Midterm 2

Soeon Park

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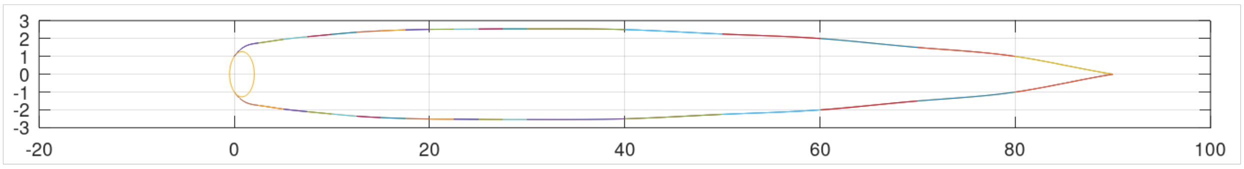
1. Graph of the spline rudder profile



2. Derivation of the spline coefficient

$S_k(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 + d_k(x - x_k)^3$
$S_{k+1}(x) = a_{k+1} + b_{k+1}(x - x_{k+1}) + c_{k+1}(x - x_{k+1})^2 + d_{k+1}(x - x_{k+1})^3$

$S_k(x_k) = y_k = a_k$ for all $k = 1, \ldots, n$
$S_k(x_k) = S_{k+1}(x_k) = a_k = a_{k+1} - b_{k+1}h + c_{k+1}h^2 - d_{k+1}h^3$ for all $k = 1, \ldots, n-1$
$S_1(x_0) = y_0 = y_1 - b_1 h + c_1 h^2 - d_1 h^3$

$S'_k(x) = b_k + 2c_k(x - x_k) + 3d_k(x - x_k)^2$
$S'_{k+1}(x) = b_{k+1} + 2c_{k+1}(x - x_{k+1}) + 3d_{k+1}(x - x_{k+1})^2$

$S'_k(x_k) = S'_{k+1}(x_k) = b_k = b_{k+1} - 2c_{k+1}h + 3d_{k+1}h^2$ for all $k = 1, \ldots, n-1$
$S'_1(x_0) = m_0 = b_1 - 2c_1 h + 3d_1 h^2$
$S'_n(x_n) = b_n$

$S''_k(x) = 2c_k + 6d_k(x - x_k)$
$S''_{k+1}(x) = 2c_{k+1} + 6d_{k+1}(x - x_{k+1})$
$S''_k(x_k) = S''_{k+1}(x_k) = 2c_k = 2c_{k+1} - 6d_{k+1}h \implies c_k = c_{k+1} - 3d_{k+1}h$ for all $k = 1, \ldots, n-1$

$y_k = y_{k+1} - b_{k+1}h + c_{k+1}h^2 - d_{k+1}h^3$
- $y_{k-1} = y_k - b_k h + c_k h^2 - d_k h^3$
$y_k - y_{k-1} = y_{k+1} - y_k - (b_{k+1} - b_k)h + (c_{k+1} - c_k)h^2 - (d_{k+1} - d_k)h^3$
$\implies 0 = y_{k+1} - 2y_k + y_{k-1} - (2c_{k+1}h - 3d_{k+1}h^2)h + (c_{k+1} - c_k)h^2 - \frac{(c_{k+1} - 2c_k + c_{k-1})}{3}h^2$
$\implies 0 = y_{k+1} - 2y_k + y_{k-1} - (2c_{k+1}h + c_k h - c_{k+1}h)h + (c_{k+1} - c_k)h^2 - \frac{(c_{k+1} - 2c_k + c_{k-1})}{3}h^2$
$\implies 0 = y_{k+1} - 2y_k + y_{k-1} - 2c_k h^2 - \frac{(c_{k+1} - 2c_k + c_{k-1})}{3}h^2$
$\implies 0 = y_{k+1} - 2y_k + y_{k-1} - \frac{(c_{k+1} + 4c_k + c_{k-1})}{3}h^2$
$\implies c_{k+1} + 4c_k + c_{k-1} = 3(y_{k+1} - 2y_k + y_{k-1})/h^2$

$a_{n-1} = a_n - b_n h + c_n h^2 - d_n h^3$, $a_{n-1} = y_{n-1}$, $a_n = y_n$, $b_n = m_n$, $c_{n-1} = c_n - 3d_n h$
$y_{n-1} = y_n - m_n h + c_n h^2 - \frac{(c_n - c_{n-1})}{3}h^2 \implies y_{n-1} = y_n - m_n h + \frac{(2c_n + c_{n-1})}{3}h^2$
$\implies 3(y_{n-1} - y_n + m_n h)/h^2 = c_{n-1} + 2c_n$ for all $2 \le k \le n-1$

$y_0 = y_1 - b_1 h + c_1 h^2 - d_1 h^3$, $m_0 = b_1 - 2c_1 h + 3d_1 h^2$, $y_1 = y_2 - b_2 h + c_2 h^2 - d_2 h^3$, $b_1 = b_2 - 2d_2 h + 3d_2 h^2$, $c_1 = c_2 - 3d_2 h$
$3y_1 - 3b_1 h + 3c_1 h^2 - 3d_1 h^3 = 3y_0$
$+\ b_1 h - 2c_1 h^2 + 3d_1 h^3 = m_0 h$
$3y_1 - 2b_1 h + c_1 h^2 = 3y_0 + m_0 h$

$y_1 = y_2 - b_2h + c_2h^2 - d_2h^3$
$+ \ b_1h = b_2h - 2c_2h^2 + 3d_2h^3$
$y_1 + b_1h = y_2 - c_2h^2 + 2d_2h^3 \Longrightarrow y_1 + b_1h = y_2 - c_2h^2 + 2h^2\frac{(c_2-c_1)}{3} \Longrightarrow b_1h = y_2 - y_1 - \frac{(2c_1+c_2)}{3}h^2$

$3y_1 - 2(y_2 - y_1 - \frac{(2c_1+c_2)}{3}h^2) + c_1h^2 = 3y_0 + m_0h \Longrightarrow 5y_1 - 2y_2 + \frac{2h^2}{3}(2c_1 + c_2) + c_1h^2 = 3y_0 + m_0h$
$\Longrightarrow 3.5c_1 + c_2 = 3(3y_0 - 5y_1 + 2y_2 + m_0h)/2h^2$

$b_k = b_{k+1} - 2c_{k+1}h + (c_{k+1} - c_k)h = b_{k+1} - c_{k+1}h - c_kh$

$y_0 = y_1 - b_1h + c_1h^2 - d_1h^3 \Longrightarrow d_1 = (y_1 - b_1h + c_1h^2 - y_0)/h^3$
$y_{k-1} = y_k - b_kh + c_kh^2 - d_kh^3 \Longrightarrow d_k = (y_k - b_kh + c_kh^2 - y_{k-1})/h^3$