TSKS01 Digital Communication

Solutions to Selected Problems from Problem Class 7

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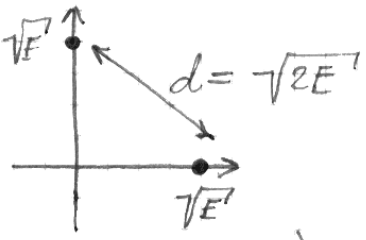
5.3 Bit-rate: $R_b = 10^6$ b/s

Noise: $\frac{N_0}{2} = 10^{-10}$ W/Hz

signal interval: $T = 1/R_b = 10^{-6}$ s.

BFSK:



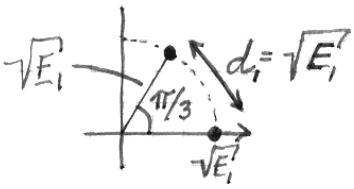
Error prob: $P_e = Q\left(\frac{d}{\sqrt{2N_0}}\right) = Q\left(\sqrt{\frac{E}{N_0}}\right) \leq 10^{-4}$

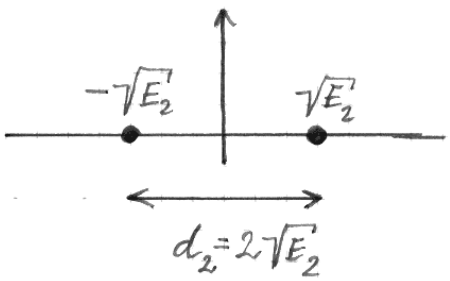
$\Rightarrow \quad \sqrt{\frac{E}{N_0}} \geq 3.71$ (Q-table)

$\Rightarrow \quad E \geq 3.71^2 \cdot N_0 = 2.75 \cdot 10^{-9}$ J

Power: $P = \frac{E}{T} \geq \frac{2.75 \cdot 10^{-9}}{10^{-6}} = 2.75$ mW

5.6 Signal constellations:





The same error probability in both cases means the same distance:

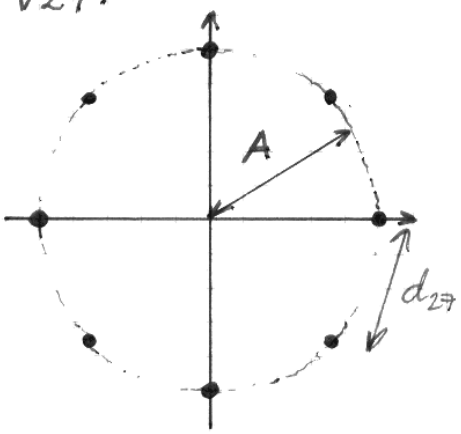
$$d_1 = d_2 \Rightarrow \sqrt{E_1} = 2\sqrt{E_2} \Rightarrow \frac{E_2}{E_1} = \frac{1}{4}$$

Expressed in dB:

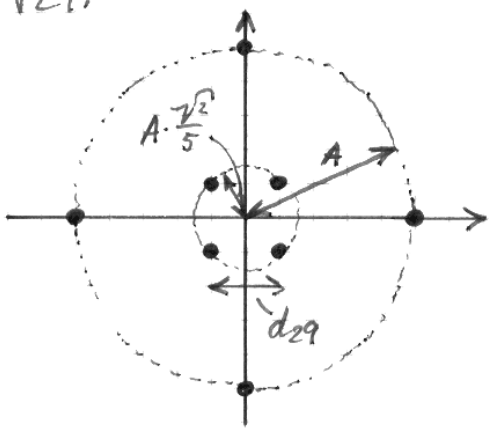
$$10 \cdot \log_{10} \frac{1}{4} = -6 \text{ dB}.$$

5.12

V27:



V29:



Large SNR $\Rightarrow$ Only the minimum distance is important for the error probability.

Minimum distances:

$$d_{27} = 2A\cdot\sin\frac{\pi}{8} \qquad \text{(This is 8-PSK)}$$

$$d_{29} = \frac{2}{5}A \qquad \text{(Obvious)}.$$

Comparison:

$$\frac{d_{27}}{d_{29}} = \frac{2\cdot\sin\frac{\pi}{8}}{2/5} = 5\sin\frac{\pi}{8} \approx 1.91$$

In dB:

$$20\cdot\log_{10}\left(5\cdot\sin\frac{\pi}{8}\right) \approx 5.6 \text{ dB}$$