

using the expressions in ② inside ① we have ②

$$y_p(t) = \int (h^I(\tau) \cos m f_c \tau - h^Q(\tau) \sin m f_c \tau) (x^I(t-\tau) \cos m f_c (t-\tau) - x^Q(t-\tau) \sin m f_c (t-\tau)) d\tau$$

$$= \int h^I(\tau) x^I(t-\tau) \cos m f_c \tau \cos m f_c (t-\tau) d\tau + \int h^Q(\tau) x^Q(t-\tau) \sin m f_c \tau \sin m f_c (t-\tau) d\tau - \int h^I(\tau) x^Q(t-\tau) \cos m f_c \tau \sin m f_c (t-\tau) d\tau - \int h^Q(\tau) x^I(t-\tau) \sin m f_c \tau \cos m f_c (t-\tau) d\tau$$

$$= \frac{1}{2} \left[\int h^I(\tau) x^I(t-\tau) d\tau \right] \cos m f_c t + \frac{1}{2} \left[\int h^I(\tau) x^I(t-\tau) \cos m f_c (2\tau - t) d\tau \right] + \frac{1}{2} \left[\int h^Q(\tau) x^Q(t-\tau) d\tau \right] [-\cos m f_c t] + \frac{1}{2} \int h^Q(\tau) x^Q(t-\tau) \cos m f_c (2\tau - t) d\tau - \frac{1}{2} \left[\int h^I(\tau) x^Q(t-\tau) d\tau \right] [\sin m f_c t] + \frac{1}{2} \int h^I(\tau) x^Q(t-\tau) \sin m f_c (2\tau - t) d\tau - \frac{1}{2} \left[\int h^Q(\tau) x^I(t-\tau) d\tau \right] [\sin m f_c t] + \frac{1}{2} \left[\int h^Q(\tau) x^I(t-\tau) \sin m f_c (2\tau - t) d\tau \right]$$

③