Project One Template

MAT350: Applied Linear Algebra

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Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.

Solution:

System of linear equations based on graphic of network links. Commented are equations before rearrangement.

```
syms x_1 x_2 x_3 x_4 x_5
%routerA = 100 == x_1 + x_1 + x_2
routerA = 2*x_1 + x_2 == 100
```

routerA = $2\,x_1 + x_2 = 100$

```
%routerB = x_1 + x_2 == x_3 + x_5
routerB = x_1 + x_2 - x_3 - x_5 == 0
```

routerB = $x_1 + x_2 - x_3 - x_5 = 0$

```
%routerC = 50 + x_1 == x_3 + x_5
routerC = x_1 - x_3 - x_5 == -50
```

routerC = $x_1 - x_3 - x_5 = -50$

```
%routerD = x_4 + x_5 == x_2 + 120
routerD = -x_2 + x_3 - x_4 +x_5 == 120
```

routerD = $x_3 - x_2 - x_4 + x_5 = 120$

```
%routerE = x_2 + x_3 + x_5 == x_4
routerE = x_2 + x_3 - x_4 + x_5 == 0
```

routerE = $x_2 + x_3 - x_4 + x_5 = 0$

```
% I am unsure how to aline Ax = b horizontally here, but this is meant to
% be in form Ax = b. Where A is the 5x5 matrix of coefficients, x is 5x1
% unknown, and b is the list of constants
```

Ax = b

```
A = [2 1 0 0 0; 1 1 -1 0 -1; 1 0 -1 0 -1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
x = [x_1; x_2; x_3; x_4; x_5]
```

x =

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$$

```
b = [100; 0; -50; 120; 0]
```

```
b = 5×1
   100
     0
   -50
   120
     0
```

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function. Write out your **reduced matrix and identify the free and basic variables of the system**.

Solution:

Coefficent matrix A

```
A = [2 1 0 0 0; 1 1 -1 0 -1; 1 0 -1 0 -1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

Column matrix of constants

```
b = [100; 0; -50; 120; 0]
```

```
b = 5×1
   100
     0
   -50
   120
     0
```

Augmented matrix [A | b]

```
Ab = [A b]
```

```
Ab = 5×6
     2     1     0     0     0   100
     1     1    -1     0    -1     0
     1     0    -1     0    -1   -50
     0    -1     0     1     1   120
     0     1     1    -1     1     0
```

Reduced matrix

```
[rowreducedAb, pivotvarsAb] = rref(Ab)
```

```
rowreducedAb = 5×6
     1     0     0     0     0    25
     0     1     0     0     0    50
     0     0     1     0     0    30
     0     0     0     1     0   125
     0     0     0     0     1    45
pivotvarsAb = 1×5
     1     2     3     4     5
```

Number of variables in the system

```
[numeqns, numvars] = size(A)
```

```
numeqns = 5
numvars = 5
```

Number of pivot variables

```
[numrows, numpivotvars] = size(pivotvarsAb)
```

```
numrows = 1
numpivotvars = 5
```

Number of free variables

```
numfreevars = numvars - numpivotvars
```

```
numfreevars = 0
```

Basic variables:

```
[x_1 x_2 x_3 x_4 x_5]
```

ans = $\begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 \end{pmatrix}$

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x**. Solve the system of equations L**y** = **b** for the unknown vector **y**.

Solution:

Coefficient matrix A

```
A = [2 1 0 0 0; 1 1 -1 0 -1; 1 0 -1 0 -1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

Matrix b constants

```
b = [100; 0; -50; 120; 0]
```

```
b = 5×1
   100
     0
   -50
   120
     0
```

LU decomposition of A, storing lower and upper matrices in L and U

```
[ L, U] = lu(A)
```

```
L = 5×5
    1.0000         0         0         0         0
    0.5000   -0.5000    1.0000    1.0000         0
    0.5000    0.5000    1.0000         0         0
         0    1.0000         0         0         0
         0   -1.0000   -1.0000   -0.5000    1.0000
U = 5×5
    2.0000    1.0000         0         0         0
         0   -1.0000         0    1.0000    1.0000
         0         0   -1.0000   -0.5000   -1.5000
         0         0         0    1.0000    1.0000
         0         0         0         0    1.0000
```

Solves system Ax=b using the LU decomposition

```
y = L\b
```

```
y = 5×1
   100
   120
  -160
   170
    45
```

```
x = U\y
```

```
x = 5×1
    25
    50
    30
   125
    45
```

```
x1 = U\y
```

```
x1 = 5×1
    25
    50
    30
   125
    45
```

Checking that the solution for x1 matches

```
x2 = A\b
```

```
x2 = 5×1
    25
    50
    30
   125
    45
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
inv(U)
```

```
ans = 5×5
    0.5000    0.5000         0   -0.5000         0
         0   -1.0000         0    1.0000         0
         0         0   -1.0000   -0.5000   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y**.

Solution:

The solution for the original system of equations:

```
x = inv(U)*y
```

```
x = 5×1
    25
    50
    30
   125
    45
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

Coefficient matrix A

```
A = [2 1 0 0 0; 1 1 -1 0 -1; 1 0 -1 0 -1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

Martix constants b

```
b = [100; 0; -50; 120; 0]
```

```
b = 5×1
   100
     0
   -50
   120
     0
```

Initializing matrices A1:A5 to equal A

```
A1 = A
```

```
A1 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A2 = A
```

```
A2 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A3 = A
```

```
A3 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A4 = A
```

```
A4 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A5 = A
```

```
A5 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
     1     0    -1     0    -1
     0    -1     0     1     1
     0     1     1    -1     1
```

Replace column 1 in A1 with the column vector of constants b

```
A1(:,1) = b
```

```
A1 = 5×5
   100     1     0     0     0
     0     1    -1     0    -1
   -50     0    -1     0    -1
   120    -1     0     1     1
     0     1     1    -1     1
```

Replace column 2 in A2 with the column vector of constants b

```
A2(:,2) = b
```

```
A2 = 5×5
     2   100     0     0     0
     1     0    -1     0    -1
     1   -50    -1     0    -1
     0   120     0     1     1
     0     0     1    -1     1
```

Replace column 3 in A3 with column vector of constants b

```
A3(:,3) = b
```

```
A3 = 5×5
     2     1   100     0     0
     1     1     0     0    -1
     1     0   -50     0    -1
     0    -1   120     1     1
     0     1     0    -1     1
```

Replace column 4 in A4 with column vector of constants b

```
A4(:,4) = b
```

```
A4 = 5×5
     2     1     0   100     0
     1     1    -1     0    -1
     1     0    -1   -50    -1
     0    -1     0   120     1
     0     1     1     0     1
```

Replace column 5 in A5 with column vector of constants b

```
A5(:,5) = b
```

```
A5 = 5×5
     2     1     0     0   100
     1     1    -1     0     0
     1     0    -1     0   -50
     0    -1     0     1   120
     0     1     1    -1     0
```

Find solution using ratios of determinants

```
x9 = det(A1)/det(A)
```

```
x9 = 25.0000
```

```
x8 = det(A2)/det(A)
```

```
x8 = 50
```

```
x7 = det(A3)/det(A)
```

```
x7 = 30.0000
```

```
x6 = det(A4)/det(A)
```

```
x6 = 125.0000
```

```
x5 = det(A5)/det(A)
```

```
x5 = 45
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

*Fill out the table in the original project document and export your table as an image. Then, use the **Insert** tab in the MATLAB editor to insert your table as an image.*

Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x_1	60	25	No Change	Network link x_1 is not close to reaching capacity, there is no need to change this link.
x_2	50	50	Remove Link	Network link x_2 is right at the capacity limit, for this reason I would suggest to remove this link.
x_3	100	30	No Change	Network link x_3 is has a large recommended capacity of 100 Mbps and it is not yet close to reaching this.
x_4	100	125	Remove Link	Network link x_4 is over capacity, this link should be removed.
x_5	50	45	Upgrade Link	Network link x_5 is very close to reaching capacity, so the recommendation is to upgrade this link before capacity is reached.