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# Modelling, Forecasting and Seasonally Adjusting Economic Time Series with the X-11 ARIMA Method

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The majority of seasonal adjustment methods, officially adapted by government statistical agencies, belong to the category of techniques based on linear smoothing filters, usually known as moving averages of length  $2m+1$ , say. These methods have often been criticized because they lack an explicit model concerning the decomposition of the original series and because their estimates, for observations in the most recent years, do not have the same degree of reliability as those of central observations.

The lack of an explicit model refers to the whole length of the series, for moving averages procedures do, of course, make assumptions concerning the time series components, but only for within the span of the set of weights of the moving average; that is, the assumptions are of a local character.

The second limitation is inherent to all linear smoothing procedures, since the  $m$  first and last observations cannot be smoothed with the same set of symmetric weights as are applied to central observations. Because of this, the estimates for current observations must be revised as more data is added to the original series. Frequent revisions, however, are not appreciated by the users of seasonally adjusted values, particularly if they are relatively large in size or if they introduce changes in the direction of the general movement of the adjusted series. In fact, policy-makers faced with the problem of controlling the level of economic activity will scarcely base their decisions on poorly seasonally adjusted data, subject to significant revisions whenever new information comes available.

The introduction of the ARIMA part into the Method X-11 variant to produce what is called X-11 ARIMA, as developed by Dagum (1975, 1976, 1977), takes into account the two constraints; it reduces the size of the revisions, substantially, and offers a model for the series.

The X-11 ARIMA basically consists of: (1) modelling the original series by integrated autoregressive moving averages processes (ARIMA models) of the Box and Jenkins (1970) type; (2) forecasting one or two

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years of raw data from the ARIMA models fitted to the original series and, then, enlarging the series with the forecasted values; (3) seasonally adjusting the enlarged original series with the filters of Methods II X-11 variant (Shiskin *et al.*, 1967).

The ARIMA part incorporated into the X-11 program plays a very important role in the estimation of seasonal factor forecasts and concurrent seasonal factors, when seasonality is moving rapidly in a stochastic manner – a phenomenon often found with key economic indicators. Because the series are enlarged with extra data, the filters applied by the X-11 to seasonally adjust current observations and to generate the seasonal forecasts are closer to the filters used for central observations. Consequently, the degree of reliability for current estimates is greater than those obtained from X-11, and the magnitude of the revisions is considerably reduced. Similar conclusions are reached when the comparisons are made with other seasonal adjustment methods based on moving averages (Kuiper, 1976; Pierce, 1978).

Generally, reductions of about 30 per cent in the bias and of 20 per cent in the absolute values of the total error, in the seasonal factor forecasts for the 12 months (4 quarters), have been found for Canadian and American series (Kuiper, 1976; Farley and Zeller, 1976; Dagum, 1978). For months (quarters) corresponding to peaks and troughs, the percentage reduction is larger than the average for the whole year.

For series whose seasonality is rather stable, a significant improvement is obtained only when the trend-cycle is growing fast or the last year of data contains a turning point. In such cases, because X-11 ARIMA permits the use of the central weights of Henderson's filter, the estimates of the trend-cycle for the last year are more reliable than those from X-11. Consequently better estimates of the seasonal and irregulars are also obtained, which then are simply averaged to produce stable seasonals.

From the viewpoint of seasonal adjustment, another important advantage of X-11 ARIMA is that it offers a statistical model for the whole length of the series. The existence of a model that fits the data well permits the fulfillment of the basic underlying principle of seasonal adjustment, namely, that the series is decomposable. If a series does not lend itself to the identification of an ARIMA model (here considering all AR, MA and ARMA as subclasses), which simply describes the general structure of the series in terms of past values and lagged random disturbances, any decomposition into trend, cycle and seasonal becomes dubious. In fact, lack of fit of an ARIMA model can well indicate that the series is practically a purely random process, or that it is so contaminated by the irregulars that its systematic movement is unidentifiable.

Finally, the X-11 ARIMA generates forecasts of the raw data. Although these forecasts are not generated for the purpose of aiding economic policy

making or decision taking, but to improve seasonal adjustment, the one step ahead forecast is a minimum mean square error forecast; and, as such, can be used as a benchmark for preliminary figures. This is particularly useful to producers of raw data obtained from incomplete returns, as is often the case with flow series.

The following sections are devoted to, respectively, ARIMA models and forecasts; the selection of the ARIMA models for X-11 ARIMA; the advantages of X-11 ARIMA over X-11; and other improvements incorporated into an automated version of X-11 ARIMA.

### **ARIMA Models and Forecasts**

One of the most fundamental considerations in the improvement of the seasonal adjustment, by the X-11 program (or equally, by any other seasonal adjustment method based on moving averages), is to decide what kind of forecasting method should be used to extend the original series. After considerable thought, for the X-11 ARIMA, the selection was based on the following requirements:

1. The forecasting method must be of the "simplest" type in terms of its description of the real world. No explanatory variables must be involved, the series should be described simply by its past values and lagged random disturbances. This requirement is necessary to ease the incorporation of the forecasting method into the X-11 program, since the procedure has to be automated.
2. The method should be effective for short term forecasting, one or two years ahead, at most, being sufficient. The forecasts are needed to extend the series only with a number of years that enables the X-11 to apply filters close to the symmetric ones, without seriously increasing the forecasting error.
3. The identified models must be robust to the incorporation of one or two years of data, and the corresponding forecasts should not change significantly with small variations in parameter values. This condition is necessary to avoid frequent changes of models, and significant revisions that confuse the users of seasonally adjusted data.
4. The method must produce forecasts that follow the intrayear movement well, although they could miss the level. This requirement reflects the fact that the forecasts are not for policy or decision making but to improve current seasonal adjustment.
5. It must generate optimum forecasts, in the sense of minimum mean square error forecasts. This condition allows the forecasts, at least the lead-one forecasts, to be used as benchmarks for preliminary data coming from incomplete returns.

6. The method must be parsimonious in the number of parameters. The main characteristics of the series are then summarized in a small number of parameters.

This set of conditions led us to choose a univariate method of forecasting and, among the several already well-developed methods, ARIMA modelling of the Box and Jenkins (1970) kind was selected, as ARIMA models have been found to be effective for forecasting large numbers of series (Newbold and Granger, 1974; Reid, 1975)

In the Box and Jenkins notation, the general multiplicative ARIMA model, for a series with seasonality of period  $s$ , is expressed as  $(P, d, q)(P, D, Q)_s$ , where  $d$  and  $D$  are, respectively, the degrees of the unit and seasonal differencing applied to the original series, in order to reduce its statistical structure to one independent of time – which provides stability.

In the context of X-11 ARIMA, the required lead time of the forecasts for monthly and quarterly series is seldom greater than 12 or 4, respectively; and model identification generally gives  $Q = 1$ , (which is true, indeed, for the three automated models discussed in the next section).

We shall not here discuss the properties and basic assumptions of ARIMA models, as these have been extensively treated in the literature; for instance by Box and Jenkins (1970), Anderson (1975) and Granger and Newbold (1977).

### **The Selection of ARIMA Models for the X-11 ARIMA**

The ARIMA models to be used in the context of the X-11 ARIMA method must fulfill the two conditions of fitting the data well and generating “reasonable” forecasts for the last three years of observed data. By “reasonable” forecasts it is meant ones with a mean absolute error smaller than 5 per cent, for well behaved series (e.g. employment adult males), and smaller than 10 per cent for highly irregular series (e.g. unemployment teenager males).

These guidelines have been tested with more than 250 economic time series. They are on the conservative side since it has been found, in several instances, that, with average forecasting error, the X-11 ARIMA still produces concurrent and forecasted seasonals with errors smaller than those from X-11.

Because the parameter values are affected by the presence of outliers and therefore, to a certain extent, these affect the forecasts, it is recommended that the identification of the ARIMA models be made to data whose extreme values have been appropriately modified. This recom-

mentation is even more relevant if the outliers fall in the most recent years, in order to avoid the rejection of good models simply because the presence of outliers, in the last three years, inflates the absolute average forecasting error above the acceptance level of the guidelines.

To determine whether or not a model fits the data well, the portmanteau test of fit developed by Box and Pierce (1970), with the variance correction for small samples, is used. The null hypothesis of randomness of the residuals is tested at a 5 per cent level of significance.

Based on the above criteria for fitting and forecasting, three ARIMA models were incorporated into the X-11 program in order to automate X-11 ARIMA. The user can supply his own model or choose the automated option. The latter checks whether one of three models passes the required guidelines. In the affirmative case, the model chosen is the one that gives the smallest average forecasting error. Then, the program automatically enlarges the series with one year of forecasts and seasonally adjusts the data.

In the case that none of the three models is acceptable, a message is printed indicating that the user should identify his own model.

The automated option chooses from the following ARIMA models:

- (a)  $(2, 1, 2) (0, 1, 1)_s$
- (b)  $(2, 0, 1) (0, 1, 2)_s$
- (c)  $\log (2, 1, 1) (0, 1, 2)_s^\dagger$

These three models were selected from a set of 12 ARIMA processes, often encountered in the modelling of economic time series, and tested with 305 series, from Canada and the United States.

The 12 models originally tested were:

- (1)  $(1, 1, 1) (1, 1, 1)_s$       (7)  $\log (2, 1, 1) (0, 1, 2)_s$
- (2)  $(2, 1, 2) (0, 1, 1)_s$       (8)  $(0, 1, 2) (1, 1, 2)_s$
- (3)  $(2, 0, 1) (0, 1, 2)_s$       (9)  $\log (0, 1, 1) (0, 1, 1)_s$
- (4)  $(2, 0, 1) (1, 2, 1)_s$       (10)  $\log (0, 1, 1) (0, 2, 2)_s$
- (5)  $(2, 0, 0) (0, 1, 1)_s$       (11)  $\log (0, 2, 2) (0, 1, 1)_s$
- (6)  $(0, 0, 2) (0, 1, 1)_s$       (12)  $\log (0, 2, 2) (0, 2, 2)_s$

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<sup>†</sup>The series is first transformed by taking logarithms, and then modelled with a  $(2, 1, 1) (0, 1, 2)_s$ .

The 305 series were obtained from the Labor Force sector, the System of National Accounts, the Real Domestic Product by Industry of Origin, the Balance of Payments, Manufacturing, Shipments and Inventories, Consumer Price Index, Wages and Salaries, Employment Indexes by Industrial Classification, Construction, Domestic Trade, Consumer Credit, Transportation and Finance.

The models were ranked according to how well they fitted the series and passed the above guidelines for forecasting. It was found that model 2 fitted and forecasted 74 per cent of the series well. Model 7 provided acceptable results for 22 per cent of the series not passed by model 2 (or 6 per cent of the total). For the remaining series, not passed by either model 2 or model 7, model 3 showed the best performance, passing an additional 1 per cent of the total number of series. Thus models 2, 3 and 7 jointly passed 81 per cent of the series. Only an additional 1 per cent could have been fitted by the other 9 models, while none of the 12 models gave acceptable results for the remaining 18 per cent.

The objective of an automated procedure is to find adequate models for a great variety of series at minimal cost; so one needs to find a small set of models that covers a large class of economic series. Inclusion of model 3 only added an extra 1 per cent to the series passed, but the overall performance of this model was found to be excellent. Whenever model 3 provided an adequate fit for a series, the three years forecast error was lower on the average than the error coming from any of the other models 2 and 7.

Because it is important that the forecasting error be relatively small, this extra model was also incorporated in the automated version. If none of the three models passes the required guidelines, the user can still identify his own model or rerun the series with prior modifications for outliers, if this has not been done on the first run. The three models chosen were later tested with 150 more Canadian and American series and passed the guidelines in 90 per cent of all these cases.

### **The Advantages of X-11 ARIMA over X-11**

The main advantage of X-11 ARIMA over the X-11 variant are:

1. The availability of a statistical model that provides relevant information on the quality of the raw data. The existence of a model that fits the original series well, even though it does not pass the guidelines for forecasting, permits the fulfillment of the most fundamental principle of seasonal adjustment, that is, that the series is decomposable. In other words, if a series does not lend itself to the identification of an ARIMA model (which simply describes the series in terms of past values

and lagged random disturbances), any decomposition into trend-cycle, seasonals and irregulars can be seriously criticized and is of dubious validity. In fact, the lack of fit by an ARIMA model can well indicate deficiencies concerning the way in which the observations are made, such as an improper choice of sampling interval.

If the series has an ARIMA model, the expected value and the variance of the original series can be calculated, and so confidence intervals can be constructed for the observations. This enables the identification of extreme values, particularly at the end of the series.

2. The lead-one forecast from an ARIMA model is a minimum mean square error forecast and can be used as a projected value, or benchmark for preliminary figures.

3. The total error in the seasonal factor forecasts and in the concurrent seasonal factors is significantly reduced for all the months.

4. If concurrent seasonal factors are applied to obtain current seasonally adjusted data, there is no need to revise the series more than twice. For many series, just one revision will give seasonal factors that are "final" in a statistical sense.

There are several reasons for the significant reduction of the error in the seasonal factor forecasts and concurrent seasonal factors. The X-11 ARIMA produces seasonal factor forecasts from the combination of two filters: (1) the filters of the ARIMA models used to forecast the raw data; and (2) the filters that Method II X-11 variant applies to obtain the first revised seasonal factors.

In this manner, the seasonal factor forecasts are obtained from the forecasted raw values, with a set of moving averages whose weights, although still asymmetric, are closer to the weights applied to central values, than is the case when using the X-11 variant.

Furthermore, the trend-cycle estimate for the last observation is made with the symmetric weights of Henderson's moving average, which can reproduce a cubic in its time span. This is relevant for years with turning points, because the X-11 can only apply the asymmetric weights of the Henderson filter – and so does no more than estimate a linear trend adequately.

Finally, by adding one or two more years of extended data (with no extreme values since they are forecasts), a better estimate of the variance of the irregulars can be obtained. This then allows a substantial improvement in the identification and replacement of outliers which, as it is well known, can severely distort the estimates obtained from linear smoothing filters. For concurrent seasonal factors, the same observations are valid, except



that the seasonal filters are closer to the central filters than those corresponding to the seasonal factor forecasts. For this reason, the number of revisions in the seasonal factor estimates is also considerably reduced.

Ideally, it would be preferable to extend the original series by a sufficient number of years to allow the X-11 program to use symmetric filters (i.e. symmetric weights, which do not introduce time lags between the input and output). Unfortunately, in the context of the X-11 method, this would require at least three years of forecasts; and the large increase in the forecasting error, for such a long lead time, generally annuls the gain introduced by the use of better filters. It has been found that one year of forecasts is the best compromise for the majority of our type of series, when using the automated option.

#### **Other Main Improvements Incorporated into an Automated Version of the X-11 ARIMA**

Besides the automatic selection of the ARIMA models, as discussed in a previous section, a set of new statistical tests, tables and graphs have been incorporated into an automated version of the X-11 ARIMA. These tests are used to assess the quality of the original series and the reliability of the seasonal adjustment. A brief description of these improvements follows.

##### *An F-test for the presence of Seasonality in Table B.1*

This test is based on a one-way analysis of variance on the SI ratios (differences), which is similar to the one already available in Method II X-11 variant for the presence of stable seasonality in Table D.8. It differs only in that the estimate of the trend-cycle is made directly from the original series, by a centred 12-term moving average.

The estimate of the trend-cycle is removed from the original series by division into (subtraction from) the raw data for a multiplicative (additive) model.

The value of the F-ratio is printed in Table B.1. The F is a quotient of two variances: (1) “between months or quarters” variance, which is mainly due to the seasonals, and (2) the “residual” variance which is mainly due to the irregulars.

Because several of the basic assumptions in the F-test are likely to be violated, the value of the F-ratio to be used, for rejecting the null hypothesis of no significant seasonality being present, is not the one obtained from the tabulated F-distribution. From experimentation with a large number of real and simulated series, it was concluded that, for a monthly series of about 10 years, an F-value less than 10 indicates that there is not enough seasonality to justify using the X-11 filters.

### *A Test for the Presence of Moving Seasonality in Table D.8*

The moving seasonality test is based on a two-way analysis of variance performed on the SI ratios (differences) from Table D.8. and was developed by Higginson (1975). It tests for the presence of moving seasonality characterized by gradual changes in the seasonal amplitude but not in the phase.

The total variance of the SI ratios (differences) is considered as the sum of:

1.  $\sigma_m^2$ , the “between months or quarters” variance, which mainly measures the magnitude of the seasonality. It is equal to the sum of squares of the difference, between the average for each month of the SI and the total average, corrected by the corresponding degrees of freedom.

2.  $\sigma_y^2$ , the “between years” variance, which mainly measures the year to year movement of seasonality. It is equal to the sum of squares of the differences between the annual average of the SI, for each year, and the total average of the SI for the whole table, corrected by the corresponding degrees of freedom.

3.  $\sigma_r^2$ , the “residual” variance, equal to the total variance minus the “between months or quarters” variance plus the “between years” variance. The F-ratio for the presence of moving seasonality is the quotient between the “between years” variance and the “residual” variance.

To calculate the variance in an additive model, the S+I are taken in absolute value – otherwise the annual average is always equal to zero. For a multiplicative model, the SI ratios are replaced by absolute deviations from 100, i.e. by  $SI - 100$ . Contrary to the previous test, for which a high value of F is a good indication for seasonal adjustment, a high value of F corresponding to moving seasonality is a bad sign. The F-test, printed in Table D.8, indicates whether moving seasonality is present or not.

The presence of moving seasonality could be taken as an indication of residual seasonality, when X-11 is used, because this program forces stable or nearly stable seasonality on the first and last three years of data. This is partially corrected for, subsequently, by the ARIMA option.

### *A Test for the Presence of Identifiable Seasonality in Table D.8*

This test combines the previous test, for the presence of moving seasonality, with the X-11 test for the presence of stable seasonality and another non-parametric test for the presence of stable seasonality, the Kruskal–Wallis chi-squared test, whose value is also printed in Table D.8.

The main purpose of this test is to determine whether the seasonality of the series is “identifiable” or not. For example, if there is little stable

seasonality and most of the process is dominated by highly moving seasonals, the chances are that the seasonals will not be well estimated because they will not be properly identified by the X-11 program.

The test basically consists of combining the  $F$  values obtained from the two previous tests as follows:

1. If the  $F_s$ -test, for the presence of stable seasonality, at the 0.1 per cent level of significance fails, the null hypothesis (that seasonality is not identifiable) is accepted.

2. If (1) passes but the  $F_M$ -test, for the presence of moving seasonality at the 5 per cent level of significance fails, then this  $F_M$  value is combined with the  $F_s$  value from (1) to give

$$T_1 = \frac{7}{F_s - F_M} \quad \text{and} \quad T_2 = \frac{3F_M}{F_s},$$

and an average of these two  $T$ 's,  $\bar{T} = (T_1 + T_2)/2$ , is next taken. If  $\bar{T} \geq 1$ , the null hypothesis of identifiable seasonality not being present is accepted.

3. If the  $F_M$ -test passes, but either  $\bar{T} < 1$  or the Kruskal – Wallis test fails at the 1 per cent level, then the program prints out that identifiable seasonality is “probably” present.

4. If the  $F_s$ ,  $F_M$  and the Kruskal – Wallis chi-squared value pass, then the null hypothesis of identifiable seasonality not being present is rejected; and the program prints “identifiable seasonality present”.

This test has been developed by Lothian and Morry (1978a), and the program automatically prints out messages mentioned above, at the end of Table D.8.

#### *A Test for the Presence of Residual Seasonality in Table D.11*

This is an  $F$ -test applied to the values of Table D.11 and calculated for the whole length of the series, including the last three years. The effect of the trend is removed by a first order difference of lag three, that is,  $\{\hat{O}_t - \hat{O}_{t-3}\}$  where  $\{\hat{O}_t\}$  are the values of Table D.11. Two  $F$ -ratios are printed at the end of the table, as well as a message saying whether residual seasonality is present or not for the last three years.

#### *A Test for the Randomness of the Residuals*

The X-11 program uses the Average Duration of Run statistic to test for autocorrelation in the final estimated residuals obtained from Table D.13. This non-parametric test was developed by Wallis and Moore (1941).

and is based on the number of turning points. It is only efficient for testing the randomness of the residuals against the alternative hypothesis that the errors,  $I_t$  say, follow a first order autoregressive process of the form  $I_t = \rho I_{t-1} + e_t$ , where  $\rho$  is the autocorrelation coefficient and  $e_t$  is a purely random process.

If a process is purely random and we have an infinite series, the ADR statistic is equal to 1.50. For a series of 120 observations, the ADR should fall between 1.36 and 1.75, 95 per cent of the time. Values greater than 1.75 indicate positive autocorrelation and values smaller than 1.36 indicate negative autocorrelation.

This test, however, is not efficient for detecting the existence of periodic components in the residuals, which can occur when relatively long series are seasonally adjusted, or when the relative variation of the seasonal component is small compared with that of the irregular. To test independence of the residuals, against the alternative hypothesis of their being periodic processes, the normalized cumulative periodogram has been incorporated (Box and Jenkins, 1970).

The normalized cumulative periodogram values are given in a table and also as a graph. By visual inspection, it is possible to determine if components with certain periodicity are present or not in the irregulars. If the residuals are the estimates of a sample realization of a purely random process, and if the size of the sample tends to infinity, then the normalized cumulative periodogram tends to coincide with the diagonal of the square in which it is drawn.

Deviations of the periodogram from that expected, assuming purely random residuals, can be assessed by use of the Kolmogorov–Smirnov test. This test is useful to determine the nature of hidden periodicities left in the irregulars, whether of seasonal or cyclical character, and complements the information provided by the test for the presence of residual seasonality.

#### *A New Table D.11.A*

This new Table D.11.A produces a modified seasonally adjusted series, where the annual totals of the seasonally adjusted values are made equal to those for the raw data. The discrepancy between the annual totals is distributed over the seasonally adjusted values of Table D.11 in such a way that minimizes the month-to-month or quarter-to-quarter movements of the originally seasonally adjusted series. The procedure is based on a quadratic minimization of the first differences of the annual discrepancies, expressed as differences or ratios. This procedure was originally developed by Huot (1975) and improved by Cholette (1978).

*A Set of Guidelines Summarized in One Statistic that will Help to Assess the Reliability of the Seasonal adjustment*

The Statistics Canada X-11 version, as developed in 1975, has two statistics called  $Q_1$  and  $Q_2$  that provide an indication of the size and nature of the irregulars and the seasonal components, respectively. A description of these statistics and their basic assumptions are discussed by Huot and de Fontenay (1973).

Considerable research has been carried out since the first set of guidelines was developed and there now is only one Q statistic, which results from a combination of several other measures (Lothian and Morry, 1978b). Most of these are obtained from the summary measures of Table F.2. Their values vary between 0 and 3, and only values less than one are considered as acceptable. A preliminary set of statistics that are under study, in order to produce the final Q-statistic, is:

1. The relative contribution of the irregulars over spans of three months, as obtained from Table F.2 and denoted by  $M_1$ .
2. The relative contribution of the irregular component to the stationary portion of the variance, denoted by  $M_2$ .
3. The value of the  $\bar{I}/\bar{C}$  ratio the ratio of the average absolute month to month (or quarter to quarter) percentage change, in the irregular, to that in the trend-cycle for the selection of the Henderson moving average, from Table D.7 and denoted by  $M_3$ .
4. The value of the average duration of run for the irregulars, from Table F.2 and denoted by  $M_4$ .
5. The MCD or QCD (the number of months or quarters it takes the average absolute change, in the trend-cycle, to dominate the average absolute change in the irregular), from Table F.2 and denoted by  $M_5$ .
6. The total  $\bar{I}/\bar{C}$  moving seasonality ratio, obtained as an average of the monthly moving seasonality ratios from Table D.9A and denoted by  $M_6$ . (It is the ratio of the average absolute year to year percentage change, in the irregulars, to that in the seasonals.)
7. The amount of stable seasonality, in relation to the amount of moving seasonality, from the tests of Table D.8 and denoted by  $M_7$ .
8. A measure of the variation of the seasonal component, for the whole series, from Table F.2 and denoted by  $M_8$ .
9. The average linear movement of the seasonal component for the whole series, denoted by  $M_9$ .
10. Same as 8, but calculated for recent years only and denoted by  $M_{10}$ .
11. Same as 9, but calculated for recent years only and denoted by  $M_{11}$ .

## REFERENCES

- ANDERSON, O. D. (1975). *Time Series Analysis and Forecasting: the Box-Jenkins Approach*. Butterworth & Co. Ltd, London and Boston, (corrected printings, 1976 and 1977).
- BOX, G. E. P. and JENKINS, G. M. (1970). *Time Series Analysis Forecasting and Control*. Holden Day, San Francisco, (revised edition, 1976).
- BOX, G. E. P. and PIERCE, D. A. (1970). Distribution of residual autocorrelations in autoregressive-integrated moving average time series models. *Journal of the American Statistical Association*, **65**, 1509-26.
- CHOLETTE, P. A. (1978). A comparison and assessment of various adjustment methods of sub-annual series to yearly benchmarks. Research paper. Seasonal Adjustment and Time Series Analysis Staff, Statistics Canada.
- DAGUM, E. B. (1975). Seasonal factor forecasts from ARIMA models. *Proceedings of the International Statistical Institute's 40th Session, Contributed Papers*, Vol. 3, pp. 206-19. Warsaw.
- DAGUM, E. B. (1976). Comments on the paper: a survey and comparative analysis of various methods of seasonal adjustment. *Proceedings of the NBER/Bureau of the Census Conference on Seasonal Analysis of Economic Time Series*, Washington, DC (to appear).
- DAGUM, E. B. (1977). The estimation of changing seasonal variations in economic time series. In *Survey Sampling and Measurement* (ed. K. Namboodiri). Academic Press, New York (to appear).
- DAGUM, E. B. (1978). Comparison and assessment of seasonal adjustment methods for labor force series. Technical paper. US National Commission on Employment and Unemployment Statistics.
- FARLEY, D. and ZELLER, S. (1976). Comments on the paper; A survey and comparative analysis of various methods of seasonal adjustment. *Proceedings of the NBER/Bureau of the Census Conference on Seasonal Analysis of Economic Time Series*, Washington, DC (to appear).
- GRANGER, C. W. J. and NEWBOLD, P. (1977). *Forecasting Economic Time Series*. Academic Press, New York.
- HIGGINSON, J. (1975). An F-test for the presence of moving seasonality when using Census Method II X-11 variant. Research paper. Seasonal Adjustment and Time Series Staff, Statistics Canada.
- HUOT, G. and DE FONTENAY, A. (1973). General seasonal adjustment guidelines, Statistics Canada version of the X-11 program. Research paper. Seasonal Adjustment and Time Series Staff, Statistics Canada.
- HUOT, G. (1975). Quadratic minimization adjustment of monthly or quarterly series to annual totals. Research paper. Seasonal Adjustment and Time Series Staff, Statistics Canada.
- KUIPER, J. (1976). A survey and comparative analysis of various methods of seasonal adjustment. *Proceedings of the NBER/Bureau of the Census Conference on Seasonal Analysis of Economic Time Series*, Washington, DC (to appear).
- LOTHIAN, J. and MORRY, M. (1978a). A test for identifiable seasonality when using the X-11 program. Research paper. Seasonal adjustment and Time Series Staff, Statistics Canada.
- LOTHIAN, J. and MORRY, M. (1978b). A set of guidelines to assess the reliability of seasonal adjustment by the X-11 program. Research paper. Seasonal Adjustment and Time Series Staff, Statistics Canada (to appear).

- NEWBOLD, P. and GRANGER, C. W. J. (1974). Experience with forecasting univariate time series and the combination of forecasts (with discussion). *Journal of the Royal Statistical Society A*, **137**, 131–65.
- PIERCE, D. (1978). Data revisions with moving average seasonal adjustment procedures. US Federal Reserve Board.
- REID, D. J. (1975). A review of short-term projection techniques. In *Practical Aspects of Forecasting*, (ed. H. A. Gorden), pp. 8–25. Operational Research Society, London.
- SHISKIN, J., YOUNG, A. H. and MUSGRAVE, J. C. (1967). The X-11 variant of Census Method II seasonal adjustment. *Technical Paper No. 15*, Bureau of the Census, US Department of Commerce.
- WALLIS, W. A. and MOORE, G. H. (1941). A significance test for time series analysis. *Journal of the American Statistical Association*, **36**, 401–9.