PA-01

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1 Newton's Method

Solving $f(x) = 0$ where $f(x) = x^2 - 2$ and $x_0 = 1$.

i) Error Analysis ($e_n = x_n - \sqrt{2}$ for $n = 0, .., 8$)

n	x	error
0	1.00000000000000000000e+00	-4.14213562373095145475e-01
1	1.50000000000000000000e+00	8.57864376269048545254e-02
2	1.41666666666666674068e+00	2.45310429357159520691e-03
3	1.41421568627450988664e+00	2.12390141474116944664e-06
4	1.41421356237468986983e+00	1.59472435257157485466e-12
5	1.41421356237309514547e+00	0.00000000000000000000e+00
6	1.41421356237309492343e+00	-2.22044604925031308085e-16
7	1.41421356237309514547e+00	0.00000000000000000000e+00
8	1.41421356237309492343e+00	-2.22044604925031308085e-16

ii) Quadratic Convergence?

In this case quadratic convergence does not exist "perfectly". As seen between steps $n = 1$ and $n = 2$ the number of significant digits does not double.

iii) Rounding Error and Numerical Convergence

The rounding error causes Newton's Method to become unstable for the given problem. While attempting to take a next step after the system shows the method has converged (via $error = 0$) a rounding error occurs such that the solution moves away from the convergence point.

iv) Solving for M_n for $n = 1, ..., 4$

The following list gives the values for M_n solved by $M_n = \frac{|e_{n+1}|}{|e_n|^2}$.

```
M1 = 3.33333333333331038872e-01
M2 = 3.52941176468276551770e-01
M3 = 3.53522384487245822093e-01
M4 = 0.00000000000000000000e+00
```

All values of M for the given case are less than 1.

2 Secant Method

Solving $f(x) = 0$ where $f(x) = x^2 - 2$ and $x_0 = 1$.

i) Error Analysis ($e_n = x_n - \sqrt{2}$ for $n = 0, .., 8$)

n	x	error
0	0.00000000000000000000e+00	-1.41421356237309514547e+00
1	1.00000000000000000000e+00	-4.14213562373095145475e-01
2	2.00000000000000000000e+00	5.85786437626904854525e-01
3	1.33333333333333348136e+00	-8.08802290397616641116e-02
4	1.40000000000000013323e+00	-1.42135623730950122479e-02
5	1.41463414634146333881e+00	4.20583968368193339415e-04
6	1.41421143847487007505e+00	-2.12389822507041969857e-06
7	1.41421356205732040578e+00	-3.15774739689800298947e-10
8	1.41421356237309536752e+00	2.22044604925031308085e-16

ii) Quadratic Convergence?

Quadratic convergence does not occur using this method. As seen between steps $n = 1$ and $n = 2$ the number of significant digits does not increase and between very few steps does the number of significant digits double.

iii) Solving for M_n for $n = 1, ..., 7$

The following list gives the values for M_n solved by $M_n = \frac{|e_{n+1}|}{|e_n|^{1.618}}$.

```
M1 = 2.43820586365651825744e+00
M2 = 1.92149900902253506496e-01
M3 = 8.31407582617175511253e-01
M4 = 4.09995753025747533549e-01
```

```
M5 = 6.16225988349691222723e-01
M6 = 4.76513332062796890476e-01
M7 = 5.22934885792810222327e-01
```

All values of M except for the first are less than 1.

3 Fixed Point Iteration

Solving $f(x) = 0$ where $x_{n+1} = h(x_n)$ and $h(x) = x - \frac{f(x)f'(x)}{f'(x)^2 - f(x)f''(x)}$.

i) Show implications

$f(x) = 0$ implies $h(x) = x$:

$$h(x) = x - \frac{0 * f'(x)}{f'(x)^2 - 0 * f''(x)}$$

$$h(x) = x - \frac{0}{f'(x)^2 - 0}$$

$$h(x) = x$$

$h(x) = x$ implies $f(x) = 0$ or $f'(x) = 0$:
If $h(x) = x$ then $h(x) = x - 0$ in some fassion. Therefor $\frac{f(x)f'(x)}{f'(x)^2 - f(x)f''(x)} = 0$ must be satisfied. To achieve this the numerator of the fraction must $=$ 0 while the denominator $\neq 0$. If $f(x) = 0$ or $f'(x) = 0$ this condition is satisfied. Further, if $f(x) = 0$ AND $f'(x) = 0$ there is an issue with division by 0:

$$h(x) = x - \frac{0 * 0}{0^2 - 0 * f''(x)}$$

$$h(x) = x - \frac{0}{0 - 0}$$

$$h(x) = x - \frac{0}{0}$$

$$h(x) = undefined$$

ii) $h'(x)$ and Stability of the Result

$$h'(x) = -\frac{f(x)(-2f(x)f''(x)^2 + f(x)f'(x)f'''(x) + f'(x)^2 f''(x))}{(f'(x)^2 - f(x)f''(x))^2}$$

Above equation taken from wolframAlpha...inserting links is harder than expected. Hopefully I remember to change this part.
$h'(x) = 0$ when $f(x) = 0$ and $f'(x) \neq 0$:

$$h'(x) = -\frac{0 * (-2 * 0 * f''(x)^2 + 0 * f'(x) * f'''(x) + f'(x)^2 f''(x))}{(f'(x)^2 - 0 * f''(x))^2}$$

$$h'(x) = -\frac{0 * (0 + 0 + f'(x)^2 f''(x))}{(f'(x)^2 - 0)^2}$$

$$h'(x) = -\frac{0}{(f'(x)^2)^2}$$

$$h'(x) = 0$$

$h'(x) = 2$ when $f(x) \neq 0$, $f'(x) = 0$ and $f''(x) \neq 0$:

$$h'(x) = -\frac{f(x)(-2f(x)f''(x)^2 + f(x) * 0 * f'''(x) + 0^2 * f''(x))}{(0^2 - f(x)f''(x))^2}$$

$$h'(x) = -\frac{f(x)(-2f(x)f''(x)^2 + 0 + 0)}{(-f(x)f''(x))^2}$$

$$h'(x) = -(-2) * \frac{f(x)(f(x)f''(x)^2)}{(f(x)f''(x))^2}$$

$$h'(x) = 2 * \frac{f(x)^2 f''(x)^2}{(f(x)f''(x))^2}$$

$$h'(x) = 2$$