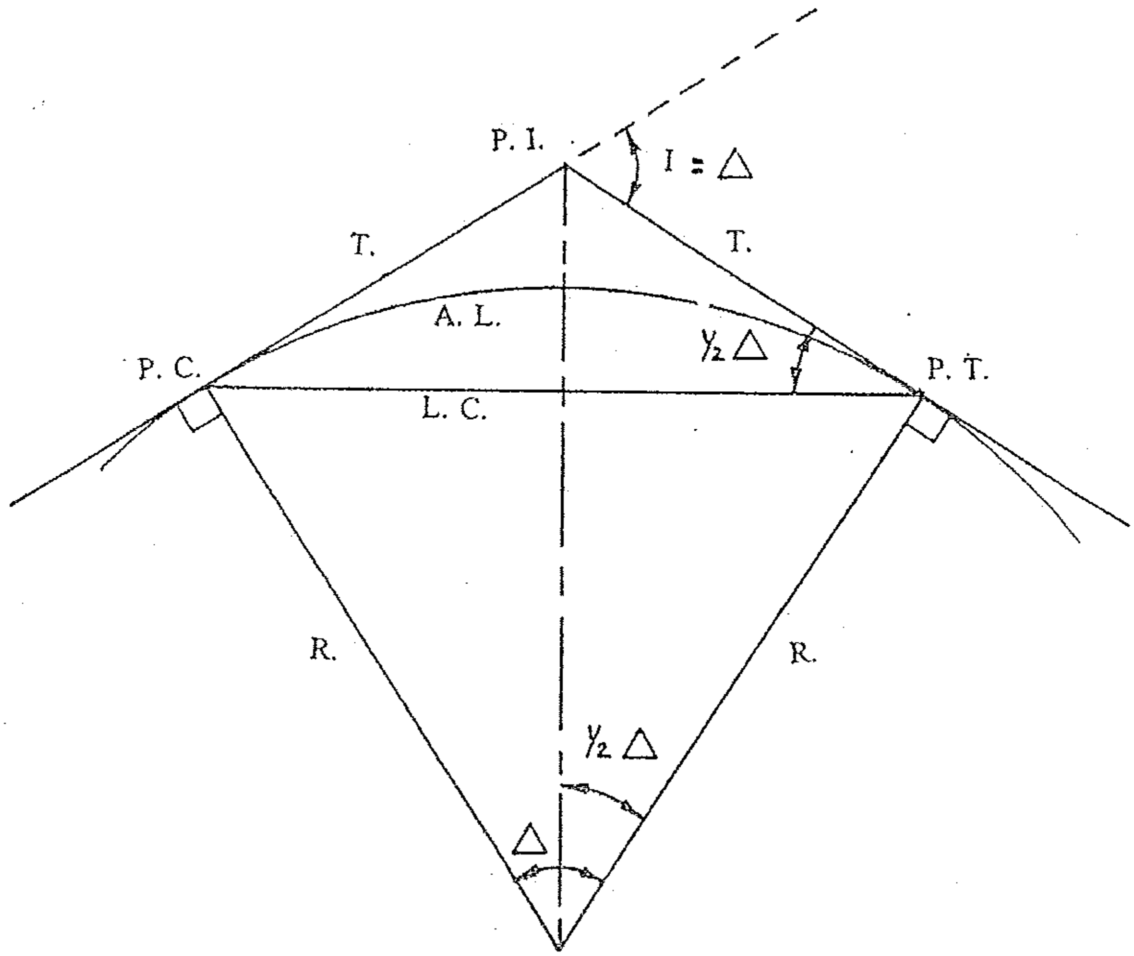
Circular curves can be drawn if a map technician knows the components of the curve and how they relate to each other. Circular curve components are shown in **Figure 1.16.**

Figure 1.16



Definitions of circular curve components are found in **Table 1.2**

Table 1.2

P.C.	(Point of curvature): The point at which the curve begins.
T	(Tangent distance): A line that touches the curve at one point and is at a right angle to the radius at the point of contact. Each curve has two tangents, always equal length; the distance from the point of curvature to the point of intersection or from the point of intersection to the point of tangency.
P.I.	(Point of intersection): The point where the two tangents meet. The deflection angle at this point is equal to the delta angle.
I	(Deflection angle): Angle created between the continuation of one tangent and the other tangent
P.T.	(Point of tangency): The point at which the curve ends.
A.L.	(Arc length): The distance measured along the line of a curve from the P.C to the P.T. **Arc length = Δ times R times (0.0174)**
L.C.	(Long chord): Straight line from the P.C. to the P.T.
R	(Radius): The distance from the tangents to the center of curve or arc. Radii are always perpendicular to the tangents at the point of curvature and the point of tangency. The two Radii are always equal in length.
△	(Delta Angle): Also, know as the central angle. The angle made by the two radii from the center of the arc to the point of curvature (P.C.) and point of tangency (P.T).
P.R.C.	Point of Reverse Curvature): A point at which a curve in one- direction ends and another curve in the opposite direction begins.

Also of interest to the property mapper are the definition of:

Forward tangent – a straight line continuation from the end of the curve (the P. T.) and a

Back tangent – The last straight line drawn before the beginning of the curve (point of curvature the P. C).

$\boxed{AL = \Delta \cdot R \cdot .0174}$

The legal descriptions in the first course "Basic Legal Descriptions" dealt only with straight property lines. However, not all property lines are straight, and it is necessary to describe these curved lines. It is important to bear in mind that curves are segments of the outer boundary of a circle.

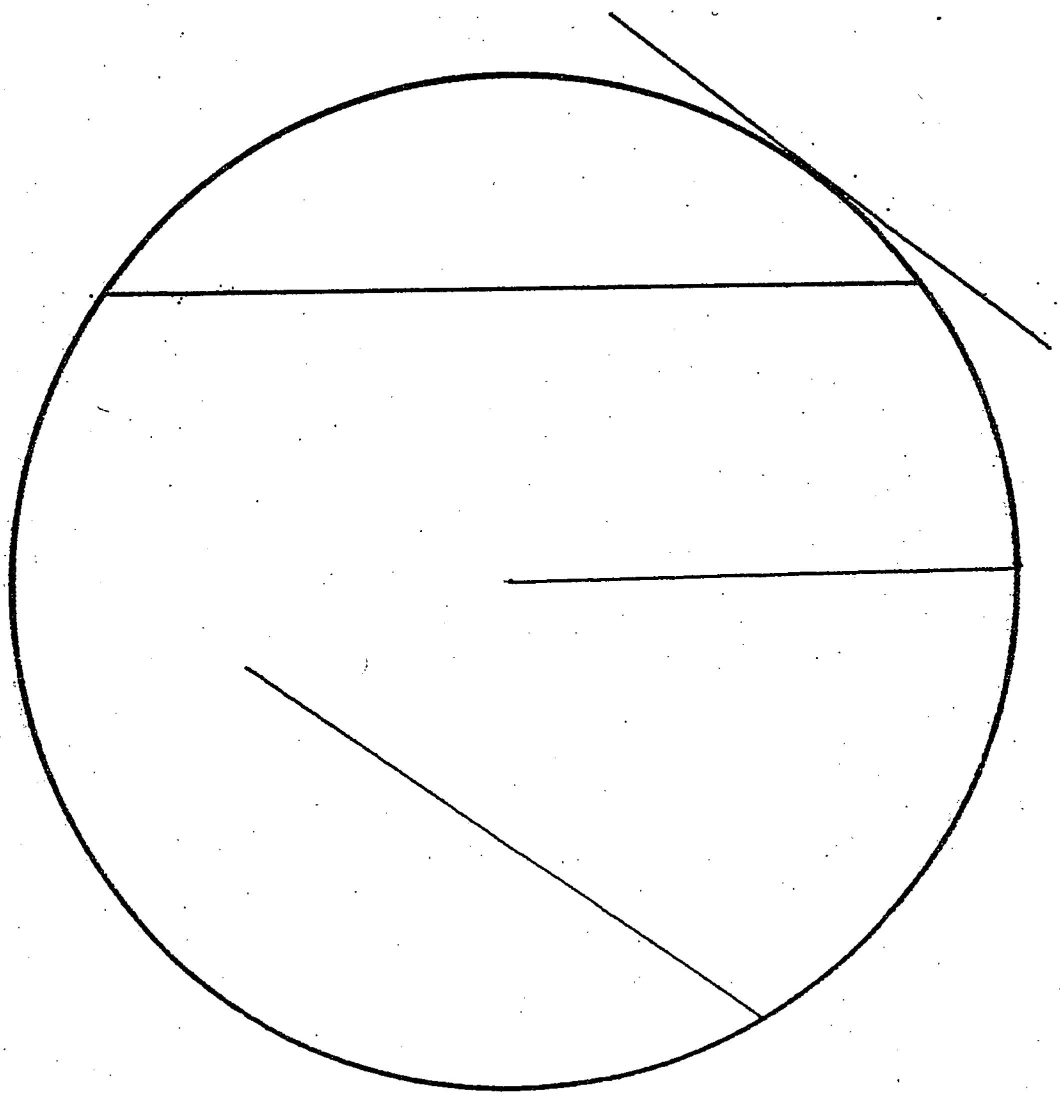
To begin this course, you should be aware of a few definitions.

CIRCUMFERENCE: The outer boundary of a circle.

CURVE: A segment of the circumference of a circle.

RADIUS or RADIAL LINE: The line between the center of the circle and its circumference.

On the circle shown below, which line is a radial line? Write "radial line" on the correct line.

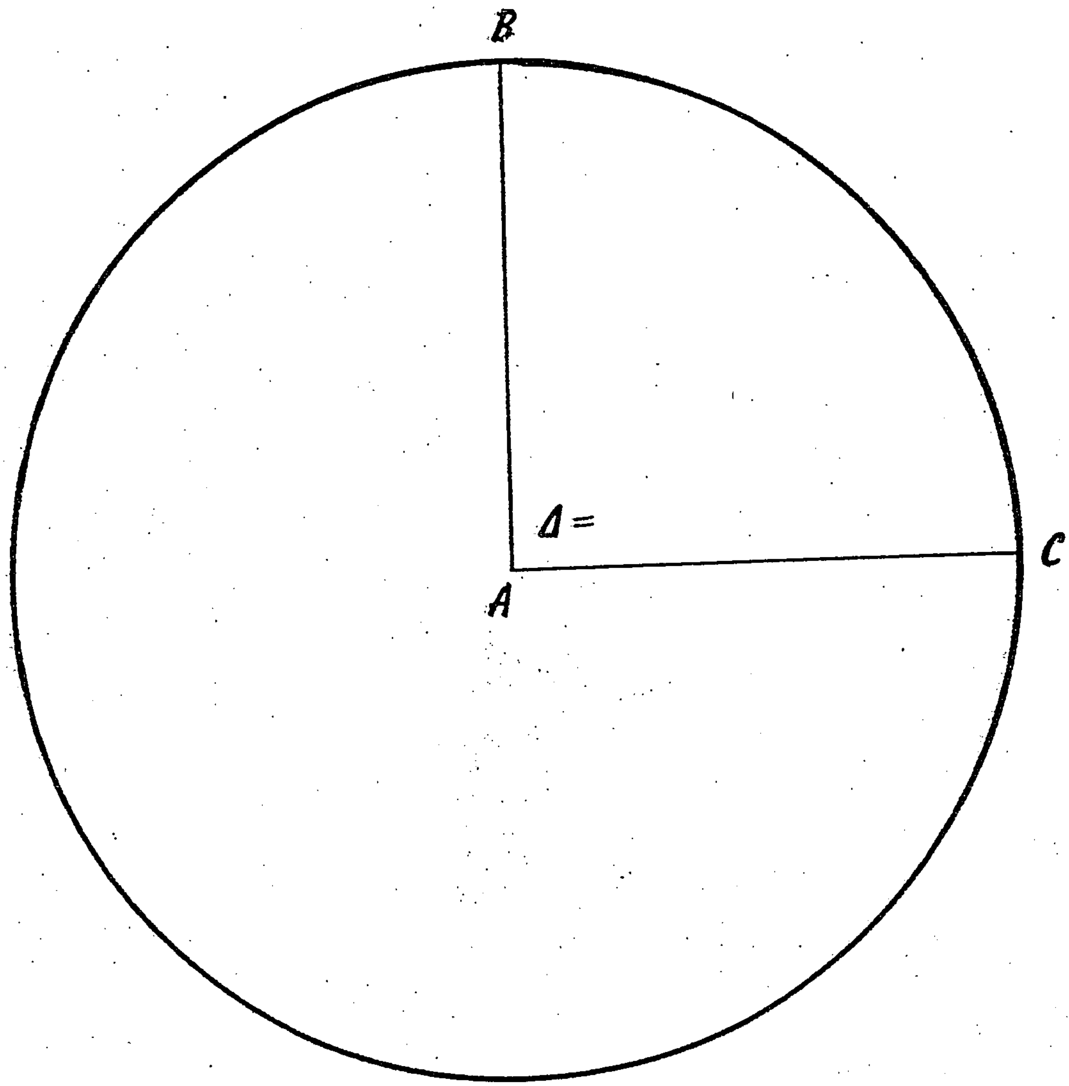


The CENTRAL ANGLE or DELTA (Δ) is the angle at the center of the circle between the radii, and is measured in degrees. A complete circle has 360° (degrees).

The RADIAL POINT is the center of the circle from which the radii originate.

On the circle below, use your protractor:

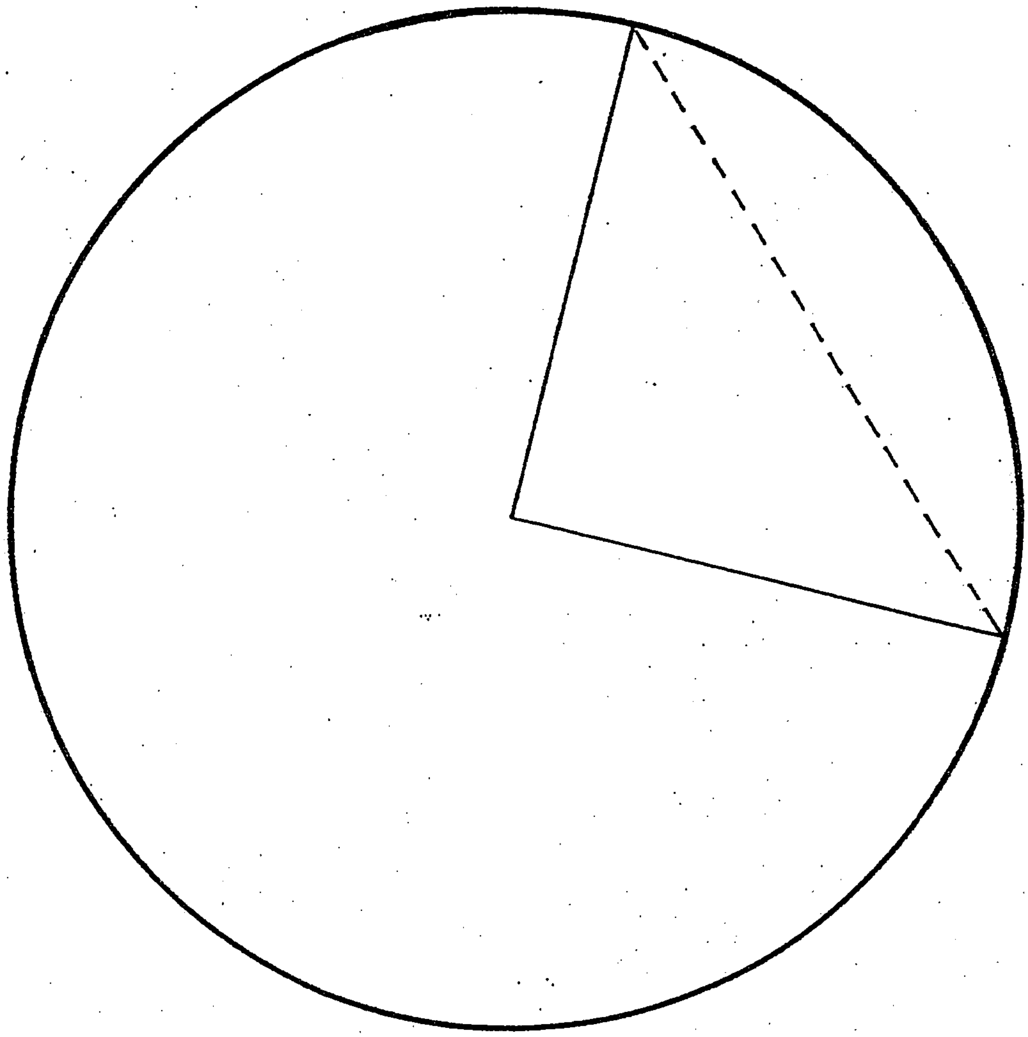
1. Measure the central angle.
2. Label the radial point (Circle A, B, or C)



The LENGTH OF THE ARC is the distance in feet, inches, meters, etc. along some portion of the circumference.

The LONG CHORD is the straight line distance between the two ends of a segment of the circumference of a circle.

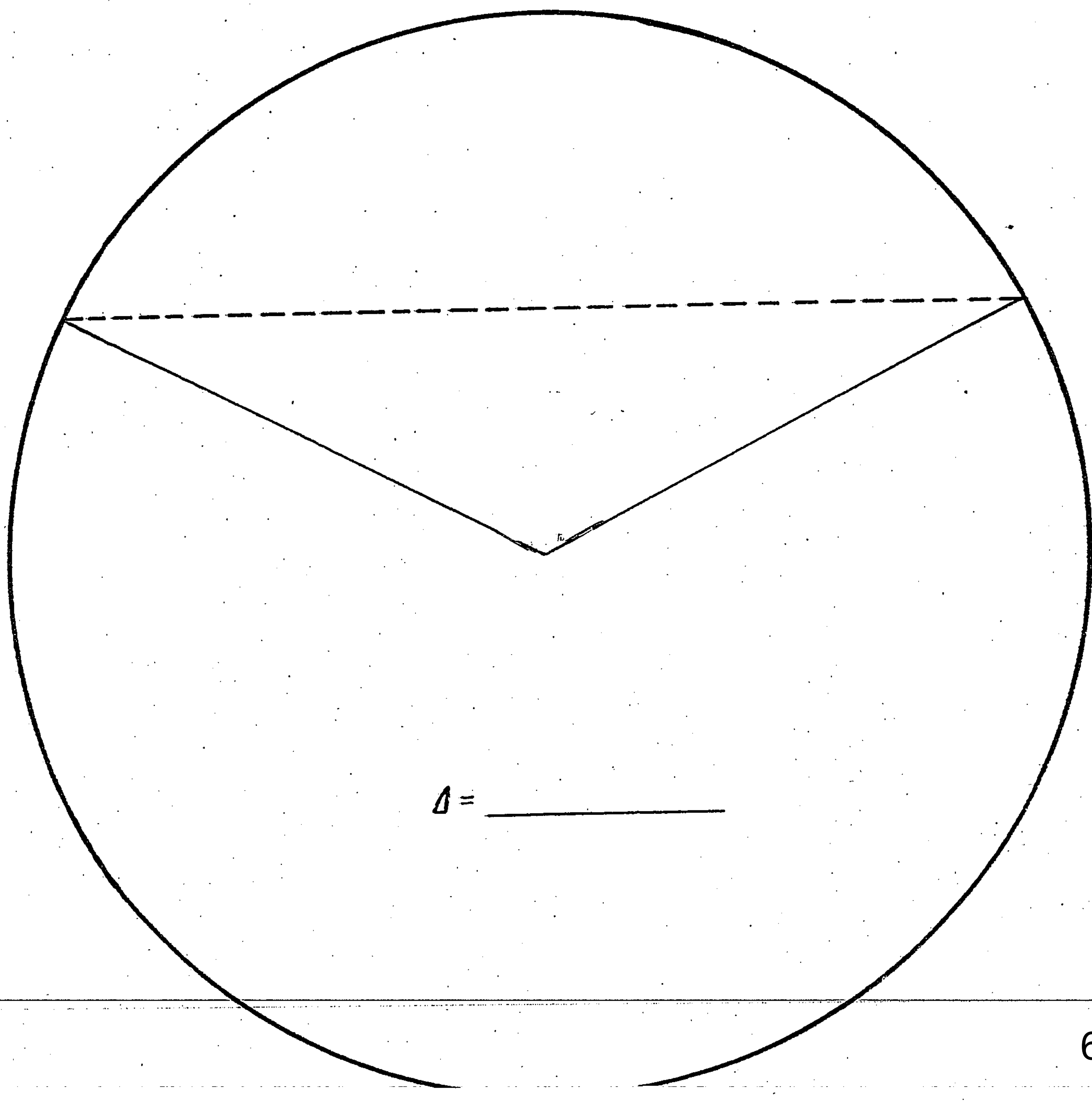
On the circle below, label the long chord and the length of the arc.



Match the parts of a circle to the illustration below. Write your answers on the illustration.

LENGTH OF THE ARC
RADIAL LINE (Label 2)
CENTRAL ANGLE
RADIAL POINT
LONG CHORD

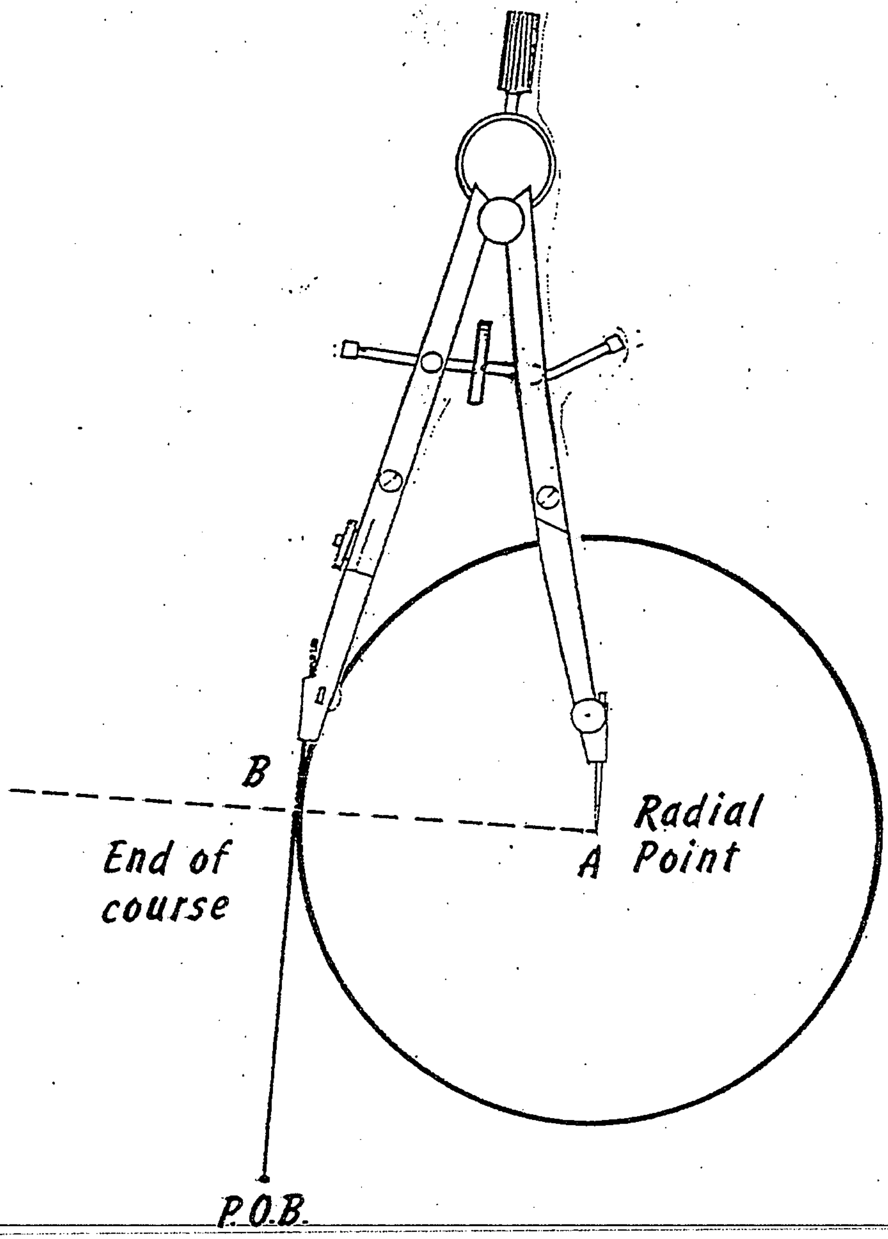
How many degrees are there in a circle?___________
Measure the delta (Δ) of the illustration below and write it.



Δ = ________________

In order to draw a curve or circle, place the metal tip of the compass on the radial point. (A). Adjust the compass so that the drawing end touches the end of the course. Hold the compass firmly and push the pencil around, making sure the metal tip stays on the radial point.

Look at the example below, then go on to the next page.



'Using the lines below as radii, adjust your compass to the length of the radius and draw a circle around each radial point.

Radial
Point

Radial
Point

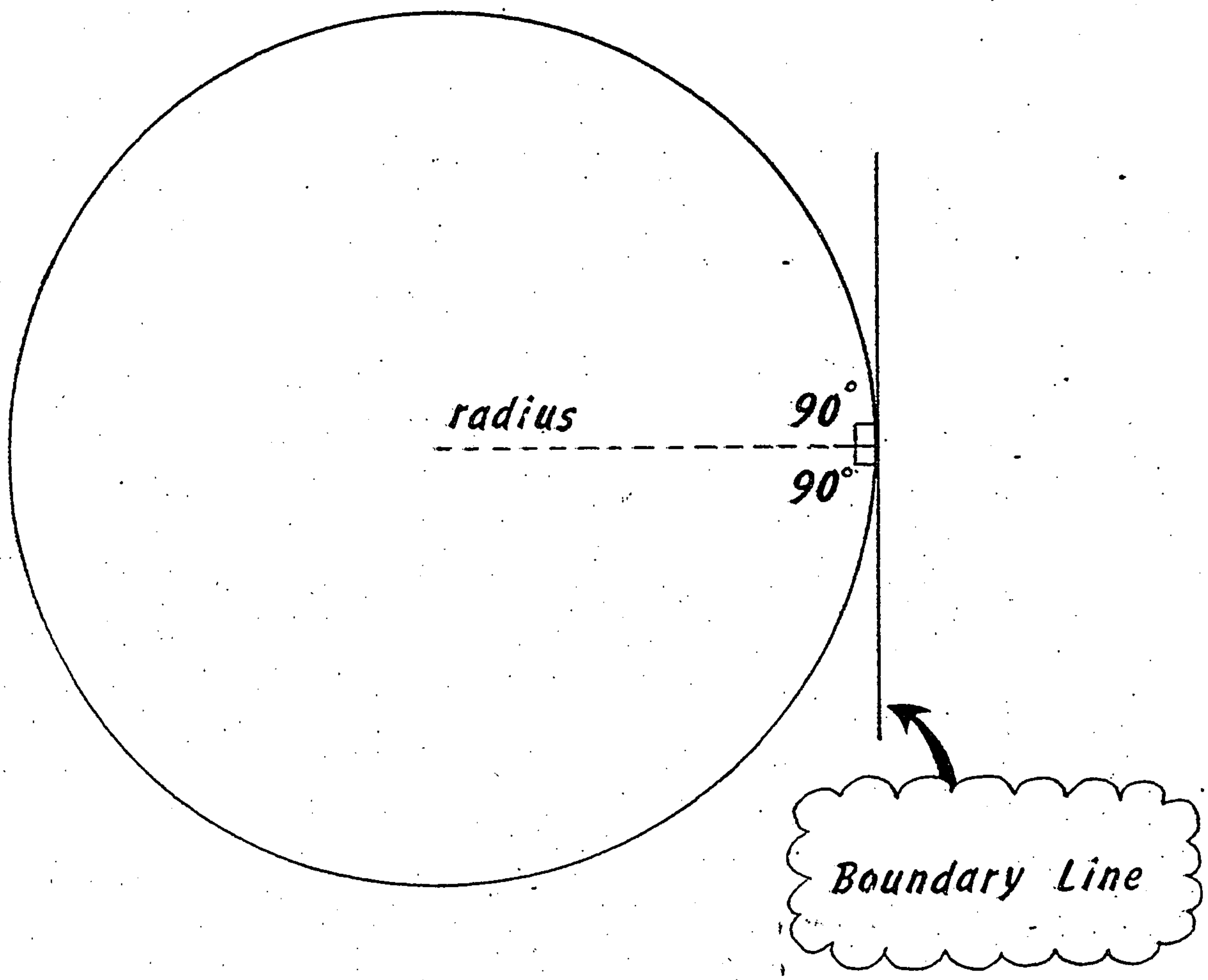
TANGENT CURVES

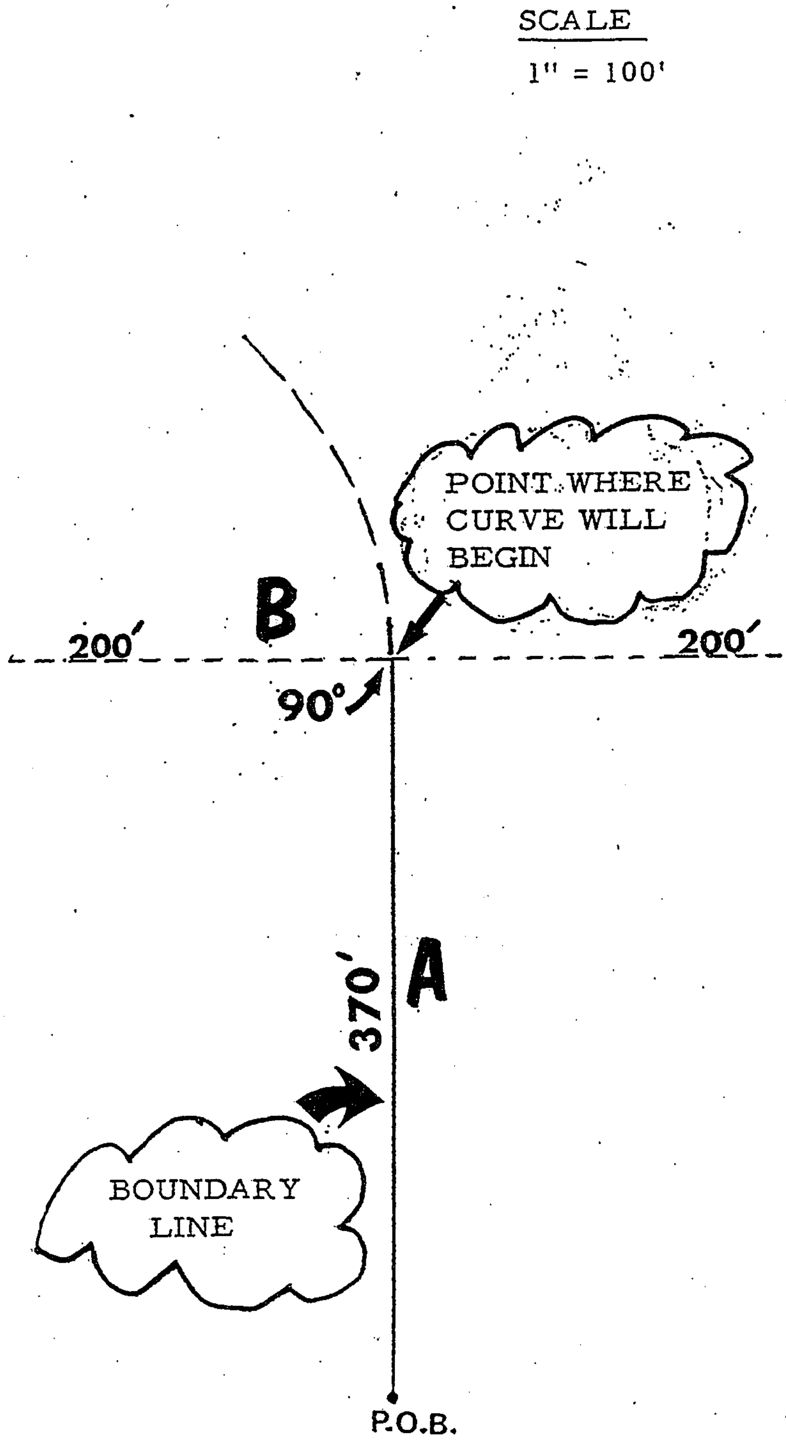
There are numerous kinds of curves. However, this course will deal only with simple tangent and non-tangent curves.

A SIMPLE CURVE is just a single segment of the circumference of a circle.

A simple tangent curve is a curve that has its radius making a 90° angle with the previous course, and the point at which it intersects is called the point of tangency.
On the illustration below, the course (boundary line) and the radius are at right angles (90°) to each other.

DRAW AN ARROW WHICH INDICATES THE POINT OF TANGENCY.




SCALE
1" = 100'
N
POINT WHERE
CURVE WILL
BEGIN
B
200'
200'
90°
A
370'
BOUNDARY
LINE
P.O.B.

The first step in drawing a tangent curve is to draw a line which is 90° to the previous course at the last point in your description.

For example: On the illustration (opposite page), the last measurement was: "From the p.o.b. thence North 370' (A) to the beginning of a tangent curve..."

At the 370' point, you will draw a line at 90° to the previous course. This line is the radius (or radial line) of the circle (B). For the purposes of this example, the radius is 200'——on each side of the course.

UNTIL THE DIRECTION OF THE CURVE IS INDICATED, THE RADIUS MAY BE ON EITHER SIDE OF THE COURSE.

Sketch this description and the 90° radial line to the course——200' on each side: "From the p.o.b. N 40° E 200' to the beginning of a tangent curve having a radius of 200'..."

SCALE

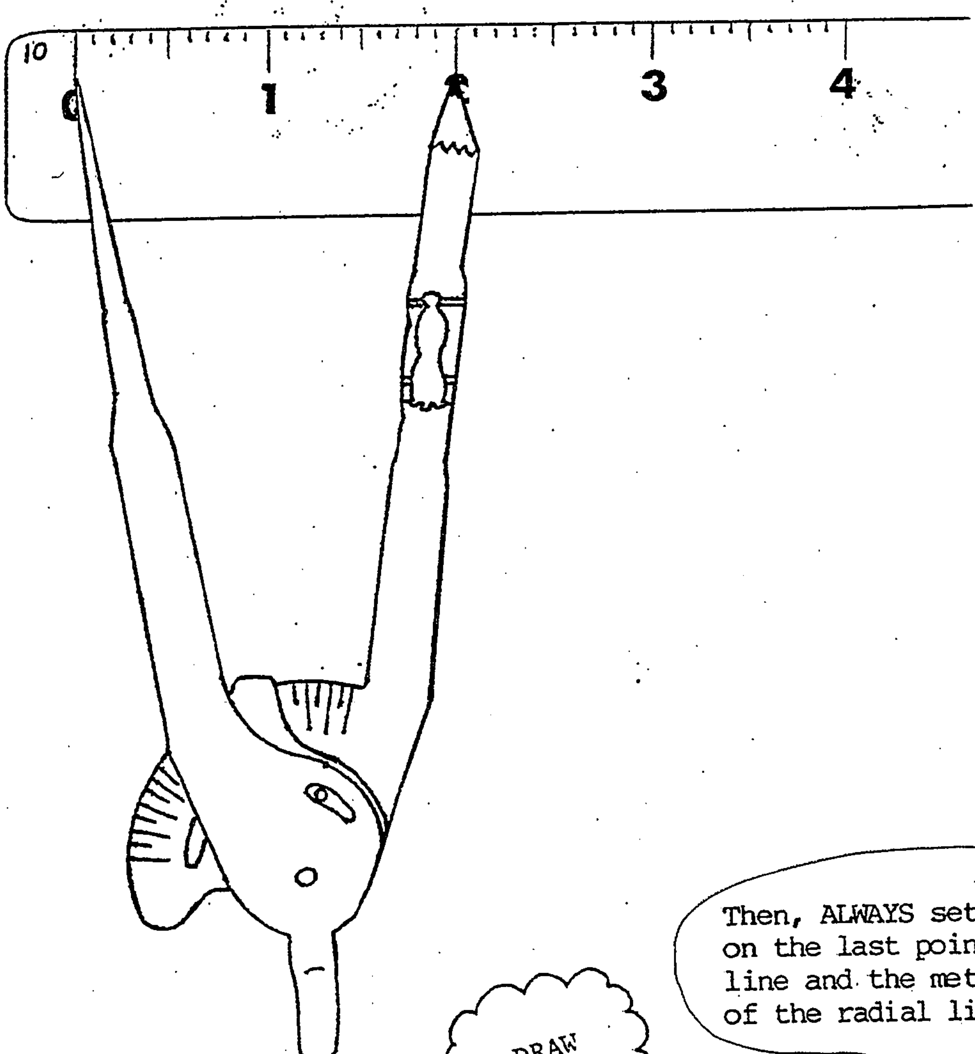
1" = 100'

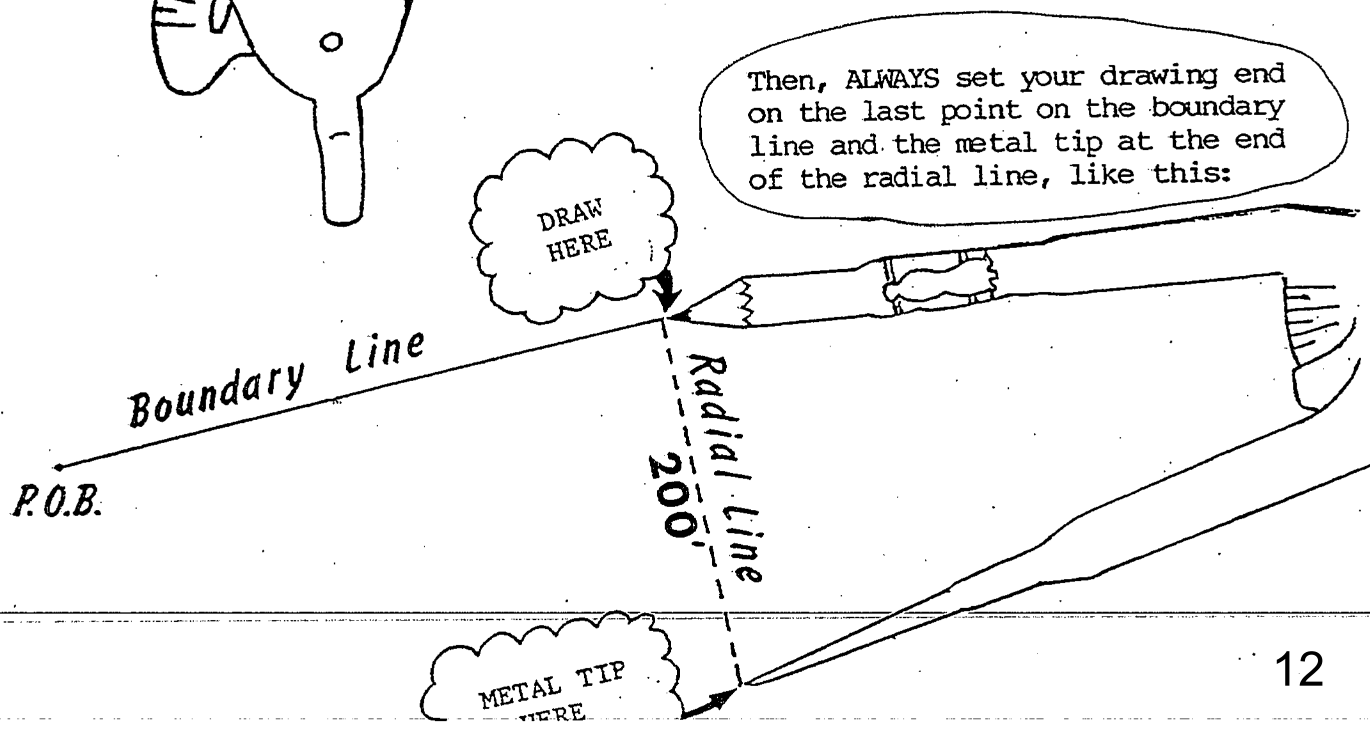
P. O. B.

READ OPPOSITE PAGE FIRST

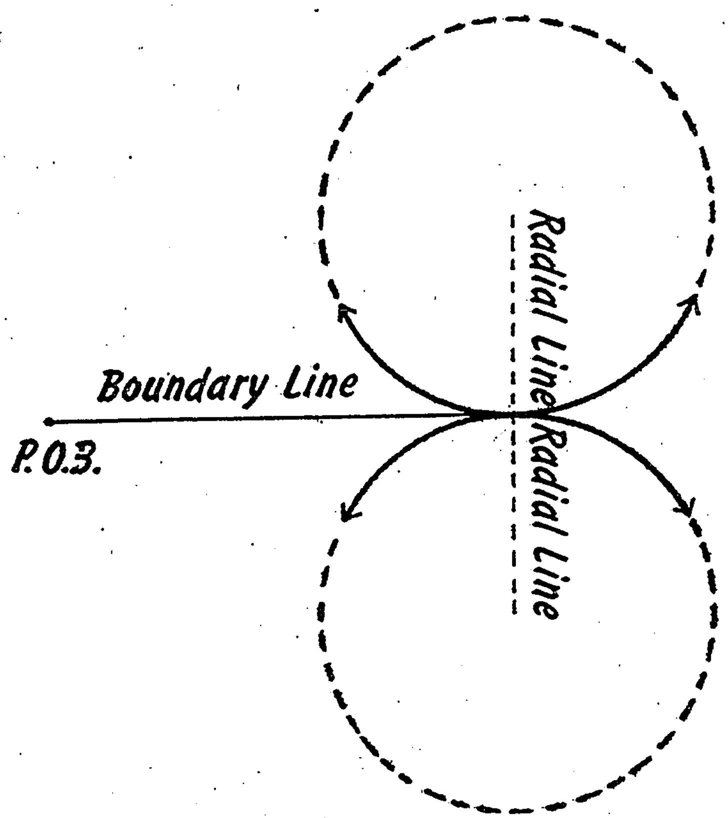
Curves are drawn by first opening your compass to the distance of the radial line using your engineer's scale.

For example, say your radial line is 200' on a scale of 1" = 100'. Set your compass like this:





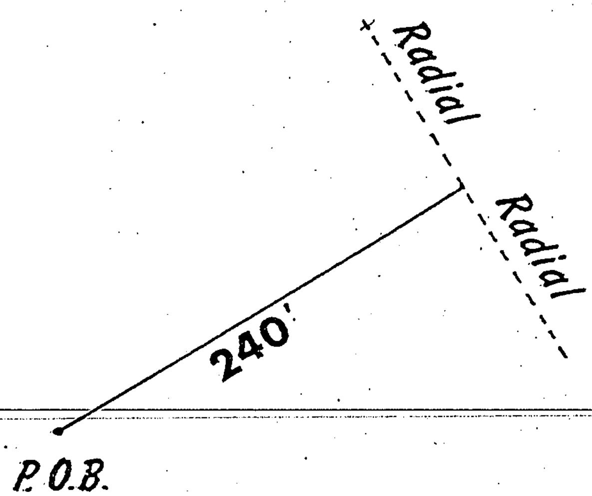
After you have drawn the 90° radial lines to the previous course at the last point in your description, you will then have the centers or radial points for two circles. The curve must begin at the end of the last call (boundary line). Look at the example below:



LOGICALLY, the curve would take one of the four directions shown. Each curve begins at the end of the last call.

NOW, READ THE OPPOSITE PAGE, THEN:

Using the radii shown below, draw two curves which might logically continue the boundary line.



The radius determines the SIZE of the curve which continues the boundary line.

Remember that the curve is a portion of a complete circle. The description for the diagram on the opposite page would read:

"From the point of beginning; thence N 40° E 300' to the beginning of a tangent curve having a radius of 200'; thence along said curve..."

The curve that continues the boundary line will be a portion of one of the circles.

TRY THIS ONE: (Use your compass and draw circles on both sides of the course.)

"From the P.O.B., thence East 150' to the beginning of a tangent curve having a radius of 100'..."

SCALE

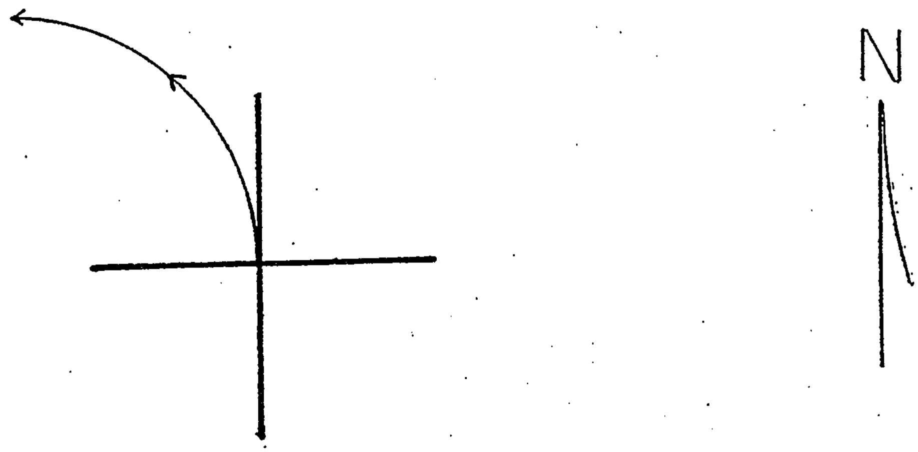
1" = 100'

P.O.B.

In order to decide which part of which circle to use, the curve must be described.

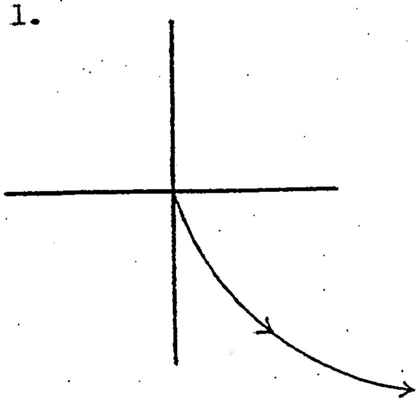
First, the curve is usually described using the quadrant or quadrants (NE, NW, SE, and SW) in which the curve travels.

For example, the curve shown below is traveling in the northwest quadrant, and would therefore be described as traveling in a northwesterly (NWrly) direction. Study this example, then answer the question below.



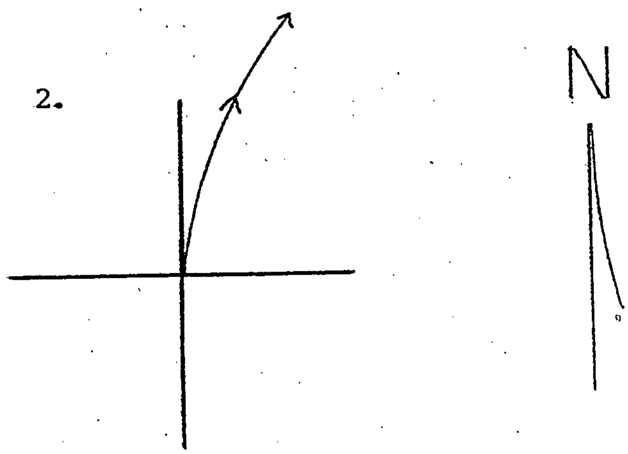
How would you describe the direction of travel of the curves shown below?

1.



____

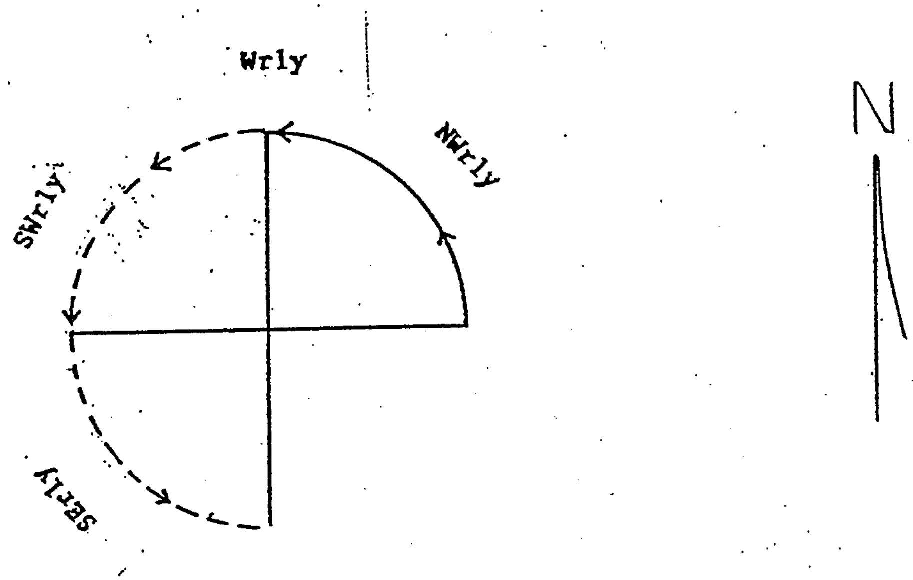
2.



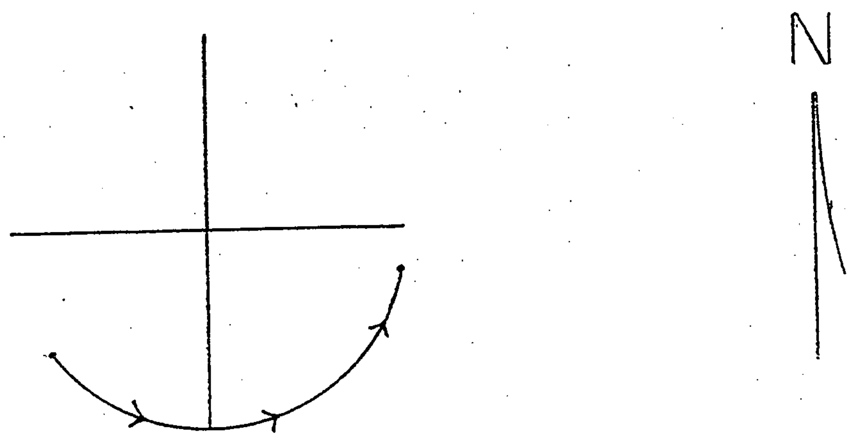
____

In some cases, a curve may be described as traveling in more than one direction. For example, the curve below might be described as traveling "northwesterly, and westerly." It begins by traveling northwest, and ends traveling almost directly westward.

If the curve went any farther along the dashed line, it might even be described as "northwesterly, southwesterly and southeasterly along a curve."



In what direction or directions does this curve travel?

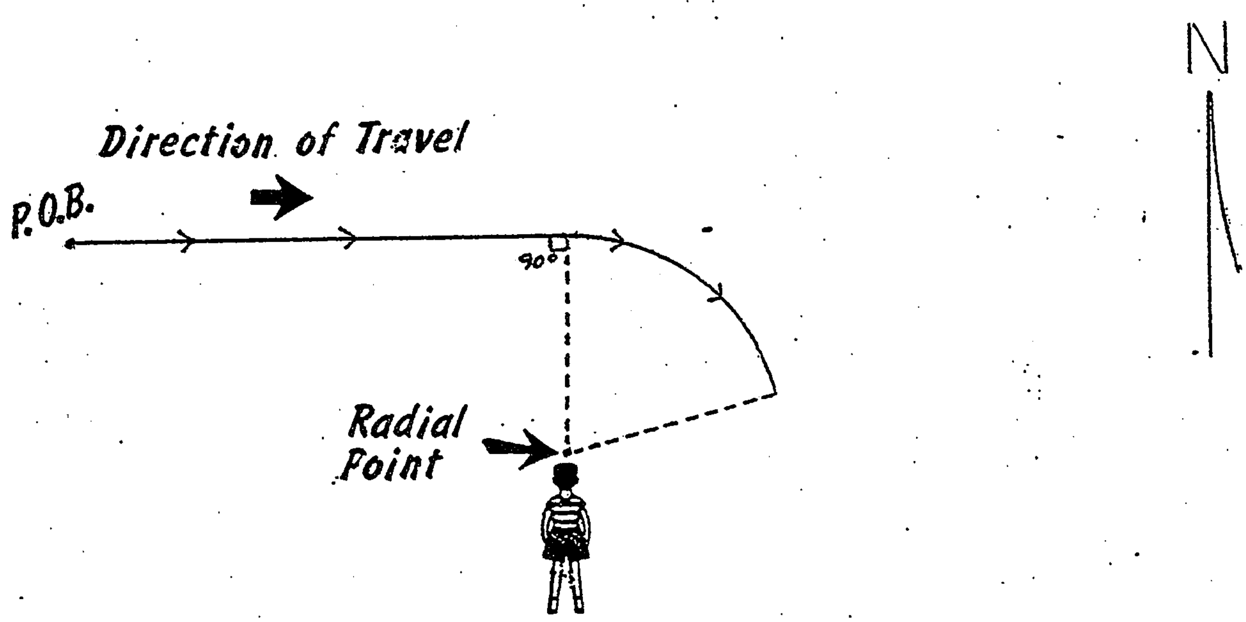


____

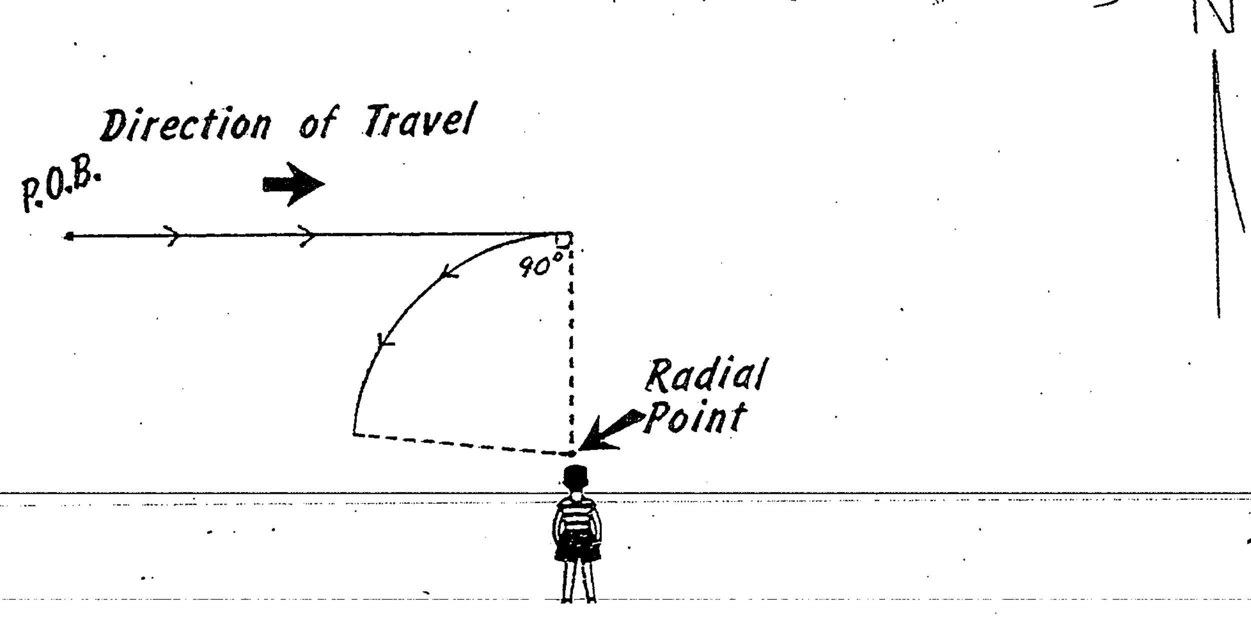
In the examples below, the direction of the curve (right or left) is determined by the direction of travel.

An easy way to determine if the curve is to the right or to the left is to place yourself at the radial point (the center of the circle or curve) and face the curve.

Southeasterly along
a curve to the RIGHT (CLOCKWISE)



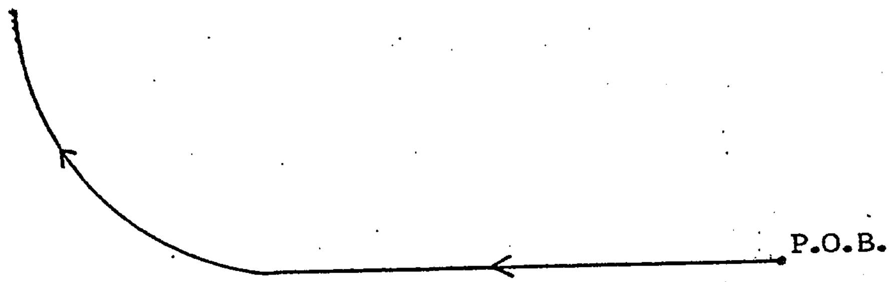
Southwesterly along
a curve to the LEFT (COUNTER-CLOCKWISE)



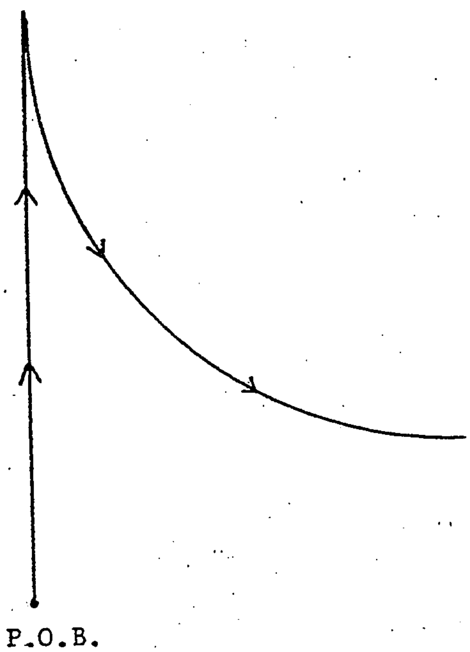
The description will also tell you if the curve is to the left or to the right. For example, a description might read: "...thence southeasterly along a curve to the left.

If you can imagine yourself standing at the radial point of each curve, you will see which way the course is traveling.

Study the examples on the opposite page, then determine the direction and bearings of the curves below.



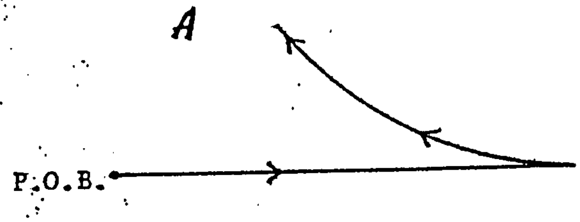
A. ______________________________

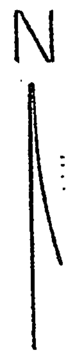


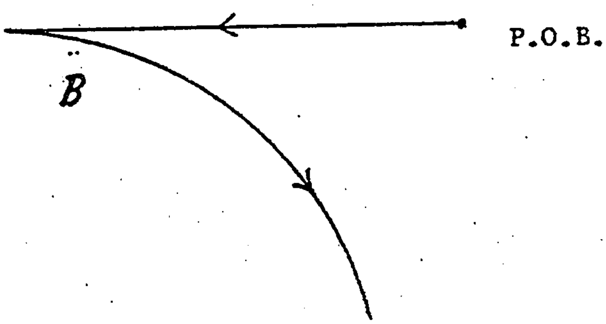
B. ______________________

These may be a little more difficult. Write in the direction of the curve below each example.

A

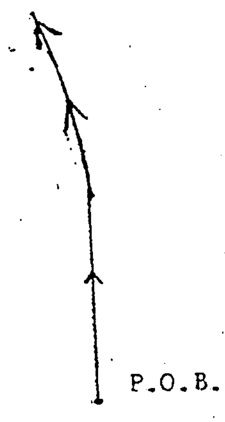






B

C

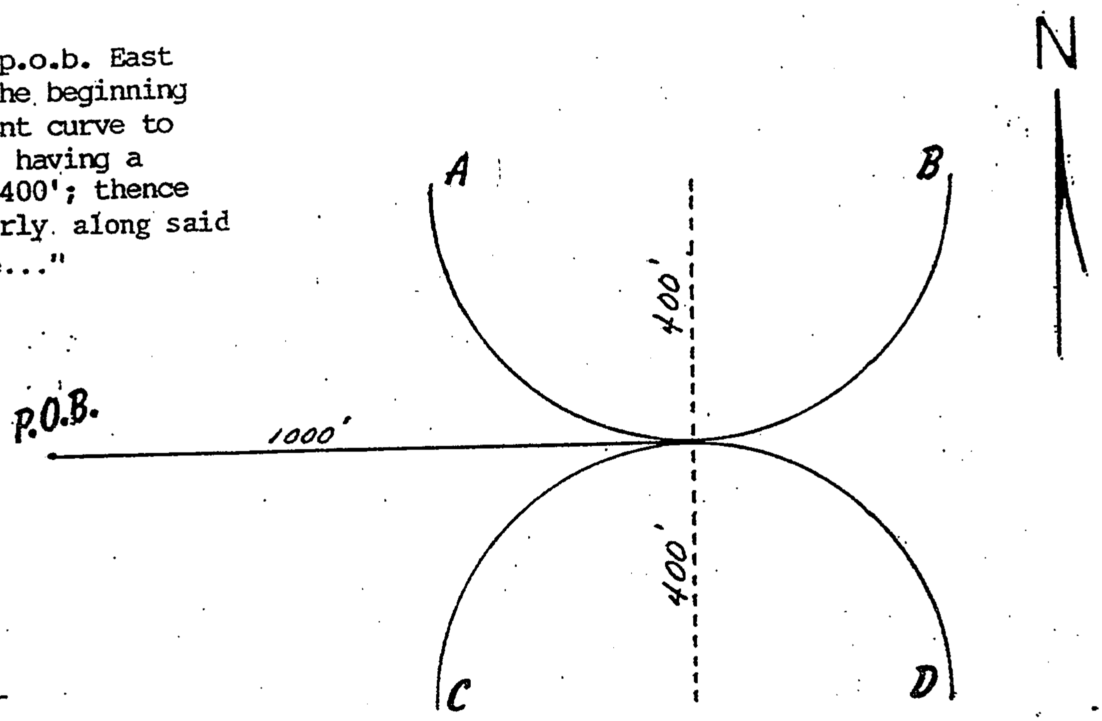


Choose the correct curve which matches the description.
The scale for both problems is 1" = 300'

"From the p.o.b. East 1000' to the beginning of a tangent curve to the right, having a radius of 400'; thence northwesterly along said said curve..."

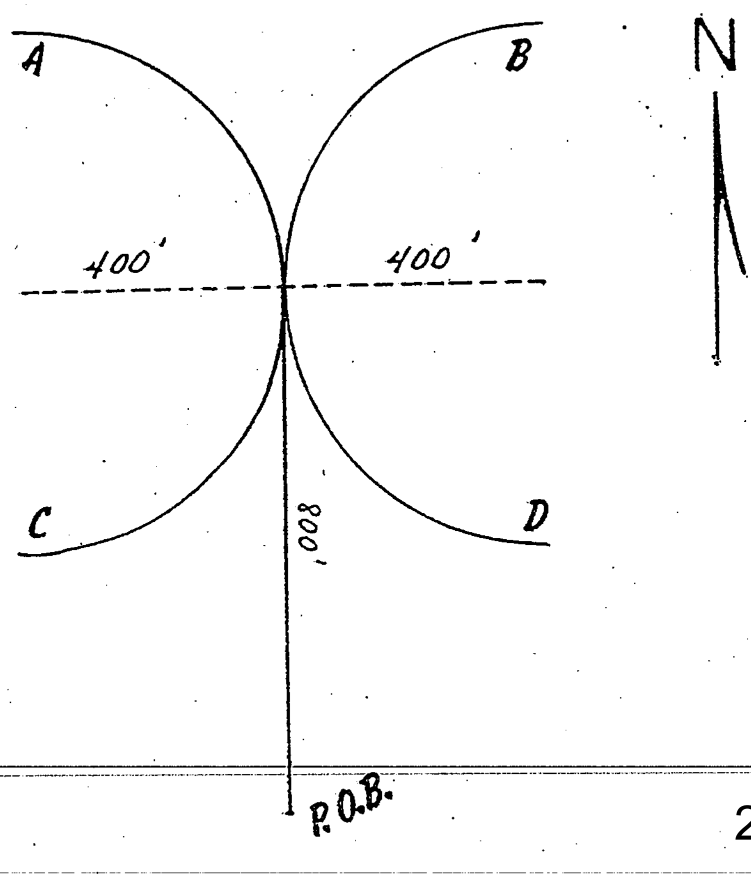
Scale

1" = 300'



1. ________

"From the p.o.b. North 800' to the beginning of a tangent curve to the right having a radius of 400'; thence southwesterly along said curve..."



2. ________

So far we have studied two steps for completing a tangent curve.

#1. Draw a radial line 90° to the previous course at the last point in the description.

#2. Determine if the curve is to the right or to the left and in which direction (quadrant) it is traveling.

REMEMBER: The curve must logically continue the boundary of the property. The curve will start at the <u>end</u> of the last point of the description.

Let's try one. Use a scale of 1" = 200'.

"From the point of beginning, East 500' to the beginning of a tangent curve to the left having a radius of 400'; thence southwesterly along said curve..."

Now that we have the direction in which the curve is traveling, we can now determine the LENGTH of the curve.

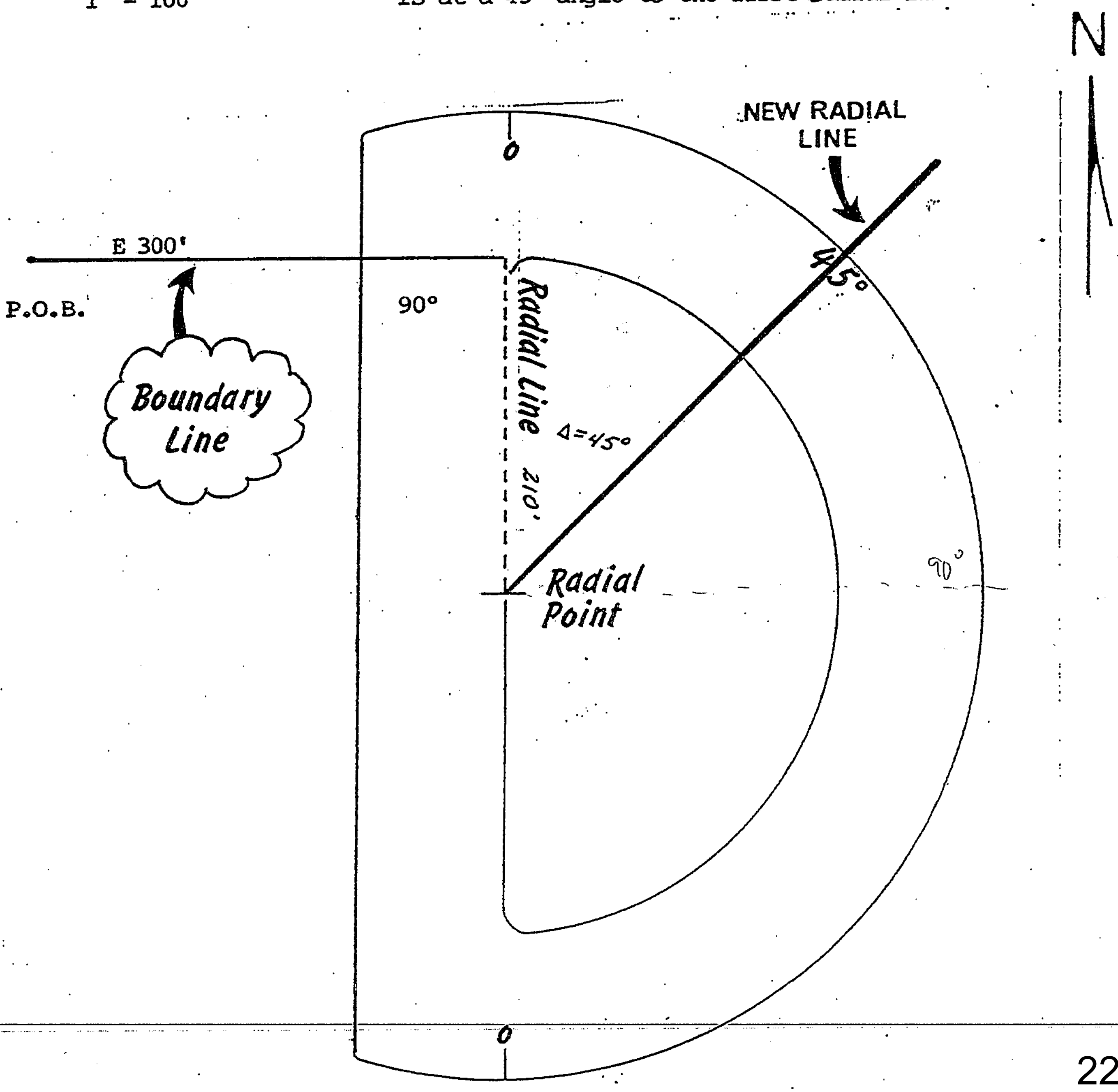
The length of the curve is determined by the size of the central angle through which the curve travels. This means you will draw an additional radial line at a specified angle to your first radial line.

For example, suppose we had the following description: From the p.o.b. East 300' to the beginning of a tangent curve to the right having a radius of 210'; thence southeasterly through a central angle of 45°...

Scale

1" = 100'

You would lay your protractor on the radial line as shown and draw the new radial line. This line is at a 45° angle to the first radial line.



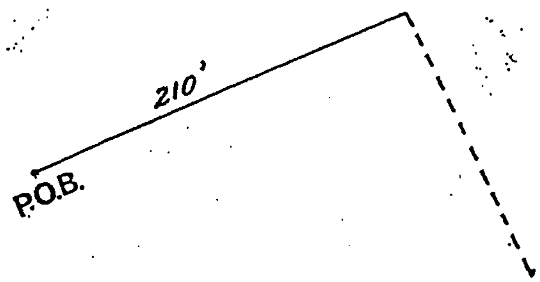
READ THE OPPOSITE PAGE FIRST
Then complete these exercises

Draw the angles for the following descriptions: (Make the new radial line the same length as the one shown.)

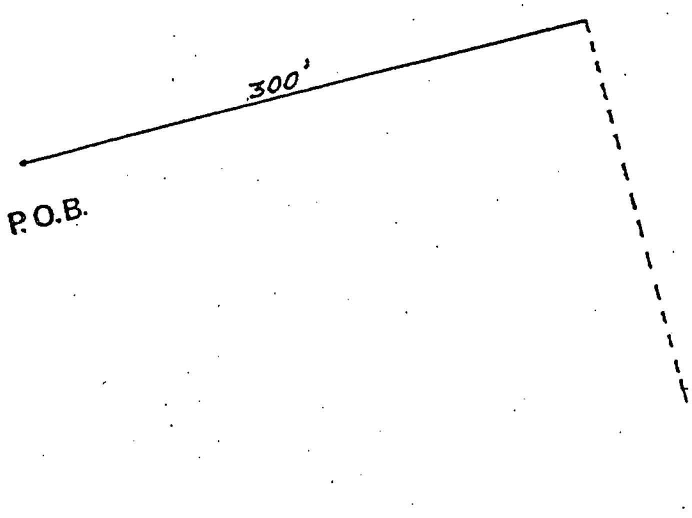
... a curve to the right, having a radius of 150'; thence southeasterly through a central angle of 50°...

<u>Scale</u>

1" = 100'



... a curve to the left having a radius of 200'; thence southwesterly through a central angle of 70°... (Remember to imagine yourself standing at the radial point.)



<u>Scale</u>

1" = 100'

Now, all that's left is to draw the curve. Let's go through the steps one more time.

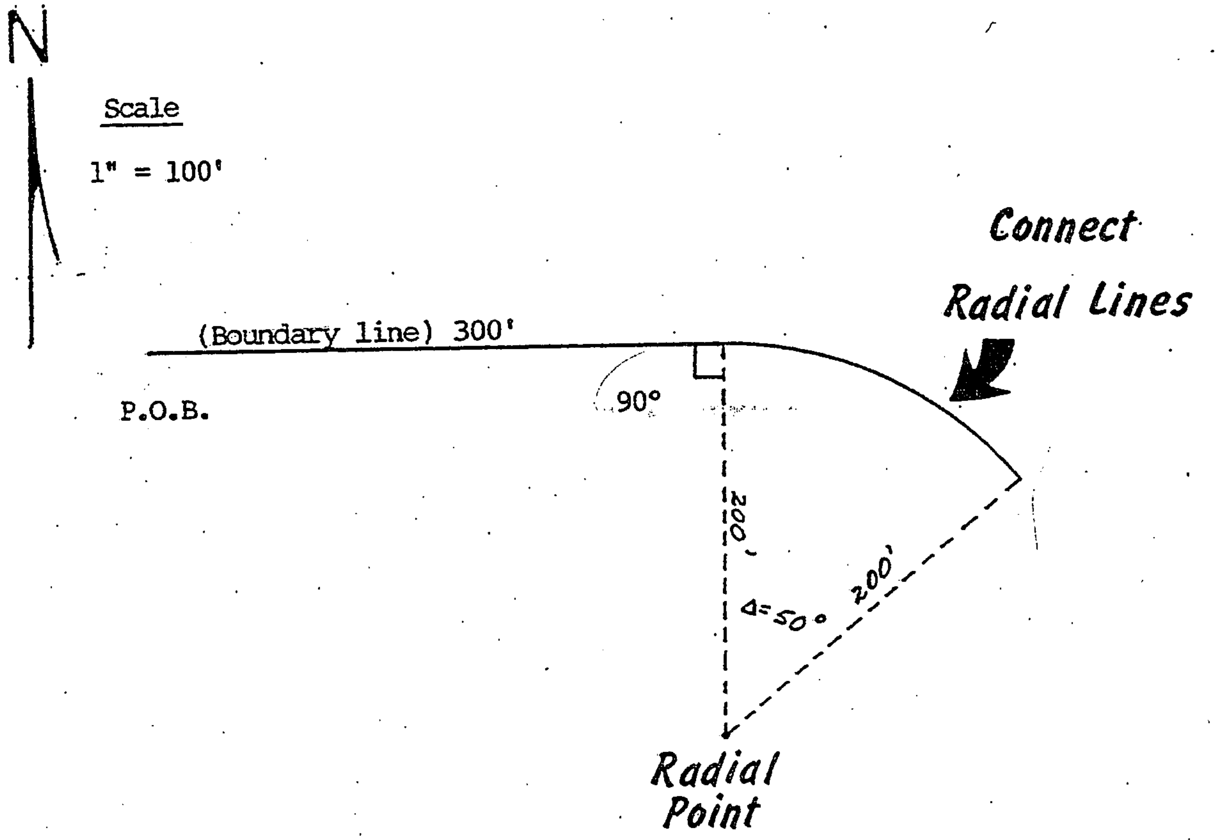
#1. Draw a radial line 90° to the previous course at the last point in the description.

#2. Determine whether the curve is to the right or left by placing yourself at the radial point, and detemine the bearing (NWrly, NErly, etc.)

#3. Using your protractor, measure a central angle, and draw the new radial line, the same length as the first radial line.

Now, the last step:

#4. Set your compass to the length of the radial lines by using your Engineer's Scale and connect the two radial lines, like this:



READ OPPOSITE PAGE FIRST

NOTE: The legal description may also tell the LENGTH OF THE ARC, or the length of the Long Chord.

FOR THE PURPOSES OF SKETCHING LEGAL DESCRIPTIONS IN THIS COURSE, YOU DO NOT NEED TO BE CONCERNED WITH THE LENGTH OF THE ARC (ARC DISTANCE).

Draw the following description:

"From the p.o.b. east 300' to the beginning of a tangent curve to the right having a radius of 200'; thence southeasterly an arc distance of 325' through a central angle of 100°; thence S70° W 310'; thence N30° W 400' to the point of beginning."

SCALE

1" = 100'

P. O. B.

That portion of the SW 1/4 of the SW 1/4 of Section 22, T4S R3E, San Bernardino Base and Meridian in the county of Pear, State of Nevada, described as follows:

Beginning at the southwest corner of said Section 22, thence East 300' to the beginning of a tangent curve to the left having a radius of 200'; thence northeasterly along said curve through a central angle of 80° an arc distance of 300'; thence N40° E 150' to the beginning of a tangent curve to the left having a radius of 200'; thence northeasterly along said curve through a central angle of 70° an arc distance of 270'; thence West 430'; thence S20° W 540' to the point of beginning.

Using the description on the opposite page, draw a map. Label the boundaries with the bearings and distances.

<u>SCALE</u>

1" = 100'

P. O. B.

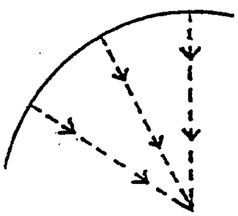
Another way to describe a curve is to define the direction in which it is concave.

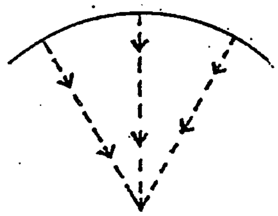
For example, a legal description might read "...to the beginning of a tangent curve, concave to the south and having a radius of 100'; thence southwesterly along said curve through a central angle of 45°..."

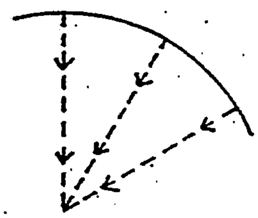
CONCAVE refers to the inside of a circle, or "towards the center." Concave to the south indicates the direction from the middle of the arc to the center of the circle.

For example, the curves below can be said to be:

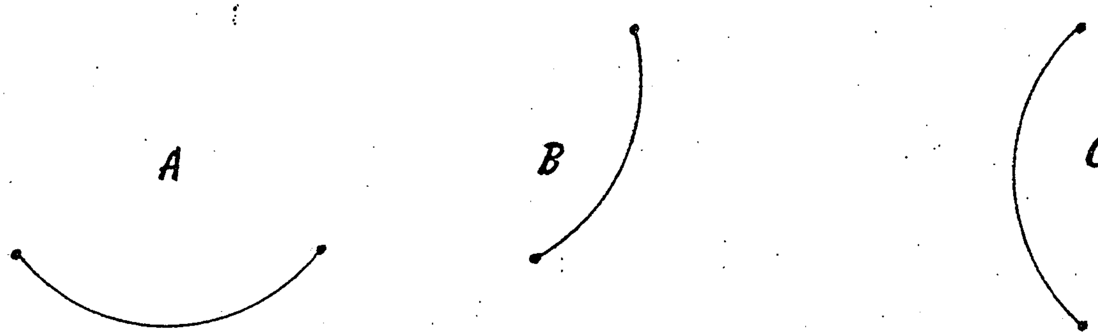
CONCAVE TO THE SOUTHEAST	CONCAVE TO THE SOUTH	CONCAVE TO THE SOUTHWEST





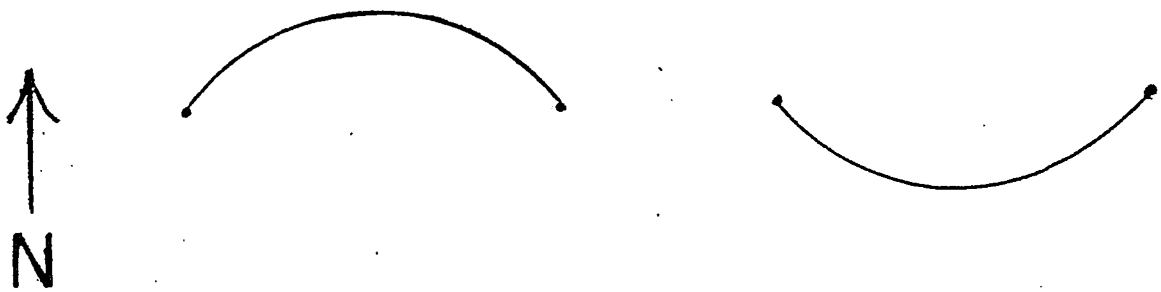


What is the direction of the concavity of each of the curves below?



(A)________________ (B)________________ (C)________________

On the lines provided, give the direction of the concavity of the arcs below.



A. ______________

B. ______________

C. ______________

D. ______________

Now, try sketching the following legal description.
Mark all the measurements and directions on your sketch.

..."From the point of beginning thence East 300' to the beginning of a tangent curve concave to the northwest and having a radius of 200'; thence northeasterly along said curve through a central angle of 100°, an arc distance of 350'; thence N63°W 400'; thence S19°W 435' to the P.O.B."

SCALE

1" = 100'

N

P. O. B.

NON-TANGENT CURVES

A non-tangent curve is any curve which has a radius at any angle other than 90° to the previous curve.

Suppose we had this description: "From the p.o.b. North 800' to the beginning of a non-tangent curve (to the left) or (concave to the northeast) having a radius of 600' and to which beginning a radial line bears N54° E; thence southeasterly..."

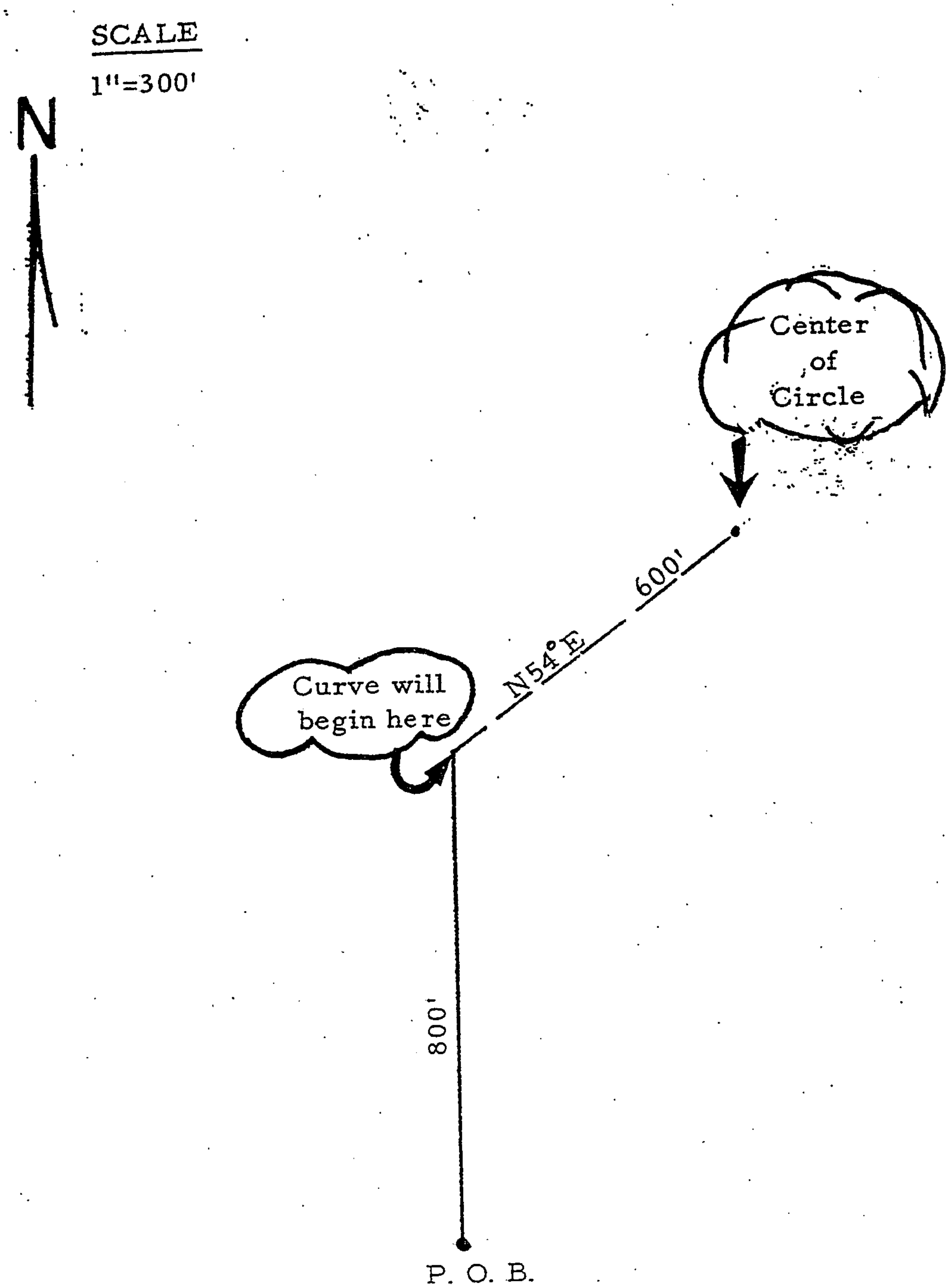
Begin at the p.o.b. shown below, draw the 800' course and the radial line. Use a dashed line to represent the radial line.

SCALE

1" = 200'

P. O. B.

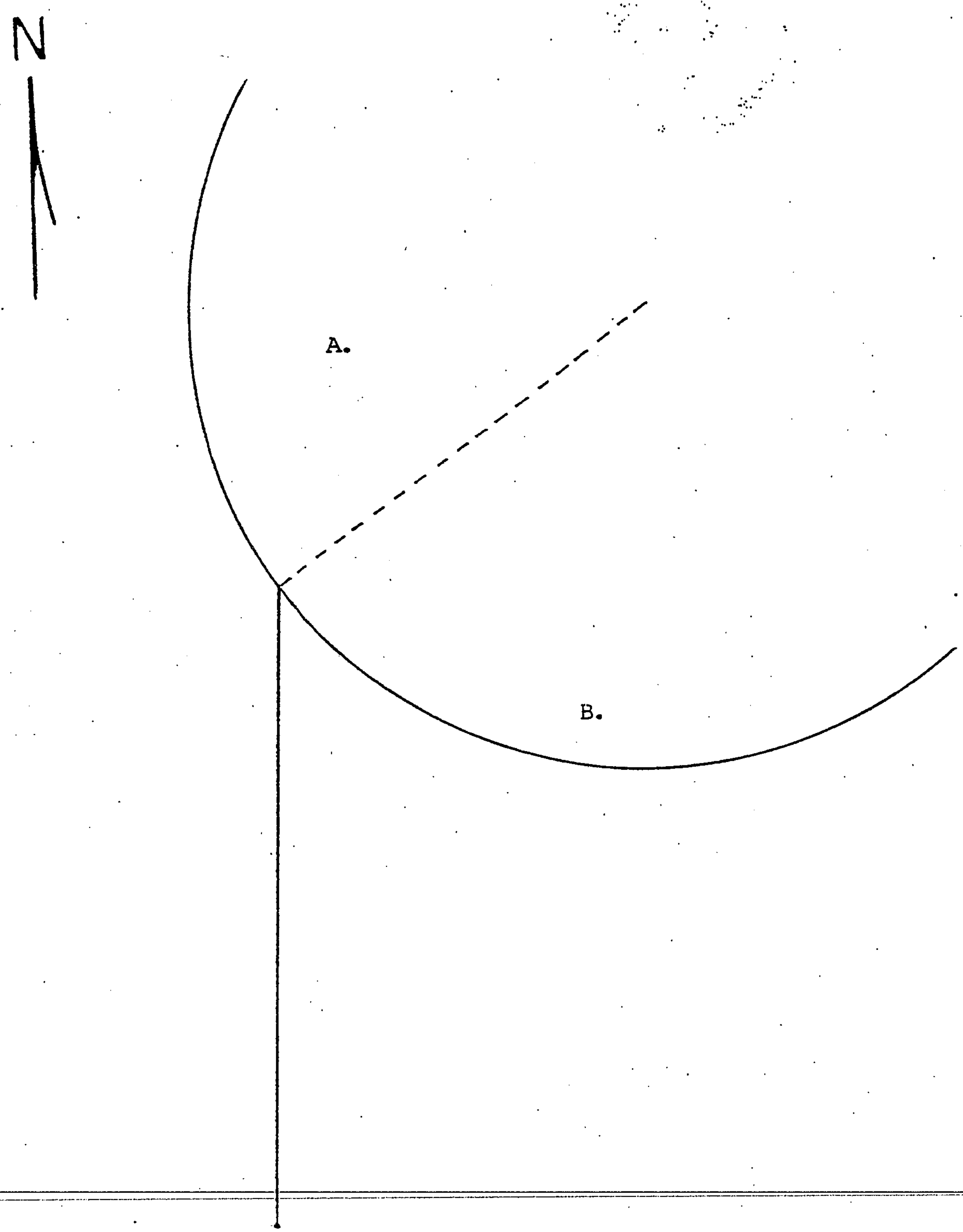
At the point where the radial line and the previous course meet is where the curve will begin. Use your compass and draw a complete circle using the radial line.



Continuing with the same description, a further call might be "...thence southeasterly along said curve..."

1. Using the illustration below, which curve is being described? ________

2. Is it a curve to the left or right? ______________

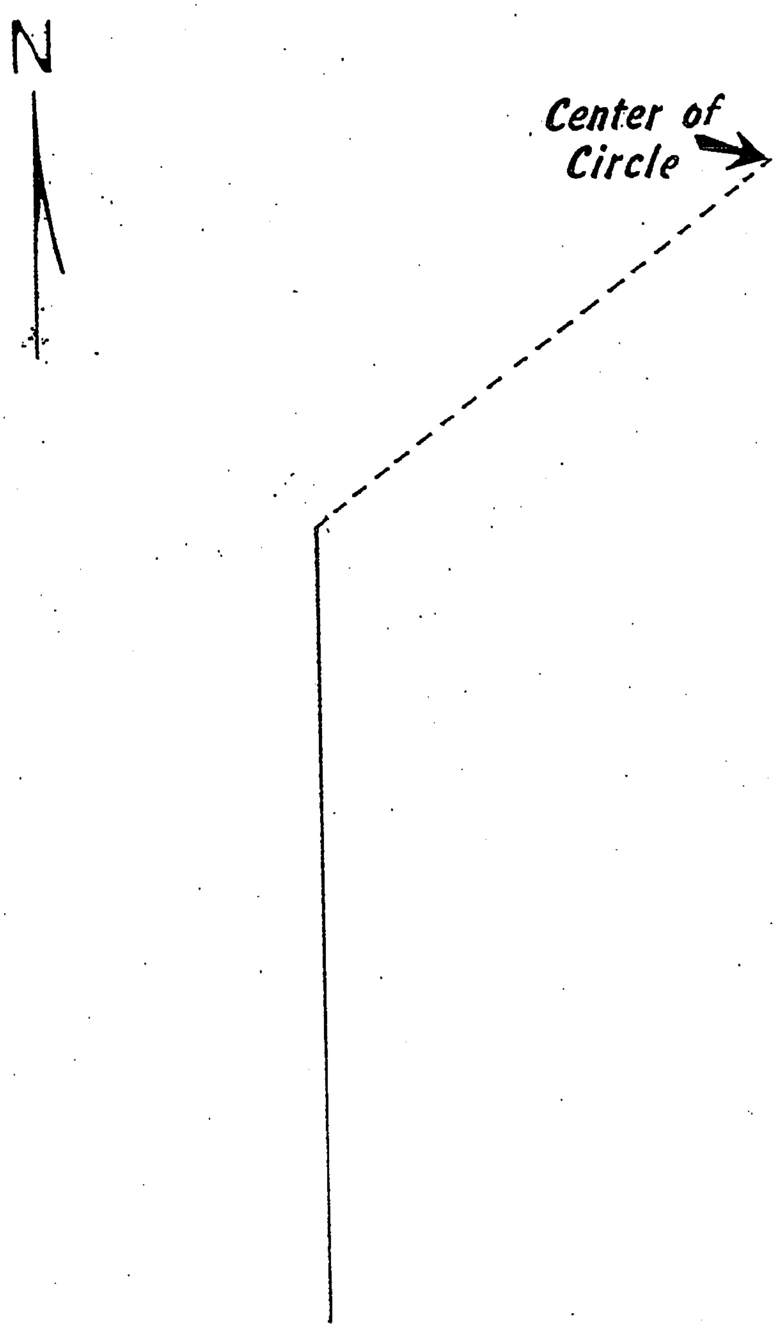


The description continues: "...thence southeasterly along a curve to the left through a central angle of 50° ..." This means you will create a 50° angle at the center of the circle.

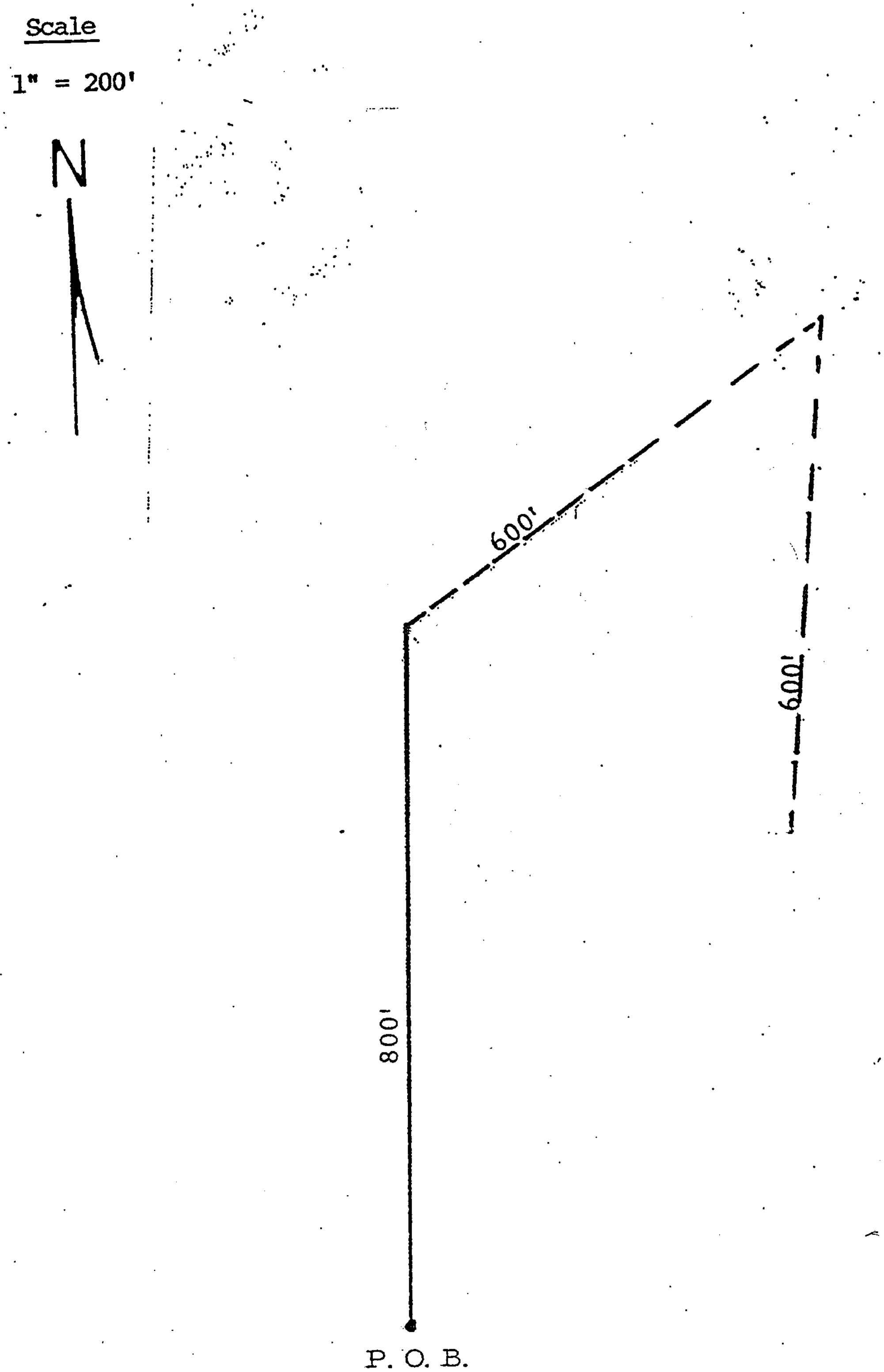
Use your protractor and draw a 50° angle. Make the new radius 600', and draw it so that you can connect the two radii with a southeasterly curve.

Scale

1" = 200'



Now the curve can be completed. Use your engineer's scale to set the compass at 600' and connect the radii.

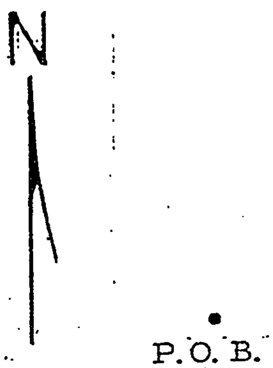


Now sketch the following description:

From the P.O.B. East 200' to the beginning of a non-tangent curve to the right and having a radius of 200' and to which beginning a radial line bears S25° E; thence southeasterly 160' along said curve through a central angle of 50°.

Scale

1" = 100'



That portion of the SW 1/4 of the SW 1/4 of Section 23, T3S R2E, San Bernardino Base and Meridian in the County of Pear, State of Nevada, described as follows:

Beginning at the NW corner of the SW 1/4 of the SW 1/4 of said Section 23, thence East 300' to the beginning of a non-tangent curve to the right having a radius of 200' and to which beginning a radial line bears S40° W; thence southeasterly 250' along said curve through a central angle of 60°; thence S 50° E 300' to the beginning of a tangent curve to the right having a radius of 300', thence southeasterly 180' along said curve through a central angle of 35°; thence S 40° W 250'; thence west 460' to the beginning of a non-tangent curve to the right having a radius of 230'; and to which beginning a radial line bears N 30° E; thence northwesterly 220' along said curve through a central angle of 53°; thence N 68° E 300' to the beginning of a non-tangent curve to the left having a radius of 230' and to which beginning a radial line bears N 70° W; thence northwesterly 310' along said curve through a central angle of 80°; thence N 44° W 200' to the p.o.b.

Using the description on the opposite page, draw a map. Label the boundaries with the bearings and distances.

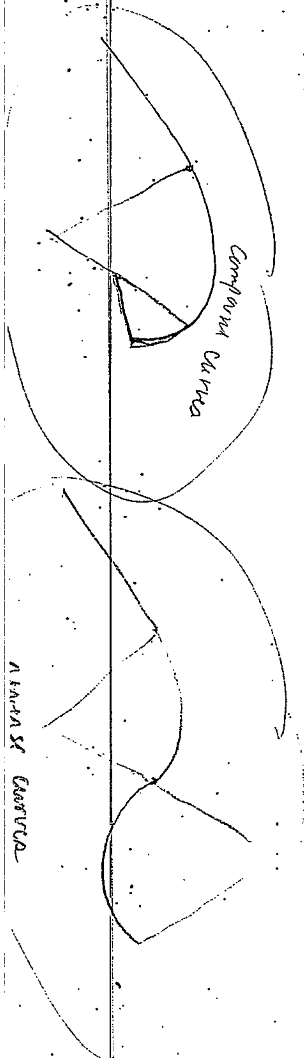
SCALE

1"=100'

N

corner
1/4 of
SW 1/4,
. 23, T3S R2E

POB



NOMENCLATURE FOR CIRCULAR CURVES

P.O.T.	Point on tangent outside the effect of any curve.
P.O.C.	Point on a circular curve.
P.O.S.T.	Point on semi-tangent *(within the limits of a curve)*.
P.I.	Point of intersection of back tangent and forward tangent.
P.C.	Point of Curvature—Point of change from back tangent to circular curve.
P.T.	Point of Tangency—Point of change from circular curve to forward tangent.
P.C.C.	Point of Compound Curve—Point common to two curves in the same direction and of different radii.
P.R.C.	Point of ~~Reserve~~ REVERSE Curve—Point common to two curves in opposite directions and with the same or different radii.
L	Total length of any circular curve measured along its arc in feet.
L_c	Length between any two points on circular curve in feet.
R	Radius of circular curve in feet.
Δ	Total intersection *(or central)* angle between back and forward tangents.
DC	Deflection angle for full circular curve measured from tangent at P.C. or P.T.
dc	Deflection angle required from tangent to a circular curve to any other point on a circular curve.
C	Total chord length, or long chord, for a circular curve in feet.
C^1	Chord length between any two points on a circular curve in feet.
Tangent	Distance along semi-tangent from the point of intersection of the back and forward tangents to the origin of curvature from that tangent in feet.
tx	Distance along semi-tangent from the P.C. *(or P.T.)* to the perpendicular offset to any point on a circular curve in feet. *(Abscissa of any point on a circular curve referred to the beginning of curvature as origin and semi-tangent as axis.)*
ty	The perpendicular offset, or ordinate, in feet, from the semi-tangent to a point on a circular curve.
E	External distance *(radial distance)* in feet from P.I. to mid-point on a simple circular curve.

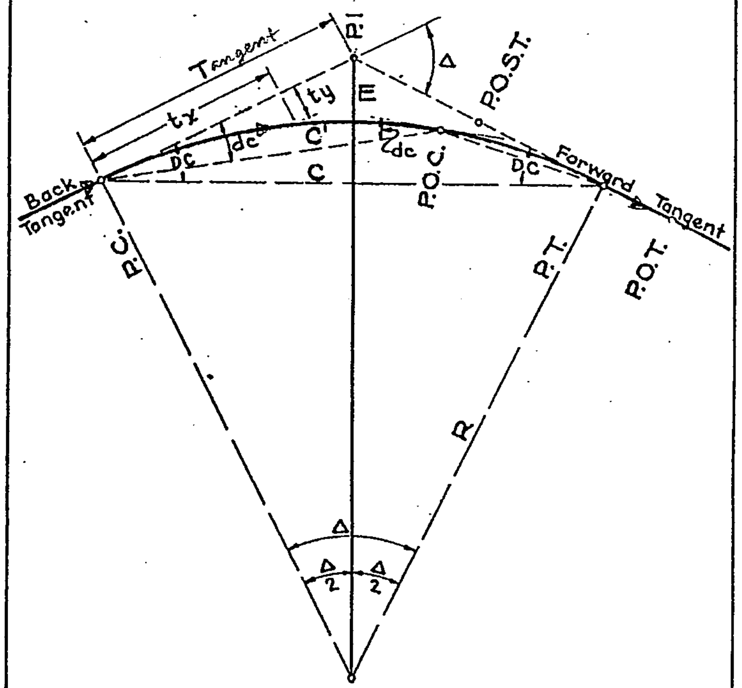


FIG. I
SIMPLE CIRCULAR CURVE

GENERAL EQUATIONS FOR CURVES

CIRCULAR CURVES

SYMBOL	EQUATION	UNIT
L	$= \frac{100\Delta}{D}$	Feet
L_c	$= \frac{200\,dc}{D}$	Feet
R	$= \frac{5730}{D}$	Feet
*T	$= R \tan\frac{\Delta}{2}$	Feet
*E	$= \frac{R}{\cos\frac{\Delta}{2}} - R$, also $= R \text{ exsec } \frac{\Delta}{2}$	Feet
DC	$= \frac{\Delta}{2}$	Degrees
dc	$= 0.3\ DL_c$	Minutes
C	$= 2R \sin\frac{\Delta}{2}$, also $= 2R \sin DC$	Feet
c'	$= 2R \sin dc$	Feet
t_x	$= (2R \sin dc)(\cos dc)$, also $= R \sin 2dc$	Feet
t_y	$= (2R \sin dc)(\sin dc)$, also $= R \text{ versin } 2dc$	Feet

*Values of T and E may be conveniently obtained from the table "Functions of a 1° Circular Curve", as explained hereinafter.

SPIRAL CURVES

SYMBOL	EQUATION	UNIT
D	$= aL_s$ (D = maximum limit of D_s)	Degrees
D_s	$= aL_s'$	Degrees
a	$= \frac{D}{L_s}$, also $= \frac{D_s}{L_s'}$	Degrees per Station
L	$= \frac{CA}{D}$	Stations
L_s	$= \frac{D}{a}$	Stations
L_s'	$= \frac{D_s}{a}$	Stations
R	$= \frac{5730}{D}$	Feet
R_s	$= \frac{5730}{D_s}$	Feet
CA	$= \Delta - 2DE$ for equal spirals $= \Delta - (DE_1 + DE_2)$ for unequal spirals	Degrees

GENERAL EQUATIONS FOR CURVES—Continued

SPIRAL CURVES (Continued)

SYMBOL	EQUATION	UNIT
DE	$= \frac{a(L_s)^2}{2}$, also $= \frac{DL_s}{2}$	Degrees
de	$= \frac{a(L_s')^2}{2}$, also $= \frac{D_s L_s'}{2}$	Degrees
DF	$= \frac{a(L_s)^2}{6} - DFk$, also $= \frac{D_s L_s}{6} - DFk$, also $= \frac{DE}{3} - DFk$, $= 10\,a(L_s)^2 - DFk$	Degrees Minutes
$\tan DF$	$= \frac{y}{x}$	
DFk	$= 0.000053\,(DE)^3$ (where DE is in degrees) Correction DFk to be applied to formula for DF when DE is 15° and over.	Minutes
df	$= \frac{a(L_s')^2}{6} - dfk$, also $= \frac{D_s L_s'}{6} - dfk$ $= 10\,a(L_s')^2 - dfk$	Degrees Minutes
dfk	$= 0.000053\,(de)^3$ (where de is in degrees) Correction dfk to be applied to formula for df when de is 15° and over	Minutes
DH	$= DE - DF$	Degrees and/or Minutes
dh	$= de - df$	Degrees and/or Minutes
dr	$= df + \frac{D_s L_s'}{6}$ in which D_s is degree at point occupied by transit and df and L_s' for the point required.	Degrees and/or Minutes
C_s	$= 100L_s - 0.000338\,a^2(L_s)^5$, also $= \sqrt{x^2+y^2}$	Feet
C_s'	$= 100L_s' - 0.000338\,a^2(L_s')^5$, also $= \sqrt{(x')^2+(y')^2}$	Feet
T	$= t + (R+Q)\tan\frac{\Delta}{2}$ (See below for unequal spirals)	Feet
t	$= 50L_s - (0.000127a^2)(L_s)^5$	Feet
x	$= 100L_s - [(0.000762a^2)(L_s)^5 + (0.0000000027a^4)(L_s)^9]$	Feet
x'	$= 100L_s' - [(0.000762a^2)(L_s')^5 + (0.0000000027a^4)(L_s')^9]$	Feet
y	$= (0.291a)(L_s)^3 - (0.00000158a^3)(L_s)^7$	Feet
y'	$= (0.291a)(L_s')^3 - (0.00000158a^3)(L_s')^7$	Feet

SPIRAL CURVES (Continued)		
SYMBOL	EQUATION	UNIT
Q	$= (0.0727a)(L_s)^3 - (0.0000002a^3)(L_s)^7$	Feet
NOTE: The last term in the equations for x, x', y, y' and Q may be omitted when the value for DE is 15° or less.		
E_s	$= (R+Q)\sec\frac{\Delta}{2} - R$, also $= R+Q \text{ exsec} \frac{\Delta}{2} + Q$ The above equations for E_s apply only where connecting spiral curves are equal. Refer to Figure III for unequal spiral connections.	Feet
u	$= x - y \cot DE$	Feet
v	$= \frac{y}{\sin DE}$, also $= y \operatorname{cosec} DE$	Feet
For curves having unequal length, spiral curves or a spiral at one end only, and Δ is less than 90°, the unequal tangent lengths will be computed from the following equations:		
UNEQUAL SPIRAL CURVES		
	For the spiral having the larger offset:	
T_1	$= t_1 + (R+Q_1)\tan\frac{\Delta}{2} - \left(\frac{Q_1 - Q_2}{\sin\Delta}\right)$ $= t_1 + (R+Q_1)\tan\frac{\Delta}{2} - (Q_1 - Q_2 \operatorname{cosec}\Delta)$	Feet
	For the spiral having the smaller offset:	
T_2	$= t_2 + (R+Q_2)\tan\frac{\Delta}{2} + \left(\frac{Q_1 - Q_2}{\sin\Delta}\right)$ $= t_2 + (R+Q_2)\tan\frac{\Delta}{2} + (Q_1 - Q_2 \cot\Delta)$	Feet
SPIRAL AT ONE END ONLY		
	For the spiralled end:	
T_1	$= \frac{R - (R+Q)\cos\Delta}{\sin\Delta} + t$	Feet
	For the end having no spiral:	
T_2	$= \frac{(R+Q) - (R\cos\Delta)}{\sin\Delta}$	Feet
No practical method of finding the external distance from the P.I. is available when spiral connecting curves are unequal. By measuring the distance M along the semi-tangent, as illustrated in Figure III, and using the smaller offset Q_2 in the equation for E_s, an accurate tie to the curve may be computed.		

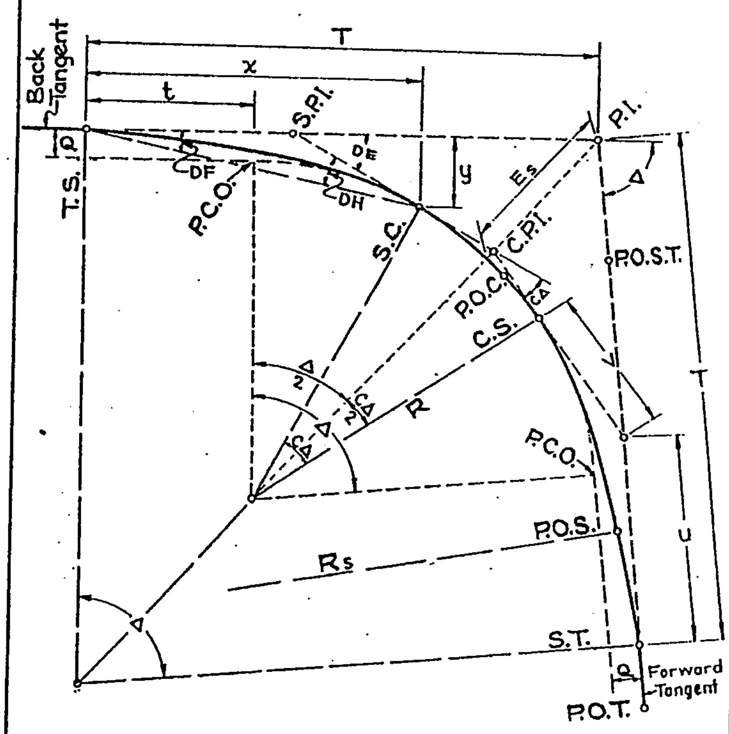


FIG. II
CIRCULAR CURVE WITH EQUAL SPIRALS