Automatic Generation of DFA Minimization Problems

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1. stelle BA vor
2. trägt Namen
3. warum die Beschäftigung
4. kurz in kleiner Motivation erläutern

Introduction

Automata theory is a classical topic in computer science curricula.
Minimization of DFAs is a typical task for students:

[1] https://www.rindlerwahn.de/zeitdiebe-besiegen-und-mehr-lebenszeit-gewinnen

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DFA-Minimierung typische Aufgabe, einfache Gründe: Alg. ist...

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- ...free up precious time for exercise constructors (if a generator is implemented)
- ...yield a deeper insight in the nature of such problems


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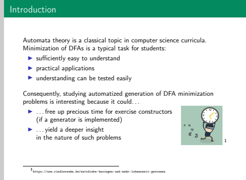

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So ergibt sich dann auch...

1. gibt Aufgabenersteller Zeit für andere Dinge
2. lässt auf interessante Einsichten hoffen
   -> Schwierigkeit

Outline

Outline

Introduction
Problem definition and approach
Generating Minimal DFAs
Extending Minimal DFAs
Live Demonstration and Conclusion

1. mit intro fertig
2. Den Ansatz in zwei Abschnitten im Detail erklären
3. im Anschluß wird es noch eine... und abschließend Resume ziehen

Problem definition

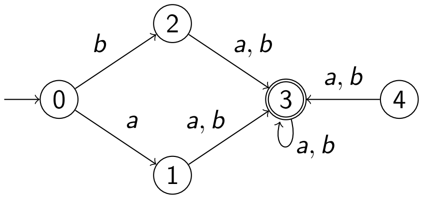
Preliminaries

We say a state q is *unreachable*, iff there is no word $w \in \Sigma^*$ such that $\delta^*(s, w) = q$.

A state pair $q_1, q_2 \in Q$ is called *equivalent*, iff $\sim_A (q_1, q_2)$ is true, where

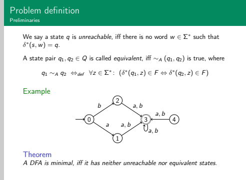
$$q_1 \sim_A q_2 \;\Leftrightarrow_{def}\; \forall z \in \Sigma^*: \; (\delta^*(q_1, z) \in F \Leftrightarrow \delta^*(q_2, z) \in F)$$

Example



Theorem

A DFA is minimal, iff it has neither unreachable nor equivalent states.



gehe davon aus, dass Def. DFA, Überführungsfkt. usw. bekannt ist

-

w, gelesen von ... aus, in ... führt

-

die folgende Relation für die beiden wahr ist

-

q1,q2 haben also das selbe Akzeptanzverhalten, egal welches Wort von den beiden Zuständen aus gelesen wird

-

weil sie schon für die selben Symbole in den jeweils selben Zustand führen

-

ein DFA ist minimal; dieses T. ist unweigerlich mit Hopcrofts Alg. verbunden

Problem definition

Hopcroft's Minimization Algorithm

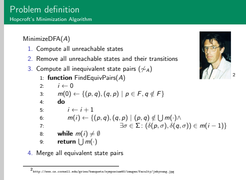
MinimizeDFA(A)

1. Compute all unreachable states
2. Remove all unreachable states and their transitions
3. Compute all inequivalent state pairs ($\not\sim_A$)

   function FindEquivPairs(A)
   $i \leftarrow 0$
   $m(0) \leftarrow \{(p,q),(q,p) \mid p \in F, q \notin F\}$
   do
   $i \leftarrow i+1$
   $m(i) \leftarrow \{(p,q),(q,p) \mid (p,q) \notin \bigcup m(\cdot) \wedge$
   $\exists \sigma \in \Sigma \colon (\delta(p,\sigma),\delta(q,\sigma)) \in m(i-1)\}$
   while $m(i) \neq \emptyset$
   return $\bigcup m(\cdot)$
4. Merge all equivalent state pairs


[2]

[2] http://www.cs.cornell.edu/gries/banquets/symposium40/images/faculty/jehyoung.jpg

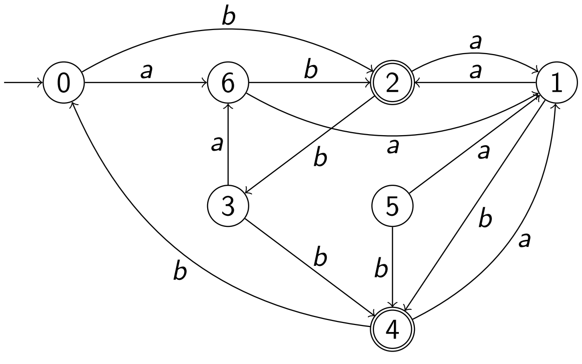


gehe auch hier davon aus
zur Erinnerung
-
wir init. eine Menge m(0) mit Zustandspaaren, von denen je genau ein Zustand akzeptierend ist
-
neue Menge m(i); Zustandspaare (p,q), (p,q) in keiner vorher ber. Menge, und Paar delta p sigma, delta q sigma ist in Vorgängermenge

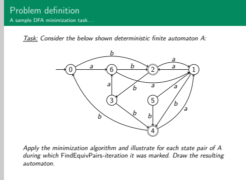
Problem definition

A sample DFA minimization task...

<u>Task:</u> Consider the below shown deterministic finite automaton A:



Apply the minimization algorithm and illustrate for each state pair of A during which FindEquivPairs*-iteration it was marked. Draw the resulting automaton.*



auf dieser Folie... Bsp DFA-Minimierungsaufgabe

Problem definition

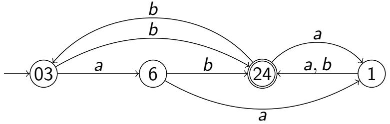
. . . and its solution

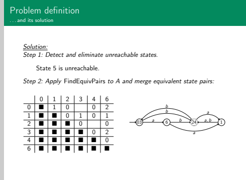
<u>*Solution:*</u>
Step 1: Detect and eliminate unreachable states.

State 5 is unreachable.

Step 2: Apply FindEquivPairs *to A and merge equivalent state pairs:*

	0	1	2	3	4	6
0	■	1	0		0	2
1	■	■	0	1	0	1
2	■	■	■	0		0
3	■	■	■	■	0	2
4	■	■	■	■	■	0
6	■	■	■	■	■	■





Lsg, die dadurch zustandekommt, dass man Hop. Min.alg. anwendet

Problem Definition and Approach

DFAMinimization

Given : A DFA A_{task}.
Task : Compute $A_{sol} = \text{MinimizeDFA}(A_{task})$.

Main question: How to generate instances of DFAMinimization?

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Eine Min.aufgabe können wir wie folgt formalisieren

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in Anlehnung an H.Min.alg.

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in meiner Arbeit... Reihe an Parametern, mit denen man versch. Eig. des gen. Problems beeinfl. kann

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Trial-and-Error-Methode

Problem Definition and Approach

DFAMinimization

Given : *A DFA* A_{task}.
Task : *Compute* A_{sol} = MinimizeDFA(A_{task}).

Main question: How to generate instances of DFAMinimization?

Idea: First generate A_{sol}, then add equivalent, then unreachable states.

Parameters

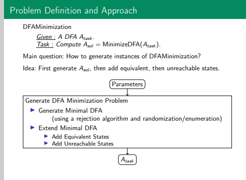
↓

Generate DFA Minimization Problem

- Generate Minimal DFA
  (using a rejection algorithm and randomization/enumeration)
- Extend Minimal DFA
  - Add Equivalent States
  - Add Unreachable States

↓

A_{task}



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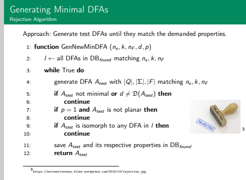
Generating Minimal DFAs
Rejection Algorithm

Approach: Generate test DFAs until they match the demanded properties.

1: **function** GenNewMinDFA (n_s, k, n_F, d, p)
2: $I \leftarrow$ all DFAs in DB_{found} matching n_s, k, n_F
3: **while** True **do**
4: generate DFA A_{test} with $|Q|, |\Sigma|, |F|$ matching n_s, k, n_F
5: **if** A_{test} not minimal **or** $d \neq \mathfrak{D}(A_{test})$ **then**
6: **continue**
7: **if** $p = 1$ **and** A_{test} is not planar **then**
8: **continue**
9: **if** A_{test} is isomorph to any DFA in I **then**
10: **continue**
11: save A_{test} and its respective properties in DB_{found}
12: **return** A_{test}

[3]

[3] https://moviewriternyu.files.wordpress.com/2015/10/rejection.jpg



1. while-Schleife
2. test DFA generieren, für die schon drei Parameter garantiert stimmen, nämlich im Enzelnen
3. restl. Eig. werden in z. 5-10, bsp. 5 minimalität
4. wenn es Fragen zu den and. get. Eig. gibt, dann kann ich hier gerne im Anschluss genauere Erläuterungen geben
5. Hier möchte.. nur auf test DFA Generierung in z.4 näher eingehen

Generating Minimal DFAs
Test DFA Generation

We will restrict ourselves to $Q = [0, n_s - 1],\ \Sigma = [0, k - 1],\ s = 0.$

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Im Rahmen dieser Arbeit

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Generation...

(a) by randomization:

$$F = random\_subset(Q)$$
$$\delta(q,\sigma) = choose\_one(Q) \qquad \forall q \in Q, \sigma \in \Sigma$$

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Ich werde hier zwei Methoden zur Gen vorstellen. Die Gen. per Randomisierung funkt. hier wie folgt

-

Für die Tr.fkt. wird für jede Komb. eines Zustands und eines Symbols ein zufälliger Zustand ausgewählt

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(b) by enumeration: An *enumeration state* $s_{n_s,k,n_F} = (F_F, F_\delta)$ has the following semantics:

$$F_F[i] = 1 \Leftrightarrow_{def} i \in F$$
$$F_\delta[i * k + j] = q \Leftrightarrow_{def} \delta(i,j) = q$$

Example state: $s_{4,2,2} = (0110)_2\ \ (10\ 13\ 22\ 03)_4$

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verwalten wir für eine gegebene Enumeration einen

-

parametrisiert mit

-

wenn i nicht akz., dann FF[i] = 0; für alle Zustände definiert

-

2. Feld speichert für alle Komb. eines Zustands und eines Symbols zu welchem Zustand man gelangt

-

Unten angedeutet; Felder als Zahlen interpretieren; durch gewisse Inkrementierungsfkt. von einem DFA zum nächsten gelangen; nicht weiter ausführen

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Kapitel der Lös.DFA Gen. abgeschlossen; widmen uns Erweiterung dieser DFAs hin zu Aufgaben-DFAs

-

Beschäftigung zunächst, gemäß vorhin vorg. Reihenfolge, mit Hinz. äqu. Z.

Extending Minimal DFAs

Adding Equivalent States (1)

We now want to add states $r_1, \ldots, r_{n_e}$ to the solution DFA, such that every r_i is equivalent to a state e in the solution DFA:

$$\forall i \in [1, n_e]: \; \exists e \in Q_{sol}: \; r_i \sim_A e$$

Whenever we add a state r_i, we will first choose an $e \in Q_{sol}$, then we add the transitions of r_i.

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Sei nE die gewünschte Anzahl an paarweise versch. äqu. Zustandspaaren.

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In diesem Verfahren werden die rI einzeln nacheinander hinzugefügt

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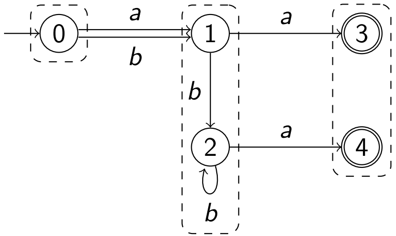

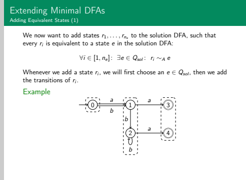

im Bsp könnten wir zunächst Zst. 2 als orig. aussuchen, und dann rI gedanklich zu dessen Äqu. hinzufügen

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dann würden wir jetzt die Tr. hinzufügen, dabei beginnen wir mit den ausgehenden Tr.

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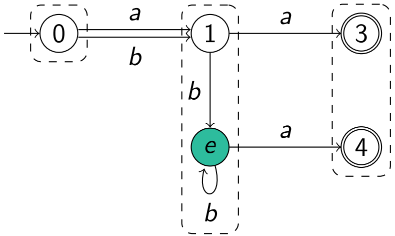

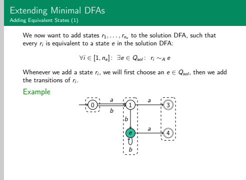

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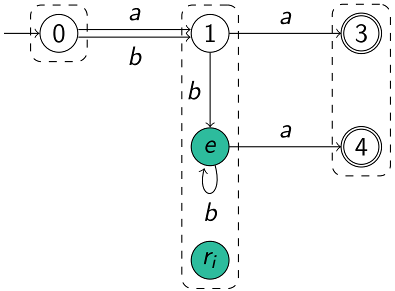

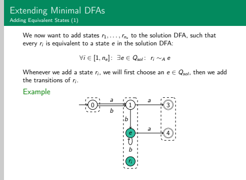

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Extending Minimal DFAs

Adding Equivalent States (2) - Outgoing Transitions

Observation:

$$r_i \sim_A e \implies \forall \sigma \in \Sigma \colon [\delta(r_i, \sigma)]_{\sim_A} = [\delta(e, \sigma)]_{\sim_A}$$

Consequently:

R1: For each symbol $\sigma \in \Sigma$ choose exactly one state $q \in [\delta(e, \sigma)]_{\sim_A}$ and set $\delta(r_i, \sigma) = q$.

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..dann sind folglich auch diejenigen Zust. äqu., wenn wir ein bel. Symbol von rI und e aus lesen

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So konnten wir mit dieser Regel erreichen, dass jede Transition von rI in dieselbe Äqu.kl. führt, die wir err. würden, wenn wir der koresp. Tr. von e folgten

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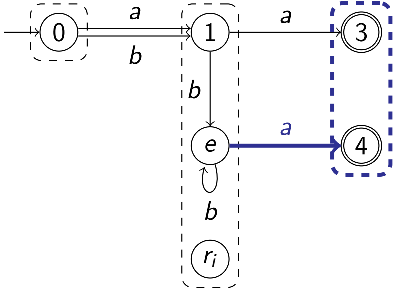

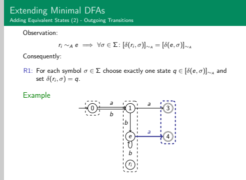

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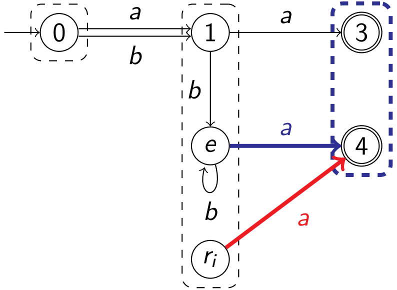

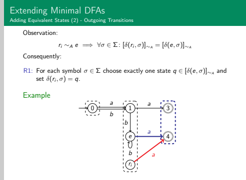

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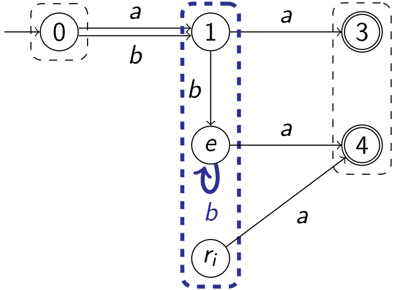
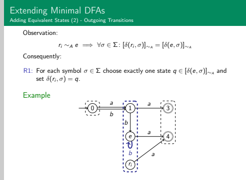
$$r_i \sim_A e \implies \forall \sigma \in \Sigma \colon [\delta(r_i, \sigma)]_{\sim_A} = [\delta(e, \sigma)]_{\sim_A}$$

Consequently:

R1: For each symbol $\sigma \in \Sigma$ choose exactly one state $q \in [\delta(e, \sigma)]_{\sim_A}$ and set $\delta(r_i, \sigma) = q$.

Example





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Extending Minimal DFAs

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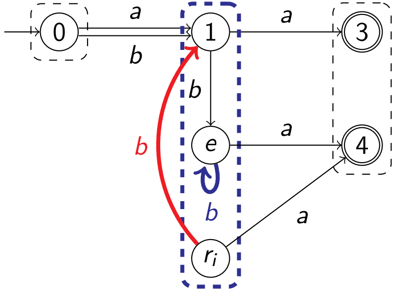

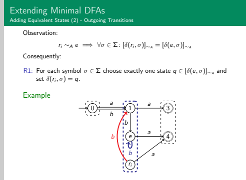

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rI kann nicht Startzustand sein, daher nicht in Erreichbarkeits-Bed. aufgenommen

-

wir müssen Tr. klauen, weil es keine Tr. mit 'losen' Enden gibt- arbeiten mit kompletten DFAs

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Extending Minimal DFAs

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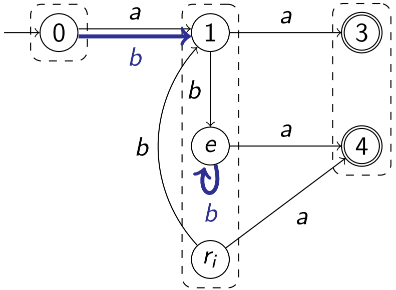

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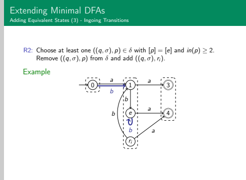

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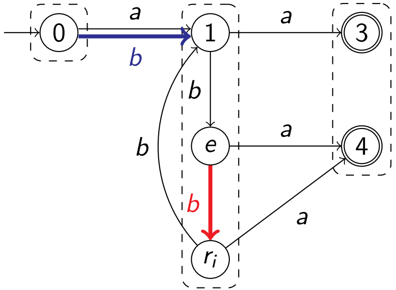

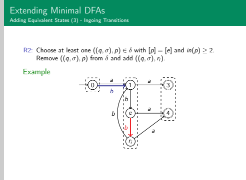

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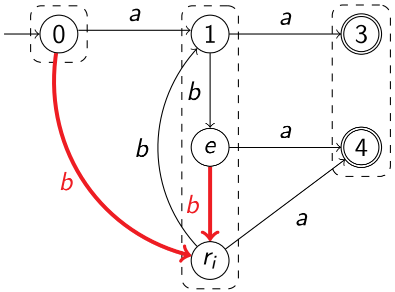

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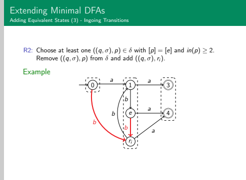
└─Extending Minimal DFAs



Reminder: We say a state q is *unreachable*, iff there is no word $w \in \Sigma^*$ such that $\delta^*(s, w) = q$.

function AddUnrStates (A, n_u, c)
    $U \leftarrow \emptyset$
    for n_u **times do**
        let q be the new state
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        add outgoing tr. to $|\Sigma|$ random states
        add q to U
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Extending Minimal DFAs
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Outline

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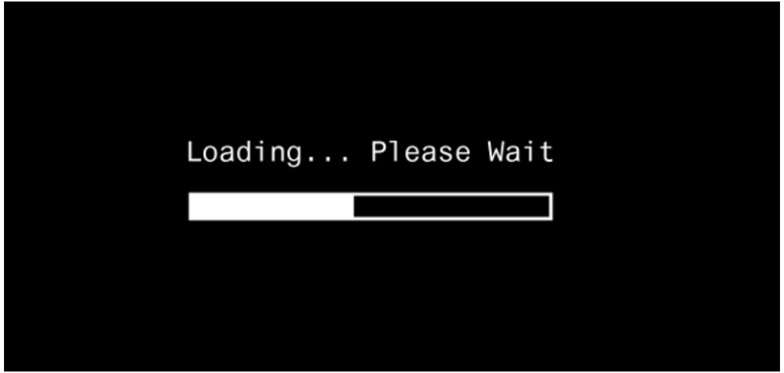
Problem definition and approach
Generating Minimal DFAs
Extending Minimal DFAs
Live Demonstration and Conclusion

Live Demonstration



[4]

4 https://sagamer.co.za/wp-content/uploads/2015/03/loading-please-wait.png

2020-07-16

Automatic Generation of DFA Minimization Problems
└Live Demonstration and Conclusion
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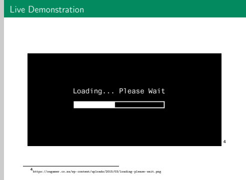

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  $\Rightarrow$ capping the max. degree?
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Bonus-Slide

$\mathfrak{D}$-Proof

Definition

We will call a word w *distinguishing word of* p, q, iff

$$\delta^*(p, w) \in F \Leftrightarrow \delta^*(q, w) \notin F$$

Lemma

Iff $(p, q) \in m(n)$*, the shortest distinguishing word of* p, q *has length* n.

Lemma

If FindEquivPairs *has done* n *iterations and terminated (so* $\mathfrak{D}(A) = n$*), then the longest word* w*, that is a shortest distinguishing word for any state pair, has length* $\mathfrak{D}(A) - 1$.

Theorem

Given two DFAs A, A'. *If both are accessible and* $L(A) = L(A')$*, then* FindEquivPairs *runs with the same number of iterations on them:* $\mathfrak{D}(A) = \mathfrak{D}(A')$.

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