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SUPERGAUGE INVARIANT EXTENSION OF QUANTUM ELECTRODYNAMICS

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A B S T R A C T

A minimal supergauge invariant extension of quantum electrodynamics is described. It contains a spinor, a scalar and a pseudoscalar field, all charged, plus the photon field and a massless Majorana spinor. A Lagrangian invariant under gauge and supergauge transformations is constructed and shown to be renormalizable in the one-loop approximation.

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1. INTRODUCTION

Recently a field theory model invariant under supergauge transformations [1),2)] has been shown to be renormalizable in a manner consistent with supergauge invariance [3)]. The model contains a scalar field, a pseudoscalar field and a Majorana spinor field. It can be considered as an extremely simplified example of how supergauge symmetry could be applied in hadronic physics. The question arises whether other supergauge invariant renormalizable field theories exist, in particular whether quantum electrodynamics can be extended, by adding suitable fields, to a renormalizable supergauge invariant theory. In this paper we show that this is indeed possible, by constructing a field theory which is both supergauge invariant and invariant under ordinary gauge transformations. This field theory describes the interaction of a vector multiplet with a complex scalar multiplet, or two real scalar multiplets. The vector multiplet contains, besides the vector field v_μ (photon), a massless Majorana spinor λ (which we could call suggestively the "neutrino", although our Lagrangian preserves parity). The scalar multiplet contains a complex scalar $A = 1/\sqrt{2}(A_1+iA_2)$, a complex pseudoscalar $B = 1/\sqrt{2}(B_1+iB_2)$ and a complex spinor field $\psi = 1/\sqrt{2}(\psi_1+i\psi_2)$, all massive. In addition there are a certain number of auxiliary fields, as usual in supergauge invariant theories, which can be eliminated by means of the equations of motion. The result of this elimination is the Lagrangian

$$
\begin{aligned}
L = &-\frac{1}{2}\left[(\partial A_1)^2+(\partial A_2)^2+(\partial B_1)^2+(\partial B_2)^2+i\bar{\psi}_1\gamma\cdot\partial\psi_1+i\bar{\psi}_2\gamma\cdot\partial\psi_2\right] \\
&-\frac{1}{2}m^2\left(A_1^2+A_2^2+B_1^2+B_2^2\right)-\frac{i}{2}m\left(\bar{\psi}_1\psi_1+\bar{\psi}_2\psi_2\right) \\
&-\frac{1}{4}v_{\mu\nu}^2-\frac{i}{2}\bar{\lambda}\gamma\cdot\partial\lambda \\
&-g\Big[v_\mu\left(A_1\partial_\mu A_2-A_2\partial_\mu A_1+B_1\partial_\mu B_2-B_2\partial_\mu B_1-i\bar{\psi}_1\gamma_\mu\psi_2\right) \\
&\quad+i\bar{\lambda}\left\{(A_1+\gamma_5B_1)\psi_2-(A_2+\gamma_5B_2)\psi_1\right\}\Big] \\
&-\frac{g^2}{2}\left[v_\mu^2\left(A_1^2+A_2^2+B_1^2+B_2^2\right)+\left(A_1B_2-A_2B_1\right)^2\right]. \qquad (1)
\end{aligned}
$$

Here $v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu$. For consistency with the rest of this paper, we have written the Lagrangian in terms of the real and imaginary parts of the fields, rather than in complex notation. Supergauge invariance implies that the masses of the scalar, the pseudoscalar and the spinor are equal and that the electromagnetic coupling constant, which we have called g, is equal to the coupling constant of the other trilinear couplings and simply related to those of the quadrilinear couplings. Although the model given here cannot be considered realistic in its present form, it is not without interest, being the first example of a theory which is invariant under combined gauge and supergauge transformations.

The paper is organized as follows. In Section 2 we give a number of formulae for combining vector and scalar multiplets and in Section 3 we use them to construct a supergauge and gauge invariant Lagrangian. This Lagrangian is an infinite power series in the coupling constant g and appears, at first sight, highly non-renormalizable. The Lagrangian is actually invariant under a more general kind of gauge transformations which are generated by taking commutators of ordinary gauge transformations and supergauge transformations. This generalized gauge invariance permits, by choosing a special gauge, to bring the Lagrangian to a simpler form which is renormalizable by power counting but is no longer manifestly supergauge invariant. This is shown in Section 4, where the remaining invariance properties of this Lagrangian are discussed. Finally, in Section 5, the renormalization of the field theory is examined in the one-loop approximation. A number of compensations among divergent diagrams occur, as expected [3] in a supergauge invariant theory. As a result, the model is shown to be renormalizable, in the one-loop approximation, in a manner consistent with gauge and supergauge invariance.

2. COMBINATION LAWS FOR MULTIPLETS

In this section we give some formulae for combining scalar and vector multiplets. These formulae will be used in the next section to construct the invariant Lagrangian. The transformation properties of a scalar and a vector multiplet can be found in Ref. 1). They are respectively Eq. (8) and (10) of that paper, with the omission of all terms with derivatives of α, since α is taken to be a constant *).

Let A_1, B_1, ψ_1, F_1, G_1 and A_2, B_2, ψ_2, F_2, G_2 be the components of two scalar multiplets, which we denote by S_1 and S_2, respectively. They can be combined symmetrically to another scalar multiplet, which we shall denote by S_1S_2, having components A', B' ψ', F', G' given by

$$
\begin{aligned}
A' &= A_1 A_2 - B_1 B_2 \\
B' &= A_1 B_2 + B_1 A_2 \\
\psi' &= (A_1 - \gamma_5 B_1)\psi_2 + (A_2 - \gamma_5 B_2)\psi_1 \\
F' &= F_1 A_2 + F_2 A_1 + G_1 B_2 + G_2 B_1 - i\bar{\psi}_1 \psi_2 \\
G' &= G_1 A_2 + G_2 A_1 - F_1 B_2 - F_2 B_1 + i\bar{\psi}_1 \gamma_5 \psi_2 \ .
\end{aligned}
\tag{2}
$$

Clearly $S_1S_2 = S_2S_1$. The two scalar multiplets can also be combined symmetrically to a vector multiplet, which we shall denote by $S_1 \times S_2 = S_2 \times S_1$, with components

$$
\begin{aligned}
C' &= A_1 A_2 + B_1 B_2 \\
\chi' &= (B_1 - \gamma_5 A_1)\psi_2 + (B_2 - \gamma_5 A_2)\psi_1 \\
M' &= F_1 B_2 + F_2 B_1 + G_1 A_2 + G_2 A_1 \\
N' &= G_1 B_2 + G_2 B_1 - F_1 A_2 - F_2 A_1 \\
v'_\mu &= B_1 \partial_\mu A_2 + B_2 \partial_\mu A_1 - A_1 \partial_\mu B_2 - A_2 \partial_\mu B_1 - i\bar{\psi}_1 \gamma_5 \gamma_\mu \psi_2 \\
\lambda' &= (G_1 + \gamma_5 F_1)\psi_2 + (G_2 + \gamma_5 F_2)\psi_1 - \partial_\mu (B_2 + \gamma_5 A_2)\gamma^\mu \psi_1 \\
&\qquad - \partial_\mu (B_1 + \gamma_5 A_1)\gamma^\mu \psi_2 \\
D' &= 2F_1F_2 + 2G_1G_2 - 2\,\partial A_1 \cdot \partial A_2 - 2\partial B_1 \cdot \partial B_2 - i\bar{\psi}_1 \gamma \cdot \partial \psi_2 - i\bar{\psi}_2 \gamma \cdot \partial \psi_1 \ .
\end{aligned}
\tag{3}
$$

*) In Ref. 1) a larger supergauge (extended) group was considered which contains the conformal group as a subgroup. In the papers of Ref. 3) and in the present paper the invariance group consists of restricted supergauge transformations having constant parameters, whose commutator is a four-dimensional translation, and of Lorentz transformations. It is interesting to observe that the generators of the larger supergauge group appear to be in some sense the quantum analogues of the classical entities which were called twistors by R. Penrose 4).

There exists another way of combining two scalar multiplets to a vector multiplet, this one antisymmetric. We denote it by $S_1 \wedge S_2 = -S_2 \wedge S_1$. Its components are

$$
\begin{aligned}
C' &= A_1 B_2 - A_2 B_1 \\
\chi' &= (A_1 + \gamma_5 B_1)\psi_2 - (A_2 + \gamma_5 B_2)\psi_1 \\
M' &= A_1 F_2 - A_2 F_1 - B_1 G_2 + B_2 G_1 \\
N' &= A_1 G_2 - A_2 G_1 + B_1 F_2 - B_2 F_1 \\
v_\mu' &= A_1 \partial_\mu A_2 - A_2 \partial_\mu A_1 + B_1 \partial_\mu B_2 - B_2 \partial_\mu B_1 - i\bar{\psi}_1 \gamma_\mu \psi_2 \\
\lambda' &= (F_2 - \gamma_5 G_2)\psi_1 - (F_1 - \gamma_5 G_1)\psi_2 + \partial_\mu (A_2 - \gamma_5 B_2)\gamma^\mu \psi_1 - \partial_\mu (A_1 - \gamma_5 B_1)\gamma^\mu \psi_2 \\
D' &= 2F_2 G_1 - 2F_1 G_2 + 2\partial A_2 \cdot \partial B_1 - 2\partial A_1 \cdot \partial B_2 + i\bar{\psi}_1 \gamma_5 \gamma\cdot\partial\psi_2 - i\bar{\psi}_2 \gamma_5 \gamma\cdot\partial\psi_1 .
\end{aligned}
\tag{4}
$$

We shall also need the symmetric composition law of two vector multiplets V_1 and V_2, of components C_1, χ_1, $v_{\mu 1}$, λ_1, D_1 and C_2, χ_2, etc., to another vector multiplet $V_1 \cdot V_2 = V_2 \cdot V_1$. It is

$$
\begin{aligned}
C' &= C_1 C_2 \\
\chi' &= C_1 \chi_2 + C_2 \chi_1 \\
v_\mu' &= C_1 v_{\mu 2} + C_2 v_{\mu 1} - \tfrac{i}{2}\bar{\chi}_1 \gamma_5 \gamma_\mu \chi_2 \\
M' &= C_1 M_2 + C_2 M_1 - \tfrac{i}{2}\bar{\chi}_1 \gamma_5 \chi_2 \\
N' &= C_1 N_2 + C_2 N_1 - \tfrac{i}{2}\bar{\chi}_1 \chi_2 \\
\lambda' &= C_1 \lambda_2 + C_2 \lambda_1 - \tfrac{1}{2}\gamma\cdot\partial C_1 \chi_2 - \tfrac{1}{2}\gamma\cdot\partial C_2 \chi_1 \\
&\quad + \tfrac{1}{2} M_1 \gamma_5 \chi_2 + \tfrac{1}{2} M_2 \gamma_5 \chi_1 + \tfrac{1}{2} N_1 \chi_2 + \tfrac{1}{2} N_2 \chi_1 \\
&\quad - \tfrac{1}{2} v_{\mu 1} \gamma_5 \gamma^\mu \chi_2 - \tfrac{1}{2} v_{\mu 2} \gamma_5 \gamma^\mu \chi_1 \\
D' &= C_1 D_2 + C_2 D_1 - \partial C_1 \cdot \partial C_2 - v_1 \cdot v_2 + M_1 M_2 + N_1 N_2 \\
&\quad - i\bar{\chi}_1 \lambda_2 - i\bar{\chi}_2 \lambda_1 + \tfrac{i}{2}\partial_\mu \bar{\chi}_1 \gamma^\mu \chi_2 + \tfrac{i}{2}\partial_\mu \bar{\chi}_2 \gamma^\mu \chi_1 .
\end{aligned}
\tag{5}
$$

This combination law is associative $(V_1 \cdot V_2)\cdot V_3 = V_1 \cdot (V_2 \cdot V_3)$.

Finally we observe that, if S is a scalar multiplet of components A, B, ψ, F, G, we can construct from it a vector multiplet of components

$$\begin{aligned} C' &= B \\ \chi' &= \psi \\ M' &= F \\ N' &= G \\ v'_\mu &= \partial_\mu A \\ \lambda' &= 0 \\ D' &= 0 . \end{aligned} \tag{6}$$

We denote this vector multiplet by ∂S.

With the notations introduced above, it is easy to verify the validity of the two relations

$$(SS_1) \times S_2 - (SS_2) \times S_1 = 2 (S_1 \wedge S_2) \cdot \partial S \tag{7}$$

and

$$(SS_1) \wedge S_1 = - (S_1 \times S_1) \cdot \partial S , \tag{8}$$

where S, S_1 and S_2 are any three scalar multiplets. We shall use these relations in the next section.

. CONSTRUCTION OF THE LAGRANGIAN

In this section we construct a Lagrangian describing the interaction of a vector and two scalar multiplets. We shall require that it be invariant under supergauge as well as ordinary gauge transformations. The two scalar multiplets can be thought of as the real and the imaginary part of a complex scalar multiplet, however, we prefer to keep the real description in this paper.

An ordinary (infinitesimal) gauge transformation is given by

$$\begin{aligned} \delta v_\mu &= \partial_\mu \Lambda \\ \delta \Phi_1 &= g \Lambda \Phi_2 \\ \delta \Phi_2 &= -g \Lambda \Phi_1 \quad , \end{aligned} \tag{9}$$

where Φ_1 and Φ_2 denote any of the fields of the two scalar multiplets S_1 and S_2 and g is the coupling constant. It is easy to see that the form (9) is not preserved by supergauge transformations. Taking successive commutators with supergauge transformations, (9) generates a kind of generalized gauge transformation, in which the role of the scalar parameter function Λ is taken by an entire scalar multiplet S (A, B, ψ, F, G). In the notation of the previous section, the generalized gauge transformation can be written as

$$\begin{aligned} \delta V &= \partial S \\ \delta S_1 &= g\, S S_2 \\ \delta S_2 &= -g\, S S_1 \end{aligned} \tag{10}$$

or, explicitly, for the vector multiplet

$$\begin{aligned} \delta C &= B \\ \delta \chi &= \psi \\ \delta M &= F \\ \delta N &= G \\ \delta v_\mu &= \partial_\mu A \\ \delta \lambda &= \delta D = 0 \end{aligned} \tag{11}$$

and for the scalar multiplets

$$\begin{aligned} \delta A_1 &= g(A A_2 - B B_2) \\ \delta B_1 &= g(A B_2 + B A_2) \\ \delta \psi_1 &= g\left[(A - \gamma_5 B)\psi_2 + (A_2 - \gamma_5 B_2)\psi\right] \\ \delta F_1 &= g\left(A F_2 + F A_2 + B G_2 + G B_2 - i\bar{\psi}\psi_2\right) \\ \delta G_1 &= g\left(A G_2 + G A_2 - B F_2 - F B_2 + i\bar{\psi}\gamma_5\psi_2\right) \\ \delta A_2 &= -g\left(A A_1 - B B_1\right) \\ \delta B_2 &= -g\left(A B_1 + B A_1\right) \\ \delta \psi_2 &= -g\left[(A - \gamma_5 B)\psi_1 + (A_1 - \gamma_5 B_1)\psi\right] \\ \delta F_2 &= -g\left(A F_1 + F A_1 + B G_1 + G B_1 - i\bar{\psi}\psi_1\right) \\ \delta G_2 &= -g\left(A G_1 + G A_1 - B F_1 - F B_1 + i\bar{\psi}\gamma_5\psi_1\right) . \end{aligned} \tag{12}$$

If the Lagrangian is invariant under supergauge transformations and gauge transformations, it is automatically invariant under the generalized gauge transformations given by (10) or (11) and (12). We now proceed to construct a Lagrangian having these properties. It will be a function of the coupling constant g which, for $g=0$, must reduce to the sum of the free Lagrangians for the three multiplets V, S_1 and S_2. Our method will consist in constructing, with V, S_1 and S_2, a vector multiplet invariant under generalized gauge transformations. Its D component will then give a Lagrangian which is also invariant under supergauge transformations.

First, with the scalar multiplets S_1 and S_2 we construct the two vector multiplets

$$V_I = \frac{1}{2}\left(S_1 \times S_1 + S_2 \times S_2 \right) \tag{13}$$

and

$$V_{II} = S_1 \wedge S_2 \quad . \tag{14}$$

Under the generalized gauge transformation (10) they transform into each other

$$\begin{aligned} \delta V_I &= g\,(S S_2) \times S_1 - g\,(S S_1) \times S_2 \\ &= -2g\,(S_1 \wedge S_2)\cdot \partial S = -2g\, V_{II} \cdot \partial S \end{aligned} \tag{15}$$

$$\begin{aligned} \delta V_{II} &= g\,(S S_2) \wedge S_2 - g\, S_1 \wedge (S S_1) \\ &= -g\,(S_2 \times S_2 + S_1 \times S_1)\cdot \partial S = -2g\, V_I \cdot \partial S \ . \end{aligned} \tag{16}$$

Here we have made use of the identities (7) and (8). The combinations

$$V_a = V_I + V_{II} \tag{17}$$

and

$$V_b = V_I - V_{II} \tag{18}$$

transform simply as

$$\delta V_a = -2g\, V_a \cdot \partial S \quad , \quad \delta V_b = 2g\, V_b \cdot \partial S \ . \tag{19}$$

Now, using the multiplication law (5) for vector multiplets, one can define the exponential of a vector multiplet. One sees from (10) that

$$\delta e^{2gV} = 2g\, e^{2gV} \cdot \partial S\,, \quad \delta e^{-2gV} = -2g\, e^{-2gV} \cdot \partial S \quad . \tag{20}$$

Therefore, we find two invariant expressions, since

$$\delta\left(V_a \cdot e^{2gV}\right) = -2g\left(V_a \cdot \partial S\right) \cdot e^{2gV} + 2g\, V_a \cdot \left(e^{2gV} \cdot \partial S\right) = 0\,, \tag{21}$$

and similarly

$$\delta\left(V_b \cdot e^{-2gV}\right) = 0\,. \tag{22}$$

We choose the particular combination

$$\frac{1}{4}\left(V_a \cdot e^{2gV} + V_b \cdot e^{-2gV}\right) \tag{23}$$

because, for $g = 0$, it gives

$$\frac{1}{4}\left(V_a + V_b\right) = \frac{1}{2} V_I \quad , \tag{24}$$

whose D component is just the sum of the free Lagrangians for the scalar multiplets S_1 and S_2. Our invariant Lagrangian consists of the D component of (23) to which one must still add a mass term and the free Lagrangian for the vector multiplet V, which are by themselves gauge and supergauge invariant. It is easy to see that, with the restrictions imposed, our Lagrangian is unique.

We do not write out the obtained invariant Lagrangian in detail. It is an infinite power series in the coupling constant g and contains all kinds of apparently non-renormalizable couplings. However, it is possible, by choosing a special gauge, to transform it into a much simpler and tractable form. This will be done in the next section.

4. SPECIAL GAUGE

Since the Lagrangian obtained in the previous section is invariant under generalized gauge transformations, we can choose a special gauge. Now, it is obvious that, by means of a (finite) transformation corresponding to (11), one can bring to zero the fields C, X, M and N of the vector multiplet V, so that only v_μ, λ and D survive. Using (5) one sees then that, for the vector multiplet V^2 only the D component survives, and equals $-v_\mu^2$, while the higher powers V^n $(n > 2)$ vanish identically in the special gauge. The expression (23) simplifies now to

$$\frac{1}{4}(V_a + V_b) + \frac{1}{2}g(V_a - V_b)\cdot V + \frac{1}{2}g^2(V_a + V_b)\cdot V^2$$
$$= \frac{1}{2}V_I + g\,V_{II}\cdot V + g^2\,V_I\cdot V^2 . \tag{25}$$

The Lagrangian in the special gauge consists of the D component of (25), plus a mass term for the scalar multiplets, plus the free Lagrangian of the vector multiplet. Written out in full, it is

$$\begin{aligned}
L = &-\frac{1}{2}\Big[(\partial A_1)^2 + (\partial A_2)^2 + (\partial B_1)^2 + (\partial B_2)^2 - F_1^2 - F_2^2 - G_1^2 - G_2^2 \\
&+ i\bar{\psi}_1\gamma\cdot\partial\psi_1 + i\bar{\psi}_2\gamma\cdot\partial\psi_2\Big] \\
&+ m\left(F_1A_1 + F_2A_2 + G_1B_1 + G_2B_2 - \frac{i}{2}\bar{\psi}_1\psi_1 - \frac{i}{2}\bar{\psi}_2\psi_2\right) \\
&-\frac{1}{4}v_{\mu\nu}^2 - \frac{i}{2}\bar{\lambda}\gamma\cdot\partial\lambda + \frac{1}{2}D^2 \\
&+ g\Big[D(A_1B_2 - A_2B_1) - v_\mu\left(A_1\partial_\mu A_2 - A_2\partial_\mu A_1 + B_1\partial_\mu B_2 - B_2\partial_\mu B_1\right. \\
&\left. - i\bar{\psi}_1\gamma_\mu\psi_2\right) - i\bar{\lambda}\left\{(A_1 + \gamma_5 B_1)\psi_2 - (A_2 + \gamma_5 B_2)\psi_1\right\}\Big] \\
&- \frac{g^2}{2}v_\mu^2\left(A_1^2 + A_2^2 + B_1^2 + B_2^2\right) .
\end{aligned} \tag{26}$$

Since we have not fixed completely the gauge, it is still invariant under the ordinary gauge transformations (9). However, the supergauge invariance is no longer manifest, a consequence of the fact that the special gauge chosen has no invariant meaning. Nevertheless the Lagrangian (26) is invariant under transformations which combine a supergauge transformation (which violates the gauge condition) with a generalized gauge transformation (which re-establishes the gauge condition). In infinitesimal form these transformations can be written as

$$\delta = \delta_S + \delta_G \quad , \tag{27}$$

where δ_S is the usual supergauge transformation on the fields given by $(i = 1,2)$

$$\begin{aligned}
\delta_S A_i &= i\,\bar\alpha \psi_i \\
\delta_S B_i &= i\,\bar\alpha \gamma_5 \psi_i \\
\delta_S \psi_i &= \partial_\mu (A_i - \gamma_5 B_i)\gamma^\mu \alpha + (F_i + \gamma_5 G_i)\alpha \\
\delta_S F_i &= i\,\bar\alpha\, \gamma\cdot\partial\, \psi_i \\
\delta_S G_i &= i\,\bar\alpha \gamma_5\, \gamma\cdot\partial\, \psi_i
\end{aligned} \tag{28}$$

and

$$\begin{aligned}
\delta_S v_\mu &= i\,\bar\alpha \gamma_\mu \lambda \\
\delta_S \lambda &= -\tfrac{1}{2} v_{\mu\nu} \gamma^\mu \gamma^\nu \alpha + D\gamma_5 \alpha \\
\delta_S D &= i\,\bar\alpha \gamma_5\, \gamma\cdot\partial\, \lambda \quad ,
\end{aligned} \tag{29}$$

while δ_G is the gauge transformation which re-establishes the special gauge, given by $(i = 1,2)$

$$
\begin{aligned}
&\delta_G A_i = \delta_G B_i = 0 \\
&\delta_G \psi_1 = -g\,(A_2 - \gamma_5 B_2)\,\gamma\cdot v\,\alpha \\
&\delta_G F_1 = -ig\left(\bar\alpha\lambda A_2 + \bar\alpha\gamma_5\lambda B_2 + v^\mu\bar\alpha\gamma_\mu\psi_2\right) \\
&\delta_G G_1 = -ig\left(\bar\alpha\gamma_5\lambda A_2 - \bar\alpha\lambda B_2 + v^\mu\bar\alpha\gamma_5\gamma_\mu\psi_2\right) \\
&\delta_G \psi_2 = g\,(A_1 - \gamma_5 B_1)\,\gamma\cdot v\,\alpha \\
&\delta_G F_2 = ig\left(\bar\alpha\lambda A_1 + \bar\alpha\gamma_5\lambda B_1 + v^\mu\bar\alpha\gamma_\mu\psi_1\right) \\
&\delta_G G_2 = ig\left(\bar\alpha\gamma_5\lambda A_1 - \bar\alpha\lambda B_1 + v^\mu\bar\alpha\gamma_5\gamma_\mu\psi_1\right)
\end{aligned}
\tag{30}
$$

and

$$\delta_G v_\mu = \delta_G D = \delta_G \lambda = 0 \quad . \tag{31}$$

These formulae are easily deduced from the supergauge transformation formulae and from the formulae (11)-(12) giving the generalized gauge transformation. Observe, for instance, that in the first of (29) the term $i\bar\alpha\partial_\mu X$ is missing because one starts from the special gauge where $X = 0$.

The commutator of two transformations of the type (27) is the combination of a four-dimensional translation and of an ordinary gauge transformation, but with a field dependent parameter. For instance

$$
\begin{aligned}
&[\delta_2,\delta_1]\,v_\mu = 2i\bar\alpha_1\gamma^\rho\alpha_2\,\partial_\rho v_\mu - 2i\,\partial_\mu\left(\bar\alpha_1\gamma^\rho\alpha_2\, v_\rho\right) \\
&[\delta_2,\delta_1]\,A_1 = 2i\bar\alpha_1\gamma^\rho\alpha_2\,\partial_\rho A_1 - 2ig\,\bar\alpha_1\gamma^\rho\alpha_2\, v_\rho\, A_2
\end{aligned}
\tag{32}
$$

and so on. One can write transformations under which the Lagrangian (26) is invariant and which satisfy exactly the usual supergauge algebra (i.e., have a commutator which is precisely a translation), by fixing the gauge completely, including the ordinary gauge of v_μ. Then to (27) one must add a third term which re-establishes the gauge of v_μ. For instance, if the gauge condition is specified in terms of a space-time operator a_μ such that [5)]

$$\partial^\mu a_\mu = -1 \tag{33}$$

by

$$a^\mu v_\mu = 0 \quad , \tag{34}$$

one must replace (27) by

$$\delta = \delta_S + \delta_G + \delta_G' \quad , \tag{35}$$

where

$$\begin{aligned} \delta_G' v_\mu &= i\,\partial_\mu \left(a^\rho \bar{\alpha} \gamma_\rho \lambda \right) \\ \delta_G' \lambda &= \delta_G' D = 0 \end{aligned} \tag{36}$$

and

$$\begin{aligned} \delta_G' \Phi_1 &= ig\, a^\rho \bar{\alpha} \gamma_\rho \lambda\, \Phi_2 \\ \delta_G' \Phi_2 &= -ig\, a^\rho \bar{\alpha} \gamma_\rho \lambda\, \Phi_1 \end{aligned} \tag{37}$$

for any field Φ of the scalar multiplets S_1 and S_2. The Lagrangian (26) is invariant under (35) (up to a total derivative) and (35) satisfy the exact supergauge algebra. Clearly at this stage we had to spoil manifest Lorentz covariance, as in ordinary electrodynamics when it is formulated in the Coulomb gauge.

5. A FIRST LOOK AT RENORMALIZATION

In the Lagrangian (26) one can eliminate the fields F_1, F_2, G_1, G_2 and D by replacing them with the values given by their own variational equations

$$\begin{aligned} F_1 &= -m A_1 \qquad & F_2 &= -m A_2 \\ G_1 &= -m B_1 \qquad & G_2 &= -m B_2 \end{aligned}$$
$$D = -g \left(A_1 B_2 - A_2 B_1 \right) . \tag{38}$$

One obtains then the Lagrangian (1) given in the Introduction. It is a Lagrangian of the usual type, except for the very particular connection among the masses and the coupling constants. The model is clearly renormalizable by power counting. The question arises, as usual, whether the renormalization is consistent with the particular connections among couplings and masses. We have checked, in the one-loop approximation, that it is. From the experience gained in the scalar model [3] one might expect that this result is valid to all orders in perturbation theory. Observe that, to all orders, the mass of the photon v_μ is zero by gauge invariance and that of the "neutrino" λ because it is related to that of v_μ by supergauge invariance or, more directly, because the Lagrangian is invariant under the discrete transformation $(i = 1,2)$

$$\lambda \to \gamma_5 \lambda, \quad A_i \to B_i, \quad B_i \to -A_i,$$
$$F_i \to G_i, \quad G_i \to -F_i. \tag{39}$$

We have calculated with the Lagrangian (1) and using for the photon propagator the form

$$\left(\eta^{\mu\nu} - f\,\frac{k_\mu k_\nu}{k^2}\right)\frac{1}{k^2}, \tag{40}$$

where f is a gauge parameter. The self-energy for the field A_1 (same for A_2) is given by the sum of a number of quadratically divergent diagrams. However, the quadratic divergence cancels in the sum. For the inverse propagator we find

$$p^2\left(1 + ig^2 f\, I\right) + m^2\left(1 + ig^2(f-4)\, I\right) \tag{41}$$

where I is the logarithmically divergent integral

$$I = \int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^4}, \tag{42}$$

Consequently, the renormalization constant Z_A and the renormalized mass m_r for the fields A_1 and A_2 are given by

$$Z_A = 1 - ig^2 f\, I, \quad m_r^2 = m^2\left(1 - 4ig^2 I\right), \tag{43}$$

where m is the unrenormalized mass.

The self-energy for the field ψ_1 (or ψ_2) is the sum of a number of logarithmically divergent diagrams. For the inverse propagator, we find

$$\gamma \cdot p \left(1 + ig^2 (f-2) I\right) - im\left(1 + ig^2 (f-4) I\right). \tag{44}$$

Consequently

$$Z_\psi = 1 - ig^2 (f-2) I \,, \quad m_r = m\left(1 - 2ig^2 I\right). \tag{45}$$

To this order the masses of the A and ψ fields remain equal.

The renormalization constants for the v_μ and λ fields are equal

$$Z_v = Z_\lambda = 1 + 2ig^2 I. \tag{46}$$

The vacuum polarization diagrams coming from the spinor loops, from the scalar loops and from the scalar sea-gulls are all formally quadratically divergent. It is amusing that all these quadratic divergences exactly cancel and the sum automatically vanishes at zero momentum (the sea-gull terms have opposite sign to the other diagrams). Our supergauge invariant model does not need the usual special treatment of the vacuum polarization diagrams (Pauli-Villars regularization or other) to ensure the gauge invariance of the result.

If one looks at the corrections to the various trilinear couplings, one finds that they are all renormalized in the same way and the renormalized coupling constant is given for all by

$$g_r = g\left(1 + ig^2 I\right). \tag{47}$$

This formula is also consistent with the renormalization of the quartic coupling

$$-\frac{g^2}{2}\left(A_1 B_2 - A_2 B_1\right)^2 \tag{48}$$

arising from the elimination of the field D. No divergent terms of the form A_1^4 are generated, the sum of all logarithmically divergent diagrams contributing to such terms adding up to a finite contribution.

Finally, let us mention that the scattering of light by light exhibits a cancellation similar to that described above for the vacuum polarization. It is well known that the contributions of the individual fields are separately finite (those of the scalar fields if one includes diagrams with sea-gulls) but do not vanish at zero momentum. The sum over all fields of the scalar multiplet, however, does vanish at zero momentum and satisfies the requirements of gauge invariance. Again, gauge invariance is "aided" by supergauge invariance.

The study of the higher orders appears more complicated, in this model, than in the purely scalar model previously studied. The reason is that the manifestly supergauge invariant Lagrangian of Section 3 is an infinite power series, while the simpler manifestly renormalizable, Lagrangian of Section 4 is only invariant under the more complicated transformations given in that same section, the corresponding Ward identities being also more complicated. Nevertheless, the results of this section provide a very strong indication that renormalization can be performed consistently with both gauge and supergauge invariance.

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