Solutions to Selected Problems
– from Madhow: Fundamentals of Digital Communication and from Johannesson & Zigangirov: Fundamentals of Convolutional Coding –

Saif K. Mohammed

INSTITUTE OF TECHNOLOGY
LINKÖPING UNIVERSITY

Department of Electrical Engineering
Linköpings universitet, SE-581 83 Linköping, Sweden

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Note: This material is prepared for the 2012 version of the Master course TSKS04 Digital Communication Continuation Course. For almost every task planned for tutorials you find either hints and answers or complete solutions. For the tasks where we give complete solutions, we have chosen to adjust the notations to concur with the notation that has been used in the lectures.
This document will evolve during the course as the tutorials go by.
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Department of Electrical Engineering
Linköpings universitet
SE-581 83 Linköping
Sweden

This document was prepared using LaTeX2ε with the aid of TeXnicCenter on an Dell PC running CentOS 5. The figures were produced using XFIG (from xfig.org). Finally, the plots were produced using MATLAB (from MathWorks, Inc.).

Contents

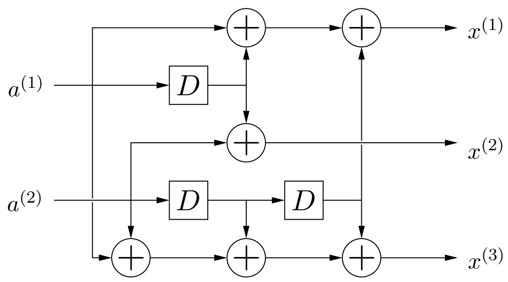
Tutorial 6

Problems from Johannesson & Zigangirov, Fundamentals of Convolutional Coding, IEEE Press 1999.

J&Z 3.2

Task

Consider the rate $R = 2/3$, memory $m = 2$, overall constraint length $\nu = 3$, convolutional encoder below.

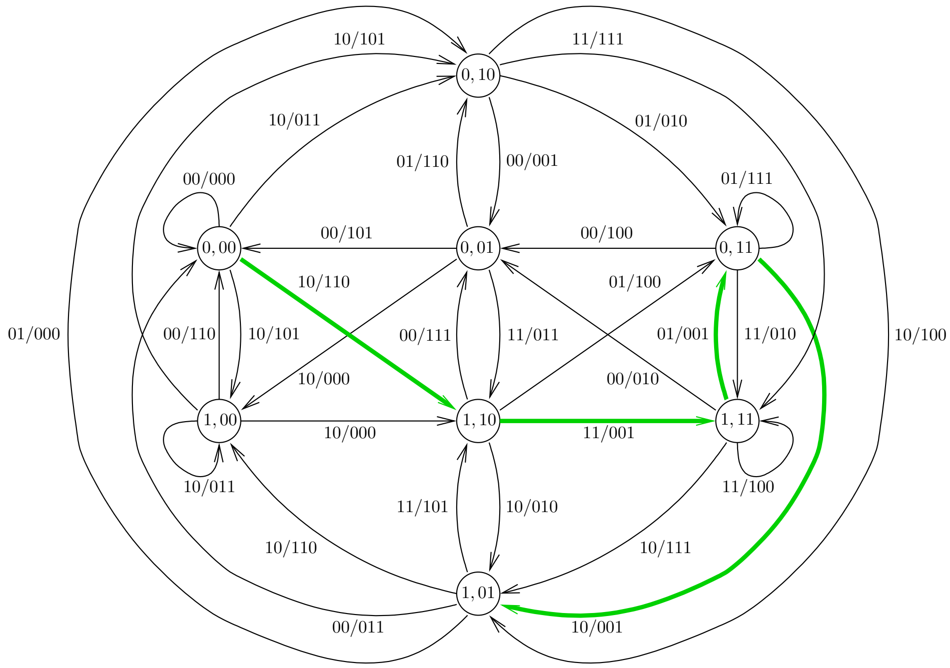


a. Draw the state-transition diagram.

b. Find the path corresponding to the input $a = 10\,11\,01\,10$ in the state-transition diagram.

Solution

a. and **b.** The state diagram of the encoder:



The path corresponding to $u = 10\,11\,01\,10$ is indicated with green lines in the state diagram above.

J&Z 3.8c

Task

Show that the two generator matrices

$$G_1(D) = \begin{pmatrix} 1+D & D & 1 \\ D^2 & 1 & 1+D+D^2 \end{pmatrix},$$

$$G_2(D) = \begin{pmatrix} 1+D & D & 1 \\ 1+D^2+D^3 & 1+D+D^2+D^3 & 0 \end{pmatrix}.$$

generate the same code.

Solution

If we can go from $G_1(D)$ to $G_2(D)$ using row operators, then they generate the same code. The two matrices have the same first row. Therefore, it is a matter of the second rows. A key to determining that is the last column of both matrices. Multiplying the first row in $G_1(D)$ by $1 + D + D^2$ gives us the row

$$\begin{pmatrix} 1+D^3 & D+D^2+D^3 & 1+D+D^2 \end{pmatrix}. \tag{1}$$

Adding that to the second row in $G_1(D)$ gives us the second row in $G_2(D)$.

Done!

7.5

Answer

a. Not included.

b. 01101(00)