$\int p(t)p^*(t-kT)dt = 0$.

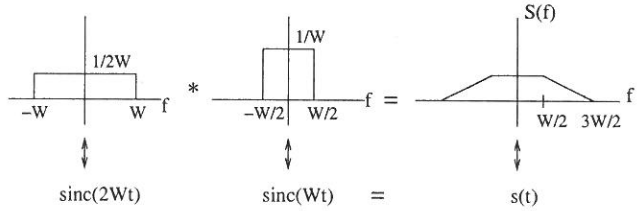

Figure 24: The product of two sinc pulses gives a trapezoidal spectrum in Problem 2.21(c).

Problem 2.21: We have

$$r_n = y((n-1)T) = \sum_{k=1}^{100} b_k s((n-1)T - (k-1)T) = \sum_{k=1}^{100} b_k s((n-k)T)$$

(a) For $r_n = b_n$, we need that $s(kT) = \text{sinc}(2WkT) = \delta_{k0}$, which requires that $2WT$ is an integer, or $T = \frac{L}{2W}$, for L a positive integer. The smallest value of T, and hence the fastest rate, corresponds to $L = 1$.
(b) Choosing $T = \frac{1}{2W}$, we have, for timing offset $0.25T$, that

$$r_n = y((n-1)T + 0.25T) = \sum_{k=1}^{100} b_k s((n-k)T + 0.25T) = \sum_{k=1}^{100} b_k \text{sinc}(n-k+0.25)$$

Thus,

$$r_{50} = \sum_{k=1}^{100} b_k \text{sinc}(50-k+0.25) = \sum_{m=-50}^{49} b_{50-m} \text{sinc}(m+0.25)$$

For $m > 0$, the sign of $\text{sinc}(m + 0.25)$ is $(-1)^m$. On the other hand, the sign of b_{50-m} is $(-1)^{50-m-1} = (-1)^{m+1}$, so that the term $b_{50-m}\text{sinc}(m + 0.25)$ has negative sign. Similarly, for $m < 0$, the sign of $\text{sinc}(m + 0.25)$ is $(-1)^{m+1}$, whereas the sign of b_{50-m} is $(-1)^{50-m} = (-1)^m$, so that the product $b_{50-m}\text{sinc}(m + 0.25)$ again has negative sign. Thus, the ISI terms for $m \neq 0$ are all negative. On the other hand, the desired term for $m = 0$, $b_{50}\text{sinc}(0.25) = \text{sinc}(0.25) = .90$ is positive. Numerical summation shows that the sum of the ISI terms is -2.06, which is significantly larger in magnitude than the desired term. We therefore obtain that $r_{50} = -1.16$ has a sign opposite to that of b_{50}.
(c) For $s(t) = \text{sinc}(Wt)\text{sinc}(2Wt)$, the two smallest values of T for which the pulse is Nyquist are $T = \frac{1}{2W}$ and $T = \frac{1}{W}$.
(d) Choosing $T = \frac{1}{2W}$, we have, for timing offset $0.25T$, that

$$r_n = y((n-1)T + 0.25T) = \sum_{k=1}^{100} b_k s((n-k)T + 0.25T) = \sum_{k=1}^{100} b_k \text{sinc}(n-k+0.25)\text{sinc}((n-k+0.25)/2)$$

which gives

$$r_{50} = \sum_{k=1}^{100} b_k \text{sinc}(50-k+0.25)\text{sinc}((50-k+0.25)/2) = \sum_{m=-50}^{49} b_{50-m}\text{sinc}(m+0.25)\text{sinc}((m+0.25)/2)$$

For $m > 0$, the sum of the ISI terms comes to -0.07. For $m > 0$, the sum of the ISI terms is -0.24. The desired term, on the other hand, is 0.8774. Thus, the sum of the ISI and desired term is $r_{50} = 0.57$, which has the same sign as b_{50}.
Note that the bit sequence in (b) no longer creates the worst-case ISI, because of the signs of the ISI coefficients are now different. If we chose the bit sequence to create the worst-case ISI, then the ISI terms for $m > 0$ would sum to -0.13, and the ISI terms for $m < 0$ would sum to -0.30, which is still not large enough in magnitude to switch the sign of the desired term. In this case, we would obtain $r_{50} = 0.45$, which is still the same sign as b_{50}. This clearly illustrates how the faster decay in the signaling pulse translates to increased robustness to timing mismatch.
(d) The spectrum of the signaling pulse in (c) is sketched in Figure 24. Clearly, the excess bandwidth is 50%. As we have seen in (c), the faster time decay associated with the excess bandwidth significantly reduces the severity of the ISI due to timing mismatch.

Problem 2.22: (a) We have