Homework Week 3: Gaussian Distribution

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August 25, 2023

Exercise 1

A probability distribution function is said to be "normalized" if the sum of all its possible results is equal to one. Physically, you can think of this as saying "we've listed every possible result, so the probability of one of them happening has to be 100%!". For the discrete probability, If you take all the probabilities, you will see that they add to 1. That is,

$$\sum_{j=1}^{M} p(x_j) = 1$$

For continuous distribution, instead of setting the sum over discrete probabilities p_j (where j is determined by the value of x), we take the integral of the probability distribution function over its domain. That is,

$$\int_D p(x)dx = 1$$

in which $p(x)$ is the probability distribution function.

(a) Prove that the Univariate Gaussian PDF is normalized.

(b) A random variable X follows Gaussian distribution (notation: $X \sim \mathcal{N}(\mu, \sigma^2)$). Prove that the expected value of X is μ and the standard deviation of X is σ.

Exercise 2

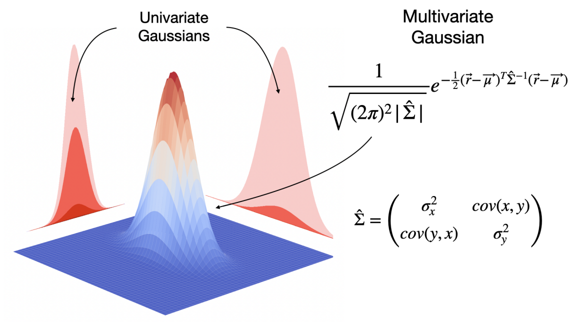

Figure 1: Multivariate Gaussian distribution is the combination of multiple Univariate Gaussian distributions

For a D-dimensional vector x, the multivariate Gaussian PDF takes the form

$$p(x \mid \mu, \Sigma) = \frac{1}{(2\pi)^{\frac{D}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x - \mu)^T \Sigma^{-1} (x - \mu) \right)$$

(a) Prove that the Multivariate Gaussian PDF is normalized.

(b) Find the formula of marginal distribution in Multivariate Gaussian distribution.

(c) Find the formula of conditional distribution in Multivariate Gaussian distribution.