Tutorial No. 3

TSKS04.

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Q.1. For the ISI channel suppose that the discrete-time information symbols and the discrete-time sampled output are related by

$$y[r] = \sum_{k=0}^{1} h[k]\, x[r-k] + N[r]$$

where $\{x[m]\}_{m=0}^{m=2}$ is the discrete-time sequence of information symbols. These symbols are known to belong to the set

$$A = \{1+j, -j-1\}$$

Further $N[r] = N^{I}[r] + j N^{Q}[r]$, where $\{N^{I}[r]\}$ and $\{N^{Q}[r]\}$ are sequences of i.i.d. real Gaussian random variables having mean zero. ~~and~~ Also, assume that $x[m]=0$ for $m<0$ and $m>2$.

The channel impulse response $\{h[k]\}$ is known to the receiver, and $h[0]=2$, $h[1]=-1$.

If the received sequence is $y[0]=1-j$, $y[1]=2$, $y[2]=3+2j$ and $y[3]=1+j$, then what is the maximum likelihood estimate for $\{x[0], x[1], x[2]\}$.

Q. 2

Consider linear modulation with the baseband signal given by

$$x(t) = \sum x[k] \operatorname{sinc}\left(\frac{t}{T} - k\right), \text{ and}$$

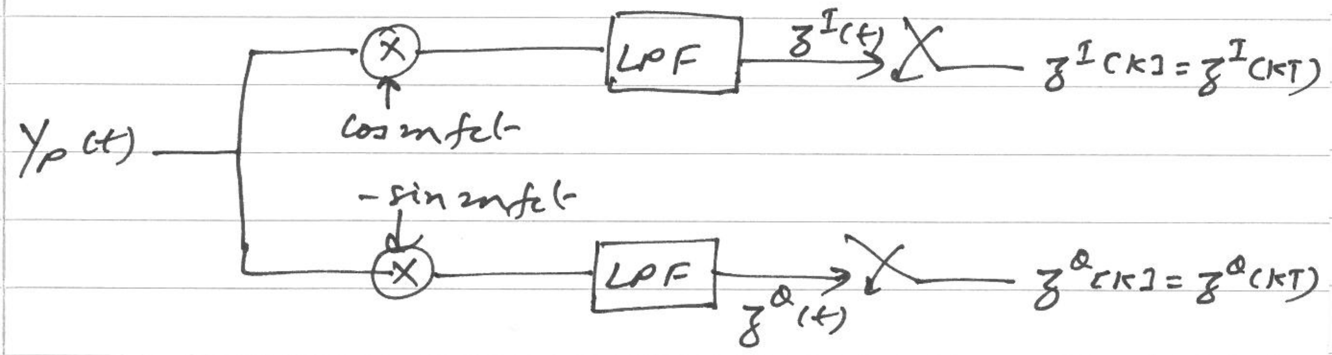
the passband signal as

$$x_p(t) = \operatorname{Re}\left[x(t)\, e^{j2\pi f_c t}\right].$$

Consider an AWGN where the received signal (passband) is given by

$$y_p(t) = x_p(t) + n_p(t).$$

The optimal receiver is then given by.



a) Derive a relation between $z[k] = z^I[k] + j z^Q[k]$ and $x[k]$.

Assume that $f_c > 1/2T$ and

$$LPF(f) = \begin{cases} 1 & |f| < 1/2T \\ 0 & \text{otherwise} \end{cases}$$

b) Consider the case of BPSK transmission. that is $x[k] \in (\sqrt{E_b}, -\sqrt{E_b})$.

Also, let $prob(x[k] = +\sqrt{E_b}) = prob(x[k] = -\sqrt{E_b})$
$= 1/2$.

let $\{x[k]\}$ be a sequence of i.i.d. BPSK symbols.

i) Find the PSD of the transmitted passband signal, and the total transmitted power. What is the energy per information bit?

ii) Assuming a transmission time from $t=0$ to $t=MT$, write down the expression for the minimum probability of error detector.
Show that each transmitted bit can be decoded separately.
Why is this so?

iii) Find the expression for the minimum probability of error for each transmitted bit.
(Assume that the sequence $\{x[k]\}$ is ergodic in autocorrelation).

iv) What is the spectral efficiency of this transmission scheme?

c) Consider another linear modulation scheme where coding is performed across two consecutive information symbols, i.e.,

$$(x[2k],\ x[2k+1]) \in \left\{ (0, +\sqrt{2E_b}),\ \left(-\sqrt{\frac{3E_b}{2}}, -\sqrt{\frac{E_b}{2}}\right), \left(\sqrt{\frac{3E_b}{2}}, -\sqrt{\frac{E_b}{2}}\right) \right\}$$

$$k = 0, 1, \ldots, \frac{(M-1)}{2}$$

$$x(t) = \sum_{m=0}^{M} x[m]\,\mathrm{Sinc}(t/T - m), \text{ and}$$

$$x_p(t) = \mathrm{Re}\left\{ x(t)\, e^{j2\pi f_c t} \right\}.$$

i) For any given sequence of information symbols $\{x[m]\}$, find the average transmitted power

$$\lim_{M \to \infty} \frac{1}{MT} \int_0^{MT} |x(t)|^2\, dt.$$

Under what conditions is the limit independent of the sequence $\{x[m]\}$?

Compare the average transmit power with the simple BPSK scheme in part b).

ii) Compare the spectral efficiency of the transmission schemes in part b) and c).

iii) At large $\frac{E_b}{\sigma^2 W}$, if the two schemes have the same average transmit power to receiver noise ~~psp ratio~~ power ratio, then which scheme has a lower error probability.

At very low error probabilities, which scheme is more power efficient, i.e., achieves the same error probability as the other scheme but at a lower value of the average transmitted power?

Do you think that lower error probability can only be achieved by sacrificing spectral/bandwidth efficiency?

Q.3. Multicarrier Communication (MC) and Orthogonal Frequency division multiplexing (OFDM).

Consider the linear modulation scheme where the transmitted passband signal is given by $(0 \le t \le T_0)$

$$x_p(t) = \sum_{k=-M'}^{M'} \left(x^I[k] \cos 2\pi \left(f_c + \frac{k}{T_0}\right)t - x^Q[k] \sin 2\pi \left(f_c + \frac{k}{T_0}\right)t \right)$$

where $x[k] = x^I[k] + j\, x^Q[k]$ are the complex information bearing symbols.

a) Assuming that $f_c \gg \frac{M'}{T_0}$, what is the bandwidth occupancy of $x_p(t)$.

b) ~~What is~~ Show that $\left\{ \cos 2\pi \left(f_c + \frac{k}{T_0}\right)t \right\}_{k=-M'}^{M'}$ ~~is~~ and $\left\{ \sin 2\pi \left(f_c + \frac{k}{T_0}\right) \right\}_{k=-M'}^{M'}$ are approximately orthogonal sets of functions ~~for $0 \le t \le T_0$~~ when ~~limited~~ limited to $t \in [0, T_0]$.

c) Consider an ISI channel with AWGN noise, i.e.,

$$y_p(t) = x_p(t) * h_p(t) + n_p(t).$$

What is the structure of the optimal receiver?

Show that the symbols $\{x[k]\}$ can be decoded separately, unlike the situation where MLSE needs to be done due to ISI.