

$$y[n] = \frac{1}{2} \int \left[ \sum_{k=-\infty}^{\infty} h[k] \operatorname{sinc}(2\omega \frac{1}{2} - k) \right] \left[ \sum_{m=-\infty}^{\infty} x[m] \operatorname{sinc}(2\omega (\frac{n}{2\omega} - z) - m) \right] dz \quad (7)$$

$$= \frac{1}{2} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h[k] x[m] \int \operatorname{sinc}(2\omega z - k) \operatorname{sinc}(n - m - 2\omega z) dz \quad (9)$$

It is known that

$$\int \operatorname{sinc}(2\omega z - k) \operatorname{sinc}(n - m - 2\omega z) dz$$

$$= \frac{1}{2\omega} \operatorname{sinc}(\cancel{m+k-n}) (m+k-n) \quad (10)$$

(This can be easily proven using the Parseval's theorem).

Using (10) in (9) we have

$$y[n] = \frac{1}{4\omega} \sum_{k=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h[k] x[m] \operatorname{sinc}(m+k-n)$$

We note that  $\operatorname{sinc}(m+k-n) = \begin{cases} 1 & m+k=n \\ 0 & m+k \neq n \end{cases}$

$$\therefore y[n] = \frac{1}{4\omega} \sum_{k=-\infty}^{\infty} \cancel{x[n-k]} h[k] x[n-k]$$

which is nothing but filtering in discrete-time baseband.