6|9|2018

$$\phi_k(t) = \frac{1}{\sqrt{T}} e^{j2\pi kt/T} \qquad 0 \le t \le T$$

$= 0$

$$x(t) = \sum_{k=-m}^{m} a_k \phi_k(t), \quad a_k = \int_0^T a_k \phi_k^*(t)\,dt \ldots$$

$$\lim_{m \to \infty} \int_0^T \left| x(t) - \sum_{m=-\infty}^{m-\infty} a_k \phi_k(t) \right|^2 dt$$

* This ~~may~~ not be zero if m is limited

$x(t)$.

Conditions are

$x(t) = 0$ for $t \notin [0, T]$

$\int_0^T |x(t)|^2 dt$

$X(f) = 0$ for $|f| > \frac{B}{2}$

or $x_m(t) = \sum_{k=-m} a_k \frac{1}{\sqrt{T}} e^{\frac{j2\pi kt}{T}}$

→

$$x_m(t) = \sum_{k=-m}^{m} \frac{a_k}{\sqrt{T}} e^{\frac{j2\pi kt}{T}}$$

$$a_k = \frac{1}{\sqrt{T}} \int_{-\infty}^{\infty} x(t) e^{\frac{-j2\pi ft}{T}} dt =$$

Changing from $-m$ to $+m$

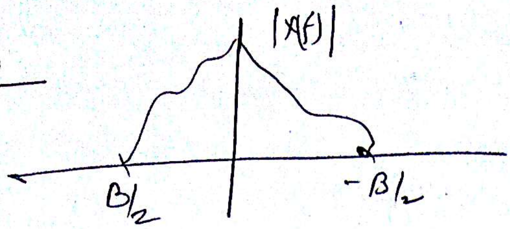
$$\text{As } \because X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

So

$$\boxed{a_k = \frac{1}{\sqrt{T}} X\left(f = \frac{k}{T}\right)}$$

So Approximation will be: $x_m(t) = \sum_{k=-m}^{m} \frac{X}{T}\left(f = \frac{k}{T}\right) e^{\frac{j2\pi kt}{T}}$

Also



$$x_m(t) = \sum_{k=-\frac{BT}{2}}^{k=BT/2} \frac{x}{T}\left(f = k/_T\right) e^{j2\pi kt/T}$$

$$\frac{K}{T} = \frac{B}{2}$$

$$K = \frac{BT}{2}$$

Suppose ~~that~~ bandwidth proved by
$\exists$ $\cancel{\phi}$ $[f_1 - B/_2 ,\ f_1 + B/_2]$

for the time duration $[0, T]$

As signal generated are mapped as $\left\{ \begin{array}{l} c_0 \\ \vdots \\ c_0(BT) \end{array} \right.$ (BT) ?,

Signal

$$S_0 = \sum_{K=-\frac{BT}{2}}^{K=BT/2} c\left(k + \frac{BT}{2}\right) e^{\frac{-j2\pi kt}{T}}$$

$\hookrightarrow$ complex no.

$\downarrow$

Next it is upconverted

Complex representes $\mathbb{C}^{BT}$ space

Same DFT $\mathbb{R}^{2BT}$

As $x(t)$ is complex baseband S_0 :- $2BT$

both time limited and Band limited So called Degrees of freedom

In General!. $y(t) = x(t) + n(t)$ or complex waveforms
$[0 \cdots T]$

Both time and Bandlimited

corresponding complex vector of x

$$x = [x_1 \quad x_2 \quad - - - \quad x_{BT}]$$

Also $\quad n = [n_1 \quad n_2 \quad - - - - \quad n_{BT}]$

~~At Trans~~ At Receiver Signal

So

$$y_k(t) = \sum_{-BT/2}^{BT/2} x_k \phi_k(t) + n(t) \quad \text{--- } (\alpha)$$

$$\left(x_{-\frac{BT}{2}} \; - - - \; x_0 \; - - - \; x_{\frac{BT}{2}}\right)$$

$$y_k \triangleq \int_0^T y(t) \cdot \phi_k^*(t)\, dt$$

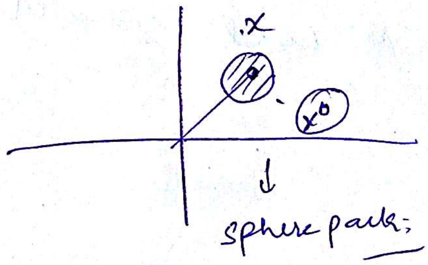
$$y_k = x_k + \underbrace{\int_0^T n(t)\, \phi_k^*(t)\, dt}_{n_k}$$

So

$$\left\{\left(y_{-\frac{BT}{2}} \; - - - \; y_0 \; - - - \; y_{\frac{BT}{2}}\right)\right.$$

At equation (α)

$$\int_0^T |x(t)|^2 dt = \sum_{k=-BT/2}^{k=BT/2} |x_k|^2$$



Date 07. 09 : 2016

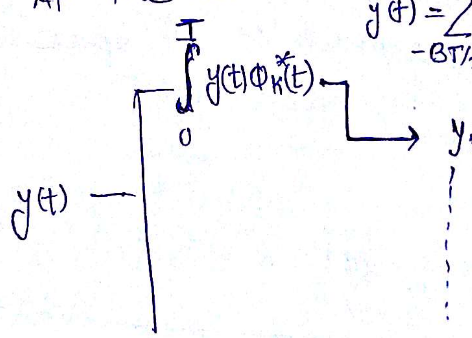
$$y(t) = x(t) + n(t)$$

$$= \sum_{k=-\frac{BT}{2}}^{k=BT/2} x_k \phi_k(t) + n(t)$$

$$y(t) = \sum_{k=-\frac{BT}{2}}^{BT/2} y_k \phi_k(t)$$

→ At the Reciever

$$y(t) = \sum_{-BT/2}^{BT/2} y_k \phi_k(t)$$



$$\int_0^T y(t)\,\phi_k^*(t)\,dt$$

$$= \int_0^T (x(t) + n(t))\,\phi_k^*(t)\,dt$$

$$= \int x(t)\,\phi_k^*(t)\,dt + \int n(t)\,\phi_k^*\,dt$$

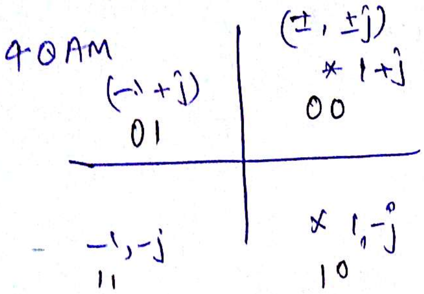
4 bits / 2^4,

Suppose $BT = 2$ → Taking 2 bits sequence x_{-1}, x_0, x_1 are generated then Transmit

$[x_{-1}\ x_0\ x_1]$

$$y(t) = \sum_{k=-BT/2}^{BT/2} x_k \phi_k(t) =$$

6 bits =



After $x_1, x_2, x_3 \dots x_K$ → $x(t)$ is generated

100 bit

For Example: $T = \frac{1}{2}$ msec, $B = 200$ kbps → So Any rule which Allow condition can be used

* If 16QAM is used Power of T_x ↑ } Trade off
Error ↑

$$y := y_{-BT/2} \dots y_{-1}\ y_0\ y_1\ y_2 \dots y_{BT/2}$$

$BT = 2$

$$y_{-1} = x_{-1} + n_{-1}$$

$$y_0 = x_0 + n_0$$

$$y_1 = x_1 + n_1$$

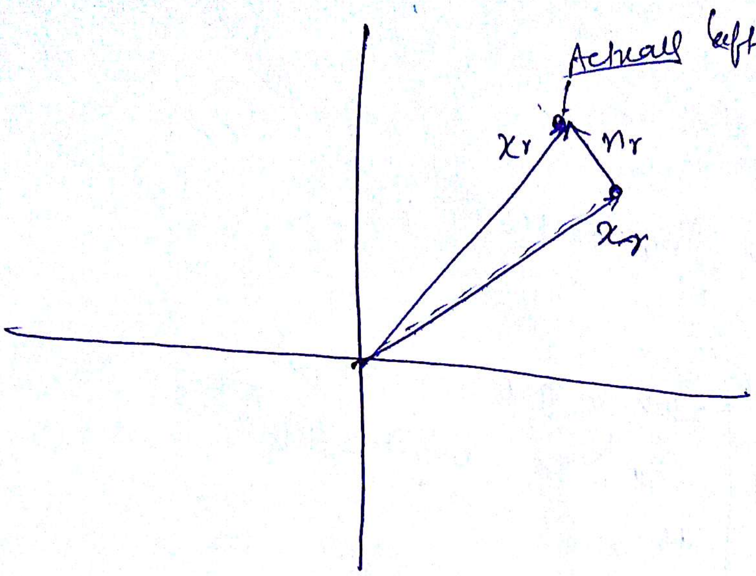
$$y = x + n$$

* {Noise random variables are orthonormal, and iid random variables}

Splitting in Real And Imaginary Vector: $y_{-1} \to y_R,\ y_I$

$$y_r \Rightarrow x_r + n_r \quad \text{or Real}$$

So Now the Dimension space is $\mathbb{R}^{2BT}$



6 bits in T secs.

$$\frac{\frac{6 \text{ bits}}{T}}{B \text{ Hz}}$$

$= \frac{6}{BT}$ b/s/Hz → Unit of spectral efficiency

Spectral efficiency : How efficient the bandwidth is used.

First scheme : 2 bits at second

$= \frac{6}{BT}$

$BT = 3$

for 6 bits ⇒ 3 bits/sec/Hz

Now if 6 bits ↓ Jointly Generated Complex numbers Shannon

0 0 0 0 0 1 → $C_0(1)$, $C_0(2)$, $C_0(3)$

0 0 0 0 0 1 1 → $C_1(1)$, $C_1(2)$, $C_1(3)$

⋮

1 1 1 1 1 1 → C_{63}

Error probability will be less in this case

In Earlier: individual mapping but here joint coding

~~Her~~ Here for the error, the noises in all 3 have to add for symbol to be in error.

Date: 16/9/2016

$y_1 = x_1 + n_1$, $y_2 = x_2 + n_2$, $y_3 = x_3 + n_3$

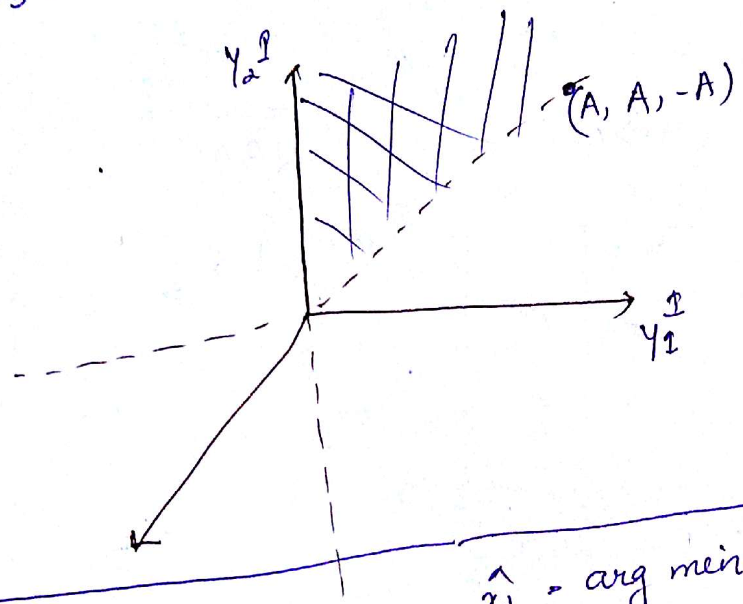
$$\arg\min |y_1 - x_1|^2 + |y_2 - x_2|^2 + |y_3 - x_3|^2$$

$x_1 \in (A, -A)$
$x_2 \in (+A, -A)$
$x_3 \in (+A, -A)$

$$\Downarrow$$

$$\arg\min = |y_1^I - x_1|^2 + |y_2^I - x_2|^2 + |y_3^I - x_3|^2$$

$$\underbrace{+ y_1^{Q^2} + y_2^{Q^2} + y_3^{Q^2}}$$ $\rightarrow$ neglected independent of x_i



$$\hat{x}_1 = \arg\min (y_1^I - x_1)^2$$
$$x_1 \in \left\{\frac{A}{2}, \frac{5A}{2}\right\}$$

If $x_1 \in \left(\frac{A}{2}, \frac{5A}{2}\right\}$

$x_2 \in \left\{\frac{-A}{2}, 3A\right\}$

$x_3 = \left\{-\frac{3A}{2}, A\right\}$

If $x_1 \in (-A, A)$

$(x_2, x_3) \in \{(+A, -A), (A, A), (-A, -A)\}$

code word: 6 ~~words~~

for Max likelihood detection: x_1 can be separately calculated but (x_2, n_3) has to be jointly characterised

Probability of Error

$$E_1 = \{x_1 \neq \hat{x}_1\}$$

$$E_2 = \{x_2 \neq \hat{x}_2\}$$

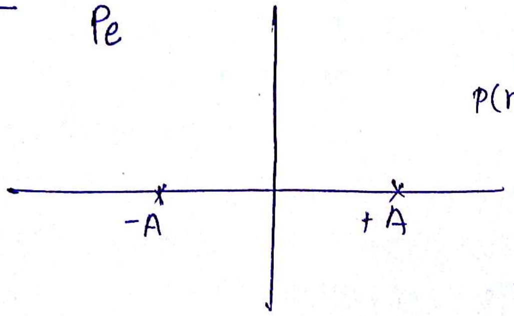
$$E_3 = \{x_3 \neq \hat{x}_3\}$$

Probability of Correct ⇒ $P_c = P(E_1^c)\, P(E_2^c)\, P(E_3^c)$

$$P_c = (1 - P(E_1))(1 - P(E_2))(1 - P(E_3))$$

So

P_e



$P(n > A)$

$$\Rightarrow \int_A^\infty \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\, dx$$

$$\Rightarrow \int_{A/\sigma}^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}}\, dt$$

So $\boxed{P_c \Rightarrow \left(1 - Q\left(\frac{A}{\sigma}\right)\right)^3}$

if $\frac{A}{\sigma} >> 1$ $Q(A/\sigma)$ goes down

$$P_e \Rightarrow 1 - \left(\underbrace{1 - Q\left(\frac{A}{\sigma}\right)}_{x}\right)^3$$

$$\Rightarrow Q(A/\sigma)\left(1 + \left(1 - Q(A/\sigma)\right) + \left(1 - Q(A/\sigma)\right)^2\right]$$

$$\Rightarrow Q(A/\sigma)\left(3 - 3Q(A/\sigma) + Q^3(A/\sigma)\right)$$

$A/\sigma >> 1$

$$\approx 3Q(A/\sigma)$$

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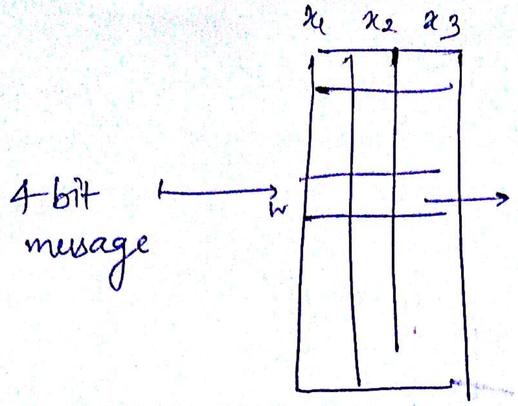
i)



3 channel
4 message

Spectral efficiency $= \frac{4}{3} = 1.33$ Mbps

ii)

2 bit message →

	x_1	x_2	
1/4	A	A	$2A^2$
1/4	A	−A	$2A^2$
1/4	−A	A	$2A^2$
1/4	−A	A	$2A^2$

Energy per bit $: 2A^2$
Spectral efficiency $= 2/1$

2 bits message →

	x_1	x_2	x_3	
1/4	A	−A	A	$3A^2$
1/8	A	2A	3A	$14A^2$
1/8	−A	−A	−A/2	$9A^2/2$
1/2	A/2	0	2A	$17A^2/4$

Spectral efficiency $= \frac{2}{3}$

(Average Energy per codeword)

$$\frac{3A^2 \times \frac{1}{4} + \frac{14A^2}{8} + \frac{9A^2}{3 \cdot 2} + \frac{17A^2}{3}}{2}$$

← Energy per bit

Reference : book.

Chap. 3 : Topic 3.4 (optimal reception..)
3.5

3.3 (Signal spaces Concept) → Grahm.

Signal Spaces Concept → Gallager → principal of digital commⁿ.
↳ chapter-5
→ 5.1
→ 5.2
→ 5.3

Chapter: 8 → 8.1
8.2
8.3

Constraint → Shannon idea

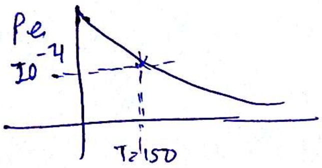
P $E[|X|^2] < P$

σ^2

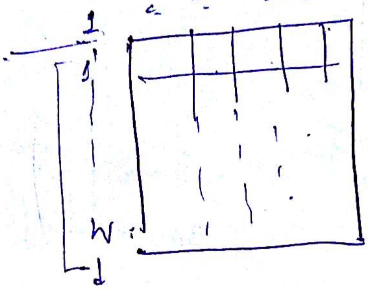
B

T

$P_e < 10^{-4}$



BT → BT complex vector



finite no of W

codebook should exist with the W num-then

↓ → Spectral Efficiency

$$\frac{\log_2(W)}{BT} < \log_2\left(1+\frac{P}{\sigma^2}\right)$$

$$\Rightarrow \boxed{W < \left(1+\frac{P}{\sigma^2}\right)^{BT}}$$

Condition for the contraints mentioned

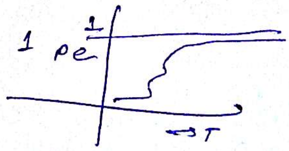
Here T is based a higher valy

Here in BT if T increased then averaging increasing results in to decrease of Pe

* if in a codebook the above constrains satisfies, the Pe will be minimised
* if T↑, spectral efficiency should be $< \log_2(1+P/\sigma^2)$
* No of waveforms finite.

* $$\lim_{T\to\infty}\left[\frac{\log_2(W(T))}{BT}\right] < \log_2\left(1+\frac{P}{\sigma^2}\right)$$

* if Spectral efficiency $> \log_2(1+P/\sigma^2)$ → then Pe → 1



* Power should be constant.

* If W↑ complexity increases

Example $B = 1$ MHz

T = 1sec

$P/\sigma^2 = 0$ dB

$\Rightarrow B\log_2(1+S/N)$

$= 10^6 \log_2(1+1)$

$= 10^6$ bits

~~But~~ still a finite value

→ can be T range

100% η

Ex for $T\uparrow \rightarrow P_e \downarrow$ $BT = 1$

$\rightarrow$ 1 bps/Hz

- $\pm A$

$P_e = Q\left(\frac{A}{\sigma}\right)$ for.

$y = x + N(0, 2\sigma^2)$ $\rightarrow$ So Code Book
$\pm A$

0	A
1	-A

If $BT = 3$

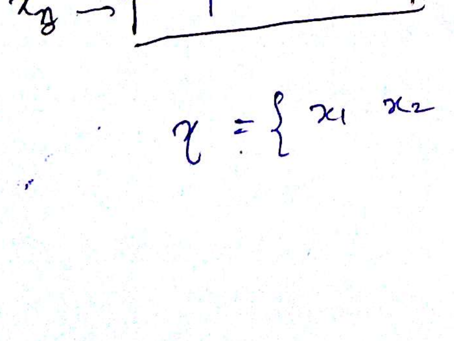
000	A	A	A
001	A	A	-A
010	A	-A	A
011	A	-A	-A
100	-A	$\cancel{A}$	A
101	-A	A	-A
110	-A	-A	A
111	-A	-A	-A

$\Rightarrow$ for this case $3Q\left(\frac{A}{\sigma}\right)$
it has increased & instead of increasing dimension.
Power is also same! as before

$\Rightarrow$ So selected code word is not valid

As N - variance reduces for other so lies S/N

To prove $\lim_{\sigma \to 0} \left[\frac{P_e^3}{Q(A/\sigma)} \right] < 1$



$BT = 3$

$y = x + n$ # received vector $\mathbb{R}^3$

$\hat{x} = \arg\min_{x \in \mathcal{X}} \|y - x\|^2$

$\mathcal{X} = \{x_1 \; x_2 \; --- \; x_8\}$
vector nota

$P_e \Rightarrow P_r(\hat{x} \neq x)$

$= \sum_{i=1}^{8} P(x_i) \; P_r(\hat{x} \neq x_i \mid x_i \text{ Trxd})$

As $P_r(x_i) = 1/8$

$= \sum_{i=1}^{8} \frac{1}{8} \; P_r(\hat{x} \neq x_i \mid x_i \text{ was } T_x)$

$\Rightarrow P(\hat{x} \neq x_i \mid x_i \text{ was Tx})$

As $y = x + n$

$= P(\hat{x} \neq x_i \mid x_i \text{ was Tx})$

$P(A_1 \cup A_2 \cdots \cup A_7) \leq P(A_1) + P(A_2) \cdots P(A_7)$

$= \cancel{1 - P(\|y - x_i\|^2 < \|y - x_j\|^2)}$ $\cancel{j \neq i,\ j = 1, 2 \ldots 8}$

$$= P\left(\bigcup_{\substack{j=1 \\ j \neq i}}^{8} \{\|y - x_j\|^2 < \|y - x_i\|^2,\ y = x_i + n\}\right)$$

$$\leq \sum_{\substack{j=i \\ j \neq i}}^{8} P\left\{\|y - x_j\|^2 < \|y - x_i\|^2 \,\middle|\, y = x_i + n\right\}$$

If we add $P(\hat{x} \neq x_i \mid x_i)$ $\rightarrow$ we will get 'upper bound'
upper union bound

$\underbrace{P\{\|y - x_j\|^2 < \|y - x_i\|^2\}}_{\text{Pair wise error probability}}$ $\{y = x + n_i\}$

as y is added ~~with~~ n so x_i closer to x_j

$x \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\|x\|^2 = x_1^2 + x_2^2 + x_3^2$

norm $\quad \|V\|^2 = V^T V$

$\Rightarrow P(\|y - x_j\|^2 < \|y - x_i\|^2)$

$\Rightarrow P(\|x_i - x_j + n\|^2 < \|n\|^2)$

$= P(\|x_i - x_j\|^2 + 2(x_i - x_j)^T n + n^2 \leq n^2)$

using As

$\|A + B\|^T$

$= (A+B)^T (A+B)$

$= (A^T + B^T)(A+B)$

$= A^T A + A^T B + B^T A + B^T B$

$= P(\|x_i - x_j\|^2 + 2(x_i - x_j)^T n < 0)$

$\Rightarrow P(2(x_j - x_i)^T n < -\|x_j - x_i\|^2)$

Let a vector $V_{ij} \Rightarrow (x_j - x_i)^T$

$\Rightarrow P(2 V_{ij}^T n < -\|V_{ij}\|^2)$

$\Rightarrow P\left(V_{ij}^T n < -\frac{\|V_{ij}\|^2}{2}\right)$

$\Rightarrow V_{ij}^T n = [V_{ij}(1) \quad V_{ij}(2) \quad V_{ij}(3)] \begin{bmatrix} n(1) \\ n(2) \\ \vdots \\ n(3) \end{bmatrix}$ iid $N(0, \sigma^2)$

$\Rightarrow V_{ij}(1) n(1) + V_{ij}(2) n(2) + V_{ij}(3) n(3)$

Here mean $= \underline{0}$

variance $\Rightarrow \|V_{ij}\| n^2$ as dot product will be zero

$\Rightarrow E[V_{ij}^2 n_1^2 + V_{ij}^2(2) n(2)^2 + V_{ij}(3)^2 n(3)^2 + 2 V_{ij}(1) V_{ij}(2) V E[n_1 n_2] + \ldots]$, with $n_1^2 \to \sigma^2$, $n(2)^2 \to \sigma^2$, $E[n_1 n_2] \to 0$

$\Rightarrow (V_{ij}(1)\sigma^2 + V_{ij}(2)\sigma^2 + V_{ij}(3)\sigma^2)$

$$\boxed{V_{ij}^T n \Rightarrow N(0, \sigma^2 \|V_{ij}\|^2)}$$

$$P\left(v_{ij}^T n \le -\frac{\|v_{ij}\|^2}{n}\right) \to d$$

$$\Rightarrow \|v_{ij}\|^2 = \frac{\|x_i - x_j\|^2}{d_{ij}^2}$$

So $\rightarrow$ $v_{ij}^T n \sim N(0, \sigma^2 d_{ij}^2)$

So $P\left(\frac{v_{ij}^T n}{\sigma d_{ij}} < -\frac{d_{ij}}{2\sigma}\right)$

So $v_{ij}^T n \sim N(0, 1)$

$$\Rightarrow \int_{d_{ij}/2\sigma}^{-d_{ij}/2\sigma} e^{-t^2/2}\,dt = Q\left(\frac{d_{ij}}{2\sigma}\right)$$

$$\Rightarrow P(\hat{x} \ne x_i / x_i) \le \sum_{\substack{j=1 \\ j \ne i}}^{8} Q\left(\frac{d_{ij}}{2\sigma}\right)$$

As

$$P_e = P_r(\hat{x} \ne x)$$

$$= \frac{1}{8} \sum_{i=1}^{8} P_r(\hat{x} \ne x_i \mid x_i \text{ was } T_x) \le \sum_{\substack{j=1 \\ j \ne 1}}^{8} Q\left(\frac{d_{ij}}{2\sigma}\right)$$

$$P_e \le \frac{1}{8} \sum_{i=1}^{8} \sum_{\substack{j=1 \\ j \ne i}}^{8} Q\left(\frac{d_{ij}}{2\sigma}\right)$$

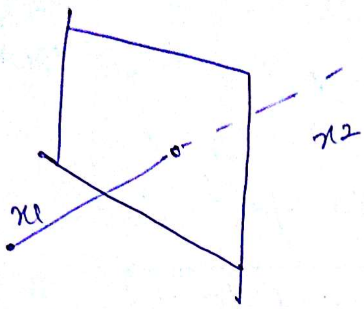
Union bound

$$\Rightarrow \text{if } P(A \cap B \cap C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\text{So } P\left(\bigcup_{\substack{j=1 \\ j \ne 1}}^{8} \left\{ \|y - x_j\|^2 < \|y - x_i\|^2 \mid y = x \right\}\right)$$

So ~~ka~~ Analysis terms

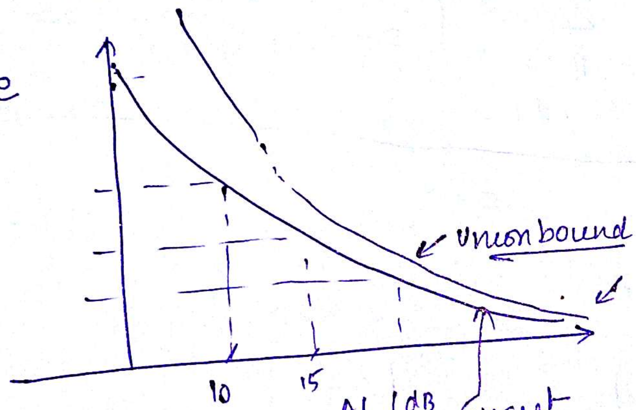
$$P\left(\{\|y-x_2\|^2 < \|y-x_1\|^2\} \cap \{\|y-x_3\|^2 < \|y_1-x_1\|^2\}\right)$$



At sufficient S/σ → $A\cap B$, $B \cap C$ components decrease

then $A \neq B, C$ so ignored

So called tight union bound



We know

$$\boxed{\frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2} < e^{-x^2/2}}$$

upper bound

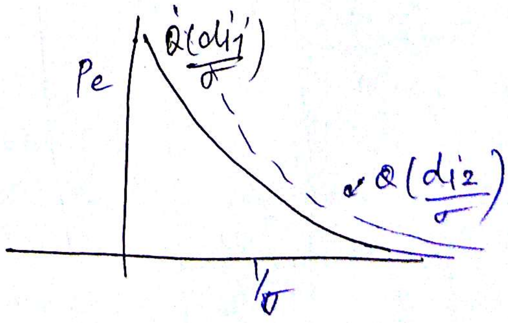
Ac

$$\Rightarrow P_e < \frac{1}{8}\sum_{i=1}^{8}\sum_{\substack{j=1\\ j\neq i}}^{8} Q\left(\frac{d_{ij}}{2\sigma}\right)$$

Let say $Q\left(\frac{d_{i1}}{\sigma}\right)$

$Q\left(\frac{d_{i2}}{\sigma}\right)$

$d_{i1} > d_{i2}$



$$\Rightarrow \lim_{\sigma\to 0} \frac{Q\left(\frac{d_{i1}}{\sigma}\right)}{Q\left(\frac{d_{i2}}{\sigma}\right)} = 0 \qquad d_{i1} > d_{i2}$$

→ Use L'hospital Rule

In $P_e < \frac{1}{8} \sum_{i=1}^{8} \sum_{j=1}^{8} Q\left(\frac{d_{ij}}{2\sigma}\right)$

Out of $Q\left(\frac{d_{ij}}{2\sigma}\right) \rightarrow$ 56 terms. effective different 28 distance

as $\sigma \uparrow \rightarrow$ all these approach 0

$\rightarrow$ slowest ~~factor~~ rate to approach 0. for which d_{ij} will be least

$\boxed{d_{min} \rightarrow \text{min distance of code}}$

At high S/N $\Rightarrow$

$$\boxed{P_e < \frac{1}{8} \sum_{i=1}^{8} \sum_{\substack{j=1 \\ j \neq i}} Q\left(\frac{d_{ij}}{2\sigma}\right) \leq 7 Q\left(\frac{d_{min}}{2\sigma}\right)}$$

In previous code $+A, -A, \ldots$

$d_{min} = 2A$

$P_e = 3Q\left(\frac{A}{\sigma}\right)$

Energy per bit $= A^2$

1	0	r
-1	0	r
0	1	r
0	-1	r
$1/\sqrt{2}$	$1/\sqrt{2}$	$-r$
$1/\sqrt{2}$	$-1/\sqrt{2}$	$-r$
$-1/\sqrt{2}$	$1/\sqrt{2}$	$-r$
$-1/\sqrt{2}$	$-1/\sqrt{2}$	$-r$

$\sqrt{\frac{3}{\left(1+\frac{1}{\sqrt{8}}\right)}}$

$r \Rightarrow \frac{1}{2^{3/4}}$

$\text{Power} = \frac{1+r^2}{3} =$ total power of any code

$-4 \mapsto$ So that we get $P_e = 3Q\left(\frac{1}{\sigma}\right)$

$P = 1$

$d_{min} = 2$

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Taking 0 and 4

$$= \left[\left(1-\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2} + \sqrt{2}\right]\frac{3}{\left(1+\frac{1}{\sqrt{8}}\right)}$$

$$\Rightarrow \frac{6}{\left(1+1/\sqrt{8}\right)} = d_{min}^2$$

d_{min} is need = 2.144

$$\text{Here probablity} = \frac{7\,Q\left(\frac{1.0052}{\sigma}\right)}{3\,Q\left(\frac{1}{\sigma}\right)} \approx \frac{7}{3}$$

Date : 21 . 09 . 2016

* Spectral Mask has to be specified by the companies to avoid interference with neighbouring signals. And the signal is supposed to be transmitted in the spectral mask only.

$\cancel{\int x(t)\phi_k(t)\,dt}$

Let say

$$x(t) \in \left[-\frac{W}{2}, \frac{W}{2}\right] \text{ Bandlimited}$$

$$\phi_k(t) = \operatorname{sinc}[Wt - k]$$

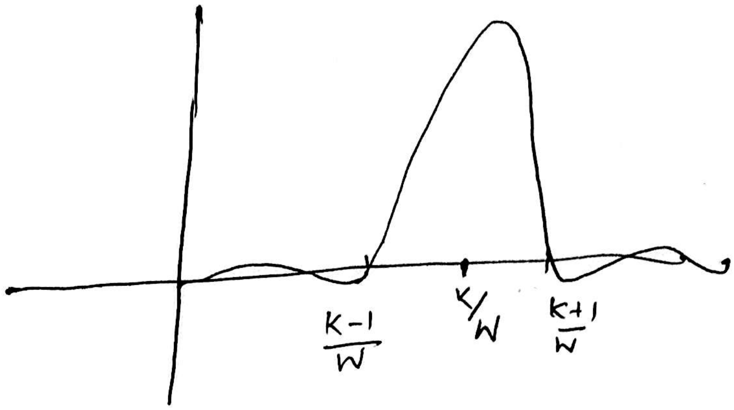
$$x(kT) = \int_{-\infty}^{\infty} x(t)\,\phi_k(t)\,dt$$

$$= \sum_{n=-\infty}^{\infty} \int x(nT) \sin(Wt-k)\operatorname{sinc}(Wt-n)\,dt$$

$x(t) \in \left[-\frac{W}{2}, \frac{W}{2}\right]$

$$\phi_k(t) = \text{sinc}[Wt - k]$$

$$x(t) = \sum_{k=-\infty}^{\infty} x[k]\, \phi_k(t)$$



Here is in the time domain

$$x(t) = \sum_{0}^{WT} x(k) \cdot \phi_k(t)$$

$$= A \sum_{0}^{WT} \phi_k(t)$$

when $\vec{x}[k]$ is constant to $\underline{A}$

$$A \sum_{0}^{WT} \text{sinc}(Wt - k]$$

Earlier in ~~time~~ frequency domain, here in time domain, total complex waveforms are $\underline{WT+1}$

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21st Sep

Practically each telecom operator is given the specific band for uplink & downlink.

But they can't be much closer because it will produce interference.

case of interference: Suppose a user is given a band to talk but his some of the energy will be definitely outside his band. Because time limited signal can be band-limited.

→ Hence there will be gaurd band

→ gaurd band size depends on the filter charac. we are using.

(filter can't be having a perfect cutoff. → order will be ∞ (ideally))

→ Filter charac. depends on how our signal is throwing its power outside its band.

→ For this spectral mass graph is drawn.

$$\frac{|X(f)|^2}{\text{Total power (ask)}} \longrightarrow \text{Band. freq}^n.$$

(f) →

extra :

$$\int_{-\infty}^{\infty} \text{sinc}(\omega t - n)\, \text{sinc}(\omega t - k)\, dt$$

$= 0 \quad , n \neq k$

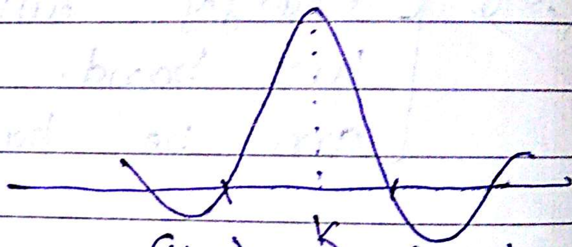
$=$ constant , $n = k$

continue

if $x(t) \rightarrow \left[-\frac{W}{2}, \frac{W}{2}\right]$

take basis as $\phi_k(t) = \text{sinc}(Wt - k)$

$$x[t] = \sum_{k=-\infty}^{\infty} X[k]\, \phi_k(t)$$



what will be k Range.

in -ve region we don't have energy

$\therefore k = 0$

now $\frac{k}{W} \approx T$ because signal is in $[0, T]$

$\therefore k \approx WT$

$$\therefore x(t) = \sum_{k=0}^{WT} x[k]\, \phi_k(t)$$

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24th Sep

Like previous class.

$[0, T]$ & $\left[-\frac{B}{2}, \frac{B}{2}\right]$

$$x(t) = \sum_{n=0}^{BT} x[n] \operatorname{sinc}(Bt - n)$$

$$\downarrow$$

$$\operatorname{sinc} B(t - n/B)$$

→ Suppose we are sending all 0's.

$$x[n] = +A \text{ for all } n.$$

$$\therefore x(t) = A \sum_{n=0}^{BT} \operatorname{sinc}\left(B\left(t - \frac{n}{B}\right)\right)$$

here $\phi(t) = \operatorname{sinc}(Bt)$.

$$\Phi(f) = \frac{1}{B}, \quad |f| \le \frac{B}{2}$$
$$= 0, \quad \text{otherwise}$$

FOR EXACT :-P

$$\therefore X(f) = \begin{cases} \dfrac{A}{B} \displaystyle\sum_{n=0}^{BT-1} e^{\frac{-j2\pi f n}{B}} & ; |f| \le \frac{B}{2} \\ 0 & ; \text{otherwise.} \end{cases}$$

$$= \begin{cases} \dfrac{A}{B} \left[\dfrac{\left(e^{-j2\pi f T} - 1\right)}{\left(e^{\frac{-j2\pi f}{B}} - 1\right)} \right] & ; |f| \le \frac{B}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

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extra :-

$$S = \sum_{n=0}^{N-1} \alpha^n = 1 + \alpha + \dots + \alpha^{N-1}$$

$$\alpha S = \sum_{n=0}^{N-1} \alpha^{n+1} = \alpha + \alpha^2 + \dots + \alpha^N$$

$$(S - \alpha S) = 1 - \alpha^N$$

$$\therefore S = \frac{1-\alpha^N}{1-\alpha}$$

continue :-

$$X(f) = \frac{e^{-j2\pi fT}\left(e^{-j\pi fT} - e^{j\pi fT}\right)\Big/ 2j}{e^{-j\pi f/B}\left(e^{-j\pi f/B} - e^{j\pi f/B}\right)\Big/ 2j}$$

$$= \frac{A}{B} \frac{e^{-j\pi fT} \sin\pi fT}{e^{-j\pi f/B} \sin\pi f/B}$$

we know that $\frac{1}{B} = T_s << T$

$$\left[-\frac{B}{2}, \frac{B}{2}\right] \rightarrow \left[-\frac{1}{2T_s}, \frac{1}{2T_s}\right]$$

now, amp. of $|\underline{X(f)}|^2$

$$\frac{A^2}{B^2} \frac{\sin^2\pi fT}{\sin^2\pi f/B}$$

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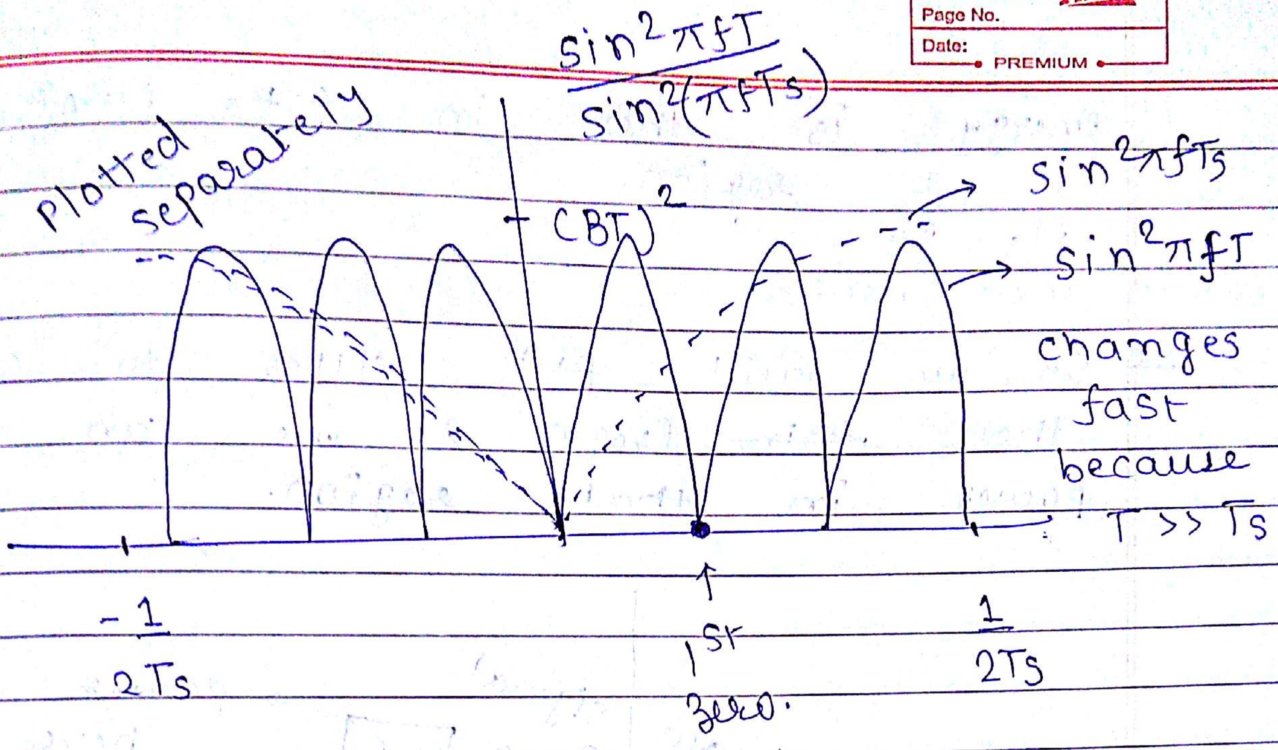
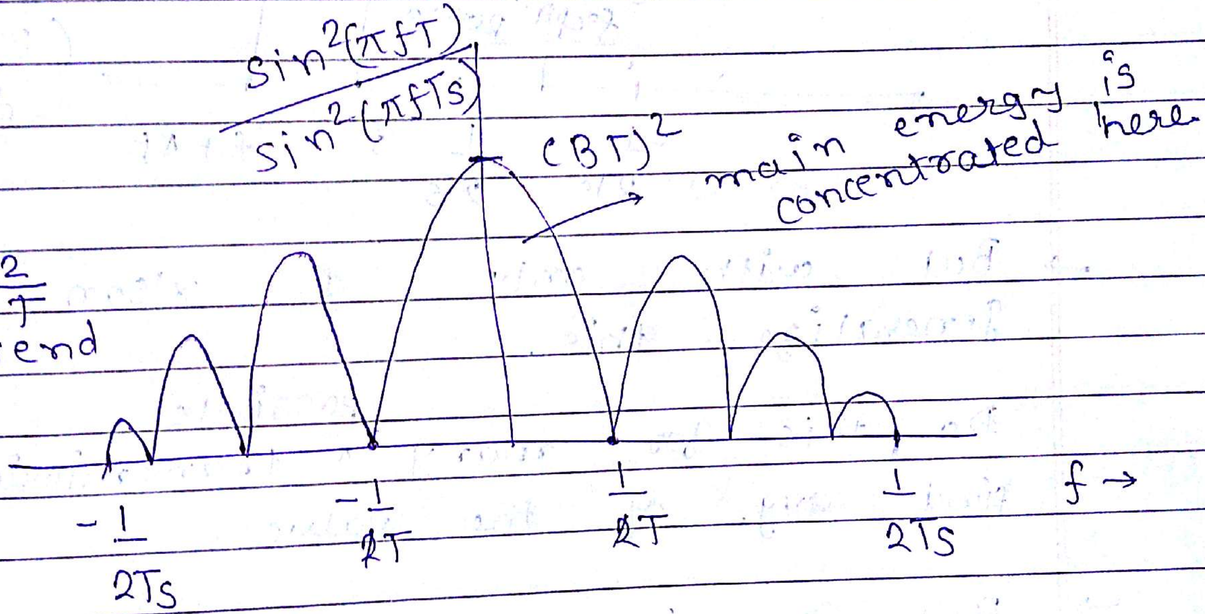




☆ B.W. is $\frac{2}{T}$ if we send all +A. it is wrong :-)

all are +A hence it is almost DC signal. (0 Hz)

info. symbols.

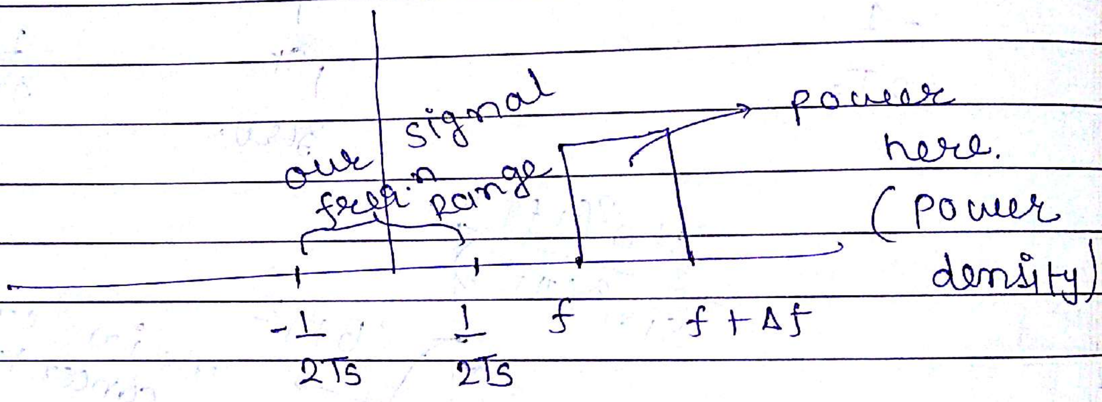
hence all the energy is stored around zero freq.n.

here some power will be spilled out of $-\frac{1}{2T_S}$, $\frac{1}{2T_S}$ also.

We want to measure that how much

energy is there in f to $f+\Delta f$ outside $-\frac{1}{2T_s}, \frac{1}{2T_s}$ region.

→ So, we will put filter to select these ~~thin~~ freq.n & we can find power in that region.



→ But with only 1 x'son we can't generalize this.

Do this for many possible transmissions & find avg. of the values.

⇒ ~~But it is not possible. take~~

suppose signal $x(t)$ $\in [0, T]$

$$\int_0^T |x(t)|^2 dt < \infty$$

energy. But now $x(t)$ is a Random Process.

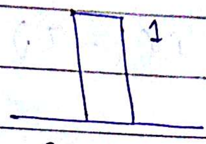
$$x(t) = \sum_{n=0}^{BT-1} x[n] \operatorname{sinc}(Bt-n)$$

→ we want energy (OR power) in $[f, f+\Delta f]$

1st way to do this

$X(t)$ → BPF → which will allow only this band.

complex Random waveform.



1

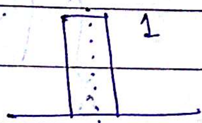
f $f+\Delta f$

$Y(t)$

OR

consider

$h(t)$ for this is.

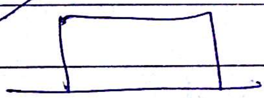


1

$f-\frac{\Delta f}{2}$ $f+\frac{\Delta f}{2}$

$f_0 \rightarrow$ center freq.^{n}

h(t) calculation

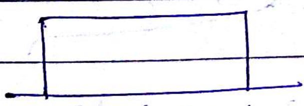


$-B/2$ $B/2$

$B\,\text{sinc}(Bt)$

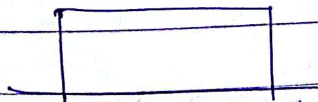
$X(f) \leftrightarrow x(t)$

$X(f-f_0) \leftrightarrow x(t)e^{j2\pi f_0 t}$



$-\Delta f/2$ $\Delta f/2$

$\Delta f\,\text{sinc}(\Delta f t)$



$f-\Delta f/2$ $f+\Delta f/2$

$\Delta f\,\text{sinc}(\Delta f t)\, e^{j2\pi f t} = h(t)$

→ now to get this energy

$$\int_0^T |Y(t)|^2\, dt$$

limits are $[0, T]$ because $X(t)$ is limited to $[0, T]$

hence we can take limits in the integration $(-\infty, \infty)$ also.

$$\rightarrow \underbrace{y_{f_0, \Delta f}(t)}_{\text{notation}} = \int_{-\infty}^{\infty} x(t-\tau_1)\, h(\tau_1)\, d\tau_1$$

$$\rightarrow \int_0^T |y(t)|^2 dt = \int_0^T \left| y_{f_0,\Delta f}(t) \right|^2 dt$$

$$\downarrow$$

$$y_{f_0,\Delta f}(t)\; y^*_{f_0,\Delta f}(t)$$

$$y^*_{f_0,\Delta f}(t) = \int_{-\infty}^{\infty} x^*(t-\tau_2)\, h^*(\tau_2)\, d\tau_2$$

$$\rightarrow \therefore \int_{-\infty}^{\infty} \left| y_{f_0,\Delta f}(t) \right|^2 dt = \int_{-\infty}^{\infty} \overbrace{\int_{\tau_1:-\infty}^{\infty} x(t-\tau_1)\, h(\tau_1)\, d\tau_1}^{y_{f_0,\Delta f}(t)} \; \underbrace{\int_{\tau_2:-\infty}^{-\infty} x(t-\tau_2)\, h(\tau_2)\, d\tau_2}_{y^*_{f_0,\Delta f}(t)}$$

→ now take E of this eq.n. which will give avg. for all the possibilities.

MIRAJ

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PREMIUM

$$= E\left[\int_{-\infty}^{\infty}\int_{\tau_1=-\infty}^{\infty}\int_{\tau_2=-\infty}^{\infty} \underbrace{x(t-\tau_1)\, x^*(t-\tau_2)}\, h(\tau_1)\, h^*(\tau_2)\, d\tau_1\, d\tau_2\, dt\right]$$

$h(t) = \Delta f \operatorname{sinc}(\Delta f\, t) e^{j2\pi f t}$

Only this part is random now. put Expectation here

$$= \int_{t=-\infty}^{\infty}\int_{\tau_1=-\infty}^{\infty}\int_{\tau_2=-\infty}^{\infty} \underbrace{E\left[x(t-\tau_1)\, x^*(t-\tau_2)\right]}\, h(\tau_1)\, h^*(\tau_2)\, d\tau_1\, d\tau_2\, dt$$

considering Random waveform $x(t)$ as WSS.

$E[x(t)] = C$

auto correlation of the R.P. X ← $E[x(t)\, x^*(t-\tau)]$

$= R_x(\tau)$ only depends on τ.

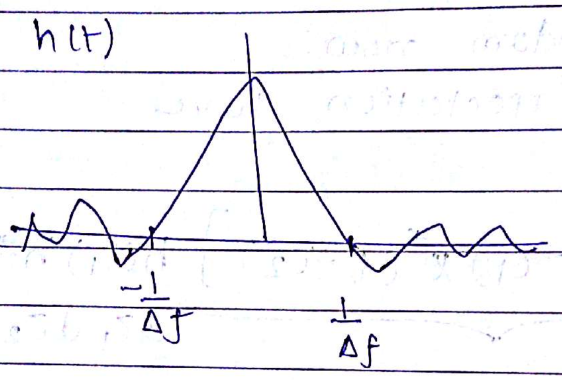
$$= \int_{t=-\infty}^{\infty}\int_{\tau_1=-\infty}^{\infty}\int_{\tau_2=-\infty}^{\infty} R_x(\tau_2-\tau_1)\, h(\tau_1)\, h^*(\tau_2)\, d\tau_1\, d\tau_2\, dt$$

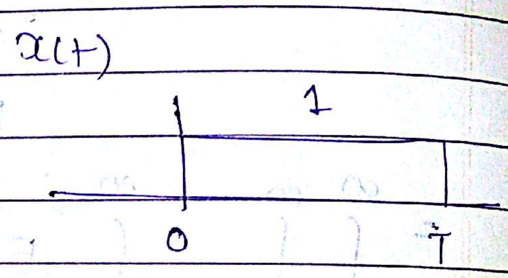
$T >> \frac{1}{B}$

$BT >> 1$ very large no. of samples.

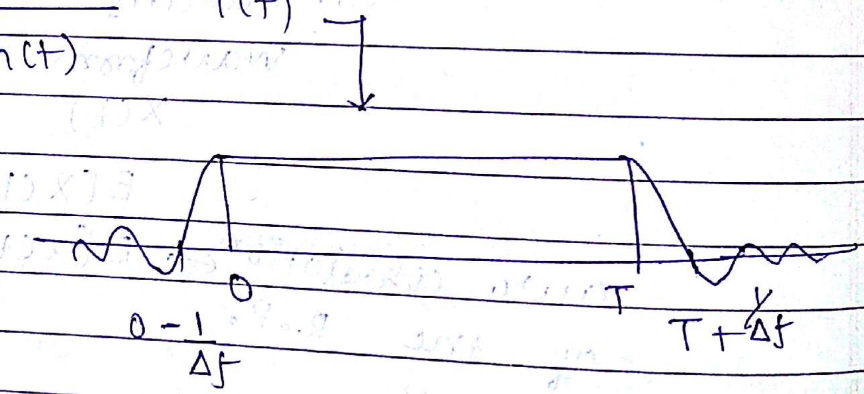
But for our BPF $T >> \frac{1}{\Delta f}$ } means while measuring $[0, T]$ we can't get precesion smaller than $1/T$
$\Delta f >> 1/T$

then we can take $t \in [0, T]$ instead of $[-\infty, \infty]$





$$x(t) \xrightarrow[h(t)]{BPF} y(t)$$



hence in o/p $y(t)$ if we take $(-\infty, \infty)$ or $[0, T]$ most energy will be concentrated there only.

$$= \int_{t=0}^{T} \int_{\tau_1=-\infty}^{\infty} \int_{\tau_2=-\infty}^{\infty} R_x(\tau_2 - \tau_1)\, h(\tau_1)\, h^*(\tau_2)\, d\tau_1 d\tau_2 dt$$

$$= T \int_{\tau_1 = -\infty}^{\infty} \int_{\tau_2 = -\infty}^{\infty} R_x(\tau_2 - \tau_1)\, h(\tau_1)\, h^*(\tau_2)\, d\tau_1 d\tau_2$$

$$\tau = \tau_2 - \tau_1 \quad \therefore \tau_2 = \tau + \tau_1$$

$$= T \int_{\tau_1 = -\infty}^{\infty} \int_{\tau = -\infty}^{\infty} R_x(\tau)\, h(\tau_1)\, h^*(\tau + \tau_1)\, d\tau_1\, d\tau \quad \text{--- (1)}$$

$$= T \sum_{\tau_1 = -\infty}^{\infty} \sum_{\tau_2 = -\infty}^{\infty} R_x[\tau_2 - \tau_1]\, h[\tau_1]\, h^*[\tau_2]$$

$$= T \sum_{\tau_1 = -\infty}^{\infty} h[\tau_1] \underbrace{\sum_{\tau_2 = -\infty}^{\infty} R_x[\tau_2 - \tau_1]\, h^*[\tau_2]}$$

here τ_1 will be constant.

$$\tau = \tau_2 - \tau_1$$

$$= T \sum_{\tau_1 = -\infty}^{\infty} h[\tau_1] \sum_{\tau = -\infty}^{\infty} R_x[\tau]\, h^*[\tau + \tau_1] \quad \text{--- (2)}$$

(1) & (2) are same

Solve eq.n (1)

$$= T \int_{\tau = -\infty}^{\infty} R_x(\tau) \left[\int_{\tau_1 = -\infty}^{\infty} h(\tau_1)\, h^*(\tau + \tau_1)\, d\tau_1 \right] d\tau$$

$$\left[f_0 - \frac{\Delta f}{2}, \; f_0 + \frac{\Delta f}{2}\right]$$

$$h(t) = (\Delta f) \cdot \mathrm{sinc}(\Delta f\, t) \cdot e^{j2\pi f_0 t}$$

$$\int_{-\infty}^{\infty} h(t)\, h^*(t+\tau)\, dt = (\Delta f)^2 \int_{-\infty}^{\infty} \mathrm{sinc}(\Delta f\, t)\, \mathrm{sinc}(\Delta f(t+\tau))\, e^{-j2\pi f_0 \tau}\, dt$$

τ is replaced by t.

$$= T(\Delta f) \int_{\tau:-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f_0 \tau} \underbrace{\left[\int_{-\infty}^{\infty} (\Delta f)\, \mathrm{sinc}(\Delta f\, t)\, \mathrm{sinc}(\Delta f(t+\tau))\, dt\right]} d\tau$$

$$\int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} X(f)\, Y^*(f)\, dt$$

use parseval's thm.

$$x(t) = \mathrm{sinc}(\Delta f\, t)$$

$$X(f) = \begin{cases} \frac{1}{\Delta f} & ; |f| < \Delta f/2 \\ 0 & ; \text{Otherwise} \end{cases}$$

$$y(t) = \mathrm{sinc}(\Delta f(t+\tau))$$

$$Y(f) = \begin{cases} \frac{1}{\Delta f} e^{j2\pi f \tau} & ; |f| < \frac{\Delta f}{2} \\ 0 & ; \text{otherwise} \end{cases}$$

$$= T(\Delta f) \int_{\tau:-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f_0 \tau} \int_{-\Delta f/2}^{\Delta f/2} \frac{\Delta f}{(\Delta f)^2}\, e^{-j2\pi f \tau}\, df\, d\tau$$

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$$= T(\Delta f) \int_{\tau:-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f_0 \tau}\, \text{sinc}(\Delta f \tau)\, d\tau$$

This is energy.

Remove $T \longrightarrow$ gives power

$$\text{power} = (\Delta f) \int_{\tau:-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f_0 \tau}\, \text{sinc}(\Delta f \tau)\, d\tau.$$

$$\underset{[-T,T]}{R_X(\tau)} = E[\underset{[0,T]}{X(t)\, X^*(t-\tau)}]$$

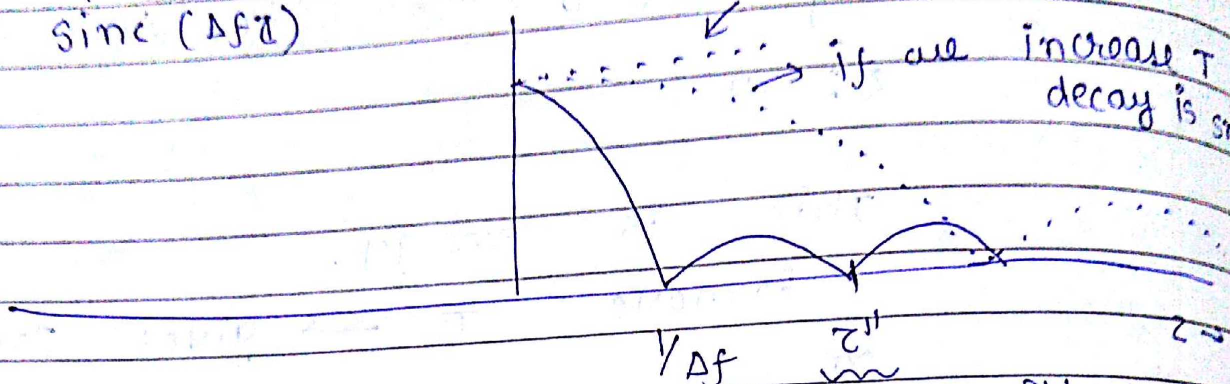
$$= (\Delta f) \int_{\tau:-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f_0 \tau}\, \text{sinc}(\Delta f \tau)\, d\tau$$

divide by $\Delta f \rightarrow$ gives power spectral density.

$$PSD = \int_{\tau:-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f_0 \tau}\, \text{sinc}(\Delta f \tau)\, d\tau$$

Take $\lim_{\tau \to \infty} R_X(\tau) = 0$ means $X(t)$ & $X(t-\tau)$ are more uncorrelated.

Plotting
→ sinc ($\Delta f \tau$)



we will consider till this only
Others neglect.

as $\underbrace{T\uparrow} >> 1/\Delta f \downarrow$ $\uparrow$ ∴

Observation window → Δf → sinc will look like
larger smaller

——— 1
constant

$$\therefore PSD = \int_{-\tau''}^{\tau''} R_x(\tau)\, e^{-j2\pi f_0 \tau}\, d\tau$$

$T \approx \underbrace{\frac{100}{\Delta f}}$
very high

→ at Rx we can do
vary T or Δf.

hence now if $\underbrace{\tau''}_{T}$ is anything

we can make Δf smaller to satisfy
the above eq.n.

→ This way we are getting away from
sinc fun.n

$$\therefore PSD = \int_{-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f_0 \tau}\, d\tau = \lim_{\Delta f \to 0} \frac{P\left(f_0 - \frac{\Delta f}{2}, f_0 + \frac{\Delta f}{2}\right)}{\Delta f}$$

$\tau'' \to \infty$

$$= S_X(f_0)$$

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27th Sep

earlier

* we know that for Random Process

☆ All the thing is going on for comp. base band signal.

$$X(t) \longrightarrow S_X(f) = \int_{-\infty}^{\infty} R_X(\tau)\, e^{-j2\pi f \tau}\, d\tau$$

$$R_X(\tau) = E\left[X(t)\, X^*(t-\tau)\right]$$

→ Suppose allowed B.W. $[-B/2, B/2]$
Hence ideally $X(t)$ should be in this range only.

→ Suppose we have two x'son schemes gives $X(t)$ & $Y(t)$ which one we will choose

$X(t)$	$Y(t)$
↓	↓
$S_X(f)$	$S_Y(f)$
total power x'mitted $\int_{-\infty}^{\infty} S_X(f)\, df$	$\int_{-\infty}^{\infty} S_Y(f)\, df$

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$R_x(-\tau) = R_x(\tau)$ This is for Real R.P.

$R_x(-\tau) = R_x^*(\tau)$ for complex. R.P.

now total power is also same

~~then~~ But we should check the fraction of power in allocated band to the total power

$$\frac{\int_{-B/2}^{B/2} S_x(f)\,df}{\int_{-\infty}^{\infty} S_x(f)\,df} \qquad \frac{\int_{-B/2}^{B/2} S_y(f)\,df}{\int_{-\infty}^{\infty} S_y(f)\,df}$$

take which-
-ever is fraction is
high.

→ But what if this fraction is also same, then.

we know at the Rx we are having filter

extra:-

$S_x(-f) = S_x(f)$ proof. True only when $x(t)$ is Real R.P.

$$S_x(f) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{-j2\pi f\tau}\, d\tau$$

$$S_x(-f) = \int_{-\infty}^{\infty} R_x(\tau)\, e^{j2\pi f\tau}\, d\tau$$

PSD can not be -ve or complex.

put $\tau' = -\tau$

$$\therefore S_x(-f) = \int_{-\infty}^{\infty} R_x(-\tau') \, e^{-j2\pi f \tau'} \, d\tau'$$

$$= \int_{-\infty}^{\infty} R_x^*(\tau') \, e^{-j2\pi f \tau'} \, d\tau'$$

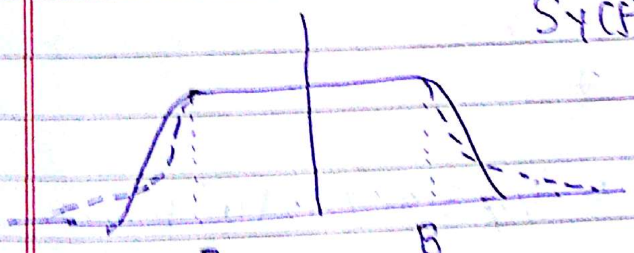
$$\boxed{S_x(f) \neq S_x(-f) \text{ always}}$$

continue

now as the ratio is same fraction of power outside the band is same.

$S_x(f)$ — line

$S_Y(f)$ --- line



$S_x(f)$ contains extra power in little band. Hence it won't affect further band.

→ while $S_Y(f)$ can affect other band.

Hence we will choose $S_x(f)$.

$S_x(f)$ has better Roll-off.

e.g. on Random Process

(*) ① $x(t)$ is a R.P.

P	Signal
1/4	$\sin 2\pi f_1 t$
1/8	$\sin 2\pi f_2 t$
1/2	$\sin 2\pi f_3 t$
1/8	$\sin 2\pi f_4 t$

Sample this at $t = t_0$.

$x(t = t_0) = z$ is a R.V.

here t_0 is fixed.

z can take $\sin 2\pi f_1 t_0$, $\sin 2\pi f_2 t_0$, $\sin 2\pi f_3 t_0$, $\sin 2\pi f_4 t_0$

with $P = \frac{1}{4}$, $1/8$, $1/2$, $1/8$

mean of R.V. z

$$= \frac{\frac{1}{4}\sin 2\pi f_1 t_0 + \frac{1}{8}\sin 2\pi f_2 t_0 + \ldots \text{4 terms}}{4}$$

is this WSS

if t_0 change mean changes

mean should be constant.

Hence it is depending on t_0.

its not WSS.

Modify $x(t)$ now

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	P	Signal	
$\rightarrow$ $x(t)$	~~1/4~~ 1/2	$\sin 2\pi f_1 t$	— 1
	~~1/4~~ 1/2	$-\sin 2\pi f_1 t$	— 2
	~~1/4~~	~~$\sin 2\pi f_2 t$~~	
	~~1/4~~	~~$-\sin 2\pi f_2 t$~~	

now mean is constant.

$E[\underbrace{x(t)\,x(t-\tau)}]$ find this.

look this combination as a R.P.

$x(t) \cdot x(t-\tau)$ is another R.P. denote it as $Z_\tau(t)$

$Z_\tau(t)$ possible waveforms.

$x(t) \rightarrow 1 \rightarrow \frac{1}{2}$ $\sin 2\pi f_1 t \sin 2\pi f_1 (t-\tau)$

$x(t) \rightarrow 2 \rightarrow 1/2$ $\sin 2\pi f_1 t \sin 2\pi f_1 (t-\tau)$ } same

$\therefore Z_\tau(t) \xrightarrow{\text{has}}$ $\sin 2\pi f_1 t \sin 2\pi f_1 (t-\tau)$
here waveform is deterministic.

$\rightarrow$ $x(t)$ P Signal.