1 Comparison of analog dynamics and Trotter expansion on a gate based processor

We define a few variables and summarize the note from Kostya regarding the gate count. let

$$r \text{ be the number of Trotter steps} \tag{1}$$

$$\epsilon \text{ the final error} \tag{2}$$

$$n \text{ the number of qubits} \tag{3}$$

We wish to answer the question: *"How many single and two qubit gates would be required to perform our analog evolution if decomposed into a Trotter expansion?"*

1.1 XY model

The simplest case is an XY model without local Z fields

$$H = J \sum_{i}^{n} X_i X_{i+1} + Y_i Y_{i+1} \tag{4}$$

We make several assumptions favorable to gates:

- Rather than using a "Textbook" gate set, we take our two qubit gate to be a small angle Givens rotion.
- We omit local Z fields from the Hamiltonian.
- We consider only $|0\rangle$ and $|1\rangle$ on each qubit.

Under these assumptions and assuming **perfect** gates Kostya computes:

$$\text{Total Trotter Error } \epsilon_T \;= \left\| \left(\prod_{k=1}^{\frac{n}{2}-1} e^{i\frac{t}{r}H_{2k,2k+1}} \prod_{k=1}^{\frac{n}{2}-1} e^{i\frac{t}{r}H_{2k-1,2k}} \right)^r - e^{-iHt} \leq \frac{nt^2}{r} \right\| \tag{5}$$

In our experiment $J = 40$MHz so that $1/J = 25$ns. Thus for 100 ns of evolution $t = 4$ and $n = 9$. Therefore in order to achieve a Trotter error of 0.1 for 100 ns evolution we require a gate depth of

$$r = \frac{9(4)^2}{0.1} = 1440 \tag{6}$$

1.2 XY model with decoherence

If we assume error from decoherence is nonzero and equal per qubit per unit time in both the analog and gate case. Kostya shows that:

- The optimal decomposition has decoherence error equal to Trotter error.
- The error in the analog case is lower than the gate decomposition by at least a factor of 14.

1.3 Including the $|2\rangle$

- You need ancilla qubits to track the dynamics of the higher levels
- For a simple nearest neighbor system where the Ancilla are ideally placed we estimate the “shuffeling" overhead to be a factor of 15
- Shuffeling overhead gets worse in full 2D systems

2 Measuring nonlocal interactions with a conditional phase experiment