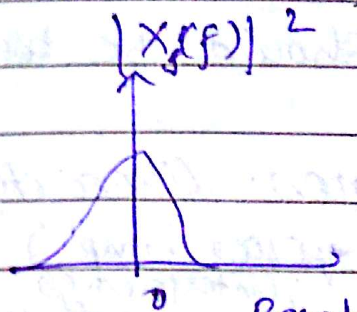
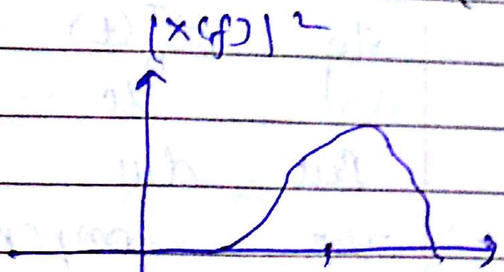
Digital Communication (ELL 712)

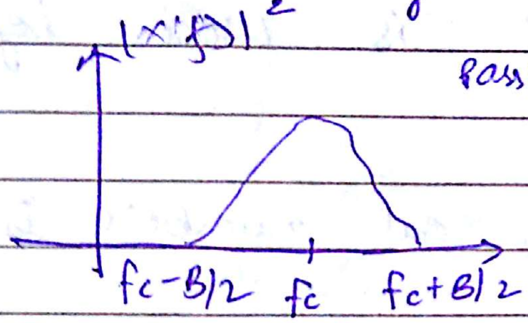
Dr. Saif Khan Mohammed (212)



Baseband s/g



Passband s/g



Passband → $x_p(t) = x^I(t) \cos 2\pi f_c t - x^Q(t) \sin 2\pi f_c t$ ← Baseband

$x^I(t)$, $x^Q(t)$ → are band-limited (base-band) s/g.

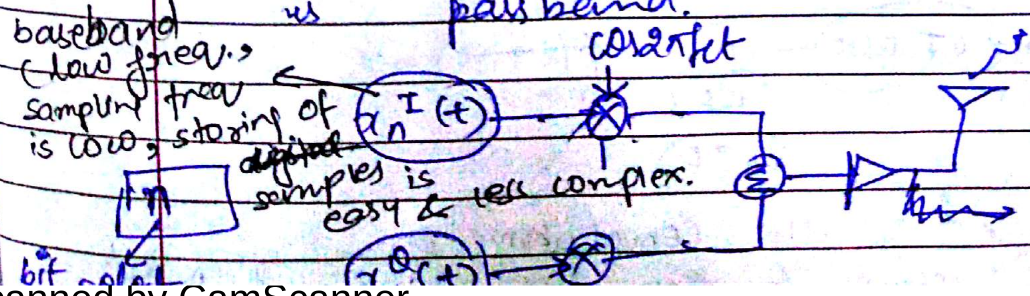
\# Our voice s/g is sent in packet of 5-10 msec as for this time, our vocal chord s/s is considered LTI s/s.

\# For two bit T_x →

Band limited & baseband

00	$Q_1(t) =$	$\boxed{x_1^I(t)} \cos 2\pi f_c t - \boxed{x_1^Q(t)} \sin 2\pi f_c t$
01	Q_2	,
10	Q_3	,
11	$Q_4 =$	$x_4^I(t) \cos 2\pi f_c t - x_4^Q(t) \sin 2\pi f_c t$

Since x_1, x_2, x_3 -- are baseband s/g, it is easy to store them & send, as compared to when s/g is passband.



Sig $x_n^I(t)$ & $x_n^Q(t)$ → should be bandlimited to $-B/2$ to $B/2$.

But due to the non-linearity of the components (like power ampr.), the Band can exceed (interference), thus transmission filter is used before antenna.

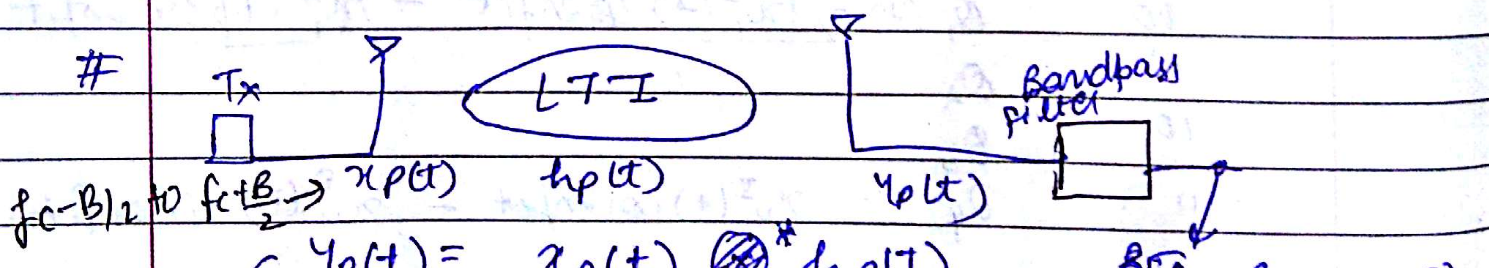
A pair of two real numbers is complex no.

$$x_p(t) = x^I(t)\cos 2\pi f_c t - x^Q(t)\sin 2\pi f_c t$$

$$x(t) = x^I(t) + j\,x^Q(t)$$

$$x_p(t) = \text{Real}\left[x(t)e^{j2\pi f_c t}\right]$$

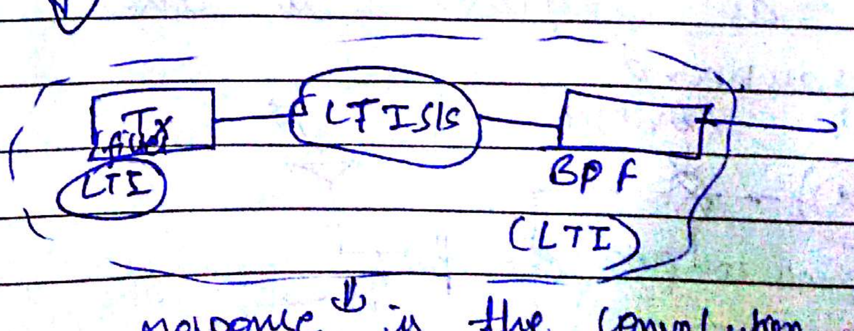
#



$$y_p(t) = x_p(t) * h_p(t)$$

$y_p(t) \to y(t)$, $x_p(t) \to x(t)$, $h_p(t) \to h(t)$

$$Y_p(f) = X_p(f)\,H_p(f)$$

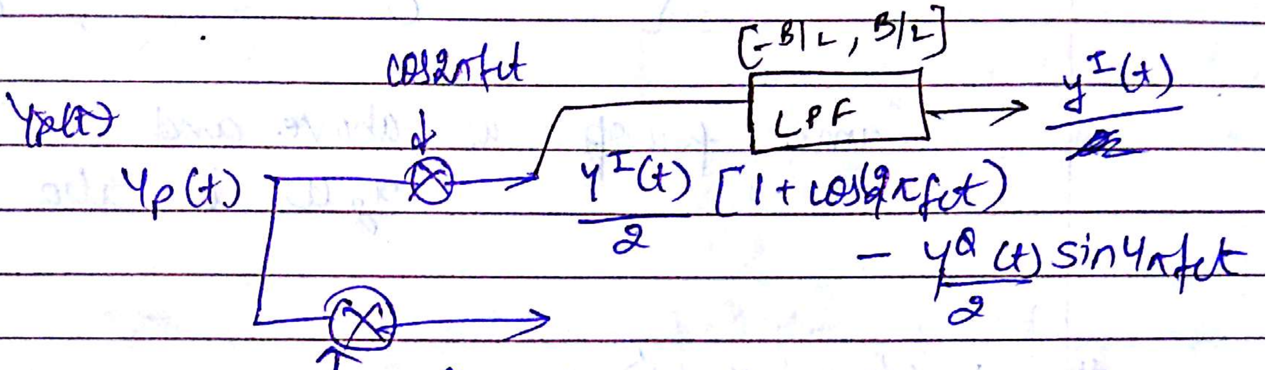


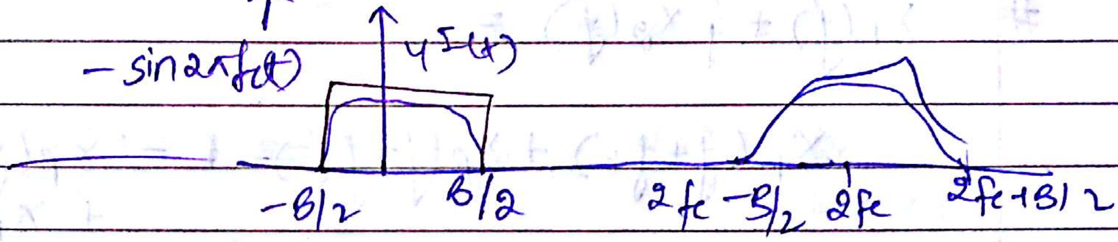
response is the convolution of each comp. & they are all band limited.

(Complex Baseband model)

$y(t) = h(t) * x(t) \longrightarrow$ (Baseband)

$$y_p(t) = y^I(t)\cos 2\pi f_c t - y^Q(t)\sin 2\pi f_c t$$





$$2f_c \mp B/2 > B/2$$

$x_p(t) \Longleftrightarrow X_p(f) = \int_{-\infty}^{\infty} x_p(t)\, e^{-j2\pi f t}\, dt$

finite energy sig.

$$x_1(t) \Longleftrightarrow X_1(f) = \begin{cases} X_p(f+f_c) + X_p(f-f_c), & |f| < \frac{B}{2} \\ 0, & \text{o.w} \end{cases}$$

$x(t)$ is real then $X(-f) = X^*(f)$

$X_1(-f) = X_1^*(f)$ (If it is true than $x_1(t)$ is real)

$X_1^*(f) = X_p^*(f+f_c) + X_p^*(f-f_c)$, $x_p(t)$ is real so, $X_p^*(f) = X_p(-f)$

$$X_1^*(f) = X_p(-f-f_c) + X_p(-f+f_c)$$

$$X_1(-f) = X_p(-f+f_c) + X_p(-f-f_c) = X_1^*(f)$$

Date :
Page No. :

$$x_2(t) \overset{F}{\Leftrightarrow} X_2(f) = \begin{cases} j\left[(X_p(f-f_c) - X_p(f+f_c)\right], & |f|<\frac{B}{2} \\ 0, & o.w. \end{cases}$$

Same prop as above and $x_2(t)$ is also real.

\# $X_1(f') + j X_2(f') =$

$$X_p(f'+f_c) + \cancel{X_p(f'-f_c)} - \cancel{X_p(f'-f_c)} + X_p(f'+f_c)$$

$$= \begin{cases} 2X_p(f'+f_c) & ; \ |f'| < B/2 \\ 0 & ; \ o.w. \end{cases}$$

Now, replace f' by $f-f_c$

$$f = f' + f_c$$

\# $X_1(f-f_c) + j X_2(f-f_c) = \begin{cases} 2X_p(f); & \text{If } |f-f_c| < B/2 \\ 0, & o.w. \end{cases}$

$$\Rightarrow X_p(f) = \begin{cases} \frac{1}{2}\left[X_1(f-f_c) + j X_2(f-f_c)\right]; & |f-f_c| < \frac{B}{2} \\ 0 & ; \ o.w. \end{cases}$$

Date :
Page No. :

$$X_p(f) = \begin{cases} \frac{1}{2}\left[X_1(f-f_c) + j\,X_2(f-f_c)\right]; & |f-f_c| < B/2 \\ \frac{1}{2}\left[X_1(f+f_c) - j\,X_2(f+f_c)\right], & |f+f_c| < B/2 \end{cases}$$

$\cancel{, |f+f_c| < B/2}$

$$\cancel{X_p(-f) = \frac{1}{2}\left[X_1(-f-f_c) + j\,X_2(-f-f_c)\right]},$$

$X_p(-f) = X_p^*(f)$ To find $X_p(f)$ in terms of $X_1(f)$ & $X_2(f)$ for $f \in \left[-f_c - \frac{B}{2}, -f_c + \frac{B}{2}\right]$

$f_c \in [f_c - B/2, f_c + B/2]$

$$X_p(-f) = \frac{1}{2}\left[X_1^*(f-f_c) - j\,X_2^*(f-f_c)\right],$$

$$X_p(-f) = \frac{1}{2}\left[X_1(-f+f_c) - j\,X_2(-f+f_c)\right]$$

$$X_p(f) = \frac{1}{2}\left[X_1(f+f_c) - j\,X_2(f+f_c)\right]$$

$$x_p(t) = \int_{-\infty}^{\infty} X_p(f)\, e^{j2\pi f t}\, df$$

$$= \frac{1}{2}\int_{-f_c-B/2}^{-f_c+B/2} \left[X_1(f+f_c) - j\,X_2(f+f_c)\right] e^{j2\pi f t}\, df + \frac{1}{2}\int_{f_c-B/2}^{f_c+B/2} \left[X_1(f-f_c) + j\,X_2(f-f_c)\right] e^{j2\pi f t}\, df$$

Date :
Page No. :

$$= \int_{-f_c-B/2}^{-f_c+B/2} \frac{1}{2}\left[X_1(f+f_c)\, e^{j2\pi f t}\, df \right.$$

$f' = f + f_c$

$df' = df$

$$= \frac{1}{2}\int_{-B/2}^{B/2} X_1(f')\, e^{j2\pi(f'-f_c)t}\, df'$$

$$= e^{-j2\pi f_c B/2} \cdot \frac{1}{2}\int_{-B/2}^{B/2} X_1(f')\, e^{j2\pi f' t}\, df'$$

$$x_p(t) = \frac{1}{2} e^{-j2\pi f_c t} \cdot x_1(t) - \frac{j e^{-j2\pi f_c t}}{2} x_2(t)$$

$$\int_{f_c-B/2}^{f_c+B/2} \frac{1}{2} X_1(f-f_c)\, e^{j2\pi f t}\, df$$

$df = df'$

$f - f_c = f'$

$$= \frac{1}{2}\int_{-B/2}^{B/2} X_1(f')\, e^{j2\pi(f_c+f')t}\, df'$$

$$= \frac{1}{2} e^{j2\pi f_c t}\, x_1(t) + \frac{j}{2} x_2(t) e^{j2\pi f_c t}$$

$$\boxed{\begin{aligned} \frac{e^{j\theta}+e^{-j\theta}}{2} &= \cos\theta \\ \frac{e^{j\theta}-e^{-j\theta}}{2j} &= \sin\theta \end{aligned}}$$

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Date:
Page No.:

$$= \frac{1}{2} e^{-j2\pi f_c t} x_1(t) - \frac{j x_2(t)}{2} e^{-j2\pi f_c t}$$

$$+ \frac{1}{2} e^{+j2\pi f_c t} x_1(t) + \frac{j}{2} x_2(t) e^{j2\pi f_c t}$$

$$= \frac{1}{2} x_1(t) \left[e^{j2\pi f_c t} + e^{-j2\pi f_c t} \right]$$

$$+ \frac{j}{2} x_2(t) \left(e^{j2\pi f_c t} - e^{-j2\pi f_c t} \right)$$

$$= \cancel{x_1(t) \cos\theta - x_2(t) \sin\theta}$$
$$= x_1(t) \cos 2\pi f_c t - x_2(t) \sin 2\pi f_c t$$

#

$$x_p(t) \longrightarrow h_p(t) \longrightarrow y_p(t) = \int_{-\infty}^{\infty} x_p(\tau) h_p(t-\tau) d\tau$$

$$\pm \left[f_c - \frac{B}{2}, f_c + \frac{B}{2} \right]$$

OR

$$\int_{-\infty}^{\infty} h_p(\tau) x_p(t-\tau) d\tau$$

$$x_p(t) = x^I(t) \cos 2\pi f_c t - x^Q(t) \sin 2\pi f_c t$$
$$h_p(t) = h^I(t) \cos 2\pi f_c t - h^Q(t) \sin 2\pi f_c t$$
$$y_p(t) = y^I(t) \cos 2\pi f_c t - y^Q(t) \sin 2\pi f_c t$$

$$y_p(t) = \int_{-\infty}^{\infty} \left[h^I(\tau) \cos 2\pi f_c \tau - h^Q(\tau) \sin 2\pi f_c \tau \right] \left[x^I(t-\tau) \cos 2\pi f_c (t-\tau) - x^Q(t-\tau) \sin 2\pi f_c (t-\tau) \right] d\tau$$

$$= \int_{-\infty}^{\infty} h^I(\tau) x^I(t-\tau) \cos 2\pi f_c \tau \cos 2\pi f_c (t-\tau) d\tau$$

$$\left[2\cos A \cos B = \cos(A+B) + \cos(A-B) \right]$$

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Parseval's theorem $\rightarrow \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \int_{-\infty}^{\infty} X(f)\, Y^*(f)\, df$

(I)

$$= \frac{1}{2}\int_{-\infty}^{\infty} h^I(\tau)\, x^I(t-\tau)\, [\cos 2\pi f_c t + \cos 2\pi f_c (2\tau - t)]\, d\tau$$

$$= \frac{1}{2}\cos 2\pi f_c t \int_{-\infty}^{\infty} h^I(\tau)\, x^I(t-\tau)\, d\tau$$

$$+ \frac{1}{2}\int_{-\infty}^{\infty} \underbrace{\cos 2\pi f_c (2\tau - t)}_{b(\tau)}\ \underbrace{x^I(t-\tau)\, h^I(\tau)}_{a(\tau)}\, d\tau$$

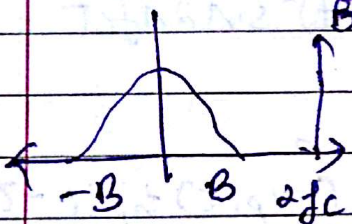
$b(\tau) \rightarrow$ is real

so $b^*(\tau) = b(\tau)$

By parseval's theorem

$$= \frac{1}{2}\int_{-\infty}^{\infty} A(f)\, B^*(f)\, df$$

impulses.



$\boxed{2f_c > B}$ $\rightarrow$ Product of $A(f) B^*(f)$ is 0

Thus,

$$\frac{1}{2}\int_{-\infty}^{\infty} A(f)\, B^*(f)\, df = 0$$

(II)

$$= -\frac{1}{2}\int_{-\infty}^{\infty} h^I(\tau)\, x^I(t-\tau)\, [2\cos 2\pi f_c \tau \sin 2\pi f_c (t-\tau)]\, d\tau$$

$$= -\frac{1}{2}\int_{-\infty}^{\infty} h^I(\tau)\, x^Q(t-\tau)\, [\sin 2\pi f_c t]\, d\tau + 0$$

$$= -\frac{1}{2}\sin 2\pi f_c t \int_{-\infty}^{\infty} h^I(\tau)\, x^Q(t-\tau)\, d\tau$$

(III)

$$= -\frac{1}{2}\int_{-\infty}^{\infty} h^Q(\tau)\, x^I(t-\tau)\, (\sin 2\pi f_c \tau \cos 2\pi f_c (t-\tau))\, d\tau$$

$$= -\frac{1}{2}\int_{-\infty}^{\infty} h^Q(\tau)\, x^I(t-\tau)\, \sin 2\pi f_c t\, d\tau$$

$2\sin A \sin B = \cos(A-B) - \cos(A+B)$

Date :

Page No. :

(IV)

$$= \frac{1}{2} - \int_{-\infty}^{\infty} h^Q(\tau)\, x^Q(t-\tau) \cos 2\pi f_c t \, d\tau$$

$$= \frac{-1}{2} \int_{-\infty}^{\infty} h^Q(\tau)\, x^Q(t-\tau) \cos 2\pi f_c t \, d\tau$$

Add (I) (II) (III) (IV)

$$= \frac{1}{2} \int_{-\infty}^{\infty} h^I(\tau)\, x^I(t-\tau) \cos 2\pi f_c t - \frac{1}{2} \int_{-\infty}^{\infty} h^Q(\tau)\, x^Q(t-\tau) \cos 2\pi f_c t$$

$$- \frac{1}{2} \int_{-\infty}^{\infty} h^Q(\tau)\, x^I(t-\tau) \sin 2\pi f_c t \, d\tau - \frac{1}{2} \int_{-\infty}^{\infty} h^I(\tau)\, x^Q(t-\tau) \sin 2\pi f_c t \, d\tau$$

$$= \frac{\cos 2\pi f_c t}{2} \overbrace{\left[\int_{-\infty}^{\infty} h^I(\tau)\, x^I(t-\tau) - h^Q(\tau)\, x^Q(t-\tau) \, d\tau \right]}^{y^I(t)}$$

$$- \frac{\sin 2\pi f_c t}{2} \left[\int_{-\infty}^{\infty} \underbrace{h^Q(\tau) x^I(t-\tau) + h^I(\tau) x^Q(t-\tau)}_{y^Q(t)} \right] d\tau$$

$$y^I(t) = \frac{1}{2} \int_{-\infty}^{\infty} \left[h^I(\tau)\, x^I(t-\tau) - h^Q(\tau)\, x^Q(t-\tau) \right] d\tau$$

$$y^Q(t) = \frac{1}{2} \int_{-\infty}^{\infty} \left[h^Q(\tau)\, x^I(t-\tau) + h^I(\tau)\, x^Q(t-\tau) \right] d\tau$$

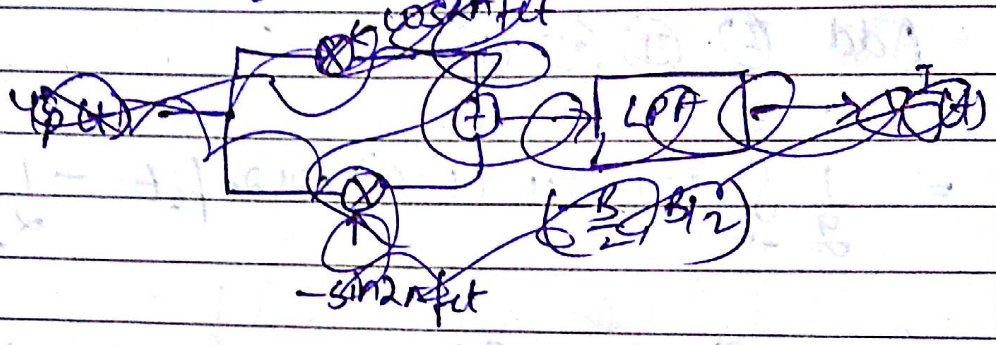
$$y(t) = y^I(t) + j\, y^Q(t)$$

$$y(t) = \frac{1}{2} \left[\int_{-\infty}^{\infty} \underbrace{\left(h^I(\tau) + j h^Q(\tau)\right)}_{h(\tau)} \underbrace{\left(x^I(t-\tau) + j\, x^Q(t-\tau)\right)}_{x(t-\tau)} d\tau \right.$$

~~al theorem + Proof ?? (read)~~

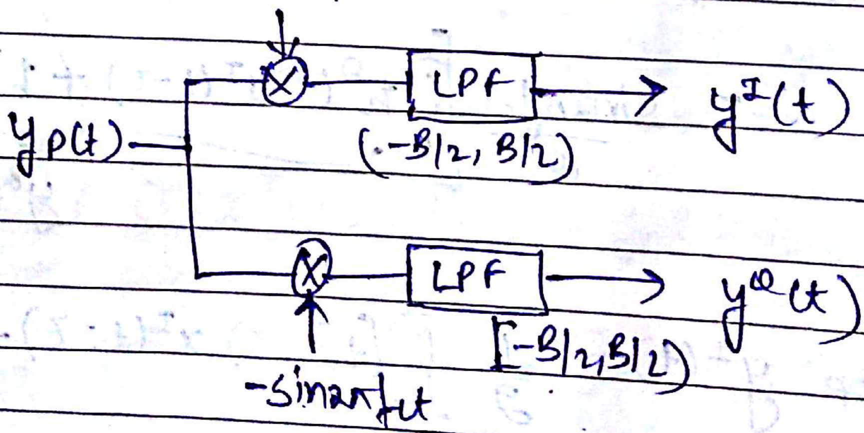
$$y(t) = \frac{1}{2}\left[\int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau\right]$$

$$y(t) = \frac{1}{2}\left[h(t) * x(t)\right]$$



∵ $y(t)$ contains all the information of $x(t)$, thus passband sig $y_p(t)$ is downconverted.

at a very high freq



Parseval's theorem $\rightarrow \int_{-\infty}^{\infty} a(t)\, b^*(t)\, dt = \int_{-\infty}^{\infty} A(f)\, B^*(f)\, df$

LTI

$a(t) \longrightarrow \boxed{h(t)} = b^*(-t) \longrightarrow a(t) * b^*(-t)$ (Proof by construction)

$$c(t) = \int_{-\infty}^{\infty} a(\tau)\, b^*[-(t-\tau)]\, d\tau$$

$$= \int_{-\infty}^{\infty} a(\tau)\, b^*(\tau - t)\, d\tau$$

$$c(t=0) = \int_{-\infty}^{\infty} a(\tau)\, b^*(\tau)\, d\tau$$

If $b(t) \overset{f}{\Longleftrightarrow} B(f)$ then $b^*(-t) \Longleftrightarrow B^*(f)$

$$C(f) = A(f)\, B^*(f)$$

I.F.T

$$c(t) = \int_{-\infty}^{\infty} \cancel{A}\, C(f)\, e^{j2\pi f t}\, df$$

$$c(t) = \int_{-\infty}^{\infty} A(f)\, B^*(f)\, e^{j2\pi f t}\, df$$

$\boxed{\text{at } t = 0}$

$$c(t=0) = \int_{-\infty}^{\infty} A(f)\, B^*(f)\, df$$

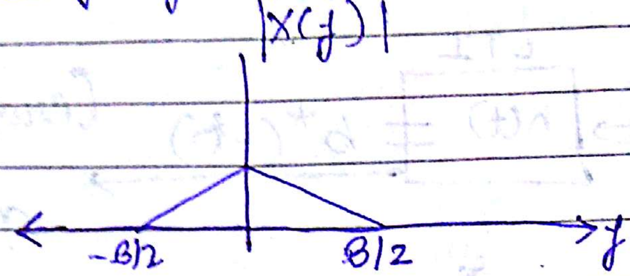
~~Thus~~, Hence proved

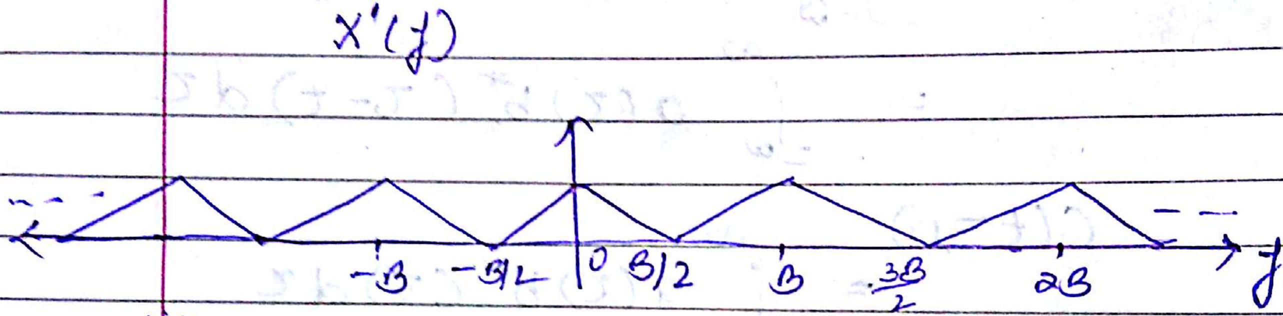
Date :

Page No. :

Proof of Sampling theorem

$$x(t) \overset{\mathcal{F}}{\Longleftrightarrow} X(f)$$





Periodic

$$X'(f) = \sum_{m=-\infty}^{\infty} X(f - mB)$$

$$X'(f) = X'(f + B)$$

If $x(t) = x(t+T)$

then,

$$x(t) = \sum_{K=-\infty}^{\infty} a_K \, e^{j\frac{2\pi}{T}Kt}$$

$$a_K = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi K t}{T}}\, dt$$

\# $$X'(f) = \sum_{K=-\infty}^{\infty} a_K \, e^{j\frac{2\pi}{B}Kf}$$

$$a_K = \frac{1}{B}\int_{-B/2}^{B/2} X'(f)\, e^{-j\frac{2\pi}{B}Kf}\, df$$

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Date :
Page No. :

for the interval $-B/2$ to $B/2$

$$a_K = \frac{1}{B}\int_{-\infty}^{\infty} X(f)\, e^{-j\frac{2\pi}{B}Kf}\, df$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi ft}\, df$$

$$a_K = \frac{1}{B}\, x(t = -K/B)$$

$$X'(f) = \sum_{K=-\infty}^{\infty} \frac{x(t=-K/B)\cdot e^{\frac{j2\pi}{B}Kf}}{B}$$

$$X(f) = \begin{cases} X'(f) & ; |f| < B/2 \\ 0 & ; \text{otherwise.} \end{cases}$$

$$X(f) = \begin{cases} \sum_{K=-\infty}^{\infty} \frac{x(t=-K/B)\, e^{j\frac{2\pi}{B}Kf}}{B} & ; |f| < \frac{B}{2} \\ 0 & ; \text{ow} \end{cases}$$

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{+j2\pi ft}\, df$$

$$= \int_{-B/2}^{B/2} \sum_{K=-\infty}^{\infty} \frac{x(t=-K/B)\, e^{\frac{j2\pi Kf}{B}}}{B} \cdot e^{j2\pi ft}\, df$$

$$= \sum_{K=-\infty}^{\infty} x\left(t = \frac{-K}{B}\right)\cdot\frac{1}{B}\int_{-B/2}^{B/2} e^{j2\pi f\left(t+\frac{K}{B}\right)}\, df$$

$\frac{e^{j\omega} - e^{-j\omega}}{2j}$ $\sin\omega$

$$= \sum_{k=-\infty}^{\infty} x\left(t = \frac{-k}{B}\right) \cdot \frac{1}{B} \left[\frac{e^{j2\pi f\left(t+\frac{k}{B}\right)}}{j2\pi\left(t+\frac{k}{B}\right)} \right]_{-B/2}^{B/2}$$

$$= \sum_{k=-\infty}^{\infty} x\left(t = -\frac{k}{B}\right) \cdot \frac{1}{B} \left[\frac{e^{j2\pi \frac{B}{2}\left(t+\frac{k}{B}\right)} - e^{-j2\pi \frac{B}{2}}}{j2\pi\left(t+\frac{k}{B}\right)} \right]$$

$$= \sum_{k=-\infty}^{\infty} x(t = -k/B) \cdot \frac{1}{\cancel{B}} \left[\frac{\sin\left\{2\pi\left[\frac{B}{\cancel{2}}\left(t+\frac{k}{B}\right)\right]\right\}}{B\pi\left(t+\frac{k}{B}\right)} \right]$$

$\operatorname{sinc} x = \frac{\sin \pi x}{\pi x}$

$$= \sum_{k=-\infty}^{\infty} x\left(t = -\frac{k}{B}\right) \operatorname{sinc}\left[B\left(t+\frac{k}{B}\right)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(t = -k/B) \operatorname{sinc}(Bt+k)$$

$k = -n$

$$\Rightarrow \sum_{-n=\infty} x(t = +n/B) \operatorname{sinc}(Bt-n)$$

$[k = -n]$

$x^I[K] = x^I\left[t = \frac{k}{B}\right]$

Date :
Page No. :

\# $\ldots 0110101\ldots$ → $x^I(n)$, $x^Q[n]$

BPSK	
0 → +1	
1 → −1	

4-PAM

$b_1 b_2$	$x^I(n)$
00	1
01	3
10	−1
11	−3

Now,

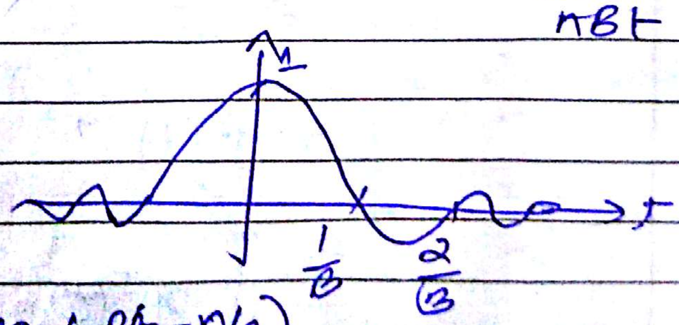
$$= \sum_{n=-\infty}^{\infty} x^I[n] \operatorname{sinc}(Bt - n) \triangleq x^I\left[\frac{t}{640}\right]$$

and

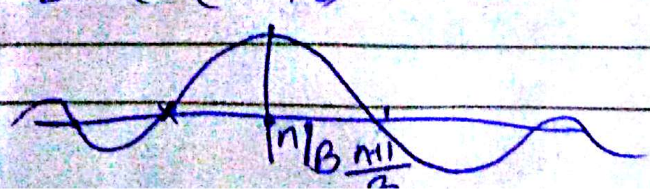
$$= \sum_{n=-\infty}^{\infty} x^Q[n] \operatorname{sinc}(Bt - n) \triangleq x^Q\left(\frac{t}{640}\right)$$

for every 20msec of s/g →
data rate = 64 kbps
Then, data = 20×64
= 1280 bits

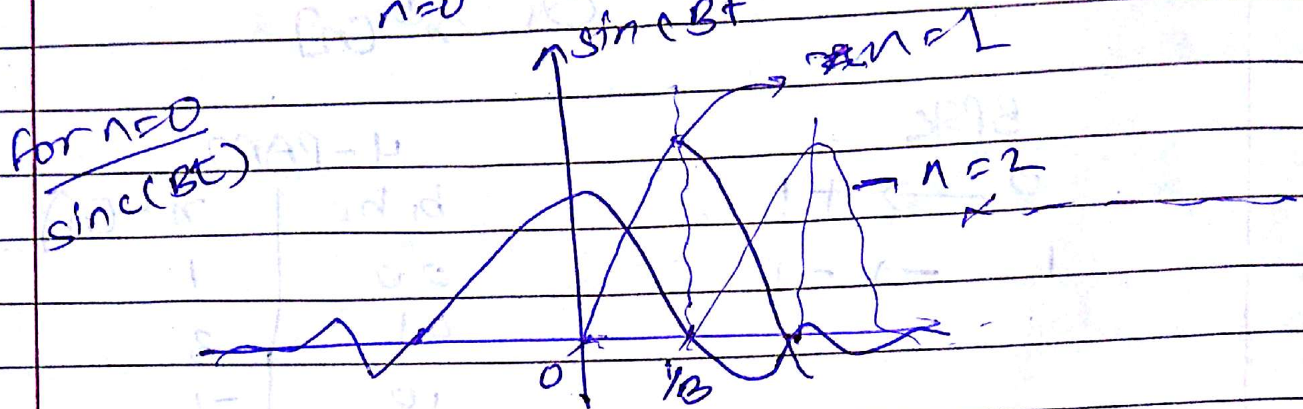
$\operatorname{sinc}(Bt - n)$, $\operatorname{sinc} Bt = \frac{\sin \pi Bt}{\pi Bt}$



$\sin(B(t - n/B))$



$$x^I(t) = \sum_{n=0}^{319} x^I[n]\,\mathrm{sinc}(Bt-n)$$

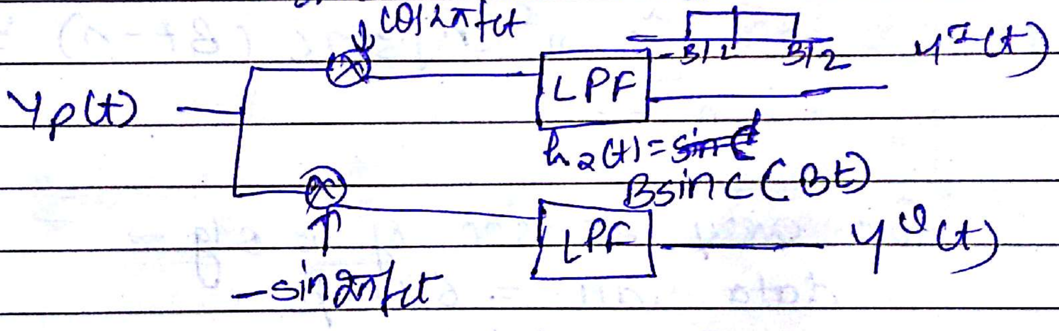


$$x^I(t)\cos 2\pi f_c t - x^Q(t)\sin 2\pi f_c t$$

$\uparrow$

$y_p(t) = h(t) * x_p(t) + n_p(t)$

let us assume no noise and $h(t) = \delta(t)$



$$y^I(t) = \left\{x^I \cos^2 2\pi f_c t - x^Q(t)\sin 2\pi f_c t \cos 2\pi f_c t\right\} * h(t)$$

$$= \frac{x^I(t)}{2}\left[1 + \underbrace{\cos 4\pi f_c t}_{\to 0}\right] - \underbrace{\frac{x^Q(t)\sin 4\pi f_c t}{2}}_{\to 0}$$

$$= \frac{x^I(t)}{2} * h_L(t)$$

$$= \int_{-\infty}^{\infty} \frac{x^I(\tau)}{2} \cdot h_L(t-\tau)\,d\tau$$

Date :
Page No. :

where

$$x^I(t) = \sum_{n=-\infty}^{\infty} x^I[n] \operatorname{sinc}(Bt-n)$$

$$= \frac{B}{2} \int \sum x^I[n] \operatorname{sinc}(B\tau - n) \operatorname{sinc}(B(t-\tau))\, d\tau$$

$$= \frac{B}{2}\, \frac{2}{2} \sum x^I[n] \int \operatorname{sinc} B\left(\tau - \frac{m}{B}\right) \operatorname{sinc} B(t-\tau)\, d\tau$$

and $x^Q(t) = \sum x^Q[n] \operatorname{sinc}(Bt-n)$

$\Rightarrow$ $y^I(t) = \frac{x^I(t)}{2} * B\operatorname{sinc}(Bt)$

$$Y^I(f) = \frac{X^I(f)}{2} \cdot H(f) = \frac{X^I(f)}{2}$$

$Y^Q(f) = X^Q(f)$

$\therefore$ $y^I(t) = \frac{x^I(t)}{2} \xrightarrow{t = \frac{K}{B}}$ $x^I(t) = \sum_{n=-\infty}^{\infty} x^I[n] \operatorname{sinc}(Bt-n)$ at $t = K/B$

$\downarrow$

$$y^Q(t) = \frac{x^Q(t)}{2}$$

$\swarrow t = K/B$

$$\frac{x^I(t)}{2} = \sum_{n=-\infty}^{\infty} \frac{x^I(n)}{2} \operatorname{sinc}(k-n)$$

$\downarrow$

$$x^Q(t) = \sum_{n=-\infty}^{\infty} x^Q[n] \operatorname{sinc}(Bt-n) \qquad \sum_{n=-\infty}^{\infty} \frac{x^I(n)}{2} \frac{\sin\pi(k-n)}{\pi(k-n)}$$

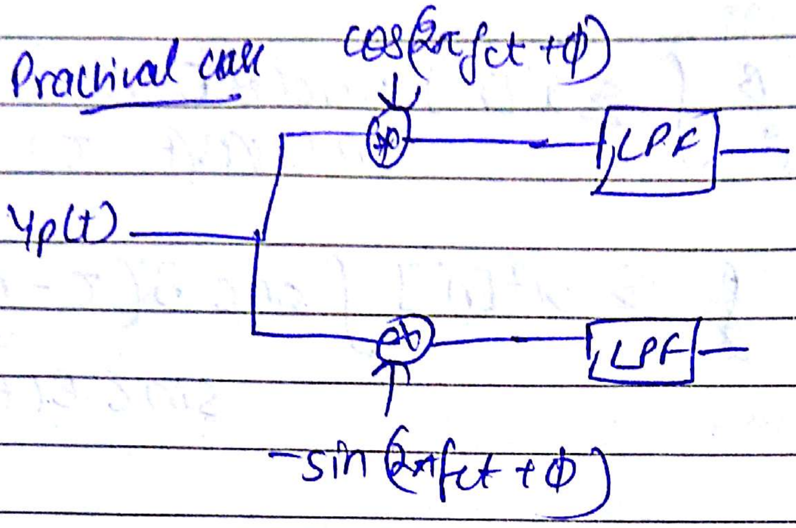
$\downarrow$ for $n=K \rightarrow$

$$\frac{x^Q(K)}{2} \cdot 1 \qquad = \frac{x^I(K) \cdot 1}{2}$$

At receiver, phase diff. is present thus PLL is used to generate same phase as Tx.

Practical case

$\Rightarrow$



$$y^I(t) = x^I(t)\cos 2\pi f_c t \cos(2\pi f_c t + \phi)$$

2cosAsinB
cost sin(A−B)
sin(A+B)

$$- x^Q(t)\sin 2\pi f_c t \cos(2\pi f_c t + \phi)]$$

$$y^I(t) = \frac{x^I(t)\cos\phi}{2} + \frac{x^Q(t)}{2}\sin\phi$$

$$y^Q(t) = -\frac{x^I(t)\sin\phi}{2} + \frac{x^Q(t)\cos\phi}{2}$$

Sample at $t = K/B$

$$y^I[K] = \frac{x^I[K]\cos\theta}{2} + \frac{x^Q[K]}{2}\sin\phi$$

$$y^Q[K] = -\frac{x^I[K]\sin\phi}{2} + \frac{x^Q[K]\cos\phi}{2}$$

Now, $y[K] = y^I[K] + j\,y^Q[K]$

$$x[K] \leftarrow \quad = \frac{(x^I[K] + j\,x^Q[K])}{2}(\cos\phi - j\sin\phi)$$

$$= \frac{1}{2}x[K]\,e^{-j\theta}$$

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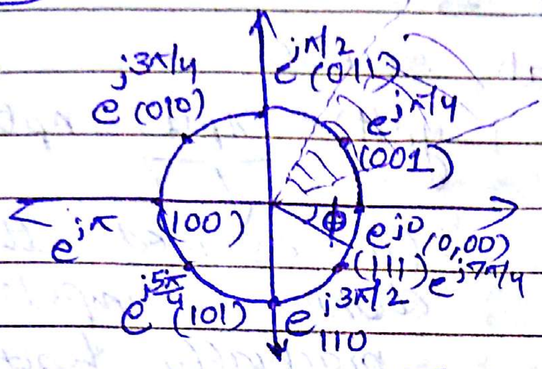
~~5 % 2~~

~~8 0 0 / 0~~

8 - PSK

3 bit



~~.......... find i ?~~

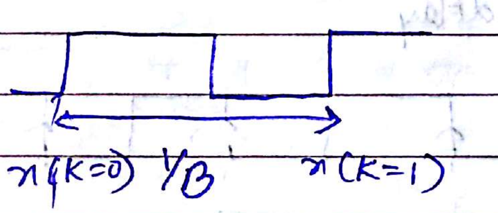
~~find i~~

Mag. at each pt. are equal → 1.

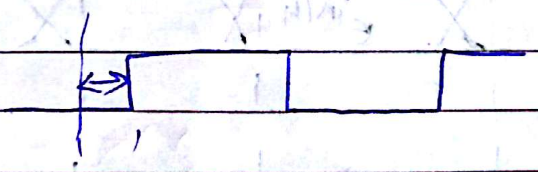
Thus, only phase is considered

Time delay

at Tx



suppose at receiver



at $t = \frac{K}{B} + \Delta$, $|\Delta| < 1/2B$

⇓

(Timing error)

$$x^I\left(t = \frac{K}{B}+\Delta\right) = \sum x^I[n]\, \mathrm{sinc}\left[B\left(\frac{K}{B}+\Delta\right)-n\right]$$

$$= \sum x^2[n]\, \mathrm{sinc}\,(B + (B\Delta) - n]$$

⇓

$-1/2 < B\Delta < 1/2$

Distorted sig

$$= x^I[K]\, \mathrm{sinc}(B\Delta) + x^I[K-1]\, \mathrm{sinc}(B\Delta+1) + x^I[K+1]\, \mathrm{sinc}(B\Delta-1)$$

Noise →

Due to noise →

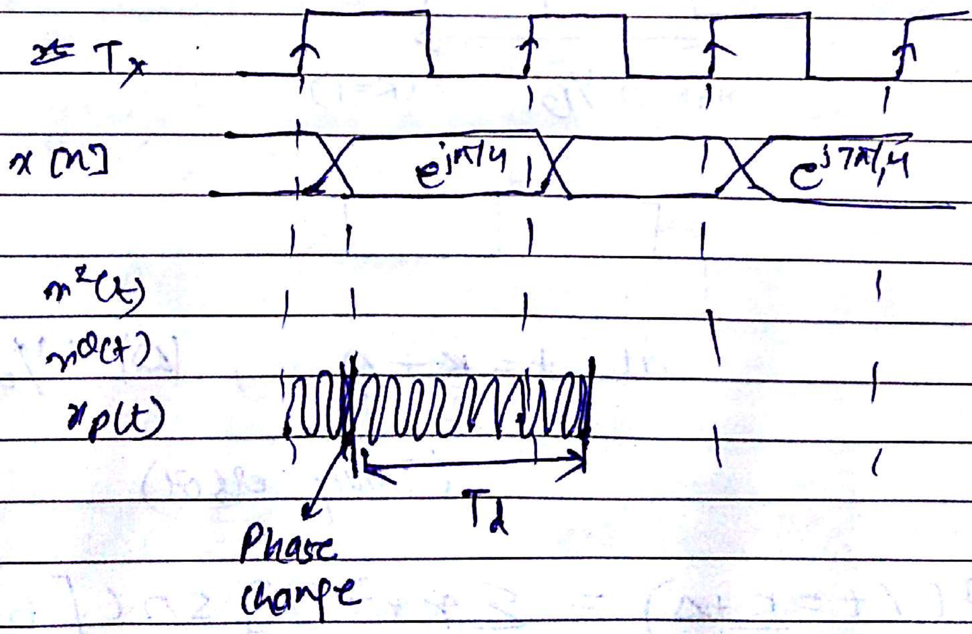
The sig at receiver →

$y_p(t) = x_p(t) + n_p(t)$

(we assumed the channel is impulse but practically $h_p(t)$ will also be present)

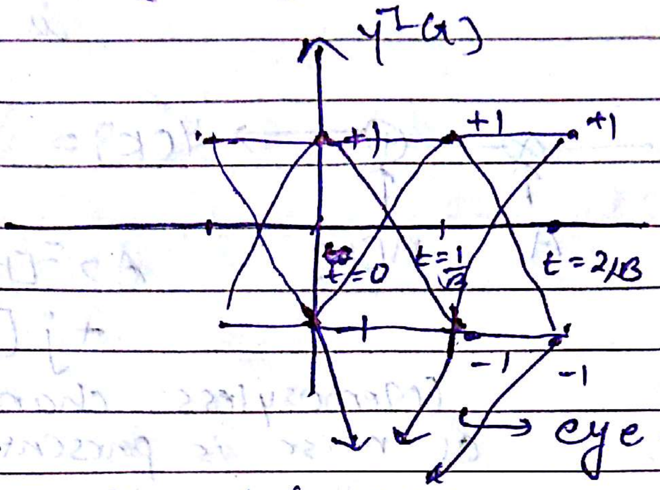
$y_p(t) = x_p(t) * h_p(t) + n_p(t)$

→ Propagation delay



$x^I(t) = \sum x^I[n] \,\text{sinc}\,(B(t - n/B))$

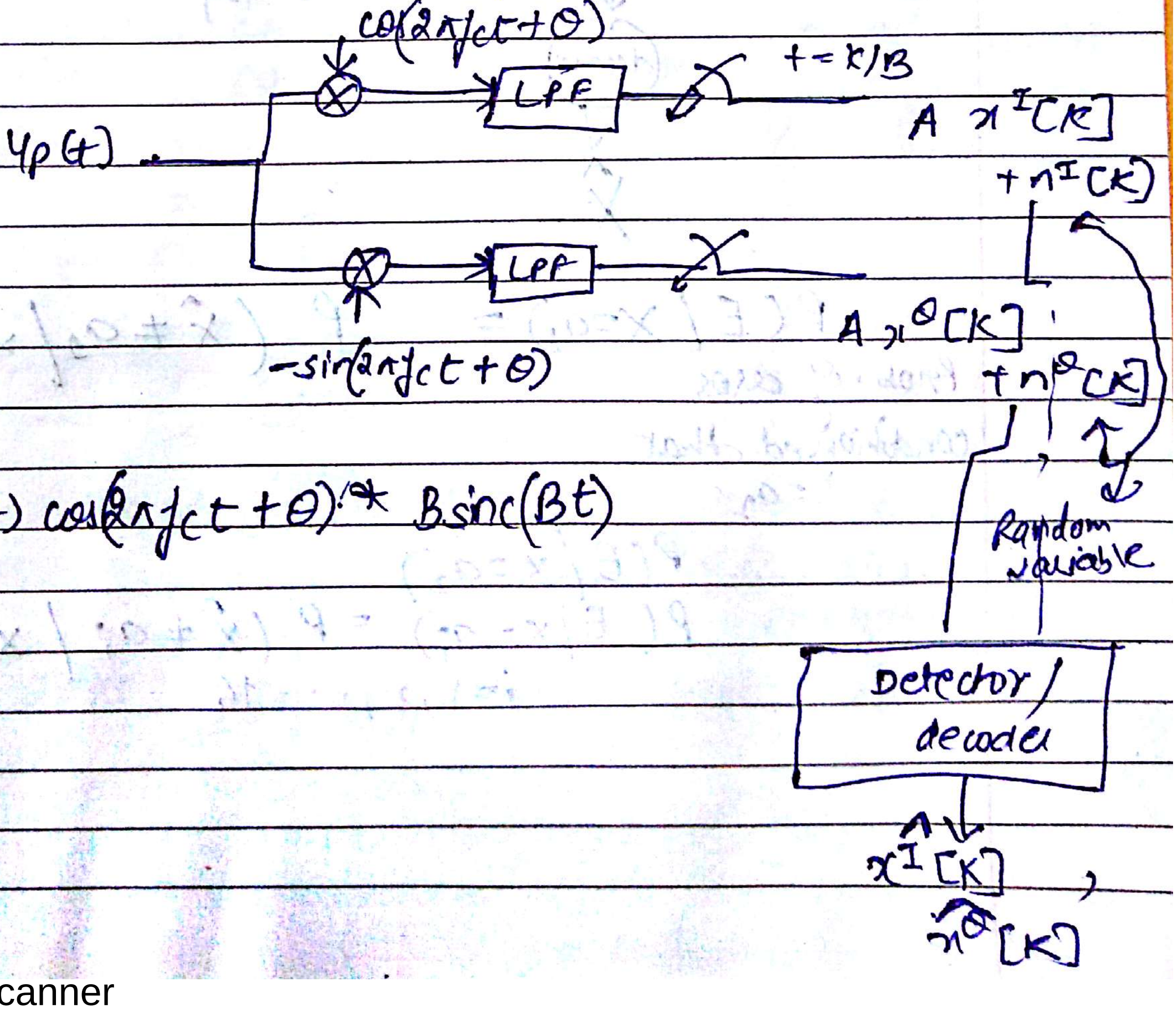
$(1 \text{ or } -1)$



Sampling should be done at this instant.

But if it is sampled at other instant, 4 values are present.

→ $y_p(t) = A x_p(t) + n_p(t)$



→ $n(t) \cos(2\pi f_c t + \theta) * B\,\text{sinc}(Bt)$

from $\hat{x}^I[K]$ and $\hat{n}^Q[K]$ $\longrightarrow$ bit stream is generated.

\# $x[K] \longrightarrow \underset{A}{\otimes} \longrightarrow \underset{n[K]}{\oplus} \longrightarrow y[K] = A x[K] + n[K]$

$$\Downarrow$$

$$A x^I[K] + n^I[K] + j\,[A x^Q[K] + n^Q[K]]$$

(Memoryless channel)

as noise is present which changes with channel

$$Y = A \underline{X} + N$$

X: 2PSK $(+1, -1)$

$$\Downarrow$$

let $X = a_1$

$Y = Aa_1 + N \longrightarrow$ decoder $\longrightarrow \hat{X}$

for 16-QAM

let

$(\pm 1, \pm 3) + j(\pm 1 \pm 3)$

$= \{a_1, a_2 \,---\, a_{16}\}$

$a_1 = 1+j$, $a_2 = 1-j$, $a_{16} = 3+3j$

$$P(E / X = a_1) = P(\hat{X} \neq a_1 / X = a_1)$$

Prob. of error conditioned that $X = a_1$

$$P(E / X = a_2)$$

$$P(E / X = a_i) = P(\hat{X} \neq a_i / X = a_i)$$

$i = 1, 2, --- 16$

$N = N_1$ (a_1 is sent)

N_2 (a_2 is sent)

⋮

N_{16} (a_{16} is sent)

$N = N_1 + N_2 + \cdots N_{16}$

E_1 (0 to N_1)

$$\frac{E_1}{N_1} = P(E/x=a_1)$$

Similarly, $\frac{E_i}{N_i} = P(E/x=a_i)$

$\Rightarrow$ $E_r = E_1 + E_2 + \cdots E_{16}$ (Total error)

$$P_e = \frac{E_r}{N} = \sum_{i=1}^{16}\left(\frac{E_i}{N}\right) = \cancel{\sum_{i=1}^{16}\left(\frac{E_i}{N_i}\right)}$$

average error prob.

$$= \sum_{i=1}^{16}\left(\frac{E_i}{N_i}\right) \times \left(\frac{N_i}{N}\right)$$

$$P_e = \sum_{i=1}^{16} P(E/x=a_i) \times P(x=a_i)$$

The decoder is called → MAP decoder (Maximum APosteriori Probability)

that gives us the least P_e.

Date :
Page No. :

MAP

$Y=y$ $\quad x=a_1$, $x=a_2$, $\vdots$, $a=a_{16}$

	cet
$P(X=a_1 \mid Y=y)$	0.1
$P(x=a_2 \mid Y=y)$	0.5
$\vdots$	0.59
$P(x=a_{16} \mid Y=y)$	0.03
	1.0

Add all $\rightarrow$ 1

max. for a_7, thus it is most ~~likely~~ likely that a_7 was Tx

$y = AX + N$ except A all are complex.

y — k^{th} rec. sample
N — " Noise "
X — " tx sample. (mod. sig.) $\sim \mathcal{X}$

$X \in \{a_1, a_2 \ldots a_{16}\} \rightarrow$ for 16 QAM

$\{\pm 1, \pm 3\} + j\{\pm 1, \pm 3\}$

$y \rightarrow \boxed{\text{Detector}} \rightarrow \hat{X}$ (estimate of X)

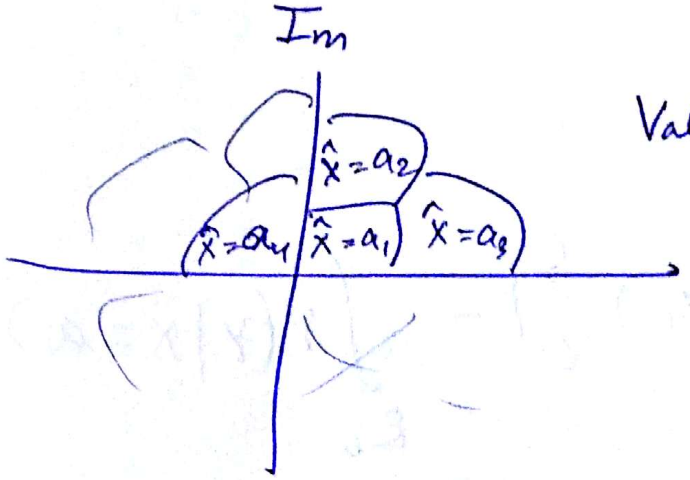
$|\mathcal{X}| \rightarrow 16$ here

$$P_e = \sum_{i=1}^{|\mathcal{X}|} P(X = a_i) \cdot P(\hat{X} \neq a_i \mid X = a_i)$$

(P_e: avg error prob)

$\hat{X}$ = funct of y

$y \longrightarrow \boxed{\text{Det.}} \longrightarrow \hat{X}$



Valid detector

non overlapping

The functn breaks this complex plane in 16 disjoint regns, corresponding to 16 mod. signals.
$(R_1, R_2, \ldots, R_{16})$

$$P(\hat{X} \neq a_i \mid X = a_i)$$

$$= \{1 - P(\hat{X} = a_i \mid X = a_i)$$

$$Y = a_i + N$$

$$P(\hat{X} = a_i \mid X = a_i) = P(Y \in R_i \mid X = a_i)$$

$$= \int_{Y \in R_i} P(Y \mid X = a_i)\, dy$$

$P(Y \mid X = a_i) \rightarrow$ Prob. of distributn of Y given a_i was Tx mitted.

$$\Rightarrow$$

$$P_e = \sum_{i=1}^{|\mathcal{X}|} P(X = a_i) \left\{ 1 - \int_{R_i} P(Y \mid X = a_i)\, dy \right\}$$

$$= 1 - \sum_{i=1}^{|\mathcal{X}|} P(X = a_i) \int_{R_i} P(Y \mid X = a_i)\, dy$$

let $Y = \frac{1}{\sqrt{3}} + j\sqrt{7}$ (arbitiary)

we now know distribution of Y.

$Y = a_i + N$

$P(X=a_1) \cdot P\left(Y = \frac{1}{\sqrt{3}} + j\sqrt{7} \;\middle|\; X = a_1\right) = p_1$

$P(X=a_2) \cdot P\left(\text{———} \;\middle|\; X = a_2\right) = p_2$

⋮

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