

# Project 1: Calorimetry

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**This report gives a description of using calorimeter data and error analysis to estimate specific heat of a sample and predict the composition of an unknown sample. The primary methods used are least squares approximations, and error propagation. The calculated values for specific heat support a sample composition of copper.**

## I. Nomenclature

$C_c$	=	specific heat of calorimeter
$C_s$	=	specific heat of sample
$M_c$	=	mass of calorimeter
$M_s$	=	mass of sample
$T_0$	=	initial temperature of calorimeter
$T_1$	=	initial temperature of sample
$T_2$	=	final temperature of calorimeter
$t_1$	=	time of sample transfer
$\sigma$	=	error of value denoted in subscript

## II. Introduction

Calorimeters are used to measure thermal properties of a sample such as specific heat, and combustion energy. In this report the process of calculating specific heat of a sample using calorimetry, the first law of thermodynamics, error propagation, and least squares extrapolation.

## III. Experimental method

The initial data includes the mass of the calorimeter, the specific heat of the calorimeter, and temperature vs time for 2 sensors in the calorimeter, the ambient air, and the water for sample preparation. The data is from a lab procedure where a sample of known mass is submerged in boiling water and let rest before being transferred to the calorimeter and let reach equilibrium with the calorimeter. The end objective is to estimate the composition of the sample using the calculated specific heat.

The first law of thermodynamics is used to calculate the specific heat of the sample by rewriting  $Q_{in} = Q_{out}$  to solve for  $C_s$  in terms of  $C_c$ ,  $M_c$ ,  $\Delta T_c$ ,  $M_s$ , and  $\Delta T_s$ .  $\Delta T$  is calculated using values from the least squares best fit of the calorimeter data before and after the sample is placed, and the temperature of the sample before being placed in the calorimeter. Both lines of best fit are calculated using formulas below[1] with  $x$  representing time, and  $y$  representing temperature averaged over both sensors.

$$A = \frac{(\sum x^2)(\sum y) - (\sum x)(\sum xy)}{(N(\sum x^2) - (\sum x)^2)} \quad B = \frac{N(\sum xy) - (\sum x)(\sum y)}{(N(\sum x^2) - (\sum x)^2)}$$

Two lines of best fit are created using the above equations are used in  $Y = A + Bx$  form to and calculated for the time the sample was placed in the calorimeter to get  $T_0$  and  $T_2$ . The value for  $T_1$  is calculated by taking the average of the boiling sample prep water temperature before the sample is transferred to the calorimeter. The specific heat of the sample is calculated from the first law of thermodynamics as the heat balance equation can be rewritten to solve for specific heat as the only process happening in the calorimeter is heat transfer.

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$$C_s = \frac{M_c C_c (T_2 - T_0)}{M_s (T_1 - T_2)}$$

Error estimates are propagated through the process starting with the error of lines of best fit. The error of the linear approximations are given below.

$$\sigma_y = \sqrt{\frac{1}{N-2} \sum (y - A - Bx)^2}$$

$$\sigma_B = \sqrt{N \sigma_y^2 / (N \sum x^2 - (\sum x)^2)} \quad \sigma_A = \sqrt{\sigma_y^2 \sum x^2 / (N \sum x^2 - (\sum x)^2)}$$

The errors from the line of best fit are propagated using quadrature to find the error in the temperature values for  $T_0$  and  $T_2$ . The error of both temperature measurements is propagated to find the error in  $C_s$  in the same method.

$$\sigma_T = \sqrt{(\sigma_A)^2 + (t_1 \sigma_B)^2}$$

$$\sigma_C = \sqrt{\left(\frac{M_c * C_c}{M_s (T_1 - T_2)} \sigma_{T0}\right)^2 + \left(\frac{M_c C_c}{M_s} \left(\frac{T_1 - T_0}{(T_1 - T_2)^2}\right) \sigma_{T2}\right)^2}$$

The resulting specific heat and error is then compared to the 4 candidate materials to find the closest match.

#### IV. Results

The calculated specific heat for the sample is 0.280 (J/gC) with an uncertainty of  $8.96 \times 10^{-4}$ . The  $C_s$  value of 0.280(J/gC) is compared to the 4 candidate material properties to find it is closest to copper with a specific heat of 0.261(J/gC). The specific heat of copper is outside of the error of the calculated sample value but is the closest of the 4 options.

#### V. Discussion

The calculated value supports that the sample is most likely to be copper. There are multiple key assumptions that impact the confidence in the result. The primary assumptions made are that all given values excluding temperature are of negligible error. In addition the assumption that ignoring error in the initial temperature of the sample as it is the result of an average of a large sample size could have an impact that would increase the error bounds, but the answer would still not be likely to change.

#### VI. Conclusion

The methods used make multiple assumptions that decrease confidence, but still produces a valid answer. A more detailed method that more accurately represents the heat loss during would increase confidence. Additionally a wider range of potential materials could find a result closer to the calculated value. In conclusion the methods used are effective for estimating the material properties and finding the sample composition from the given list.

#### Appendix

Error Derivations:

From first Law of Thermodynamics

$Q_{in} = Q_{out}$  Can be expanded to

$$C_c M_c (T_2 - T_0) = C_s M_s (T_1 - T_2)$$

$$C_s = \frac{M_c C_c (T_2 - T_0)}{M_s (T_1 - T_2)}$$

Error by quadrature.

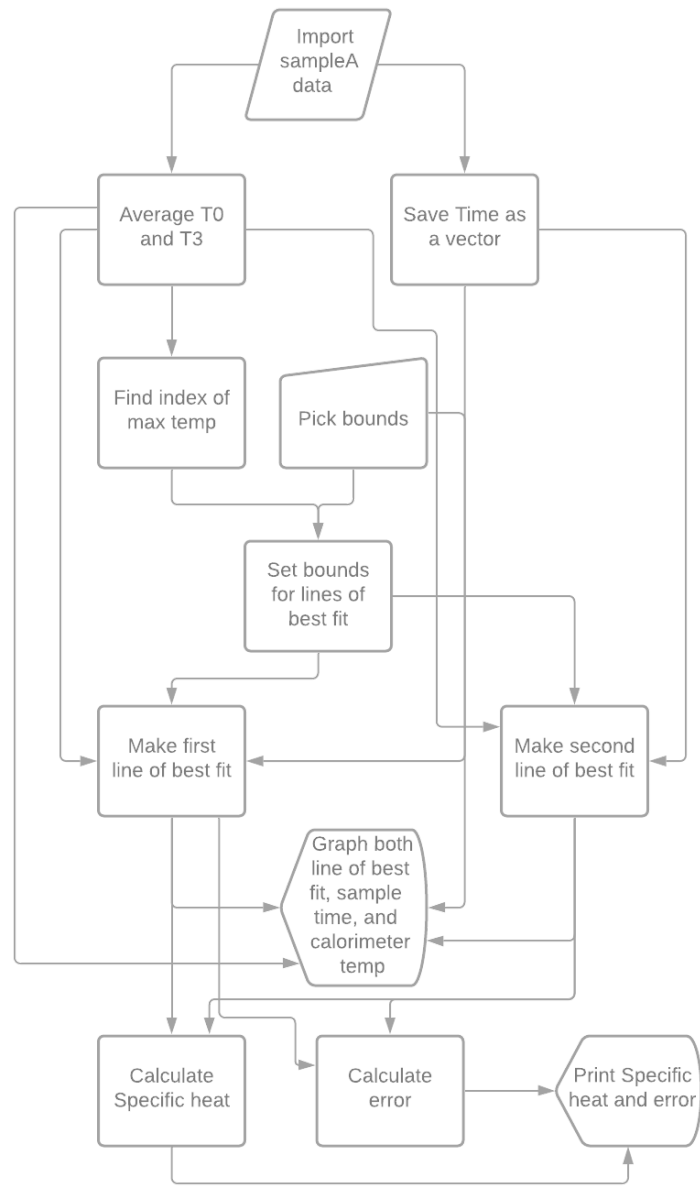
$$\sigma_C = \sqrt{\left(\left(\frac{dC}{dT_0}\right)\sigma_{T0}\right)^2 + \left(\left(\frac{dC}{dT_2}\right)\sigma_{T2}\right)^2}$$

$$\frac{dC}{dT_0} = \frac{M_c C_c}{M_s (T_1 - T_2)}$$

$$\frac{dC}{dT_2} = \frac{M_c C_c}{M_s} * \left(\frac{T_1 - T_0}{(T_1 - T_2)^2}\right)$$

$$\sigma_C = \sqrt{\left(\frac{M_c * C_c}{M_s (T_1 - T_2)}\sigma_{T0}\right)^2 + \left(\frac{M_c C_c}{M_s} \left(\frac{T_1 - T_0}{(T_1 - T_2)^2}\right)\sigma_{T2}\right)^2}$$

### Code Flowchart:



### References

[1] Taylor J. R., "An introduction to error analysis" The Study of Uncertainties in Physical Measurements, Mill Valley, CA, 1982, pp. 153-162.