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Developing an Approximation to the Heidler Function - With an Analytical Transformation into the Frequency Domain

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***Abstract*—IEC 62305 utilises the Heidler function as the standardised lightning current waveshape because it mimics the properties of a real lightning stroke. There is no analytical solution to the integral of the Heidler function and this means that it is not possible to obtain an expression for the Heidler function in the frequency domain. There are several approximations that are used to overcome this shortcoming. This paper proposes a new approximation that is designed in the Laplace domain. Initial results show that the amplitude differs with that of the Heidler function by no more than 3.7%. Furthermore, the first derivative of the approximated current waveshape, dI/dt, differs with that of the Heidler function by 10.5%. This however can be explained by the steepness factor not being calibrated correctly for the same shape Heidler function. The approximation is created in the Laplace domain and therefore it is trivial to plot the frequency spectrum of the approximation. Frequency analysis shows that the approximation agrees with those of other researchers. It is concluded that the initial results are evident of a promising approximation to the Heidler function.**

***Index Terms*—Lightning Channel Base Current; LEMP; IEC 62305; Heidler; Laplace; Frequency Domain; Approximation.**

I. Introduction

Lightning current data studies have shown that although lightning currents have random waveshapes, there are certain characteristics that each lightning current has [1]–[3]. IEC 62305 defines these characteristics for different types of lightning strokes. In order to simulate the behaviour of a lightning stroke, the standard recommends the use of the Heidler function as it meets the criteria outlined in the standard [4].

This function is ideal for current amplitude and other parameters in the time domain such as the charge and change in current. There is however no analytical transform of the Heidler function into the frequency domain because it cannot be integrated. This leads to problems when trying to obtain an accurate power spectral density plot of a particular lightning current. With a power spectral density plot, it is possible to analyse the effects of particular frequencies on a system. Furthermore, the integral of a lightning channel base current is required when carrying out Lightning Electromagnetic Pulse (LEMP) calculations. Having a function that has an analytical expression to its integral simplifies these LEMP calculations [5].

Therefore an approximation to the Heidler function that is transformable into the frequency domain is proposed. This can be used to show the effects of lightning current frequency components on systems.

This paper details the process of developing such an approximation by first detailing some background into the use of the Heidler function and its limitation with respect to Laplace/Fourier transform. Next, the Laplace domain approximation is developed and some preliminary results are given to show the viability of the approximation. Finally, the direction of the work is outlined and the paper is concluded.

II. Background

The IEC standard on Lightning Protection, IEC 62305, discusses different types of lightning currents such as a first return stroke, subsequent strokes, etc. [4]. In this standard the shape of a typical lightning stroke is described along with properties (of a lightning current) that must be used in the design of lightning protection systems. An adaptation of the waveshape shown in IEC 62305 standard is shown in Figure 1 [4]. This figure shows how the different stroke currents, such as the 1.2/50, 8/20 and 10/350 are composed. T_1 is the rise time (number before the '/') and T_2 is the fall time (number after the '/'). It is important that any lightning current shape can be obtained in order to test against different scenarios and lightning protection levels. Therefore the Heidler function is used in the IEC 62305 because it contains different factors allowing for the rise time, fall time, amplitude and steepness factors to be customised.

The Heidler function is defined as in *Equation 1* [3]

$$i(t) = \frac{I_0}{\eta} \frac{\left(\frac{t}{\tau_1}\right)^n}{1 + \left(\frac{t}{\tau_1}\right)^n} e^{-\frac{t}{\tau_2}} \tag{1}$$

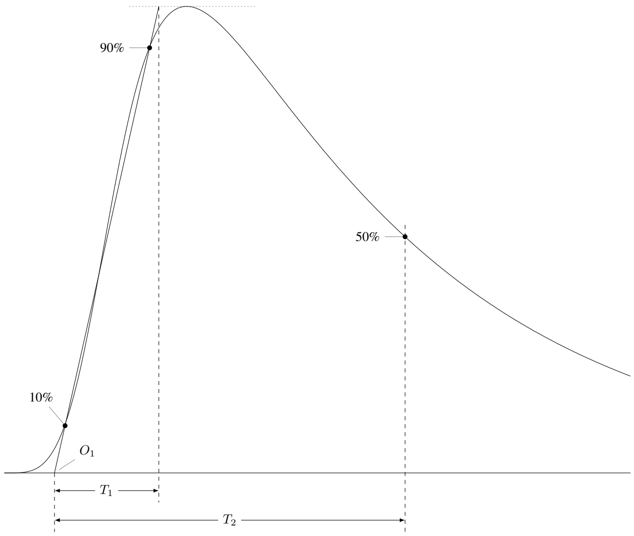

Fig. 1. Definitions of short stroke parameter adapted from [4]

Where:

I_0	=	Peak current [kA]
η	=	Correction factor of peak current
τ_1	=	Rise time constant [s]
τ_2	=	Fall time constant [s]
n	=	Steepness factor

Equation 1 is often written as $i(t) = \frac{I_0}{\eta}x(t)y(t)$. Where $x(t)$ and $y(t)$ are the rise and fall equations represented in *Equations 2* and *3* respectfully.

$$x(t) = \frac{\left(\frac{t}{\tau_1}\right)^n}{1+\left(\frac{t}{\tau_1}\right)^n} \tag{2}$$

$$y(t) = e^{-\frac{t}{\tau_2}} \tag{3}$$

Equation 3 is the decay function and can be integrated. Therefore this function is left as is in the approximation. Furthermore, the constant, $\frac{I_0}{\eta}$ can also be included directly in the approximation. *Equation 2* on the other hand, is an S-curve and the steepness of the rise as well as the front duration of waveshape can be adjusted by modifying n and τ_1 respectfully. This allows for a configurable rise time and maximum dI/dt.

The rise time equation in *Equation 2* cannot be transformed analytically (using Laplace or Fourier transforms) into the frequency domain because it cannot be integrated [6]. This creates a problem when accurate analyses of the frequency components of a lightning strike are required. For example, if the amplitude of the 11 kHz component of a lightning stroke is required to evaluate if lightning would introduce noise into a system operating at 11 kHz, it would not be possible to plot an analytical power spectral density from the Heidler function.

Furthermore, the numerical analysis of the LEMP fields is related to the lightning channel base current. This can be seen in *Equation 4*

$$i(z',t) = u(t - z'/v_f)P(z')i(0, t - z'/v) \tag{4}$$

where $i(z',t)$ is the return stroke current and $i(0, t - z'/v)$ is the lightning channel base current. In order to obtain the vertical and horizontal electric fields, $i(z',t)$ is integrated when solving Maxwell's equations. When carrying out these calculations, the use of an equation with an analytical solution in the frequency domain is preferred as this simplifies the mathematics [5].

Because of these limitations, several researchers have tried to approximate the transform of the Heidler function [6]. Most of these approximations have one or another limiting factor that changes the Heidler function enough so that it is no longer highly customisable: not all of the parameters remain variable. Many approximations are for a set steepness factor or entire waveshape [6]. In [6], the authors create an approximation that can be used with any factors but it is extremely complex.

Another example is where Delfino et al in [7] describe how the lightning channel base current can be approximated in terms of a Prony series. The limitation with this approach is the large amount of computation required for each approximation.

Therefore the purpose of this research is to formulate a lightning channel-base current function that has "customisable" parameters and can be transformed analytically into the frequency domain for use in frequency design and analysis as well as LEMP calculations. The approximation is simple to use and understand. It is also closely related to the Heidler function and so most of the parameters stay the same and the structure of the equation is the same (i.e. $i(t) = \frac{I_0}{\eta}x(t)y(t)$). This function is approximating the Heidler function and the parameters outlined in the IEC 62305 and therefore any errors are with respect to the Heidler function.

III. Heidler Function Approximation

A different approach is taken to approximate the Heidler function. A transfer function is defined in the Laplace domain and then an inverse Laplace transform is carried out to obtain the Heidler function approximation. With this approach, the equation is already in the frequency domain and therefore an analytical transform is no longer required.

A. Creating the Heidler Function Approximation

The Heidler function is not analytically transformable into the frequency domain because of the rise function, $x(t)$ in *Equation 2*. This function is merely an S-curve and therefore the step response of an n-th order, real and negative, pole is used to approximate this part of the function. The start of the approximation is seen in *Equation 5*.

$$X(s) = \frac{1}{s\left(\frac{s}{\omega_0}+1\right)^n} \tag{5}$$

Where:

ω_0	=	Rise time constant (rad/s)
n	=	Steepness factor

By varying the values of n and ω_0, the shape of the S-curve that is produced by the inverse Laplace transform of *Equation 5* can be "customised". The inverse Laplace transform of *Equation 5* can be seen in *Equation 6*.

$$x(t) = 1 - e^{-\omega_0 t}\left(\sum_{i=0}^{n} \frac{\omega_0^i t^i}{i!}\right) \tag{6}$$

The Heidler function also has a decay function as shown in *Equation 3*. There is no need to modify this function as it behaves exactly as required and is analytically transformable into the frequency domain. By including the constant and the decay function from *Equation 1* in *Equation 6*, the complete time domain approximation is obtained as in *Equation 7*

$$i(t) = \frac{I_0}{\eta}\left(1 - e^{-\omega_0 t}\left(\sum_{i=0}^{n} \frac{\omega_0^i t^i}{i!}\right)\right) e^{-t/\tau_2} \tag{7}$$

where I_0, η and τ_2 are the same as those in the Heidler function.

Equation 8 shows the complex shifting property of the Laplace transform.

$$\mathcal{L}\left\{e^{-at} f(t)\right\} = F(s+a) \tag{8}$$

By applying this property to *Equation 5* and using *Equation 3* as the model, an overall approximation to the Heidler function in the frequency domain can be obtained, as shown in *Equation 9*. The constants I_0 and η have also been included in *Equation 9* as they are real constants and are only required for changing the peak amplitude of the impulse current.

$$I(s) = \frac{I_0}{\eta} \frac{1}{s + \frac{1}{\tau_2}} \frac{1}{\left(\frac{s + \frac{1}{\tau_2}}{\omega_0} + 1\right)^n} \tag{9}$$

B. Frequency Domain

As the function is developed in the Laplace domain, simply replacing the s with $j\omega$ in *Equation 9* gives rise to the frequency domain equation as seen in *Equation 10*.

$$I(j\omega) = \frac{I_0}{\eta} \frac{1}{j\omega + \frac{1}{\tau_2}} \frac{1}{\left(\frac{j\omega + \frac{1}{\tau_2}}{\omega_0} + 1\right)^n} \tag{10}$$

This can be used directly in LEMP equations or the modulus and argument can be found and the power spectral density and phase of the lightning current can be analysed. Alternatively a bode plot of *Equation 9* can be plotted directly to obtain a frequency response.

IV. Results

There are several aspects of importance with this approximation.

1) The amplitude and other properties such as the derivative need to be comparable to the Heidler function.
2) The parameters outlined in the IEC62305 must be comparable to the approximation (intended as future research).
3) A frequency analysis must be carried out to show the frequency spectra of the function.

A. Comparison to Heidler

The proposed approximation is with respect to the Heidler function and therefore tested against it to determine accuracy of the approximation. The two aspects looked at in this paper are the amplitude of the current waveshape as well as the instantaneous derivative.

1) Amplitude: A 4 kA, 10/350 lightning current waveshape is used to test the approximation against the Heidler function. To obtain this current with both functions the parameters in *Table I* are used. It is clear that the parameters are similar

TABLE I
TABLE OF PARAMETERS USED IN CREATING EQUATIONS FOR *Figure 2*.

	Heidler	**Approximation**
τ_1 (μs)	19	-
τ_2 (μs)	485	485
n	10	33
ω_0 (rad/s)	-	1700000
I_0 (kA)	4	4
η	0.9341	0.9341

for the two functions with the notable differences being the steepness factor (n) and the rise time constants (τ_1 and ω_0). The reason for this is that the approximation's parameters are defined in the Laplace domain and this leads to the steepness factor having to be larger. Moreover, a number in rad/s is required for the approximation while the Heidler function expects a rise time constant in μs. However as the decay function as shown in *Equation 3* is unchanged, τ_2 is identical to that used in the Heidler function. Moreover, the shape of the rise function (S-curve) is the same as that of the Heidler function and therefore the same I_0 and η are used to achieve the required peak current.

Figure 2 shows the plots of both the Heidler function and the approximation with a 4 kA current and a 10/350 waveshape. The parameters used in the approximation are found by brute-force trial-and-error. The authors made some educated guesses and then optimised the values by hand until the curves looked very similar. A better method is required for this (see *Section VI*). The absolute value of the difference between the two functions shows that there is a maximum error of about 3.7% of the maximum amplitude of the Heidler waveshape.

2) Derivative: The derivative of a lightning current is important when determining the effects of inductive elements in a system. Therefore the approximation must represent a similar dI/dt graph to that of the Heidler function. *Figure 3* shows the first derivative of both the Heilder function and the approximation. There is a fairly large discrepancy in the peak values which reaches about 10.5%. This is a large error however the peak amplitude of the approximation function could be increased when carrying out these kinds of analyses. Moreover, this discrepancy results from the steepness factor being too small. This can be rectified by finding the correct correlation between the steepness factors of both functions (see *Section VI*).

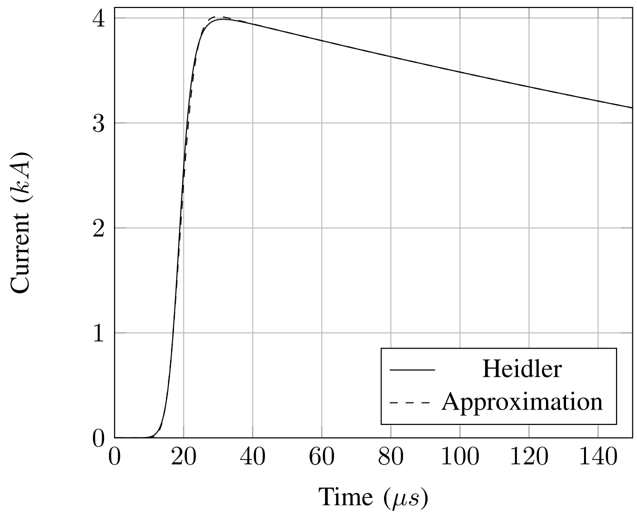

Fig. 2. Comparison between the approximation and the Heidler functions (tails shortened to show a more detailed comparison).

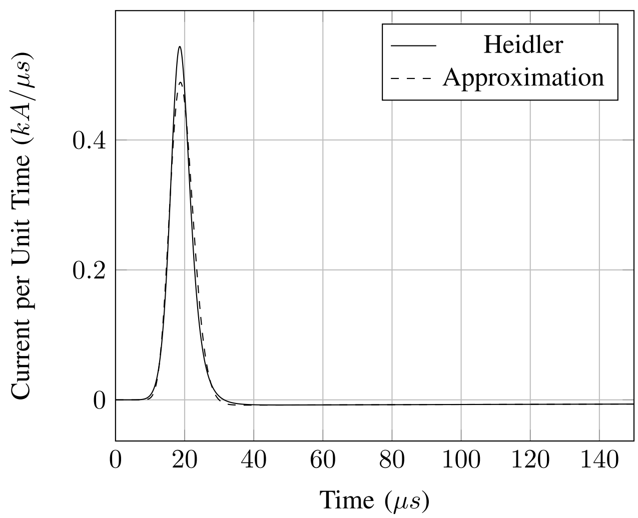

Fig. 3. Comparison between the first derivative of the approximation and the Heidler functions.

B. Frequency Analysis

A frequency analysis of lightning currents is useful to determine the effects that a particular frequency has on a system. Having a function that can be analytically transformed into the frequency domain allows for such an analysis. The power spectral density of the approximation can be plotted and hence the amplitude of the lightning current at a specific frequency can be determined.

Furthermore, having an expression for the lightning channel base current that is integrate-able is useful for carrying out LEMP calculations. The approximation is developed in the Laplace domain and is therefore trivially transformed into the frequency domain ($s = j\omega$) as expressed in *Equation 10.*

Preliminary analysis of the approximation in the frequency domain gives rise to the bode plot seen in *Figure 4.* The results

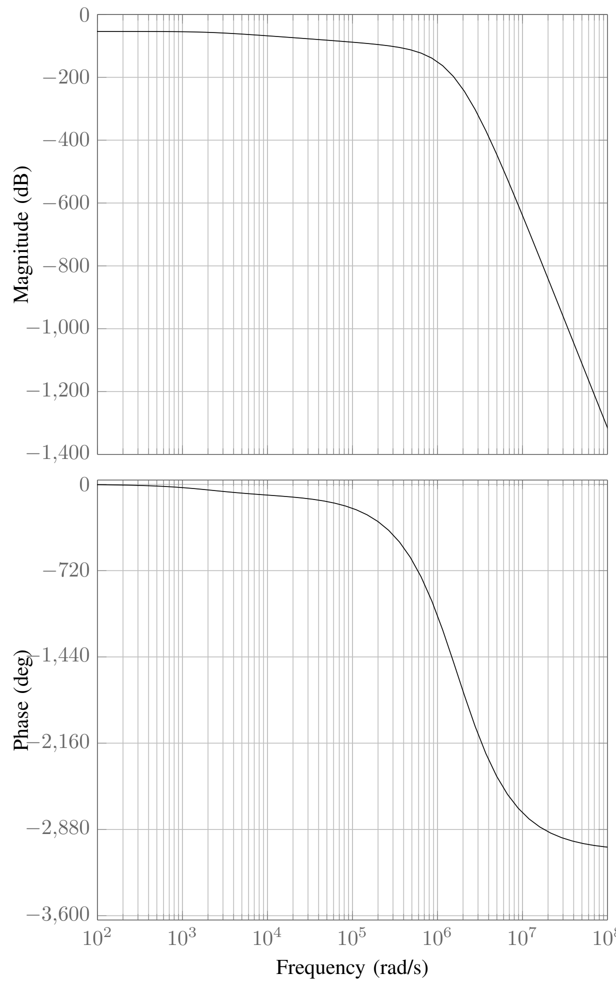

Fig. 4. Bode plot of the Heidler function approximation.

thus far look promising as they are aligned with the work done by Vujevic et al in [6] and [8] as well as that of Heidler et al in [9].

V. Discussion

This new proposed approximation to the Heidler function shows some promising preliminary results. Further work is required to fully understand how this function behaves with changing parameters (see *Section VI*).

The initial modelling has been carried out using a 4 kA, 10/350 lightning current waveshape. The parameters used in plotting the approximation are based on a trial-and-error brute-force process (see *Section III*). With these values the maximum error in the approximation with respect to the Heilder function is 3.7%. This is an acceptable error percentage in most engineering applications. However the instantaneous change in current is as much as 10.5% inaccurate. This can be easily explained by the steepness factor being incorrect. Therefore some optimisation and design is required to make sure that the parameters used in the approximation are the correct ones for the shape of the Heidler function used.

Because the approximation is developed in the Laplace domain, an expression for the approximation in the frequency domain is obtained by substitution rather than calculation. The initial plot of the frequency spectra shows that this approximation agrees with previous research.

VI. Future Research

There are several analyses required to determine the validity of such an approximation. In future work, the areas to be focused on are:

1) Correlating the parameters to those of the Heidler function.
2) Carrying out analyses on different waveshapes (such as the 1.2/50 and 8/20).
3) Comparing the characteristics of the approximation with those outlined in the IEC 62305 standard.
4) Further frequency analyses.

The parameters used in the approximation are similar to those used in the Heidler function. τ_2, I_0 and η are the same for both functions. n and ω_0 in the approximation are utilised in the Laplace domain and therefore are not necessarily correlated with n and τ_1 of the Heidler function. Further work is required to determine whether or not there is a relationship between these parameters in the approximation and in the Heidler function. If there is no obvious relationship then a table of parameter values for common waveshapes should be obtained.

Moreover, analyses should be carried out on different waveshapes. The analysis in this paper is done by modelling a 4 kA, 10/350 lightning current waveshape. It is still necessary to determine whether or not this approximation holds value for other lightning current waveshapes with varying amplitudes.

In order to make sure that this function can be used and is compliant with the standards, further work is required to determine whether or not this approximation holds the same characteristics as those outlined by the IEC for lightning currents.

Finally, more work is required to determine the accuracy of the power spectral density of this approximation.

VII. Conclusion

Lightning current waveshapes are utilised in analyses that can vary from instantaneous current change to frequency analysis. It is therefore important to have a standardised lightning current waveshape that can be used to perform these analyses. The IEC endorses the Heidler function for this use and although it shares the characteristics with those of a typical lightning current, it is limited in that an analytical integral cannot be obtained. An integral of such a function could be required in any scenario where an analysis is required on the frequency components of a lightning current or the effects thereof on a particular system. It is also required in calculating electric fields in LEMP analyses. Therefore approximations have been proposed throughout the years to solve this problem. No one approximation is perfect as all of them are limited in one or another aspect. Therefore another approximation is proposed that is developed in the Laplace domain, alleviating the need to transform into the frequency domain by taking the integral of the function.

This new approximation shows that it does a good job of approximating the Heidler function in the time domain. It also has characteristics (such as dI/dt) that are similar to the Heidler function. An initial frequency analysis shows that the approximation gives a similar frequency response to that of other researchers.

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