Journal of Forecasting, Vol. 1, 173–187 (1982)

Revisions of Time Varying Seasonal Filters

ESTELA BEE DAGUM
Chief of the Seasonal Adjustment and Time Series Analysis Staff, Statistics Canada, Ottawa, Ontario, Canada

ABSTRACT

The main purpose of this study is to analyse the magnitude and the nature of the revisions that the time varying seasonal filters of the X-11 and X-11-ARIMA methods introduce in the current seasonally adjusted series.

The total revision is measured by the mean absolute difference of the transfer functions corresponding to the forecasting and the concurrent seasonal filters with respect to the central 'final' seasonal filter. To take into consideration the fact that the spectrum of a typical economic time series peaks at the low and seasonal frequencies, the revision measures are calculated for selected frequency intervals associated to the trend-cycle, seasonal variations and the irregular component.

KEY WORDS Seasonal adjustment revisions X-11 X-11-ARIMA Current forecasting Time varying linear filters Transfer function

The current seasonal adjustment of economic time series is very important for policy making at any level of the economic activity. The seasonally adjusted data are mainly useful to assess the stages of the business cycle at which the economy stands. Because of this, current seasonally adjusted series subject to frequent and high revisions are disliked by policy makers, particularly, if the revisions show a change in the direction of the cyclical movement.

From an empirical point of view, the problem of revisions of seasonal factors[1] generated by the Census Method II, has been studied extensively by Young (1960) who analysed its causes and magnitude for selected American economic indicators. From a theoretical viewpoint, the nature of the revisions that result from moving averages seasonal adjustment procedures has been studied extensively by Geweke (1978), Cholette (1979) and Pierce (1980), who also obtained some empirical results for selected methods and series.

In the context of the Census Method II-X-11 Variant (Shiskin, Young and Musgrave, 1967) and its extended version X-11-ARIMA (Dagum, 1980a), the revisions of the current seasonally adjusted series are due to: (1) differences in the moving averages or smoothing linear filters applied to the same observation as it changes its position in time, i.e. to changes in the filtering procedures, and (2) the fact that new information enters into the series when extended with new observations.

[1] In this study, the expression seasonal factor refers to the estimation of the seasonal variations independently of the decomposition model assumed (multiplicative or additive).

0277–6693/82/020173–15$01.50

Received September 1981

The main purpose of this study is to measure that part of the revisions due to changes in the filters applied for 'current' and 'historical' seasonal adjustment. A current seasonally adjusted value can be obtained by either applying a seasonal factor forecast or a concurrent seasonal factor. This latter is generated from a series that includes the last available observation whereas the former is usually obtained from a series that ended one year before. The historical seasonally adjusted value is generated by applying the 'final' seasonal factor; 'final' in the sense that it will not change significantly when more observations are added to the series.

Section 1 deals with the calculation of the X-11 seasonal filters and introduces a measure of the total revision of the asymmetric filters. This measure is calculated for selected frequency intervals associated with the trend-cycle, the seasonality and the irregular component.

Section 2 deals with the filter revisions of the X-11-ARIMA program. The cases analysed correspond to the three program-supplied ARIMA models and selected parameter values.

Section 3 compares the effects of the asymmetric filters of X-11 and X-11-ARIMA on the estimation of the trend-cycle, seasonality and irregulars. Finally, the conclusions are given in Section 4.

1. REVISIONS OF THE FORECASTING AND CONCURRENT SEASONAL FILTERS OF THE X-11 PROGRAM

1.1. The X-11 time varying seasonal filters

The seasonal adjustment filters of the X-11 method were first calculated by Young (1968) considering only a linear approximation of this technique. Young showed that 141, 145 and 165 monthly observations must be available (depending on the length of the Henderson detrending filter) for the seasonal filter to be symmetric. Under the condition that the seasonal estimates must add to zero (approximately) over any 12-month period, Wallis (1974) then showed that 24 more points are needed to estimate the central observation with a two-sided filter (i.e. 165, 169 or 189). Both authors concluded, however, that a good approximation to these long filters can be obtained with only one half of their spans because the values of the weights given to distant observations are very small. For the standard option of the X-11 program (and the X-11-ARIMA as well) this means 7 years of monthly data.

The weights of the filters applied by the X-11 method for the seasonal adjustment of a given observation result from the combination of various linear filters used for the estimation of the trend cycle and the seasonal components. The main steps of this program default option to obtain a seasonally adjusted series are:

(1) Compute the ratios (differences) between the original series and a centred 12-months moving average (2×12 m.a., that is, a 2-months average of a 12 months average) as a first estimate of the seasonal and irregular components.

(2) Apply a weighted 5 term moving average (3×3 m.a.) to the seasonal irregular ratios (differences) for each month separately, to obtain an estimate of the seasonal factors. Compute a 2×12 m.a. of the preliminary factors for the entire series. To obtain the six missing values at either end of this average, repeat the first (last) available value six times.

(3) Adjust these seasonal factors (differences) to sum to 12 (0.000 approximately) over any 12-month period by dividing (subtracting) the centred 12-months average into (from) the factors.

(4) Divide (subtract) the seasonal factors (differences) into (from) the seasonal irregular ratios (differences) to obtain an estimate of the irregular component.

(5) Compute a moving 5-year standard deviation (σ) of the estimates of the irregular component and test the irregulars in the central year of the 5-year period against 2.5σ. Remove values beyond 2.5σ as extreme and recompute the moving 5-year σ.
(6) Assign a zero weight to irregulars beyond 2.5σ and a weight of one (full weight) to irregulars within 1.5σ. Assign a linearly graduated weight between 0 and 1 to irregulars between 2.5σ and 1.5σ.
(7) For each separate month apply a weighted 5-term moving average (3×3 m.a.) to the seasonal-irregular ratios (differences) with extreme values replaced by the corresponding values in (6), to estimate preliminary seasonal factors.
(8) Repeat step (3).
(9) To obtain a preliminary seasonally adjusted series (subtract) (8) into (from) the original series.
(10) Apply a 13-term Henderson moving average to the seasonally adjusted series and divide (subtract) the resulting trend-cycle into (from) the original series to obtain a second estimate of the seasonal-irregular ratios (differences).
(11) Apply a weighted 7-term average (3×5 m.a.) to the seasonal-irregular ratios (differences) for each month separately to obtain a second estimate of the seasonal component.
(12) Repeat step (3).
(13) Divide (subtract) (11) into (from) the original series to obtain a seasonally adjusted series.

Using matrix notation, the estimations of the seasonal factors, one for each observation of a given time series of length m, may be expressed as follows:

$$\hat{S} = WZ \tag{1}$$

where $\hat{S}$ is a $m \times 1$ vector of estimated seasonal factors, W is the seasonal linear filters matrix ($m \times m$) and Z is the $m \times 1$ vector of original data. Each row vector of W is a seasonal linear filter and the W matrix is obtained as follows:

$$W = DM_2(I - H(I - DM_1D)) \tag{2}$$

where $D = I - (2 \times 12)$ is a matrix whose row vectors are the complement of the centred 12-term filter (2×12 m.a.) used in steps (1) and (3) above, M_1 and M_2 are matrices with row vectors given by the 5-term (3×3) and 7-term (3×5) seasonal moving averages, respectively; H is a matrix whose row vectors are the 13-term Henderson filters, applied by the X-11 program for the final estimation of the trend-cycle (step 13) and finally, I denotes the identity matrix.

1.2. Differences between the transfer function of the time varying and time invariant seasonal filters of the X-11

The central seasonal filter of X-11 which corresponds to the observations falling in the fourth year of a seven year series is time invariant (its weights do not vary from one time to another) and two-sided (its weights are symmetric). In contrast, the end-points seasonal filters which correspond to the observations of the first and the last three years are time-varying (their weights change from one point to another) and one-sided filters (their weights are asymmetric).

The most important time-varying seasonal filters are those applied for current seasonal adjustment. The current seasonal adjustment can be made by applying a concurrent seasonal estimate or a seasonal forecast (usually one-year ahead forecast). The concurrent seasonal filter corresponds to the last available observation, i.e. *the 84th point of a seven-year series*. The forecasting seasonal filters (one for each month) are applied to observations 85 to 96 as they become available. These twelve (four) forecasting seasonal filters for a monthly (quarterly) series correspond to the last 12 (4) row vectors of the seasonal matrix W, now of dimension $m + l \times m$, where l is the span of the forecasts. Their weights are calculated by taking into consideration that in the X-11 method, the seasonal factor forecasts are generated by summing the corresponding seasonal factors for the current year plus one-half of the difference between these latter and the seasonal factors for the prior year. Denoting by w^*_{tj} the jth element of the row vector t which defines the t forecasting seasonal filter of a monthly series, we then have,

$$w^*_{t,j} = w_{t-12,j} + 1/2(w_{t-12,j} - w_{t-24,j}) \qquad j = 1, 2, \ldots, m \qquad t = m+1, \ldots, m+12 \tag{3}$$

The properties of the seasonal filters can be studied by analysing their transfer functions and phase diagrams as done by Wallis (1974) for the two-sided X-11 filters of monthly series, and by Laroque (1977) for quarterly series.

The frequency response function completely describes the effects of time invariant linear filters and may be expressed by (see Jenkins and Watts, 1968, p. 41):

$$L(\lambda) = G(\lambda)\,e^{i\phi(\lambda)} \tag{4}$$

where $G(\lambda)$ is called the gain of the filter and measures the amplitude of the output for a sinusoidal input of unit amplitude; $\phi(\lambda)$ is called the phase function and shows the shift in phase of the output compared with the input.

For two-sided filters the phase function is zero or $\pm\pi$ and for the one-sided filters, it can take any value between $\pm\pi$; being undefined at those frequencies where the gain function is zero.

The square of the gain function $|G(\lambda)|^2 = H(\lambda)$ is called the transfer function, which relates the power spectra $f_Z(\lambda)$ and $f_Y(\lambda)$ of the input and the output, respectively, by

$$f_Y(\lambda) = H(\lambda) f_Z(\lambda) \tag{5}$$

Thus, the transfer function measures the effect of the filter on the total variance of the input at different frequencies.

To measure that part of the revision in the current seasonally adjusted observation due to filtering changes, mean absolute differences between the transfer functions corresponding to the one-sided filters (forecasting and concurrent) and the two-sided (central) seasonal filters are calculated. These measures are obtained over all the frequencies and for the frequency intervals associated with the trend-cycle, the seasonal variations and the irregulars. Of these, the differences between the various filters corresponding to the seasonal frequencies are the most important for the input is always a seasonal series (thus, a large concentration of the total variance is expected in these frequencies) and the output is a seasonally adjusted series.

The phase shifts of the one-sided filters of X-11 and X-11-ARIMA are small and their patterns very similar.[2] Therefore, the main difference in the revisions of the seasonal factors are due to the differences in the transfer functions of the filters as discussed below.

The angular frequency in this study is defined as $\lambda_j = 2\pi f j$ where $f = 1/180$ is the rotational frequency with periodicity of 180 months and $j = 1, 2, \ldots 90$. Thus,

$$R^{\mathrm{f}} = \sum_{j=1}^{90} |H_{\mathrm{f}}(\lambda_j) - H(\lambda_j)|/90 \qquad 0 \le \lambda_j \le \pi \tag{6}$$

measures the mean absolute difference between the transfer functions of the forecasting seasonal filter $H_{\mathrm{f}}(\lambda_j)$ and the final seasonal filter $H(\lambda_j)$. There are twelve R^{f} measures, one for each month.

[2] Phase shifts up to two months are observed only for the seasonal frequencies around the fundamental annual cycle and its first harmonic.

A partition of all the frequencies λ_j (expressed in degrees) is then as follows:

(a) $\lambda_j = 0°, 2°, 4°, 6°, \ldots, 18°$. These frequencies correspond to long periodicities, from 180 up to 20 months and are passed without modification by the final trend-cycle filter of X-11.

(b) $\lambda_j = 30°$, $60°$, $90°$, $120°$, $150°$ and $180°$. These frequencies correspond to the seasonal periodicities of one year up to two months. To allow for stochastic moving seasonality as implicitly assumed by the final seasonal filter of the X-11, we include those frequencies falling in an interval of $\pm 4°$ around each one of these six main seasonal frequencies.

(c) The remaining λ_j frequencies are associated with the irregular variations. Using equation (6) and the partition given above, we obtain the total revision measures $R^{\mathrm{f}}_{\mathrm{c}}$, $R^{\mathrm{f}}_{\mathrm{s}}$ and $R^{\mathrm{f}}_{\mathrm{i}}$ of the forecasting filters for the trend-cycle, the seasonality and the irregular variations, respectively.

To evaluate the total revision of the concurrent seasonal filter, we define

$$R^{\mathrm{c}} = \sum_{j=1}^{90} |H_{\mathrm{c}}(\lambda_j) - H(\lambda_j)|/90 \qquad 0 \leq \lambda_j \leq \pi \tag{7}$$

where $H_{\mathrm{c}}(\lambda_j)$ is the transfer function of the concurrent seasonal filter. Using the same partition of frequencies of the forecasting filter we obtain $R^{\mathrm{c}}_{\mathrm{c}}$, $R^{\mathrm{c}}_{\mathrm{s}}$ and $R^{\mathrm{c}}_{\mathrm{i}}$.

The results for all the forecasting and concurrent seasonal filters are shown in Table 1, overleaf. We observe that R^{c} is significantly smaller than any of the twelve R^{f}, in particular, $R^{\mathrm{c}}_{\mathrm{s}}$ is nearly one half of any of the $R^{\mathrm{f}}_{\mathrm{s}}$. This means that we expect the revisions to be much smaller if a concurrent seasonal factor is applied for current seasonal adjustment instead of a seasonal factor forecast. This result was empirically observed by Kenny and Durbin (1981).

2. REVISIONS OF THE FORECASTING AND CONCURRENT SEASONAL FILTERS OF THE X-11-ARIMA PROGRAM

2.1. The X-11-ARIMA time varying seasonal filters

The X-11-ARIMA program extends the observed series $Z(t)$ with extrapolated values $\hat{Z}(t + \tau)$ from an ARIMA model which may be expressed as a weighted linear combination of observed Zs. Thus, the linear seasonal filters applied to the extended series for a given observation t, result from the combinations of the w weights of the X-11 filters with the weights used by the ARIMA extrapolation function. The influence of these latter is strong for the forecasting and concurrent seasonal filters, and decreases rapidly as we depart from the end of the series to approach to the central observations. The extrapolation weights do not affect the central seasonal filter which is the same for X-11 and X-11-ARIMA.

The end points seasonal adjustment filters of X-11-ARIMA change with the ARIMA model chosen and its parameter values. These filters have been calculated by Dagum (1980b) for the three built-in ARIMA models of X-11-ARIMA and various sets of parameter values.

Following the approach of Section 1.1 we now write,

$$S = WZ^* \tag{8}$$

Where S is a vector of seasonal estimates of dimension $n = m + l$ which corresponds to the m observations plus the l forecast values, W is the $m \times m$ seasonal matrix of fixed weights defined in equation (1) and Z^* is the vector of the extended original series of dimension $n = m + l$. The Z^* vector may be expressed as follows:

$$\hat{Z}^*_{n \times 1} = \begin{bmatrix} Z_{m \times 1} \\ \hat{Z}_{l \times 1} \end{bmatrix} \tag{9}$$

Table 1. Various revision measures of the X-11 seasonal filters

Mean absolute difference between forecasting and central seasonal filters: $R^{\mathrm{f}} = \sum_{j=1}^{N} |H_{\mathrm{f}}(\lambda_j) - H(\lambda_j)|/N$

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
R^{f} Absolute average over all frequencies, $N = 90$	48	48	48	48	48	48	48	49	49	49	51	56
$R^{\mathrm{f}}_{\mathrm{c}}$ Absolute average over trend-cycle frequencies	1	1	1	2	1	1	1	1	1	1	2	4
$R^{\mathrm{f}}_{\mathrm{s}}$ Absolute average over seasonal frequencies	84	85	86	86	86	86	92	91	91	91	95	104
$R^{\mathrm{f}}_{\mathrm{i}}$ Absolute average over irregular frequencies	36	36	35	35	35	36	33	34	34	34	35	38

Mean absolute difference between the concurrent and central seasonal filters: $R^{\mathrm{c}} = \sum_{j=1}^{N} |H_{\mathrm{c}}(\lambda_j) - H(\lambda_j)|/N$

	84th point
R^{c} Absolute average over all frequencies, $N = 90$	26
R^{c} Absolute average over trend-cycle frequencies	3
$R^{\mathrm{c}}_{\mathrm{s}}$ Absolute average over seasonal frequencies	39
$R^{\mathrm{c}}_{\mathrm{i}}$ Absolute average over irregular frequencies	22

Where $\hat{Z}$ denotes the vector of forecast values. Because these forecast values are generated from ARIMA models they may be expressed as a function of previous observations by

$$\hat{Z}_{l\times 1} = \Pi_{l\times m} Z_{m\times 1} \tag{10}$$

The general term of the Π matrix is (see Box and Jenkins, 1970, p. 142)

$$\pi_j^{\tau} = \pi_{j+\tau-1} + \sum_{h=1}^{\tau-1} \pi_h \pi_j^{(\tau-h)} \qquad j = 1, 2, \ldots \tag{11}$$

Replacing Z^* in (8) we obtain,

$$S_{n\times 1} = W^*_{n\times m} Z_{m\times 1}$$

Where W^* is the matrix of the combined weights of the X-11-ARIMA filters.

2.2. Differences between the transfer functions of the time varying and time invariant seasonal filters of the X-11-ARIMA

The same statistical measures of revisions discussed in Section 1.2 are used to assess the seasonal filters changes of X-11-ARIMA. The analysis is made for the three program-supplied ARIMA models, i.e. $(0, 1, 1)\ (0, 1, 1)_{12}$, $(0, 2, 2)\ (0, 1, 1)_{12}$ and $(2, 1, 2)\ (0, 1, 1)_{12}$ and selected parameter values that represent various degrees of flexibility of the trend-cycle and seasonal components.

Tables 2 to 6 show the total revisions for each one of the forecasting seasonal filters and the concurrent seasonal filter corresponding to the above models.

The results in these tables indicate that independently of the ARIMA model chosen and its parameter values, there is a large reduction (around 50 per cent) in the size of the revisions of the concurrent seasonal filter with respect to any of the forecasting seasonal filters. This reduction, larger in X-11-ARIMA than in X-11, is observed for the total revisions over all the frequencies and every one of the selected frequency intervals.

3. COMPARISONS BETWEEN THE X-11 AND X-11-ARIMA TIME VARYING FILTER REVISIONS

Various empirical studies have shown that the use of the ARIMA options of X-11-ARIMA reduces significantly the magnitude of the revisions of the seasonal factor forecasts for a large number of series (see, among others, Dagum 1975, 1978; Dagum, Mayes and McKenzie, 1979; Farley and Zeller 1979; Kuiper 1979, 1981).

The differences in the time varying filters of X-11 and X-11-ARIMA analysed in the two previous sections should give us some insights of the nature of the revisions from a theoretical viewpoint. Because the power spectrum of the seasonally adjusted series is equal to the transfer function of the filter times the power spectrum of the original series, for practical cases, the magnitude of the revisions also depends on the spectrum of the original series. Given the difference between the transfer functions of the filters for a frequency band, the total revisions of the seasonally adjusted series will be larger, the larger the power spectrum of the original series in the corresponding frequency interval.

Keeping the above considerations in mind, we proceed to comparing the revisions of the time varying filters of both methods. The R^f measures of Tables 1 to 6, shows that, often, the values for X-11-ARIMA are larger than for X-11. This means that the mean absolute difference between the forecasting and central filters, over all the frequencies, is smaller for X-11 than X-11-ARIMA. If the spectrum of the original series is constant over frequency, as it is in the case of a purely random

Table 2. Various revision measures of the X-11-ARIMA seasonal filters for a (0, 1, 1) $(0, 1, 1)_{12}$ model, $\theta = 0.60$, $\Theta = 0.70$

Mean absolute difference between forecasting and central seasonal filters: $R^{f} = \sum_{j=1}^{N} |H_{f}(\lambda_j) - H(\lambda_j)|/N$

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
R^{f} Absolute average over all frequencies, $N = 90$	48	48	48	48	48	48	48	48	48	49	49	50
R^{f}_{c} Absolute average over trend-cycle frequencies	5	5	6	6	7	6	5	3	2	4	8	11
R^{f}_{s} Absolute average over seasonal frequencies	62	62	63	63	63	63	64	65	65	66	69	73
R^{f}_{i} Absolute average over irregular frequencies	47	47	47	46	46	46	46	46	46	46	45	44

Mean absolute difference between the concurrent and central seasonal filters: $R^{c} = \sum_{j=1}^{N} |H_{c}(\lambda_j) - H(\lambda_j)|/N$

	84th point
R^{c} Absolute average over all frequencies, $N = 90$	17
R^{c}_{c} Absolute average over trend-cycle frequencies	1
R^{c}_{s} Absolute average over seasonal frequencies	22
R^{c}_{i} Absolute average over irregular frequencies	16

Table 3. Various revision measures of the X-11-ARIMA seasonal filters for a $(0,1,1)(0,1,1)_{12}$ model, $\theta = 0.40$, $\Theta = 0.60$

Mean absolute difference between forecasting and central seasonal filters: $R^{\mathrm{f}} = \sum_{j=1}^{N} |H_{\mathrm{f}}(\lambda_j) - H(\lambda_j)|/N$

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
R^{f} Absolute average over all frequencies, $N = 90$	50	50	50	50	50	50	50	50	50	51	52	52
$R^{\mathrm{f}}_{\mathrm{c}}$ Absolute average over trend-cycle frequencies	3	4	5	6	6	6	5	4	2	3	6	10
$R^{\mathrm{f}}_{\mathrm{s}}$ Absolute average over seasonal frequencies	62	62	62	63	63	63	64	64	65	66	69	77
$R^{\mathrm{f}}_{\mathrm{i}}$ Absolute average over irregular frequencies	51	51	50	50	50	50	49	50	50	50	49	45

Mean absolute difference between the concurrent and central seasonal filters: $R^{\mathrm{c}} = \sum_{j=1}^{N} |H_{\mathrm{c}}(\lambda_j) - H(\lambda_j)|/N$

	84th point
R^{c} Absolute average over all frequencies, $N = 90$	16
$R^{\mathrm{c}}_{\mathrm{c}}$ Absolute average over trend-cycle frequencies	2
$R^{\mathrm{c}}_{\mathrm{s}}$ Absolute average over seasonal frequencies	24
$R^{\mathrm{c}}_{\mathrm{i}}$ Absolute average over irregular frequencies	14

Table 4. Various revision measures of the X-11-ARIMA seasonal filters for a (0, 1, 1) $(0, 1, 1)_{12}$ model, $\theta = 0.30$, $\Theta = 0.40$

Mean absolute difference between forecasting and central seasonal filters: $R^{f} = \sum_{j=1}^{N} |H_{f}(\lambda_j) - H(\lambda_j)|/N$

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
R^{f} Absolute average over all frequencies, $N = 90$	54	53	53	53	53	53	53	53	53	54	55	55
R^{f}_{c} Absolute average over trend-cycle frequencies	3	4	5	6	6	6	6	4	2	4	7	11
R^{f}_{s} Absolute average over seasonal frequencies	58	58	59	59	59	59	61	61	61	62	66	77
R^{f}_{i} Absolute average over irregular frequencies	59	58	58	57	57	57	55	56	56	57	57	50

Mean absolute difference between the concurrent and central seasonal filters: $R^{c} = \sum_{j=1}^{N} |\overline{H_{c}}(\lambda_j) - H(\lambda_j)|/N$

	84th point
R^{c} Absolute average over all frequencies, $N = 90$	16
R^{c}_{c} Absolute average over trend-cycle frequencies	4
R^{c}_{s} Absolute average over seasonal frequencies	21
R^{c}_{i} Absolute average over irregular frequencies	15

Table 5. Various revision measures of the X-11-ARIMA seasonal filters for a $(0, 2, 2)\ (0, 1, 1)_{12}$ model, $\theta_1 = 1.10$, $\theta_2 = -0.30$, $\Theta = 0.60$.

Mean absolute difference between forecasting and central seasonal filters: $R^{\mathrm{f}} = \sum_{j=1}^{N} |H_{\mathrm{f}}(\lambda_j) - H(\lambda_j)|/N$

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
R^{f} Absolute average over all frequencies, $N = 90$	50	50	50	50	50	50	50	50	50	51	54	54
$R^{\mathrm{f}}_{\mathrm{c}}$ Absolute average over trend-cycle frequencies	4	3	3	3	3	3	3	3	2	2	3	7
$R^{\mathrm{f}}_{\mathrm{s}}$ Absolute average over seasonal frequencies	62	62	63	63	63	63	65	65	65	66	70	75
$R^{\mathrm{f}}_{\mathrm{i}}$ Absolute average over irregular frequencies	51	51	51	51	51	51	50	50	50	51	52	55

Mean absolute difference between the concurrent and central seasonal filters: $R^{\mathrm{c}} = \sum_{j=1}^{N} |H_{\mathrm{c}}(\lambda_j) - H(\lambda_j)|/N$

	84th point
R^{c} Absolute average over all frequencies, $N = 90$	18
$R^{\mathrm{c}}_{\mathrm{c}}$ Absolute average over trend-cycle frequencies	6
$R^{\mathrm{c}}_{\mathrm{s}}$ Absolute average over seasonal frequencies	30
$R^{\mathrm{c}}_{\mathrm{i}}$ Absolute average over irregular frequencies	14

Table 6. Various revision measures of the X-11-ARIMA seasonal filters for a $(2, 1, 2)\ (0, 1, 1)_{12}$ model, $\phi_1 = -0.85$, $\phi_2 = -0.60$, $\theta_1 = -0.25$, $\theta_2 = -0.25$, $\Theta = 0.60$

Mean absolute difference between forecasting and central seasonal filters: $R^{f} = \sum_{j=1}^{N} |H_{f}(\lambda_j) - H(\lambda_j)|/N$

	Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
R^{f} Absolute average over all frequencies, $N = 90$	51	51	51	50	50	50	50	50	51	51	52	54
R^{f}_{c} Absolute average over trend-cycle frequencies	3	3	3	6	5	5	6	3	2	3	6	9
R^{f}_{s} Absolute average over seasonal frequencies	63	63	63	63	63	63	64	64	64	66	71	77
R^{f}_{i} Absolute average over irregular frequencies	52	52	52	50	50	50	50	50	52	51	49	49

Mean absolute difference between the concurrent and central seasonal filters: $R^{c} = \sum_{j=1}^{N} |H_{c}(\lambda_j) - H(\lambda_j)|/N$

	84th point
R^{c} Absolute average over all frequencies, $N = 90$	18
R^{c}_{c} Absolute average over trend-cycle frequencies	3
R^{c}_{s} Absolute average over seasonal frequencies	23
R^{c}_{i} Absolute average over irregular frequencies	18

process, then we would have smaller revisions from X-11 as compared to X-11-ARIMA, for the majority of the forecasting filters (one exception being the forecasting filter for the 96th point corresponding to December). However, there are no economic time series which have a constant spectrum. The majority of these series have a typical spectral density dominated by low and seasonal frequencies (see examples in Granger (1964) and Nerlove, Grether and Carvalho (1980)). Therefore, to perform an evaluation with practical relevance, all the frequencies have been partitioned in intervals associated with the three main components, trend-cycle, seasonality and irregulars and the corresponding measures R_c^f, R_s^f and R_i^f have been calculated. The partitions associated with the trend-cycle and the irregulars are less precise because these components are difficult to define. But on the other hand, for the purpose of evaluating seasonal adjustment filters, these two components are relatively less important than the seasonal variations. The seasonal frequency intervals are better defined and they agree with those of the two-sided final filters of X-11 and X-11-ARIMA (default option). The width of these bands corresponds to a 7-years moving seasonality which, in fact, results from the application of a 3×5 moving average to smooth the seasonal-irregular ratios.

Tables 1 to 6 show that the R_s^f measures of X-11-ARIMA are smaller as compared to X-11, for any of the models and parameter values selected. The fact that the discrepancy between the forecasting filters and the final filter for the seasonal frequencies is smaller in X-11-ARIMA than X-11 agrees with the empirical results of the various studies mentioned above. The gain in the revisions reduction will be larger, the larger the contribution of the seasonal variations to the total variance of the original series.

On the other hand, we observe that the R_i^f are always higher for the forecasting filters of X-11-ARIMA as compared to those of X-11.

The X-11-ARIMA forecasting filters tend to amplify the spectrum of the output at the frequencies corresponding to the irregular variations whereas, at the same time, they eliminate better than X-11, the seasonal variations.

Finally, Tables 1 to 6 show that for both methods, R_c^f absolute revisions over the trend-cycle frequencies are very small relative to the other two measures R_s^f and R_c^f (though the revisions of the X-11-ARIMA filters are larger than those of X-11).

If the current seasonally adjusted data are obtained by applying concurrent seasonal factors, both methods have a large reduction (50 per cent or more) in the revision of the filters at the seasonal frequencies. Furthermore, we observe that the various revision measures of the X-11-ARIMA concurrent filters are much smaller than those of X-11 with only few exceptions. This means that from the viewpoint of the total revisions, the X-11-ARIMA concurrent seasonal factors are better than those of X-11 and, in general, that concurrent seasonal factors are to be preferred to seasonal factor forecasts.

4. CONCLUSIONS

This study has analysed the magnitude and the nature of the revisions of the concurrent and forecasting seasonal filters of the X-11 and X-11-ARIMA programs applied for current seasonal adjustment.

The total revisions due to changes in the filters is measured by the mean absolute difference between the transfer functions of the various time-varying filters and the time invariant filter.

The results obtained (see Tables 1 to 6) indicate that: (1) the revisions of the concurrent and forecasting seasonal filters of X-11-ARIMA are from 30 per cent up to 50 per cent smaller than those of X-11 and (2) the revisions for the concurrent seasonal filter are nearly one-half those of the forecasting filters for both methods. The latter means that, from the viewpoint of revisions, the use of current seasonal estimates is preferable to seasonal forecasts. In some instances, producers of seasonally adjusted data can only apply these latter, in which case, the results show that the forecasting filters for the first six months are smaller as compared to the last six months.

The reduction in revisions due to the better properties of the X-11-ARIMA filters can be smaller or larger for real data, depending on: (1) the power spectrum of the original series at the seasonal frequencies and (2) the presence of outliers. Series with small seasonal variations or with a repetitive seasonal pattern will be equally well seasonally adjusted by X-11 and X-11-ARIMA. The results for series strongly affected by outliers are difficult to predict. Unless the outliers are properly identified and temporarily replaced, the extrapolations with the wrong ARIMA model may even introduce larger revisions than the X-11.[3] With an exception of these two cases, the use of the X-11-ARIMA filters will reduce significantly the revisions of current seasonally adjusted data.

REFERENCES

Box, G. E. P., and Jenkins, G. M., *Time Series Analysis, Forecasting and Control*, San Francisco: Holden Day, 1970.

Cholette, P.-A., 'Spectral diagnosis and marginal improvements of the X-11 seasonal adjustment method for short series', Research Paper, Seasonal Adjustment and Time Series Staff, Statistics Canada (unpublished), 1979.

Dagum, E. B., 'Seasonal factor forecasts from ARIMA models', *Proceedings of the International Statistical Institute*, 40th Session, **3** (1975), 206–219.

Dagum, E. B., *Comparison and Assessment of Seasonal Adjustment Methods for Labor Force Series*, U.S. Government Printing Office, Washington, D.C., Stock No. 052-003-00603-1, 1978.

Dagum, E. B., *The X-11-ARIMA Seasonal Adjustment Method*, Ottawa, Statistics Canada Publication, Catalogue No. 12-564E, 1980a.

Dagum, E. B., 'Properties of the linear facilities of the X-11-ARIMA method for current seasonal adjustment', (forthcoming) *Canadian Journal of Statistics*, 1980b.

Dagum, E. B., Mayes, F. Jr., and McKenzie, S. K., 'ARIMA modelling and seasonal adjustment: applications in U.S. foreign trade data *Proceedings of the American Statistical Association*, Business and Economic Statistics Section, 1979, 328–332.

Farley, D., and Zeller, S. H., Comments in Zeller, Arnold (ed.) *Seasonal Analysis of Economic Time Series*, U.S. Government Printing Office, Washington, D.C., Stock No. 003-024-01655-9, 1979, pp. 464–467.

Geweke, J., 'The revision of seasonally adjusted time series', *SSRI Report No. 7803*, Department of Economics, University of Winsconsin, 1978.

Granger, C. W., *Spectral Analysis of Economic Time Series*, Princeton, New Jersey: Princeton University Press, 1964.

Jenkins, G. M., and Watts, D. G., *Spectral Analysis and its Applications*, San Francisco: Holden Day, 1968.

Kenny, P. B., and Durbin, J., 'Local trend estimation and seasonal adjustment of economic and social time series', *Journal of the Royal Statistical Society* (forthcoming), 1981.

Kuiper, J, 'A survey and comparative analysis of various methods of seasonal adjustment' in Zeller, Arnold (ed.), *Seasonal Analysis of Economic Time Series*, U.S. Government Printing Office, Washington, D.C., Stock No. 003-024-01655-9, 1979, pp. 59–76.

Kuiper, J., 'The treatment of extreme values in the X-11-ARIMA program', in (Anderson, O. and Perryman, M. R., eds.) *Time Series Analysis and Forecasting*, Amsterdam: North Holland Publishing Co., 1981.

Laroque, G., 'Analyse d'une méthode de désaisonnalisation: le programme X-11 du U.S. Bureau of Census, version trimestrielle', *Annales de L'I.N.S.E.E.*, **28** (1977), 105–127.

Nerlove, M., Grether, D. M., and Carvalho, J. L., *Analysis of Economic Time Series*, New York: Academic Press, 1979.

[3] The X-11-ARIMA program has an option that automatically identifies and replaces outliers before the final parameter estimation of the model.

Pierce, D., 'Data revision with moving average seasonal adjustment procedures', *Journal of Econometrics* (Annals of Applied Econometrics 1980–83), **14** (1980) 95–114.
Shiskin, J., Young, A. H., and Musgrave, J. C., 'The X-11 variant of the census method II seasonal adjustment program', *Technical Paper No. 15*, U.S. Bureau of the Census, 1967.
Wallis, K. F., 'Seasonal adjustment and relations between variables', *Journal of the American Statistical Association*, **69** (1974), 18–31.
Young, A. H., 'Seasonal factor revisions in UNIVAC method II', *Proceedings of the American Statistical Association*, Business and Economic Statistics Section (1960), 44–52.
Young, A. H., 'Linear approximations to the census and BLS seasonal adjustment methods', *Journal of the American Statistical Association*, **63** (1968), 445–457.

Author's biography:
Estela Bee Dagum is the Chief of the Seasonal Adjustment and Time Series Research Staff of Statistics Canada, Ottawa, Canada. She graduated from the National University of Cordoba, Argentina and continued her education at the London School of Economics and Princeton University.

Dr. Dagum is a Fellow of the American Statistical Association and the first recipient of the Julius Shiskin Award granted by the Washington Statistical Society.

She has published over fifty articles in scientific journals, co-authored two books in the field of Econometrics and developed the X-11-ARIMA seasonal adjustment method widely applied in Canada, the United States and various other countries.

She is editorial collaborator of several scientific journals and has lectured in Canada, the United States and other European and Latin American universities.

Author's address:
Estela Bee Dagum, Chief, Seasonal Adjustment and Time Series Analysis Staff, 25-A Coats Building, Statistics Canada, Ottawa, Ontario, Canada K1A OT6.