1)

$$\max xy \quad \text{s.t.} \quad 2x + 3y <= 300$$
$$x >= 0$$
$$y >= 0$$

$$\mathcal{L} = xy - \lambda_1(2x + 3y - 300) - \lambda_2(-x) - \lambda_3(-y)$$

$$\frac{\partial \mathcal{L}}{\partial x} = y - \lambda_1 . 2 - \lambda_2 . (-1) = 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = x - \lambda_1 . 3 - \lambda_3 . (-1) = 0 \quad (2)$$

$$\lambda_1 \frac{\partial \mathcal{L}}{\partial \lambda_1} = -\lambda_1(2x + 3y - 300) \quad (3)$$

$$\lambda_2 \frac{\partial \mathcal{L}}{\partial \lambda_2} = -\lambda_2(-x) = 0 \quad (4)$$

$$\lambda_3 \frac{\partial \mathcal{L}}{\partial \lambda_3} = -\lambda_3(-y) = 0 \quad (5)$$

(1) $y - 2\lambda_1 + \lambda_2 = 0$

(2) $x - 3\lambda_1 + \lambda_3 = 0$

(3) $-\lambda_1(2x + 3y - 300) = 0$

$\lambda_1 = 0$ ou $2x + 3y = 300$

1) CASO $\lambda_1 = 0$

(1) $y - \cancel{2\lambda_1}^{0} + \lambda_2 = 0 \Rightarrow y + \lambda_2 = 0$

$-y = \lambda_2$

(2) $x - \cancel{3\lambda_1}^{0} + \lambda_3 = 0 \Rightarrow x + \lambda_3 = 0$

$-x = \lambda_3$

(4) $-\lambda_2(-x) = 0 \Rightarrow -(-y)(-x) = 0$

$-xy = 0$

⑤ $-\lambda_3(-y) = 0 \Rightarrow -(-x)(-y) = 0$

$-xy = 0$

$x = 0$ ou $y = 0$

1.1) CASO $x = 0$

② $\lambda_3 = 0$

③ $\cancel{2x}^{0} + 3y - 300 \leqslant 0$

$3y \leq 300$

$y \leqslant 100$

De uma das restrições $\Rightarrow y \geqslant 0$

Então $y \in [0, 100]$

Pontos críticos : $\{(x,y) \in \mathbb{R}^2 \mid x = 0$ e $0 \leq y \leq 100\}$

c/ $\lambda_1 = 0$ e $\lambda_3 = 0$

1.2) CASO $y = 0$

① $\lambda_2 = 0$

③ $2x + \cancel{3y}^{0} - 300 \leqslant 0$

$x \leqslant 150$

De uma das restrições $\Rightarrow x \geqslant 0$

Então $x \in [0, 150]$

Pontos críticos : $\{(x,y) \in \mathbb{R}^2 \mid y = 0$ e $0 \leq x \leq 150\}$

c/ $\lambda_1 = 0$ e $\lambda_2 = 0$

2) CASO $2x + 3y = 300$

④ $\lambda_2 x = 0$

$\lambda_2 = 0$ ou $x = 0$

2.1) CASO $x=0$

$2x+3y=300$

$y=100$

Se $y=100$, de 5 $\lambda_3=0$

1 $y-2\lambda_1+\lambda_2=0 \Rightarrow 100-2.0+\lambda_2=0$

$\lambda_2=-100$

2 $\cancel{x}^{0}-3\lambda_1+\cancel{\lambda_3}^{0}=0 \Rightarrow \lambda_1=0$

Ponto crítico : $x=0,\ y=100,\ \lambda_1=\lambda_3=0,\ \lambda_2=-100$

2.2) CASO $\lambda_2=0$

5 $\lambda_3\, y=0$

$\lambda_3=0$ ou $y=0$

2.2.1) CASO $y=0$ ($\lambda_2=0$ e $2x+3y=300$)

1 $\cancel{y}^{0}-2\lambda_1+\cancel{\lambda_2}^{0}=0 \Rightarrow \lambda_1=0$

2 $x-\cancel{3\lambda_1}^{0}+\lambda_3=0 \Rightarrow x=-\lambda_3$

$2x+\cancel{3y}^{0}=300 \Rightarrow x=150$

Ponto crítico : $x=150,\ y=0,\ \lambda_1=\lambda_2=0,\ \lambda_3=-150$

2.2.2) CASO $\lambda_3=0$ ($\lambda_2=0$ e $2x+3y=300$)

1 $y-2\lambda_1+\cancel{\lambda_2}^{0}=0 \Rightarrow y=2\lambda_1$

$\lambda_1=\frac{y}{2}$

2 $x-3\lambda_1+\cancel{\lambda_3}^{0}=0 \Rightarrow x-3.\frac{y}{2}=0$ (*)

$$2x + 3y = 300$$
$$3y = 300 - 2x$$
$$y = \frac{300 - 2x}{3}$$

De (*) :

$$x - \frac{3}{2}\left(\frac{300 - 2x}{3}\right) = 0$$
$$x - 150 + x = 0$$
$$2x - 150 = 0$$
$$x = \frac{150}{2} = 75$$

$$y = \frac{300 - 2x}{3} \Rightarrow y = \frac{300 - 2.75}{3} = \frac{150}{3} = 50$$
$$y = 50$$

Ponto crítico : $x = 75$, $y = 50$, $\lambda_3 = \lambda_2 = 0$, $\lambda_1 = 25$

Conjunto de restrições formam conjunto compacto (limitado e fechado), $-x \le 0$, $-y \le 0$, $2x + y \le 300$ weierstrass garante que $\exists$ max/min dentre P.C:

Pontos críticos	$f(x,y) = xy$	
$\{(x,y) \in \mathbb{R}^2 \mid x = 0 \text{ e } 0 \le y \le 100\}$	0	mín
$\{(x,y) \in \mathbb{R}^2 \mid y = 0 \text{ e } 0 \le x \le 150\}$	0	mín
$x = 0$, $y = 100$, $\lambda_1 = \lambda_3 = 0$, $\lambda_2 = -100$	0	mín
$x = 150$, $y = 0$, $\lambda_1 = \lambda_2 = 0$, $\lambda_3 = -150$	0	mín
$x = 75$, $y = 50$, $\lambda_3 = \lambda_2 = 0$, $\lambda_1 = 25$	$75 . 50$	max

mesmo ponto

mesmo ponto

max é dado por $(x, y) = (75, 50)$ e $f(x,y) = 75.50$

2) $\min x^2 + y^2 \quad \text{s.t.} \quad 2x + 3y = 300$

$$\mathcal{L} = x^2 + y^2 - \lambda(2x + 3y - 300)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 2x - \lambda(2) = 0 \Rightarrow \lambda = x \quad (1)$$

Se $x = 0 \Rightarrow \lambda = 0$

$$\frac{\partial \mathcal{L}}{\partial y} = 2y - \lambda(3) = 0 \Rightarrow \lambda = \frac{2y}{3} \quad (2)$$

Se $y = 0 \Rightarrow \lambda = 0$

De (1) e (2) $\Rightarrow x = \frac{2y}{3}$ (se $x \neq 0$ ou $y \neq 0$)

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 2x + 3y - 300 = 0$$

$$2x + 3y = 300$$

$$2\left(\frac{2y}{3}\right) + 3y = 300$$

$$\frac{4y}{3} + 3y = 300$$

$$\frac{4y + 9y}{3} = 300$$

$$13y = 900$$

$$y = \frac{900}{13}$$

$$x = \frac{2y}{3} \Rightarrow x = \frac{2}{\cancel{3}} \cdot \frac{\overset{3}{\cancel{900}}}{13} = \frac{600}{13}$$

Compacto + WGIE... $\Rightarrow$ dentro e no limite do intervalo

P.C	$f(x,y) = x^2 + y^2$	
$\frac{600}{13}, \frac{900}{13}$	$\left(\frac{600}{13}\right)^2 + \left(\frac{900}{13}\right)^2 = 6.923,08$	mín
$0, 100$	$0^2 + (100)^2 = 10.000,00$	
$150, 0$	$(150)^2 + 0^2 = 22.500,00$	máx

3) $\max x + y \quad \text{s.t.} \quad x^2 + y^2 <= 4$

3) $\max x + y \quad \text{s.t.} \quad x^2 + y^2 <= 4$

$$\mathcal{L} = x + y - \lambda(x^2 + y^2 - 4)$$

$$\frac{\partial \mathcal{L}}{\partial x} = 1 - \lambda . 2x = 0 \Rightarrow \lambda = \frac{1}{2x}, \text{ se } x \neq 0 \quad (1)$$

$$\frac{\partial \mathcal{L}}{\partial y} = 1 - \lambda . 2y = 0 \Rightarrow \lambda = \frac{1}{2y}, \text{ se } y \neq 0 \quad (2)$$

De (1) e (2), temos:

$$\frac{1}{2x} = \frac{1}{2y} \Rightarrow x = y$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = x^2 + y^2 - 4 = 0 \Rightarrow x^2 + y^2 = 4$$

$$x^2 + x^2 = 4$$
$$2x^2 = 4$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$
$$y = \pm\sqrt{2}$$

PC	$f(x,y) = x+y$	
$\sqrt{2}, \sqrt{2}$	$2\sqrt{2} \approx 2{,}9$	max
$-\sqrt{2}, -\sqrt{2}$	$-2\sqrt{2} \approx -2{,}9$	min

Contour plot: