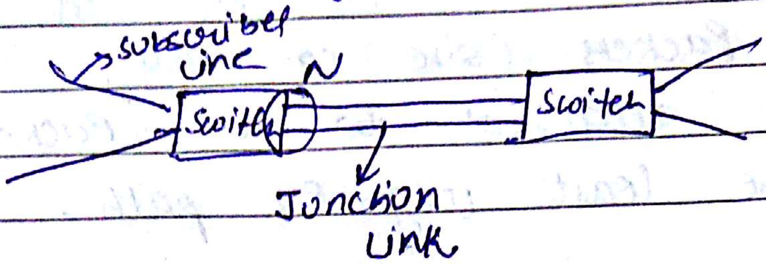
Tele traffic Theory

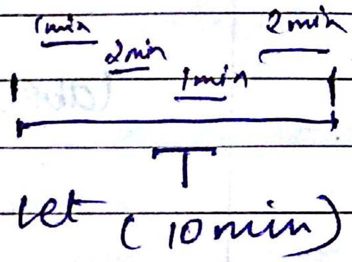


[What should be the fanout of the switch?] → scope of this chapter.

Traffic measurement

$$A = \frac{C \times h}{T}$$

(Traffic Intensity)



~~where C =~~

h → average call duration

$6/4 = 1.5$

C → no. of calls

T → Total observation time

ex → $T = 10\,min$, $h = \frac{6}{4} = 1.5$, $C = 4$

$$A = \frac{\frac{6}{4} \times 4}{10} = \frac{6}{10} = 0.6 \text{ Erlang}$$

(dimensionless)

A (Traffic intensity) = 6/10 erlang
↓
[For a total duration of 10 min, 6 min line is busy].

time is always 1 hr.

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If $A = 1E$ → link is always busy.
$A = 2E$ → 2 lines are required.
$\underline{A = 4E}$ → 4 " "
or
on an avg no. of sources that are on are 4.

★ Other units of Traffic measurement →
① EBHC → (Equated Busy Hour Call)

$$1 \text{ EBHC} = 2 \text{ min}$$
(1 call of 2 min)

$$1 \text{EBHC} = \frac{2\text{min} \times 1 \text{ call}}{60 \text{min}}$$

If we see link for 60 min, we see 1 call for 20 min.

$$1 \text{ Erlang} = 30 \text{ EBHC}$$

★ ② Cs (Call seconds) = 1 call × 1 sec.

③ Ccs (Call hundred sec) = 1 call × 100 sec

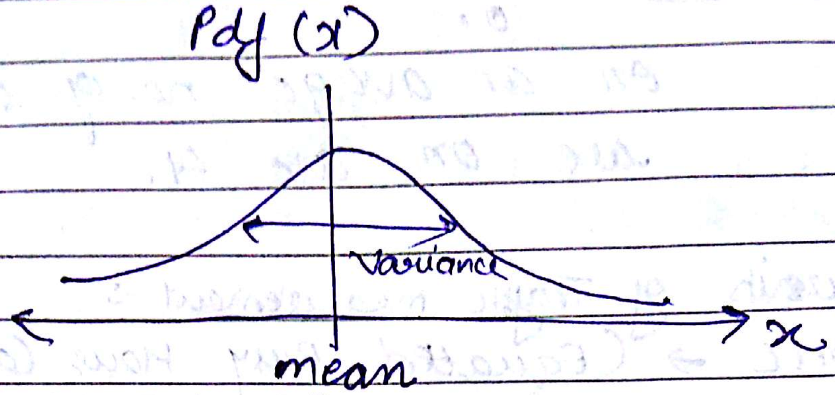
GRADE OF SERVICE (B) (f)

$$f = \frac{\text{No. of calls lost (in } 1^{st} \text{ attempt)}}{\text{No. of calls offered}}$$

eg → if 1 call is lost in 100 calls made to 100 diff. subscribers

$$\frac{1}{100} \Rightarrow gos \Rightarrow f = 0.01$$
(it should be)

Prob Pdf of traffic intensity



α (variance to mean ratio) < 1 (Traffic is smooth) eg → Binomial distribution

$\alpha = 1$ (Random traffic) eg. (Poisson)

$\alpha > 1$ (Rough traffic) eg → (Pareto traffic)

~~Almost~~ Deterministic traffic (smooth traffic)

$\alpha = 0$

as variance $= 0$.

Bernoulli Distribution

If n sources are present.
probability that source is on is p

Q- What is the prob. that x sources $(x < n)$ are on?

$\rightarrow \; {}^nC_x \cdot p^x (1-p)^{n-x}$

$p = p_{on}$
$(1-p) = p_{off}$

Mean → np Variance → $np(1-p)$

$$\alpha = \frac{np(1-p)}{np} = 1-p < 1$$

(smooth traffic).

<u>Poisson traffic</u> (n is very large) — n → no. of sources

$n \to \infty$

$np = A$ prob. for sources to be on ← p is very small

$p \to 0$

A: mean no. of sources to be ON.

$$= \frac{n!}{x!\,(n-x)!}\left(\frac{A}{n}\right)^{x}\left(1-\frac{A}{n}\right)^{n-x}$$

$$= \frac{\overbrace{n(n-1)\cdots\cancel{(n-x)!}}^{n^{x}}}{\cancel{(n-x)!}\;x!}\left(\frac{A}{n}\right)^{x}\underbrace{\left(1-\frac{A}{n}\right)^{n}}_{e^{-A}}\underbrace{\left(1-\frac{A}{n}\right)^{-x}}_{1}$$

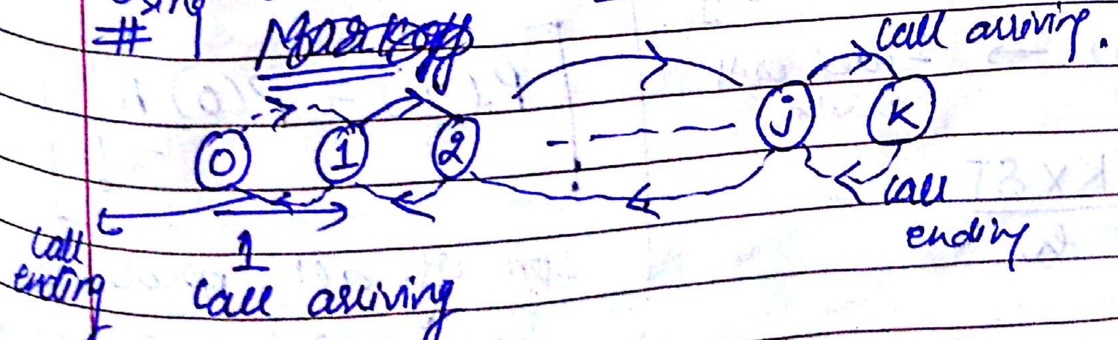
= If n is very large

$$\frac{\cancel{A^{x}}}{(n-x)!\,x!} \times \frac{A^{x}}{\cancel{x}\cancel{A^{x}}} \times e^{-A}$$

$$= \boxed{\frac{A^{x}\cdot e^{-A}}{x!}}$$

<u>Markov chain</u>

Using # ~~Poisson~~ ~~Markoff~~



(Birth - death process)

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No. of birth = No. of death.

$\boxed{K = j+1}$

$$\boxed{j \rightleftarrows K} \longrightarrow \text{in statistical equilibrium}$$

$$P(j)\,P(j,K) = P(K)\,P(K,j)$$

$P(j)$ → Prob. of j

$P(j,K)$ → Prob. of process pushing it from j to K.

$P(K,j)$ → from K to j

$$A = \frac{C \times h}{T}$$

No. of calls: $C = \dfrac{A \times T}{h}$

when $T \to$ very small

$$P(j,K) = C = \frac{A \times \delta t}{h} << 1 \quad \text{(assumed)}$$

Ex:→ $\dfrac{A(\delta t)}{h} = 0.2$

It means in δt interval → 0.2 calls arrive

as calls are integer, it corresponds to no call.

Thus, another δt is taken

$\delta t \;\; \delta(t) \;\; \delta(t) \;\; \delta(t) \;\; \delta(t) = 0.2 \times 5 = 1.$

1/5 call will arrive

In $5\,\delta(t)$ → 1 call will arrive.

in δt: $P(K,j) = \dfrac{K \times \delta T}{h}$

thus it is the prob.

$$P(j)\,\frac{A \times \delta t}{h} = P(K) \cdot \frac{K\,\delta(t)}{h}$$

$$P(K) = P(j)\,\frac{A}{K}$$

$$\boxed{P(1) = P(0)\left(\frac{A}{1}\right)}$$

$$P(2) = P(1)\left(\frac{A}{2}\right)$$

$$\boxed{P(2) = P(0)\,\frac{A^2}{2!}}$$

$$P(3) = P(2)\left(\frac{A}{3}\right)$$

$$\boxed{P(3) = P(0)\,\frac{A^3}{3!}}$$

$$\boxed{P(x) = P(0)\,\frac{A^x}{x!}}$$

Sum of all prob = 1

$$P(0) + P(1) + P(2) \cdots = 1$$

$$P(0)\left[1 + \frac{A}{1!} + \frac{A^2}{2!} \cdots\right] = 1$$

$P(x) \rightarrow$ Prob. of x calls in the s/s when mean $= A$.

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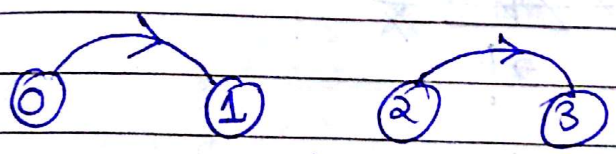
$\rightarrow P(0) \cdot e^{A} = 1$

$$\boxed{P(0) = e^{-A}}$$

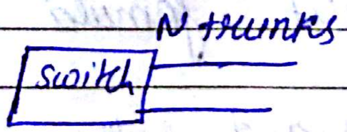
Thus, $$\boxed{P(x) = e^{-A} \cdot \frac{A^x}{x!}}$$

<u>For 3 sources</u>



For the traffic intensity to be always A irrespective of the state of s/s, it is only possible for ∞ source.

let there is a switch, N trunks



~~Total~~ ~~A~~ No. of sources can be N.

⓪ ① - - - - Ⓝ

$$P(x) = \frac{A^x}{x!} P(0) = \frac{A^x}{x!} P(0)$$

$P(0) + P(1) + \cdots P(N) = 1$

$$P(0)\left[1 + \frac{A}{1!} + \frac{A^2}{2!} + \cdots \frac{A^2}{N!}\right] = 1$$

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$$P(0) = \frac{1}{\sum_{x=0}^{N} A^x / x!}$$

Prob. of a call drop $= P(N) =$ grade of service

all trunks are busy. so any call will be dropped.

~~$P(N) = 1 / A^x / x!$~~

(for first attempt) $P(N) = \frac{A^N}{N!} \times \frac{1}{\left[\sum_{x=0}^{N} \frac{A^x}{x!}\right]}$ = grade of service

Erlang B formula
Or Erlang [Lost call s/s formula]
or Erlang first formula.

Table is provided for the above formula →

	A
N_1	P_1
N_2	P_2
N_3	P_3

mean no of calls.

\# $E_{1,N}(A) = \frac{A^N}{N!} \times \frac{1}{\sum_{x=0}^{N} \frac{A^x}{x!}}$

erlang first

No. of trunks

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$$E_{1,N-1}(A) = \frac{\frac{A^{N-1}}{(N-1)!}}{\sum_{x=0}^{N-1}\left(\frac{A^x}{x!}\right)}$$

$$* \quad \sum_{x=0}^{N}\left(\frac{A^x}{x!}\right) = \sum_{x=0}^{N-1}\frac{A^x}{x!} + \frac{A^N}{N!}$$

$\bigg\downarrow$

$$\frac{A^N}{N!\,E_{1,N}(A)} = \frac{A^{N-1}}{(N-1)!\,E_{1,N-1}(A)} + \frac{A^N}{N!}$$

$$\Rightarrow \boxed{E_{1,N}(A) = \frac{A\,E_{1,N-1}(A)}{N + A\,E_{1,N-1}(A)}}$$

Q→

$$E_{1,1}(A) = \frac{A^1}{1!} \times \frac{1}{1+A}$$

$$= \frac{A}{\cancel{2}A} = \frac{1}{\cancel{2}} \quad \frac{A}{1+A}$$

$$E_{1,2}(A) = \frac{A^2}{2!} \times \frac{1}{1+A+\frac{A^2}{2}}$$

$$= \frac{A^2}{(A^2+2A+2)}$$

OR

$$E_{1,2}(A) = \frac{A \times \frac{A}{1+A}}{2 + A \times \frac{A}{1+A}}$$

$$= \frac{A^2}{2+2A+A^2}$$

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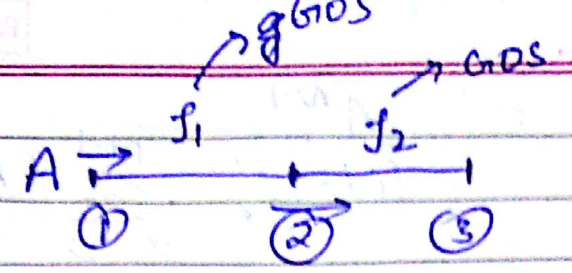
Q1 →



find overall GOS?

\# Af_1 → lost traffic

Traffic at (2) →

$$A(1-f_1)$$

Lost traffic at (3)

$$Af_2(1-f_1)$$

So, traffic at 3 →

$$A(1-f_2)(1-f_1)$$

$$\cancel{f_{net} = (1-f_2)(1-f_1)\frac{A}{A}}$$

$$= \cancel{(1-f_2)(1-f_1)}$$

$$= \cancel{B\, f_1 + f_2 - f_1 f_2} \rightarrow 0$$

Traffic lost $= \cancel{f_1 + f_2}$

$$= A[1-(1-f_1)(1-f_2)]$$

$$= A[\cancel{1}-\cancel{1}+f_1+f_2-\underbrace{f_1 f_2}_{0}]$$

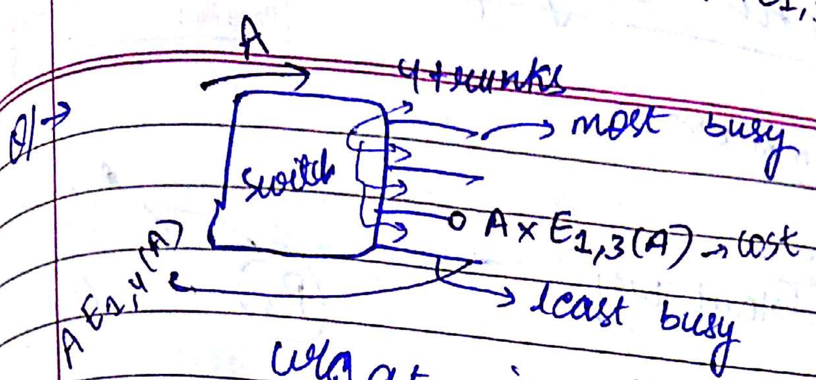
$$= A(f_1+f_2)$$

Total traffic $= A$

\# GOS $= \dfrac{A(f_1+f_2)}{A} = \underline{f_1+f_2}$

Net traffic at 4 link = $A[E_{1,3}(A) - E_{1,4}(A)]$

Q/→



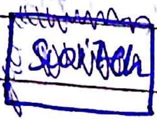
prob. of traffic cost = $\frac{A}{1+A}$

1 link T.L. = $A^2/1+A$

Traffic carried = $1 - A^2/1+A$

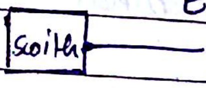
What is the traffic carried by each trunk?

Ans →



\# When a switch has just 1 trunk

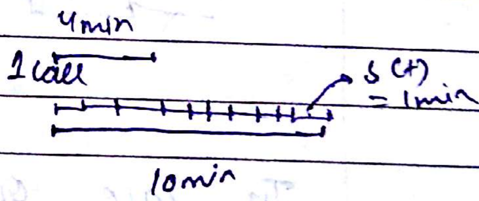
$E_{1,1}(A) = P_1$



→ B. for other cases P_1 & $E_{1,1}(A)$ can be diff.

Example:- $A = \frac{C \times h}{T}$

let, T = 10 min,



$A = \frac{1 \times 4}{10} = \frac{2}{5}$

$A = 0.4$

\# $C = \frac{A\,\delta(t)}{h} = \frac{0.4 \times 1}{4}$ let $\delta(t) = 1$ min

$= 0.1$

Prob.

OR

Total no. of observations = 10,

No. of calls = 1

$\text{Prob.} = \frac{1}{10} = 0.1$

In minor → Switch Designing

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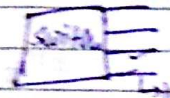


$P_3 \xrightarrow{A/4} P_2$

This approach is wrong

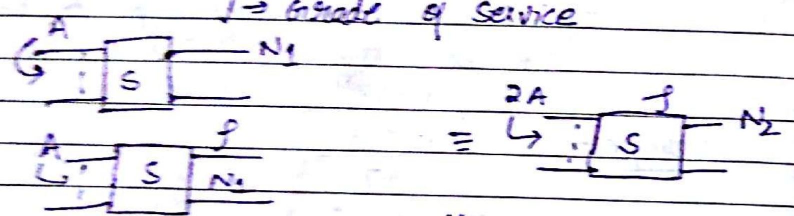
Traffic lost $= \frac{A}{4} \times A E_{1,3}(A)$

Traffic carried $= A \times E_{1,3}(A) - A \times E_{1,3}(A) \times \frac{A}{4}$

~~To~~ ~~to~~

\# Find out what is wrong in above?

$f \Rightarrow$ Grade of service



Here,

$N_2 < 2N_1$

$\hookrightarrow$??

In case of traffic overshoot

If $A \rightarrow A + 0.01A$ | A then in big switch 2

$f \rightarrow f + \Delta f_1$

$2A \rightarrow 0.02A$

then $f = f + \Delta f_2$

Here,

$\Delta f_2 > \Delta f_1$

⇓

disadvantage of bigger switch.

→ ~~Also~~ Advantage of big switch: less no. of trunks are ~~re~~ require

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Queuing Systems

In this sls, calls are not lost.

GOS = 0 (No call lost)

Markov chain

calls are N

(0) (1) - - - - - - (N) | (N+1) (N+2) - - - (∞)

$j \rightarrow k$

$P(x) = \frac{A^x P(0)}{x!}$

$P(j) \frac{A\delta(t)}{h} = P(k) \frac{N\delta(t)}{h}$

$P(K) = P(K-1) \frac{A}{N}$

$P(N+1) = P(N) = \frac{A}{N}$

$P(N) = \frac{A^N}{N!} P(0)$, $P(N+1) = \frac{A^N P(0)}{N!} \frac{A}{N}$

$P(N+2) = \frac{A^2}{N^2} \times \frac{A^N}{N!} \times P(0)$

If $x \geq N$

$P(x) = \frac{A^x}{N^{x-N}} \times \frac{1}{N!} P(0)$

$$\# \sum_{x=0}^{N-1} \frac{A^x}{x!} P(0) + \sum_{x=N}^{\infty} \frac{A^x}{N!} N^N \left(\frac{A}{N}\right)^x P(0) = 1$$

$$P(0) \left[\sum_{x=0}^{N-1} \frac{A^x}{x!} + \sum_{x=0}^{\infty} \frac{N^N}{N!} \frac{A^N}{N^N} \left(\frac{A}{N}\right)^x \right] = 1$$

$$P(0) = \left[\sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{A^N}{N!} \left(\frac{N}{N-A}\right) \right]^{-1}$$

$(1 + a + a^2 + \ldots)$

⇓

$1/(1-a)$

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$$P(0) = \frac{1}{\left[\sum_{x=0}^{N-1}\left\{\frac{A^x}{x!} + \frac{N}{N-A} \times \frac{A^N}{N!}\right\}\right]}$$

Probability of Delay → Prob. of Queuing =

This prob. is for the first attempt.

$$\sum_{x=N}^{\infty} \cancel{\frac{A^N}{N!}} \frac{N^N}{N!}\left(\frac{A}{N}\right)^x P(0)$$

$$= \sum_{x=0}^{\infty} \frac{A^N}{N!}\left(\frac{A}{N}\right)^x P(0)$$

$$= \frac{N}{N-A} \times \frac{A^N}{N!} P(0)$$

$$P(D) = \frac{A^N}{N!} \times \frac{N}{N-A}$$

$$E_{2,N}(A) = \frac{\frac{A^N}{N!} \times \frac{N}{N-A}}{\left[\sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{N}{N-A} \times \frac{A^N}{N!}\right]}$$

→ Erlang C formula

[Erlang second formula]

[Erlang. Queuing s/s formula]

→ Can a recurrence relation be made?

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→ This s/s is not practical as infinite queue are present

Finite Queuing s/s. (Q → Queue size)

$$P(0)\left[\sum_{x=0}^{N-1} \frac{A^x}{x!} + \sum_{x=0}^{Q} \frac{A^N}{N!}\left(\frac{A}{N}\right)^x\right] = 1$$

$$\frac{A^N}{N!}\left[\frac{1-(A/N)^{Q+1}}{1-(A/N)}\right]$$

$$P(D) = \sum_{x=0}^{Q} \left(\frac{A^N}{N!}\right)\left(\frac{A}{N}\right)^x . P(0)$$

Here, call be be lost thus Grade of service is calculated →

when $P(x \geq N+Q)$ (starting from infinite q s/s.)
approximate GOS

$$P(x \geq N+Q) = \left(\frac{A}{N}\right)^Q E_{2,N}(A)$$

Prove = ??