Tutorial 5

Q.1). Non coherent communication.

let the received complex discrete-time symbols be given by

$$Y[k] = A e^{j\Phi} X[k] + N[k], \quad k = 1, 2, 3$$

where $(X[1], X[2], X[3])$ is the transmitted vector which ~~can~~ belongs to the set of vectors

$$\mathcal{X} = \left\{ (1, b_1, b_2) \;\middle|\; \begin{array}{l} b_1 \in [+1, -1] \\ b_2 \in [+1, -1] \end{array} \right\}$$

There are therefore only four possible vectors.

A is the random unknown channel loss $(A > 0)$ and $\Phi \in [-\pi, \pi)$ is the random unknown phase offset between the local oscillators at the transmitter and the receiver. $N[k]$ is AWGN. $\{N[k]\}_{k=1}^{3}$ is an i.i.d sequence of complex Gaussian random variables with independent real and imaginary components having mean 0.

a) Formulate and derive an Expression for the Generalized likelihood ~~decto~~ detector for the transmitted vector.

b) What is the most likely transmitted vector if the received vector is

$$(y[1], y[2], y[3]) = \left(1+\frac{j}{2},\ 1-j/2,\ -2-j\right)$$

c) Is there a problem if we Extend the set $\mathcal{X}$ to also include vectors of the form

$$(-1, \pm 1, \pm 1).$$

Q.2. (Channel coding)

Consider the following channel coding scheme:

Message	$x[1]$	$x[2]$	$x[3]$	$x[4]$
$m=1$	$-1+j$	$1-j$	$-1-j$	$2+j$
$m=2$	$-1-j$	$2+j$	$1+j$	$-1+j$
$m=3$	$1-j$	$-1+j$	$1+2j$	$1-j$
$m=4$	$1+j$	$-1-j$	$-1-j$	$2-j$

The transmitted complex baseband signal is given by.

$$x(t) = \sum_{k=1}^{4} x[k]\sqrt{2W}\,\mathrm{sinc}(2Wt-k)$$

and

$$x_p(t) = \sqrt{2}\,\mathrm{Re}\left(x(t)e^{j2\pi f_c t}\right)$$

a) What is the value of E_b (energy per information bit), P (average transmitted power), and the spectral efficiency η? (assume that all messages are equally likely to be transmitted).

b) If the channel is AWGN, with one sided PSD (σ^2), then what is the minimum value of $\left(\frac{E_b}{\sigma^2}\right)$ for which reliable communication is possible ~~for the spe~~ at a spectral efficiency equal to that of the given channel code?

Q.3. J & Z Q. 3.2 (page 159)

Q.4. J & Z Q. 3.8c (page 160).

A family of extended distances was introduced for the class of unit memory (UM), that is, $m = 1$, convolutional codes by Thommesen and Justesen [ThJ83]; see also [JTZ88]. They were generalized to $m > 1$ convolutional codes by Höst, Johannesson, Zigangirov, and Zyablov and presented together with the corresponding bounds in 1995 [HJZ95][JZZ95]; they are closely related to the active distances [HJZ99].

PROBLEMS

3.1 Consider the convolutional encoding matrix (cf. Problem 1.25)

$$G = \begin{pmatrix} 11 & 10 & 01 & 11 & & \\ & 11 & 10 & 01 & 11 & \\ & & \ddots & \ddots & \ddots & \ddots \end{pmatrix}$$

(a) Draw the state-transition diagram.
(b) Find the path $\boldsymbol{u} = 11001$ in the state-transition diagram.
(c) Find the lowest weight path that leaves the zero state and returns to the zero state.

3.2 Consider the rate $R = 2/3$, memory $m = 2$, overall constraint length $\nu = 3$, convolutional encoder illustrated in Fig. P3.2 (cf. Problem 1.27).

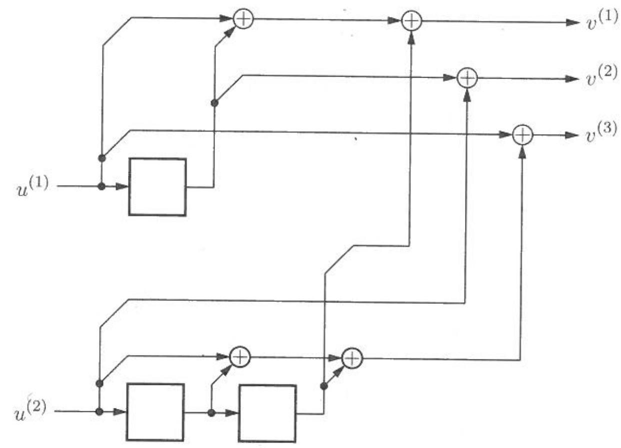

Figure P3.2 Encoder used in Problem 3.2.

(a) Draw the state-transition diagram.
(b) Find the path $\boldsymbol{u} = 10\,11\,01\,10$ in the state-transition diagram.

3.3 Consider the rate $R = 1/2$ convolutional code with encoding matrix $G(D) = (1 + D + D^2 \quad 1 + D^2)$.
(a) Find the column distances $d_0^c, d_1^c, \dots, d_\infty^c$.
(b) Find the distance profile $\boldsymbol{d}^p$.
(c) Find the row distances $d_0^r, d_1^r, \dots, d_\infty^r$.

3.4 Consider the rate $R = 1/2$ convolutional code with encoding matrix $G(D) = (1 + D + D^2 + D^3 \quad 1 + D^2 + D^3)$ and repeat Problem 3.3.

3.5 Consider the rate $R = 1/3$ convolutional code with encoding matrix $G(D) = (1 + D + D^2 \quad 1 + D + D^2 \quad 1 + D^2)$ and repeat Problem 3.3.

3.6 Consider the rate $R = 2/3$ convolutional code with encoding matrix

$$G(D) = \begin{pmatrix} 1 + D & D & 1 + D \\ 1 & 1 & D \end{pmatrix}$$

and repeat Problem 3.3.

3.7 Consider the rate $R = 1/2$ convolutional code with encoding matrix $G(D) = (1 + D + D^2 \quad 1 + D^3)$ (cf. Problem 2.4).

(a) Draw the state-transition diagram.

(b) Find an infinite-weight information sequence that generates a codeword of finite weight.

(c) Find d^c_∞ and d^r_∞.

3.8 Find the distance profile and the free distance for the rate $R = 2/3$ convolutional code with encoding matrix

(a)

$$G_1(D) = \begin{pmatrix} 1+D & D & 1 \\ D^2 & 1 & 1+D+D^2 \end{pmatrix}$$

(b)

$$G_2(D) = \begin{pmatrix} 1+D & D & 1 \\ 1+D^2+D^3 & 1+D+D^2+D^3 & 0 \end{pmatrix}$$

(c) Show that $G_1(D)$ and $G_2(D)$ encode the same code.

3.9 Consider the rate $R = 1/2$ systematic convolutional encoding matrix $G(D) = (1 \quad 1+D+D^2)$.

(a) Draw its controller canonical form.

(b) Draw the state-transition diagram.

(c) Draw the signal flowchart.

(d) Find the extended path enumerator $T(W, L, I)$.

3.10 Consider the rate $R = 1/2$ convolutional encoding matrix $G(D) = (1+D+D^2+D^3 \quad 1+D^2+D^3)$.

(a) Draw the state-transition diagram.

(b) Draw the signal flowchart.

(c) Find the path weight enumerator $T(W)$.

3.11 Consider the rate $R = 2/3$ convolutional encoding matrix

$$G(D) = \begin{pmatrix} 1+D & D & 1+D \\ 1 & 1 & D \end{pmatrix}$$

Find the extended path enumerator $T(W, L, I)$.

3.12 Consider a rate $R = 1/2$ convolutional code with a memory $m = 4$ encoding matrix.

(a) Calculate the Heller bound.

(b) Calculate the Griesmer bound.

Remark. The optimum time-invariant code with an encoding matrix of memory $m = 4$ has $d^c_\infty = d_{\text{free}} = 7$, but there exists a catastrophic time-invariant encoding matrix with $d^r_\infty = 8$.

3.13 Repeat Problem 3.12 for rate $R = 1/2$ and memory $m = 5$.

3.14 Repeat Problem 3.12 for rate $R = 1/2$ and memory $m = 28$.

3.15 Consider the rate $R = 2/3$ convolutional encoding matrix

$$G(D) = \begin{pmatrix} 1 & 1 & 0 \\ D & 1+D & 1 \end{pmatrix}$$

(a) Find the column distances $d^c_0, d^c_1, \ldots, d^c_\infty$.

(b) Find the extended path enumerator $T(W, L, I)$.

3.16 Consider the rate $R = 1/3$ convolutional code with encoding matrix $G(D) = (1 \quad 1+D+D^2 \quad 1+D^2)$.
Find the spectral component n_{16}.

3.17 Prove the Costello bound for periodically time-varying, rate $R = b/c$ convolutional codes with polynomial, systematic generator matrices (Theorem 3.31).
Hint: Modify the proof of Theorem 3.29 by using the idea from the proof of Theorem 3.23.