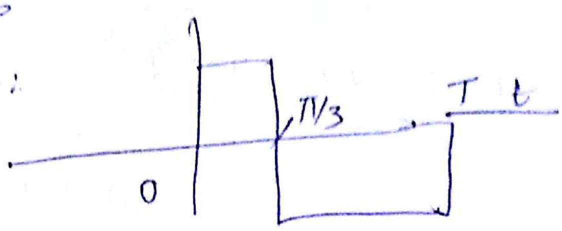
Let for $\tau = 0$

$$E[x(t)\,x^*(t-\tau)] \Rightarrow \sum_{-\infty}^{\infty} p(t-KT)^2$$

Let say pulse the :

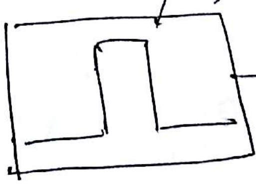


Date 4-10th 2016

$$\int_{-T}^{T} \underbrace{|x(t)|^2 dt}_{2T} = \int_{-\infty}^{\infty} \underbrace{|X(f)|^2 df}_{2T}$$

$h(t)$

$x(t)$ ⟶



⟶ $\underbrace{\int_{-\infty}^{\infty} |x_{f_0,\Delta f}(t)|^2 dt}_{2T}$

$f_0+\frac{\Delta f}{2} \to f_0-\frac{\Delta f}{2}$

If $x(t)$ which is from $[-T, T]$ and passed from $h(t)$ i.e band pass

At the output $\int_{-\infty}^{\infty} |x_{f_0,\Delta f}(t)^2|\, dt$, gives the gives energy with frequency component in $f_0-\frac{\Delta f}{2},\ f_0+\frac{\Delta f}{2}$

• And if we devide the $\int_{\infty}^{\infty} x_{f_0,\Delta f}(t)^2 dt$ 1 by 2T

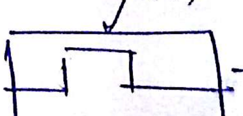
then we will get average energy

from parseval theoram $\int_{\infty}^{\infty} x_{f_0,\Delta f}(t)^2 dt \Rightarrow \int_{\infty}^{a} (X(f)^2)\, df$ $f, \Delta f$

Now

$h(t)$

$x(t)$ ⟶



⟶ $\int_{-\infty}^{\infty} |x_{f_0,\Delta f}(t)|^2 dt = \int_{f_0-\Delta f/2}^{f_0+\Delta f/2} |X(f)|^2 df \,/\, 2T$

These results are for only deterministic signals

So .

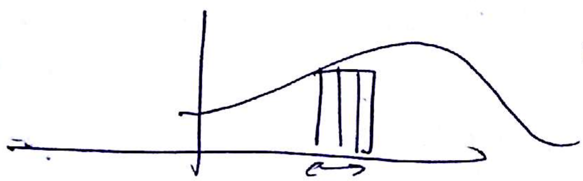
$$P((f_0 - \frac{\Delta f}{2}, f_0 + \frac{\Delta f}{2}) \Rightarrow \int_{f_0 - \frac{\Delta f}{2}}^{f_0 + \frac{\Delta f}{2}} |X(f)|^2 df \Big/ 2T$$

To find PSD divide it by Δf :

$$\Rightarrow \int_{f_0 - \frac{\Delta f}{2}}^{f_0 - \Delta f/2} \frac{|X(f)|^2 df}{2T\Delta f}$$

As deterministic signal $2T$ is fixed

if $G(f) \triangleq |H(f)|^2$



if we shrink the interval $f_0 - \frac{\Delta f}{2}$ $f_0 + \Delta f/2$

then $|H(f)|^2 \Rightarrow |H(f_0)|^2$

using Same ~~reat~~ point

or by Taylor Expansion

$$G(f) = G(f_0) + \Delta f G'(\xi)$$

So using result

$$= \int_{f_0 + \Delta f/2}^{f_0 - \Delta f/2} \frac{|X(f)|^2 df}{2T\Delta f} = \frac{X(f_0)^2 (f_0 + \Delta f/2 - f_0 + \Delta f/2)}{2T\Delta f}$$

$$= \frac{X|f_0|^2}{2T}$$

Doubt

\# As T becomes larger $\frac{X|f_0|^2}{2T}$ will converge to a random value

Each time T' Button is pressed 2N+1 i,i.d sequence of $x(n)$ ~~X(t)~~ are generated

$$x(t) = \sum_{n=-N}^{n=N} x(n)\, p(t-nT) \quad \text{i,i,d seq}$$

$$x(t) \Rightarrow \sum_{n=-N}^{n=N} x(n)\, p(t-nT)$$

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f_0 t} dt$$

$$|X(f)|^2 \Rightarrow (X(f))(X^*(f)) \Rightarrow \int_{-\infty}^{\infty} \left(x(t_1) e^{-j2\pi f_0 t_1} dt_1\right)\left(x^*(t_2) e^{j2\pi f_0 t_2} dt_2\right)$$

$$\Rightarrow \iint_{-\infty}^{\infty} \sum_{n=-NT}^{NT} \sum_{m=-NT}^{m=mT} |x[n]\, x^*[n]|\; p(t_1-nT)\, p^*(t_2-mT)\, e^{-j2\pi f(t_1-t_2)} dt_1\, dt_2$$

$$\Rightarrow \sum_{n=-NT}^{NT} \sum_{m=-NT}^{m=+NT} x(n)\, x^*(m) \int_{t_1=-\infty}^{\infty}\int_{t_2=-\infty}^{\infty} p(t_1-nT)\, p^*(t_2-mT)\, e^{-j2\pi f(t_1-t_2)} dt_1\, dt_2$$

$$= \sum_{n=-NT}^{NT} \sum_{m=-NT}^{m=NT} x(n)\, x^*[m] \left[\int_{-\infty}^{\infty} p(t_1-nT) e^{j2\pi f t_1} dt \int_{-\infty}^{\infty} p^*(t-mT) e^{j2\pi f t_2} dt\right]$$

$$\left[\; p(f)\, e^{-j2\pi f nT} \quad p^*(f)\, e^{jm f mT}\right.$$

$$\Rightarrow \sum_{n=-NT}^{NT} \sum_{m=-NT}^{m=NT} x(n)\, x^*[m]\, |p(f)|^2\, e^{-j2\pi f(n-m)T}$$

$$\Rightarrow \sum_{n=-NT}^{NT} \sum_{m=-NT}^{m=NT} a[n]\, b[n]\, c\,(n-m)$$

$N = M-1$

$c(k+1)$

$$= \sum_{n=-N}^{N} \sum_{m=-N}^{N} a(n)\, b(n)\, c\,((n-m)$$

$\underline{n=m}$ $\quad c(0) \sum_{k=-N}^{N} a[k]\, b[k]$

$n = m+1 \quad c(1) \sum_{k=-N}^{N} a[k]\; b[k-1]$

$n = m-1 \quad c[-1] \sum_{k=-N}^{N-1} a[k]\, b[-k]$

$n = m+2N \Rightarrow c[2N]\; x[n]\, x^*[-N]$

for $n = m+i$ $\quad c[i] \sum_{k=-(N-i)}^{N} x[k]\, x^*[k-i]$

$\to$ or $\quad c(0) \sum_{k=-N}^{k=N} x[k]\, x^*[k]$

$\to c[1] = \sum_{k=-(N-1)}^{N} x[k]\, x^*[k-1]$

$c[-1] \sum_{k=-N}^{N-1} x[k]\, x^*[k+1]$

$\to \overset{n=m-i}{c[-i]} \sum_{k=-N}^{N-i} x[k]\, x^*[k+i]$

Using them in double summation sequence

$$\sum_{m=d}^{N} \sum_{n=-N}^{N} x[n]\, x^*[m]\, |p(f)|^2\, e^{-j2\pi f(n-m)T}$$

$$= \frac{|P(f)|}{T} \sum_{i=1}^{2N} e^{-j2\pi f i T}\left(\sum_{k=-(N-i)}^{N} x[k]\, x^*[k-i]\right)\Big/ 2N$$

$$+ \frac{|P(f)|^2}{T} \sum_{i=-2N}^{-1} e^{j2\pi f i T}\left(\sum_{k=-N}^{N-i} x[k]\, x^*[k+i]\Big/ 2N\right)$$

Continued..

Date: 5/10/2016

$$\frac{|X(f)|^2}{2\pi T} = \frac{|P(f)|^2}{T} \sum_{i=-2N} e^{j2\pi f i T} \left(\frac{\sum_{K=-N+i}^{N} |x(K)|^2}{2N} + \sum_{i=1}^{2N} e^{-j2\pi f i T} \right.$$

$$\left(\frac{\sum_{K=-(N-i)}^{N} x[K]x^*[K-i]}{2N} \right) + \sum_{i=-2N}^{-1} e^{j2\pi f i T} \left(\frac{\sum_{K=-N}^{N+i} x[K]x^*[K+i]}{2N} \right)$$

$$= \frac{|P(f)|^2}{T} \left[\frac{\sum_{K=-N}^{N} |x(K)|^2}{2N} + \sum_{i=1}^{2N} e^{-j2\pi f i T} \left(\sum_{x=-(N-i)}^{N} x(K)x^*[K-i] \right) \right.$$

$$\sum_{n=-N}^{N}\sum_{m=-N}^{N} x[n]x^*[m] e^{-j2\pi f(n-m)T} + \sum_{i=-2N}^{-1} e^{-j2\pi f i T} \left(\frac{\sum_{K=-N}^{N+i} x[K]x^*[K-i]}{2N} \right)$$

For the integer part:

$$\frac{1}{2N} \sum_{k=-(N-i)}^{N} x[K]x^*[K-i]$$

$$= \frac{\sum_{K_1=m_1}^{K_2=m_2} x[K]x^*[K-p]}{m_2-m_1}$$

$\{x[n]\}_{n\to\infty}^{\infty}$ is a discrete time random process

$$= \frac{\sum_{K_1=m_1}^{m_2} x[K]x^*[K-p]}{m_2-m_1}$$

we take lim

$m_2 \to \infty$

$m_1 \to \infty$

$m_2 - m_1 \to \infty$

$$\frac{\sum_{K_1=m_1}^{K_2=m_2} x[K]x^*[K-p]}{m_2-m_1} \Rightarrow$$ As each time, it will be same as $m_2 - m_1 = \infty$

No matter what sequence is there

So let

$$\frac{\sum_{K=m_1}^{m_2} x[K]x^*[K-p]}{m_2-m_1} = R[p] \quad -①$$

Let $\{\hat{x}[n]\} = \{+A, -A\}$ with equal probability $\frac{1}{2}$ & $\frac{1}{2}$

To check for IIDs

Let say outputs are generated multiple times from a machine as 1^{st}, 2^{nd} ... n^{th} .... m^{th}

		$x[n]$	$x[m]$
Trial	#1	+A	+A
Trial	#	−A	+A

Let say

$P(x(n) = +A, x(m) = +A) = 1/9$

$P(-A, +A) = 2/9$

$P(+A, -A) = 2/9$

$P(-A, -A) = 4/9$

And let be tested

$P(x(n)=A) = 1/3$

$P(x(n) = -A) = 2/3$

$P(x(m) = -A) = 1/3$

$P(x(m) = +A) = 2/3$

No independency can be checked by $f_{x,y}(x,y) = f(x)f_y(y)$

Definition

$P(x(n) = a, x(m) = b) = P(x(n=a)\ x(m=b))$

* Toss example

Now coming back to expression $-①$

Let $\{x[n]\}_{-\infty}^{\infty} = \{0, A, 2A\}$ i.i.d

Law of large numbers

if $x[n]$ ~ i.i.d

$$\boxed{\lim_{N\to\infty} \left(\frac{\sum_{n=1}^{N} X[n]}{N} \right) = E[X]}$$

It converges to with probablity 1.

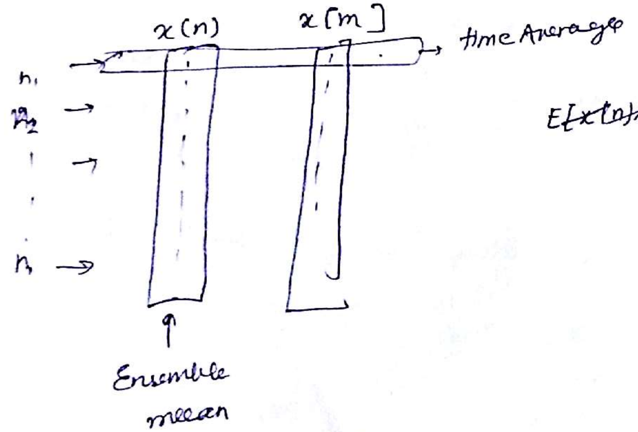
Evaluating

$$\lim_{N\to\infty} \left(\frac{\sum_{n=1}^{N} |X[n]|^2}{N} \right) = E[|X|^2]$$

So $\frac{|P(f)|^2}{T} E[|X|^2]$

If $\{x[n]\}_{n=-\infty}^{\infty} = \{0, A, 2A\}$ is a WSS discrete time sequence

~~Ergodic~~



$E[x(n)x^*[m]]$

For a discrete time, random process

* Ensemble means should be constant

And

② $E[x[n]x^*[m]]$ should be independent of t or n

Time Averaged correlation $\Rightarrow \lim_{N\to\infty} \sum_{n=-N}^{n=N} x[k]x^*[k+1] / N$ for a erg. equ

Ensemble correlation $\rightarrow E[x(n)x^*[n]]$

for WSS - , Ergodic in mean, autocorrelation

$\Rightarrow$ These results infer

$$\sum_{k=m_1}^{m_2} \frac{x[k]x^*[k-p]}{m_2 - m_1} = R[p] = E[x[n]x^*[n-p]]$$

O

for $p \neq 0 \rightarrow$ terms will be zero

Similarly

$$e^{-j2\pi f i T} \sum_{k=-N}^{N+i} \frac{x[k]x^*[k-i]}{2N} = R_i$$

~~So terms we go to~~

$$\Rightarrow \frac{|P(f)|^2}{T} E[|x^2|]^2 \frac{e^{j2\pi f \cdot 0 \cdot T}(R_0)}{\sum_{i=1}^{2N} e^{-j2\pi f i T} R[i]}$$

&

$$e^{-2\pi f i T} \sum_{k=-N}^{N+1} \frac{x(k)x^*[k-i]}{2N}$$

And for $N \rightarrow \infty$

$$S(f) = \frac{|P(f)|^2}{T} \sum_{i=\infty}^{\infty} R[i] e^{-j2\pi f i T} \rightarrow E[|X|]^2$$

for the i.i.d case only terms, which is non-zero

Question !

Q5 Let $\{b_n\}_{}^{\infty}$ be a sequence of i.i.d information bits (+1 and −1) are equally likely

bits → modulation symbol

$b[n-1]$	$b[n]$	$s[n]$
+1	+1	$A+jA$
+1	−1	$2A-jA$
−1	+1	$-A+jA$
−1	−1	$-A-j2A$

Complex baseband is represented by $x(t) = \sum_{n=-\infty}^{n=\infty} s[n]\, \text{sin}c(Bt-n)$

Find the PSD of $x(t)$

⇒ first to show $s[n]$ is an ergodic process or not

$E[s(n)] = \cancel{x[n]}\ x[n]\, p(t-nT)$

$E[s(n)] \Rightarrow [A+jA]\,P[s[n] = A+jA]$
$+ (2A-jA)[P[s[n] = 2A-jA]]$
$+ (A+jA)[P\, s[n] = -A+jA]]$
$+ (-A-j2A)\, P[s[n] = -A-j2A]$

$E[s[n]] = E(A+jA)\ P\dfrac{[\cancel{s[n]=A}]P}{P[b(n-1)\ b[n]]}$

$P(b(n-1))\ P[b(n)]$

$\Rightarrow \frac{1}{4}\Big[(\cancel{A}+jA) + \cancel{2A} - j\cancel{A} + \cancel{-A}+j\cancel{A} - \cancel{A} - j\cancel{2A}\Big]$

$\dfrac{[A-jA]}{4} = \dfrac{A(1-j)}{4}$

$k=0$

W' $E[s[n]s^*[n-1]] \Rightarrow$

$= E[\,[s_j^*]^2]P[B_i]$

$\cancel{A^2} \times \frac{1}{4}\,[2A^A + 5A^2 + 2A^2 + 8A^2]$

$(4A)^2 + A^2$

$= \frac{14A^2}{4} = \frac{7}{2}A^2$

$E[s[n]s^*[n-1]] =$ $s[n] = b_n, b_{n+1}$ b_{n+1}, b_{n-2}

$\Rightarrow \neq 0$

$b_n,\ b_{n-1},\ b_{n-2}$

	b_n	b_{n+1}	b_{n-2}	S_n	S_{n-1}
1/8	ϕ	ϕ	1	$A+jA$	$A-jA$
		1	−1	$A+jA$	$2A+jA$
1/8	1	−1	1		
1/8	1	−1	−1		
1/8	1	ϕ	1		
1/8	−1	1	−1		
1/8	−1	−1	1		
1/8	−1	−1	−1		

$E[s(n)s^*(n-1)] \Rightarrow \frac{1}{8} \times\ s(n)(s_{n-1})$

$E[s(n)s^*(n+1)] = E[s(n+1)s^*(n)]^*$

$= \dfrac{A^2}{8}[1+13j]$

$\Rightarrow \dfrac{A^2}{8}(1-j13)$

$S_x(f) = \sum R_s(k) e^{j2\pi f c t} \dfrac{|P(f)|^3}{T} = \dfrac{1}{B}\Big(\sum R_s[k]e^{-j2\pi f k/B}\Big)\dfrac{|P(f)|^2}{T}$

$= R_x(k)\, e^{-j2\pi f k/B}$

~~$P(f) = \text{sinc}$~~

$P(f) = \text{sinc}\,Bt \quad T = 1/B$

=

17/10/2016

Minor 1 3rd question

$\underline{\underline{Q}}$ $y = x + N$

$N = N^I + jN^Q$

$E[n^I n^Q] = \gamma\sigma^2$

$|\gamma| \leq 1$

$\text{cov}\left(\begin{bmatrix} n^I \\ n^Q \end{bmatrix}\right)$

if $A \in R^{n \times n}$
A is PSD (semi) iff
for any $x \in R^n$
$x^T A x \geq 0$ — for PD Positive definite > 0 strictly

if complex
$x^H A x \geq 0$

for $R_N = E\left\{\begin{bmatrix} N^I - E(n^I) \\ N^Q - E(n^Q) \end{bmatrix}\left[(N^I - E(N^I))(n^Q - E(n^Q))\right]\right\}$

To prove PSD

$x^T R_N x \geq 0$ — As random ness is due to N here

$E\left\{[x_1 \quad x_2]\, E\begin{bmatrix} n^I - E(n^I) \\ n^Q - E(n^Q) \end{bmatrix}\left[(n^I - E(n^I))(n^Q - E(n^Q))\right]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right\}$

$\Rightarrow E\left[\left[x_1(n^I - E(n^I)) + x_2(n^Q - E(n^Q))\right]\left[x_1(n^I - E(n^I)) + x_2(n^Q - E(n^Q))\right]\right]$

$= E\left[\left(x_1(n^I - E(n^I)) + x_2(n^Q - E(n^Q))\right)^2\right]$

= So Above term of is positive semidefinite ≥ 0

$\Rightarrow$ R_N covariance matrix is symmetric also

$R_N = E\left[\begin{bmatrix} N^I - E(n^I) \\ n^Q - E[n^Q] \end{bmatrix}\left[(n^I - E(n^I))(n^Q - E(n^Q))\right]\right]$

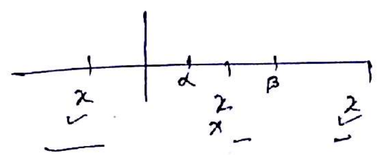
$R_N^T = R_N$ → for symmetric case.

$= E\begin{bmatrix} \underbrace{E[(n^I - E(n^I))^2]}_{\text{covariance of } n^I \rightarrow \text{non negative}} & E[(n^I - E(n^I))(n^Q - E(n^Q))] \\ E[(n^I - E(n^I))(n^Q - E(n^Q))] & \underbrace{E[(n^Q - E(n^Q))^2]}_{\text{covariance of } n^Q \rightarrow \text{positive definite}} \end{bmatrix}$

Symmetric — can't say Above +ve or -ve.

Let say take an example $\begin{bmatrix} x & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ 1 \end{bmatrix} \geq 0$

Analysis condition.



$(ax + c)x + (cx + b) \geq 0$

$ax^2 + 2cx + b \geq 0$

$x^2 + \frac{2c}{a}x + \frac{b}{a} \geq 0$

Can be interpreted as $(x-\alpha)(x-\beta) \geq 0$

Both α and β either be real or complex conjugate

D has to ≤ 0

$\left(\frac{2c}{a}\right)^2 - \left(\frac{4b}{a}\right) \leq 0$

$c^2 \leq ab$

$ab - c^2 \geq 0$

Scanned by CamScanner

$\Rightarrow$ <u>2×2 → Symmetric matrix</u>

Condition

1) Diagonal elements $a \geq 0$ $b \geq 0$

2) and Discriminant $\underline{\leq 0}$

$y = x + N$

· Need to find out optimal detector. but here equally likely so ML is also suitable

$$y \longrightarrow \boxed{\text{MLD}} \longrightarrow \hat{x}$$

(y^I, y^Q)

Jointly Gaussian

$y^I = N^I + 1$

$y^Q = N^Q$

$P_1(y) = P(x=+1) P(y|x=+1)$

$P_2(y) = P(x=-1) P(y|x=-1)$

$\rightarrow R_y = \sigma^2 \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}$

$$P(y/x=+1) \Rightarrow \frac{1}{(2\pi)|K_{y/x+1}|^{1/2}} e^{-\frac{[(y^I-1)\,y^Q]\, R_{y|x=+1}^{-1} \begin{bmatrix} y^I-1 \\ y^Q \end{bmatrix}}{2}}$$

covariance doesn't change

if $P(x=+1) = P(x=-1) = \frac{1}{2}$

Now if

$$e^{-\frac{((y^I+1),\, y^Q)(R_{y/x=-1}^{-1})(\;)}{2}} \underset{\hat{x}=+1}{\overset{\hat{x}=-1}{\lessgtr}} e^{-((y^I-1)^T (R_{y/x=1}^{-1}) \cdot \begin{pmatrix} y^I-1 \\ y^Q \end{pmatrix})}$$

if $<$ then $x \Rightarrow +1$

if $>$ then $x \Rightarrow \underline{-1}$

19.10.2016

$$\underline{\text{Part-II}}$$

$$[(y^I-1)\,y^Q]\, R_N^{-1} \begin{bmatrix} y^I-1 \\ y^Q \end{bmatrix} \underset{\hat{x}=-1}{\overset{\hat{x}=+1}{\lessgtr}} [(y^I+1)\,y^Q]\, R_N^{-1} \begin{bmatrix} y^I+1 \\ y^Q \end{bmatrix}$$

$$R_N = \sigma^2 \begin{bmatrix} 1 & r \\ r & 1 \end{bmatrix}$$

$$R_N^{-1} \Rightarrow \frac{1}{\sigma^2(1-r^2)} \begin{bmatrix} 1 & -r \\ -r & 1 \end{bmatrix}$$

$$= [(y^I-1)\,y^Q] \begin{bmatrix} 1 & -r \\ -r & 1 \end{bmatrix} \begin{bmatrix} y^I-1 \\ y^Q \end{bmatrix}$$

$$= [(y^I-1) - ry^Q \quad -r(y^I-1)+y^Q] \begin{bmatrix} y^I-1 \\ y^Q \end{bmatrix}$$

$$= [(y^I-1) - ry^Q](y^I-1) + y^Q(-r(y^I-1)+y^Q)]$$

$$= [(y^I-1) - ry^Q] + y^Q(-r(y^I-1)+y^Q)]$$

$$= [(y^I-1) - ry^Q]^2$$

$$\Rightarrow (y^I-1)^2 - 2r(y^I-1)y^Q + (y^Q)^2$$

$$(y^I-1)^2 - 2r(y^I-1)y^Q + y^{Q^2} \underset{\hat{x}=+1}{\overset{\hat{x}=-1}{\gtrless}} (y^I+1)^2 - 2r(y^I+1)y^Q + y^{Q^2}$$

$$0 \underset{\hat{x}=+1}{\overset{\hat{x}=-1}{\gtrless}} (y^I - ry^Q)$$

$$\hat{x} = \text{sign}(y^I - ry^Q)$$

P_e expression

As $P_e = P(X=+1)\,P(\text{Error}/X=+1) + P(X=-1)\,P(\text{Error}/X=-1)$

$P_e = P(Y^I - rY^Q < 0 \mid X=+1)$

$Y^I = 1 + N^I$

$Y^Q = N^Q$

$Y^I - rY^Q = 1 + (N^I - rN^Q)$

$= P((N^I - rN^Q) < -1 \mid X=+1)$

Here N^I and N^Q are independent of X so above expression simplifies to

$\Rightarrow P((N^I - rN^Q) < -1)$

As N^I and N^Q are Gaussian R.V

$E[W] = 0$

$E[W - E(W)^2] = E[W^2] = E[(N^I - rN^Q)^2]$

$= E[N^I] + r^2E[rN^Q] + 2r\,E[N^I N^Q]$

$= \sigma^2 + r^2\sigma^2 - \sigma\sigma^2 2r$

$\Rightarrow \sigma^2(1+r^2) - 2r^2\sigma^2$

$= \sigma^2(1-r^2)$

So pdf would be

$$= \int_{-\infty}^{-1} \frac{1}{\sqrt{2\pi\sigma^2(1-r^2)}}\, e^{-\frac{W^2}{2\sigma^2(1-r^2)}}\,\partial\omega \quad = e^{-x^2/2}$$

$\Rightarrow$

$P_e \Rightarrow Q\left(\frac{1}{\sqrt{\sigma^2(1-r^2)}}\right)$

$= \sigma^2(1-r^2)$

As r is correlation coefficient

$r > 0 \qquad \frac{E[N^I N^Q]}{\sqrt{E(N^{I2})\,E(N^{Q2})}} = r \qquad \text{var}(N^I - rN^Q)$

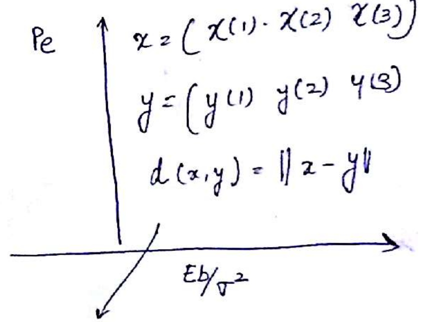
$P_e = Q\left[\frac{1}{\sqrt{\sigma^2(1-r^2)}}\right]$

By subtracting, It is decreasing variance of noise N^I and N^Q

As $1 + (N^I - \alpha N^Q) \rightarrow 0$ if $r = 1$ fully correlated

Minor Q. 1 3rd part solution

$1+j$	$-1+j$	0	
$-1+j$	0	$-1-j$	



$x = (x(1)\; x(2)\; x(3))$

$y = (y(1)\; y(2)\; y(3))$

$d(x,y) = \|x - y\|$

$d_{min} = \sqrt{(x(1)-y(1))^2 + (x(2)-y(2))^2 + (x(3)-y(3))^2}$

Q-

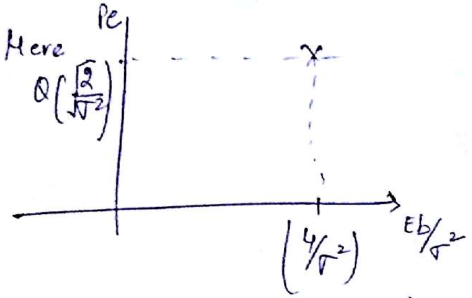
$x = [x(1)\; x(2)\; x(3)]$

Energy of a codeword $= (|x(1)|^2 + |x(2)|^2 + |x(3)|^2)$

Net Ene $= \frac{1}{2}\times 4 + \frac{1}{2}\times 4 = 4$

$E_b = \frac{4}{1}$ / No of Information bits

Here



To find curve here we have to multiply code book by constant to vary E_b

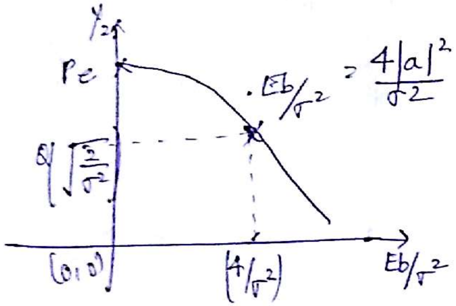
$a(1+j)$	$(-1+j)a$	0	$4\lvert a\rvert^2$
$a(-1+j)$	0	$(-1-j)a$	$4\lvert a\rvert^2$

$d_{min} = 8|a|^2$

$Q \cdot Q\left(\sqrt{\frac{d_{min}^2}{4\sigma^2}}\right)$

$E_{avg} = \frac{1}{2}\times 4 + \frac{1}{2}\times 4 \Rightarrow Q\left(\sqrt{\frac{2}{\sigma^2}}\right)$

$E_b \quad \frac{4}{\#\text{ info bits}} = 4$



$P_e \Rightarrow Q\left(\sqrt{\frac{d_{min}}{2\sigma^2}}\right) = Q\left(\sqrt{\frac{2|a|^2}{\sigma^2}}\right)$

$\frac{a^2}{\sigma^2} = \frac{E_b}{4\sigma^2}$

$P_e = Q\left(\sqrt{\frac{E_b^A}{2\sigma^2}}\right)$

Similar

$P_e^{(A)} \leq 3Q\left(\sqrt{\frac{d_{min}^2}{4\sigma^2}}\right)$

$d_{min} \Rightarrow \underline{\sqrt{10}}$

if $\times$ be $|a|$

$d_{min}^2 = 10|a|^2$

$E_{avg} = 6|a|^2$

$E_b = \frac{6|a|^2}{2} = 3|a|^2$

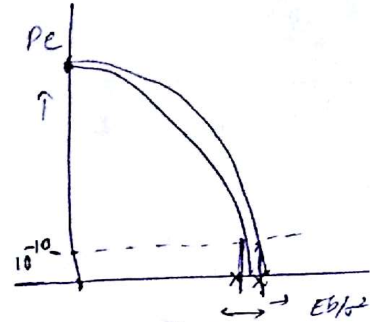
$P_e^{(A)} \leq 3Q\left(\sqrt{\frac{10|a|^2}{4\sigma^2}}\right)$

$\frac{E_b}{\sigma^2} = \frac{3|a|^2}{\sigma^2}$

$\frac{|a|^2}{\sigma^2} = \frac{E_b}{3\sigma^2}$

$P_e^{(A)} \leq 3Q\left(\sqrt{\frac{10.5}{4}\times\frac{E_b}{3\sigma^2}}\right)$

$$\boxed{P_e^A \leq 3Q\sqrt{\left(\frac{5E_b^B}{6\sigma^2}\right)}}$$



In the high SNR regime

$\frac{E_b}{\sigma^2}$ requirement of B is more than A

So Code Book A is more efficient.

Plotting both Pe on the Same Graph

$P_e^A = P_e^B$

$\frac{5}{6}\frac{E_b^A}{\sigma^2} = \frac{E_b^B}{2\sigma^2} \quad \frac{E_b^A}{E_b^B} = \underline{\frac{3}{5}}$

Coding Gain $10\log_{10}\left(\frac{E_b^B}{E_b^A}\right)$

$= 10\log_{10}\left(\frac{5}{3}\right) = 7 \Rightarrow 4.8 = \underline{2.2\text{ dB}}$

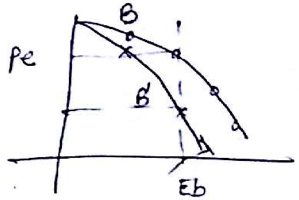
Date 19.10.2016

Code B

$1+j$	$-1+j$	0
$-1+j$	•	$-1-j$

$$P_e = Q\left(\sqrt{\frac{E_b^B}{2\sigma^2}}\right)$$

For Improving efficiency keeping E_b same, d_{min} same = P_e



Code B' will be more power efficient than code B if B's curve is below B

$d_{min} = 8$

$$d_{min}^B = \sqrt{4+4+2+2} = \sqrt{12}$$

$\boxed{d_{min}^{2\,B'} = 12}$

So d_{min} increases if a is changed to $-1-j$

If

$1+j$	$-1+j$	0
$-1+j$	0	$-1-j$

$d_{min} = 12$

⇒

$1+j$	$-1+j$	0
~~$1+j$~~	$1-j$	0

$d_{min} = 16$ ↓ if removed ↓ More B and need L efficien

$\boxed{\frac{1}{2}}$ =

Bit rate $= B/2$

Spectral efficien $= 1/2$

Bit Rate $= B/2$

Spectral efficiency $= 1/2$

1 bit in $\frac{2\,sec}{B}$

$R_b = \frac{n}{T_s} \Rightarrow 2$

$R_b = \frac{n}{T_s}$, $R_b = \frac{n}{T_b}$, $T_b = \frac{T_s}{n} \Rightarrow \frac{T_s}{2} = T_b$, $\frac{2}{T_s}$

Q.2

$$x(t) = \sum_{n=-\infty}^{n=\infty} s(n)\,\mathrm{sinc}(Bt-n)$$

b[n] i.i.d

b[n-1]	b[n]	s[n]
+1	+1	A(1+j)
+1	-1	A(-1+j)
-1	+1	A(1-j)
-1	-1	A(-1-j)

n = b0 b1 -2 -3 -4 - - - -

b[n] +1 -1 +1 +1 +1

n=1 n=2 n=3

s[n] A(-1+j) A(1-j)

For 100 bits $= \frac{100}{B}$ seconds are taken ~~are transmitted~~ and 101 information bits are transferred.

(1) $\boxed{\text{bit rate} = \frac{10001}{10000/B}}$ if 10000 bits transferred

$= B$ bits/sec

$n = 10000$

(2) Spectral Efficiency $= \frac{B \text{ bits/sec}}{B \text{ Hz}}$ ~~10000~~

$= 1$ bits/sec-Hz

Scanned by CamScanner

Irrespective of bit sequence, the symbol energy is $2A^2$

If so total Avg Energy $\Rightarrow \dfrac{\overbrace{2A^2 \times 10000}^{\text{Total Energy}} \rightarrow \text{total no. of sequences}}{10001 \rightarrow \text{No of Information bits}}$

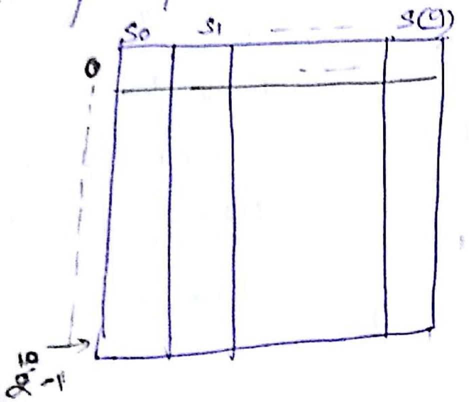
Sequence can also be transmitted as

+1	1+j	-1+j	0
-1	-1+j	0	-1+j

→ Only symbols have to transmitted

S(0) 0 →

Now Taking sequence in chunks of 10.



Average codeword Energy

$E_{cw} = \dfrac{99 \times 2A^2}{100}$

$= 2A^2$

If N symbols has to transmit $b(n-1)\ b(n)\ \ s(n)$

then N is divided in to four chunks

$\frac{N}{4} \times 2A^2 + \frac{N}{4} \times 2A^2 + \frac{N}{4} \times 2A^2 + \frac{N}{4} \times 2A^2$

$= \dfrac{\frac{N}{4} \times 2A^2}{N+1 \rightarrow \text{No of Info bit}}$

$= \dfrac{11A^2}{4}$

Express $E[s(n)\, s^*(n-k)]$

$E[|s(n)|^2] \Rightarrow E[|s(n)|^2]$

$$\begin{cases} A^2, & k=0 \\ jA^2, & k=1 \\ jA^2, & k=-1 \\ 0, & |k| \geq 2 \end{cases}$$

for $k=-1$ $E(s(n)\, s^*(n-1))$

As $E[s[n] \times s^*(n-1)] \Rightarrow s[n]$

$= A^2[b(n) + j\, b(n-1)]\ (b(n-1) - j(b(n-2))$

$\Rightarrow \cancel{A^2\, b(n)\, b(n-1)}$

$\Rightarrow A^2 E\Big[b(n)\, b(n-1) - j\, b(n)\, b(n-2) + j\, b(n-1)^2 + b(n-1)\, b(n-2)\Big]$

Real part $= A^2 E[b(n) b(n-1) + b(n-1) b(n-2)]$

$= A^2 E[\underbrace{b(n)\, b(n-1)}_{\text{As Independent}}] + A^2 E[b(n-1)\, b(n-2)]$

$\Rightarrow 0 + 0$

$= \underline{0}$

Imaginary part $= A^2 E[j[b(n-1)]^2 - b(n)\, b(n-2)]$

$= A^2 j \cdot E[\underbrace{b(n-1)^2}_{1}] - 0$

$\Rightarrow jA^2$

PSD =

$S_x(f) = \dfrac{|P(f)|^2}{T} \sum_{k=\infty}^{-\infty} R_s(k)\, e^{-2\pi k f T}$

21-10-2016

Linear Modulation

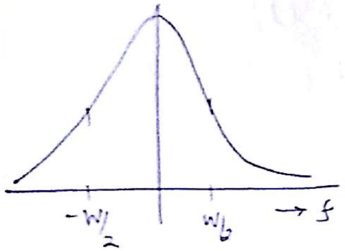
$$x(t) = \sum_{n=-\infty}^{\infty} x[n]\, p(t-nT)$$

$x[n]$ are iid zero mean r.v and $x[n] \in A$

R.r

$$S_x(f) = \frac{|P(f)|^2}{T} \sum_{k=-\infty}^{\infty} R_x[n]\, e^{-j2\pi f kT}$$

$$S_x(f) \Rightarrow \frac{|P(f)|^2}{T} R_x[0] \qquad \text{as iid}$$



$$\Rightarrow \frac{\int_{-W/2}^{W/2} S_x(f)\,df}{\int_{-\infty}^{\infty} S_x(f)\,df} \qquad \text{fraction power}$$

To find out $W =$ such that above expression ≥ 0.99.

It is defined as 99% bandwidth

Let

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$ (LTI)

$$y(t) = x(t) * h(t)$$

$$\Rightarrow \left(\sum_{n=-\infty}^{\infty} x[n]\, p(t-nT)\right) * h(t)$$

Let $h(t) = \delta(t)$

$$y(t) = \left(\sum_{n=-\infty}^{\infty} x[n]\, p(t-nT)\right)$$

$x(t) \longrightarrow \boxed{LTI} \longrightarrow y(t) \longrightarrow \boxed{LPF}\ q(t) \longrightarrow z(t) \xrightarrow{t=mT} y[m]$

$h(t) = \delta(t)$

$$z(t) \Rightarrow x(t) * q(t)$$

$$z(t) = \left[\underbrace{\left(\sum_{n=-\infty}^{\infty} x[n]\, \delta(t-nT)\right)}_{s(t)} * p(t)\right] * q(t)$$

$$\Rightarrow \cancel{\sum_{n=-\infty}^{\infty} x[n]} \quad [\, s(t) * p(t) * q(t)$$

$$= s(t) * [p(t) * q(t)]$$

$$c(t) = p(t) * q(t)$$

$$z(t) = s(t) * c(t)$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, c(t-nT)$$

$$\Rightarrow z(t) = \sum_{n=-\infty}^{\infty} x[n]\, c(t-nT) \qquad c(t) = p(t) * q(t)$$

$$y[m] = z(t = mT)$$

$$= A[m] = z(t = mT)$$

$$= \sum_{n=-\infty}^{\infty} x[n]\, c(m-nT)$$

Let.

$y(t) \longrightarrow \boxed{LPF}\ q(t) \longrightarrow z(t) \xrightarrow{t=mT} A[m]$

Here we need. Above expression should be $= x[m]$ m^{th} sample

$$\Rightarrow x[m]c[0] + x[m-1]c(t) \ldots + \ldots$$

Here to

$x[m]\,c[0] \rightarrow$ information part

$x[m-1]\,c(t) \longrightarrow$ intersymbol interference

All the interference terms should be 0

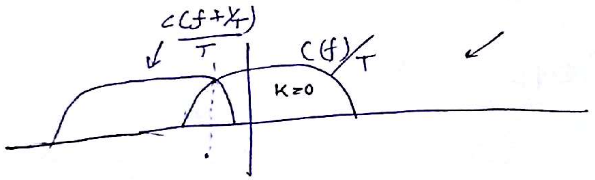
$c(t) = q(t) * p(t)$

$$c(kT) = \begin{cases} 1, & k = 0 \\ 0, & k = 0 \end{cases}$$

$\Rightarrow$ Above condition can be satisfied by sinc function

In frequency domain

$$c(kT) = \begin{cases} 1, & k = 0 \\ 0, & k \neq 0 \end{cases} \quad \text{if} \quad f\text{ domain} \quad \frac{1}{T}\sum_{k=-\infty}^{\infty} c\left(f + \frac{k}{T}\right) = 1$$



Taking $c(t) = \operatorname{sinc}(t/T)$

$$c(f) \Rightarrow \begin{cases} T, & |f| < \frac{1}{2T} \\ 0, & 0 \end{cases} \quad \text{iff} \quad \frac{1}{T}\sum_{k=-\infty}^{k=\infty} c\left(f + \frac{k}{T}\right) = 1$$