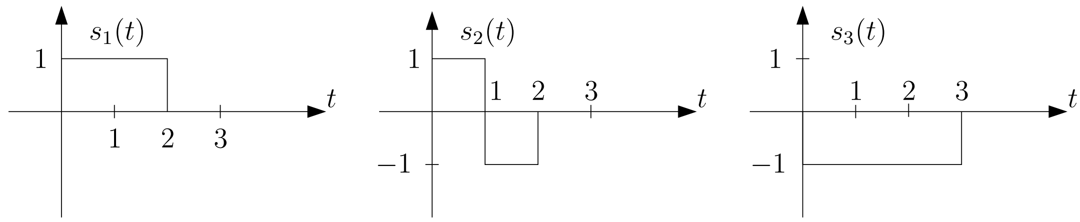
TSKS01 Digital Communication

Extra Tasks for Tutorial 3

1. Consider the following three signals.



   (a) Determine an ON basis for this signal constellation and express the signals as vectors with respect to this basis.
   (b) Determine the minimum distance of the signal constellation.
   (c) Determine the maximum distance of the signal constellation.

2. Create a two-dimensional signal constellation with four signals, such that the minimum distance is at least 1 and the maximum signal energy does not exceed 2. To make this problem interesting, do not use any of the signal constellations in Chapter 6 in the course book, or any of those used in the problems in the problem material.

   (a) Give the signals both as functions of time and as vectors with respect to a basis.
   (b) Give the minimum distance and maximum energy of your signal constellation.
   (c) Determine the average energy of your signal constellation.

3. Consider the following two signals.

$$s_1(t) = \cos\left(\frac{\pi t}{T}\right), \quad 0 \le t < T, \qquad s_2(t) = \cos\left(\frac{\pi t}{2T}\right), \quad 0 \le t < T.$$

   (a) Determine the lengths of the corresponding vectors.
   (b) Determine the angle between the signals.
   (c) Determine the distance between the signals.

Hints

1. (a) The obvious ON basis is perfectly OK. No need for Gram-Schmidt.

   (b) $d_{ij} = ||s_i - s_j|| = \sqrt{\int\limits_0^3 (s_i(t) - s_j(t))^2 \, dt}$

   (c) As in b.

2. Place signal vectors in two dimensions.

3. (a) $||s_i|| = \sqrt{\int\limits_0^3 s_i^2(t) \, dt}$

   (b) $(s_i, s_j) = ||s_i|| \cdot ||s_j|| \cdot \cos(\alpha_{ij})$

   (c) $d_{ij} = ||s_i - s_j|| = \sqrt{\int\limits_0^T (s_i(t) - s_j(t))^2 \, dt}$

Answers

1. (a) Basis:
   $\phi_1(t) = 1, \quad 0 \le t < 1,$
   $\phi_2(t) = 1, \quad 1 \le t < 2,$
   $\phi_3(t) = 1, \quad 2 \le t < 3.$
   Vectors: $\overline{s_1} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \overline{s_2} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \quad \overline{s_3} = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

   (b) $d_{\min} = 2.$

   (c) $d_{\max} = 3.$

2. There are infinitely many solutions to this problem. Here is one example.

   (a) Vectors:

   $$\overline{s_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \overline{s_2} = \frac{1}{2}\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}, \quad \overline{s_3} = \frac{1}{2}\begin{pmatrix} \sqrt{3} \\ -1 \end{pmatrix}, \quad \overline{s_4} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

   Basis:

   $$\begin{aligned} \phi_1(t) &= 1/\sqrt{T}, \quad 0 \le t < T, \\ \phi_2(t) &= \begin{cases} -1/\sqrt{T}, & 0 \le t < T/2, \\ 1/\sqrt{T}, & T/2 \le t < T. \end{cases} \end{aligned}$$

   Signals:

   $$\begin{aligned} s_1(t) &= 0 \\ s_2(t) &= \frac{\sqrt{3}}{2}\phi_1(t) + \frac{1}{2}\phi_2(t) = \begin{cases} \frac{\sqrt{3}-1}{2\sqrt{T}}, & 0 \le t < T/2, \\ \frac{\sqrt{3}+1}{2\sqrt{T}}, & T/2 \le t < T, \end{cases} \\ s_3(t) &= \frac{\sqrt{3}}{2}\phi_1(t) - \frac{1}{2}\phi_2(t) = \begin{cases} \frac{\sqrt{3}+1}{2\sqrt{T}}, & 0 \le t < T/2, \\ \frac{\sqrt{3}-1}{2\sqrt{T}}, & T/2 \le t < T, \end{cases} \\ s_4(t) &= -\phi_2(t) = \begin{cases} \frac{1}{\sqrt{T}}, & 0 \le t < T/2, \\ -\frac{1}{\sqrt{T}}, & T/2 \le t < T. \end{cases} \end{aligned}$$

   (b) $d_{\min} = 1, \qquad E_{\max} = 1$

   (c) $E = 3/4$

3. (a) $||s_1|| = ||s_2|| = \sqrt{T/2}$

   (b) $\alpha_{12} = \arccos(4/3\pi) \approx 1.13 \text{ rad} \approx 65°$

   (c) $d_{ij} = \sqrt{T\left(1 - \frac{4}{3\pi}\right)} \approx 0.76\sqrt{T}.$