Solutions to Selected Problems
– from Madhow: Fundamentals of Digital Communication and from Johannesson & Zigangirov: Fundamentals of Convolutional Coding –

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Linköping 2012

Note: This material is prepared for the 2012 version of the Master course TSKS04 Digital Communication Continuation Course. For almost every task planned for tutorials you find either hints and answers or complete solutions. For the tasks where we give complete solutions, we have chosen to adjust the notations to concur with the notation that has been used in the lectures.
This document will evolve during the course as the tutorials go by.
This version of the document was compiled February 28, 2012.

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This document was prepared using LaTeX2ε with the aid of TeXnicCenter on an Dell PC running CentOS 5. The figures were produced using XFIG (from xfig.org). Finally, the plots were produced using MATLAB (from MathWorks, Inc.).

Contents

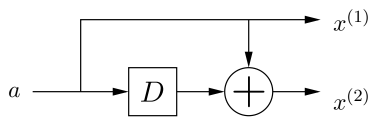
Tutorial 7

Problems from Johannesson & Zigangirov, Fundamentals of Convolutional Coding, IEEE Press 1999.

J&Z 1.24

Task

Consider the following convolutional encoder.



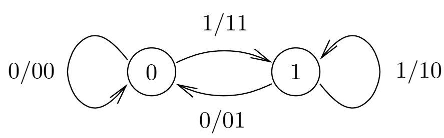
a. Draw the trellis corresponding to four information digits and $m = 1$ dummy zero.

b. Find the number of codewords represented by the trellis.

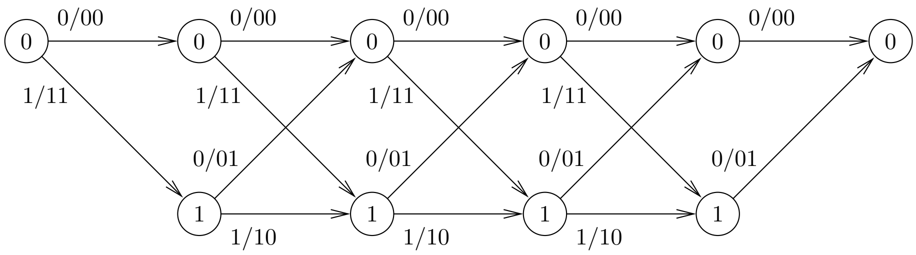
c. Use the Viterbi algorithm to decode when the sequence $r = 11\,01\,10\,10\,01$ is received over a BSC with error $0 < p < 1/2$.

Solution

a. The state diagram is given below, where we label the edges by $a_i/x_i^{(1)}x_i^{(2)}$.

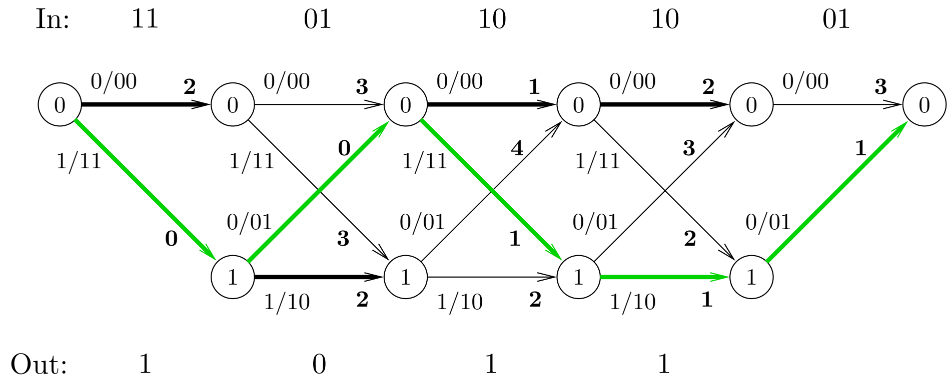


We draw the trellis of the code:



b. We have 4 information bits. That gives us $2^4 = 16$ codewords. This can also be found from the trellis. At each of the first four time instances, each state has two outgoing branches.

c. We run the Viterbi algorithm based on the received sequence.



The bold edges are the survivors up to that point. The final best path is indicated with green edges, and it corresponds to the bits 1011.

J&Z 3.10

Answer

a, b. Omitted.

c. $T(W) = \frac{W^6+W^7-W^8}{1-2W-W^3}$

J&Z 4.1

Answer

a. Omitted.

b. 10101(0) (Assume that 5 info bits are followed by 1 dummy zero to terminate the

convolutional code.)
c. Number of errors: 3

J&Z 4.2

Answer
a. Omitted.
b. 1011(00) (Assume that 4 info bits are followed by 2 dummy zeros to terminate the convolutional code.)
c. The decoded bits are the same as the transmitted bits, and the corresponding path weight in the trellis is 2, therefore, 2 channel errors are correct.

Problems from UMadhow, Chapter 7.

7.1

Answer

a. The coefficient of X^7 is $4I^3$. Therefore, the minimum number of differing input bits must be 3 to produce codewords that have a Hamming distance 7.
b. Input sequences of weight 4 correspond to the term containing I^4. The coefficient is $8X^8$, which means that the maximum output weight is 8.
c. $P_{e,\text{pairwise}} = Q(\sqrt{2E_b R d_H(c_1, c_2)/N_0}) = Q(\sqrt{35}) = 1.58 \times 10^{-9}$
d. $T(W, L) = (L^3W^5)/(1 - LW - L^2W) = L^3W^5 \sum_{k=0}^{\infty}(L + L^2)^k W^k$
e. We look for a term like $W^{100}L^{190}$ and find out there are $\begin{pmatrix} 95 \\ 92 \end{pmatrix}$ such error events.

7.2

Hints

a. The implementation is given on page 298 in the textbook. For the transition diagram, each state is represented by $y[k-1]y[k-2]$ and each branch is labeled by $u[k]/u[k]v[k]$.

Answer

a,b. Omitted.
c. The transfer function is

$$\begin{aligned} T(I, X) &= (I^3X^5 + I^2X^6 - I^4X^6)/[1 - (2IX + X^2 - I^2X^2)] \\ &= (I^3X^5 + I^2X^6 - I^4X^6)(1 + Y + Y^2 + Y^3 + ...), \end{aligned}$$

where $Y = (2IX + X^2 - I^2X^2)$.

7.3

Answer

a,b. Omitted.
c. $T(I, X) = IX^4(I + X - IX^2)/(1 - IX - IX^3 + I^2X^4 - I^2X^2)$
d. For determinate the free distance, we only need to consider the powers of X. We can set $I = 1$ in the transfer function $T(I, X)$. This gives

$$T(X) = X^4(1 + X - X^2)(1 + Y + Y^2 + ...),$$

where $Y = X + X^2 + X^3 - X^4$. Then it is clear that the lowest power of X in the expansion of $T(X)$ is 4, i.e., the free distance is 4.

7.4

Answer

a,b. Omitted.

c.

$$
\begin{aligned}
T(X) &= X^3(3 + 4X - 3X^2 - 5X^3 + 3X^4 + 2X^5 - X^6)/(1 - (X - 4X^2 - 3X^4 + X^6)) \\
&= X^3(3 + 4X - 3X^2 - 5X^3 + 3X^4 + 2X^5 - X^6)(1 + Y + Y^2 + Y^3 + ...)
\end{aligned}
$$

where $Y = X - 4X^2 - 3X^4 + X^6$.

Then, we know that the free distance is 3 and the number of error events of weight 5 is 16.