TSKS01 Digital Communication

Solutions to Selected Problems from Problem Class 9

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6.8 The code is cyclic, which means that the code is linear and cyclic shifts of codewords are codewords. First we list all cyclic shifts of the given codeword:

$$\begin{matrix} 1010001 \\ 1101000 \\ 0110100 \\ 0011010 \\ 0001101 \\ 1000110 \\ 0100011 \end{matrix}$$

These four codewords (rows 2–5) are linearly independent. Thus they can form a generator matrix.

This gives us the generator matrix

$$G_1 = \begin{pmatrix} 1&1&0&1&0&0&0 \\ 0&1&1&0&1&0&0 \\ 0&0&1&1&0&1&0 \\ 0&0&0&1&1&0&1 \end{pmatrix}$$

We would like to have parity check matrix of the code. If our generator matrix would be on systematic form, that would be easy. So, we create a new generator matrix by performing row operations on G_1. If we could get zeros in tead of the three circled ones, we would have a generator matrix on systematic form. Thus, replace row 3 by the sum of rows 1 and 3, and replace row 4 by the sum of rows 1, 2 and 4. The resulting generator matrix is

$$G = \begin{pmatrix} 1&1&0&1&0&0&0 \\ 0&1&1&0&1&0&0 \\ 1&1&1&0&0&1&0 \\ 1&0&1&0&0&0&1 \end{pmatrix} = (\underbrace{\;}_{P} \mid \underbrace{\;}_{I_4}) \Rightarrow H = (I_3, P^T) = \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&1&1&1 \end{pmatrix}$$

We were given the received vector (1000101). Calculate the syndrome

$$\bar{s} = \begin{pmatrix} 1&0&0&1&0&1&1 \\ 0&1&0&1&1&1&0 \\ 0&0&1&0&1&1&1 \end{pmatrix} \begin{pmatrix} 1\\0\\0\\0\\1\\0\\1 \end{pmatrix} = \begin{pmatrix} 1\\0\\0 \end{pmatrix} + \begin{pmatrix} 0\\1\\1 \end{pmatrix} + \begin{pmatrix} 1\\0\\1 \end{pmatrix} = \begin{pmatrix} 0\\1\\0 \end{pmatrix}$$

The estimated error vector: $\hat{\bar{e}} = (0100000)$

The estimated codeword: $\hat{\bar{c}} = \bar{r} + \hat{\bar{e}} = (1100101)$

6.9 Given parity check matrix:

$$H = \left(\begin{array}{cccccc:cc} 1 & & & & & & 1 & 0 \\ & 1 & & & & & 0 & 1 \\ & & 1 & & & & 1 & 0 \\ & & & 1 & & & 0 & 1 \\ & & & & 1 & & 1 & 1 \\ & & & & & 1 & 1 & 1 \end{array}\right)$$

Two solutions:

1. Linearly dependent columns in H:

   We go through all linear combinations of the last two columns in H (those not in the identity part of H), and add as many columns needed from the identity part of H to get the result $\bar{0}$. The number of columns used is then a weight of a codeword.

   Each such column has weight 4, and thus we have to add four columns from the identity part to them to get $\bar{0}$. Totally we have added 5 columns.

   The sum of those two columns has weight 4, and again we have to add four columns from the identity part of H to get $\bar{0}$. Totally 6 columns.

   The smallest number found is 5, which is our minimum distance.

2. Determine the generator matrix and generate all codewords.

   $$G = \left(\begin{array}{cccccc:cc} 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 \end{array}\right)$$

$\bar{m}$	$\bar{c} = \bar{m}G$	$w_H(\bar{c})$
0 0	0 0 0 0 0 0 0 0	0
0 1	0 1 0 1 1 1 0 1	5
1 0	1 0 1 0 1 1 1 0	5
1 1	1 1 1 1 0 0 1 1	6

   Smallest 5.

   The smallest numbe found is 5, which is our minimum distance.