Project 1

Soeon Park

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Method 1: Gradient descent using the Distance between two circles

I obtained the following results as the intersection point of given two circles by minimizing the distance between the circles using the Gradient descent method. Multiplying each of x, y, z by the Earth's mean radius, 6371.000 km, we get the corresponding x, y, z coordinates on the Earth. Then the coordinates can be used to calculate the latitude and longitude. Figure 1 shows that the Gradient descent method work: the trajectory (red points) heads to a minimum of the distance between two circles. The Octave codes are attached as a zip file.

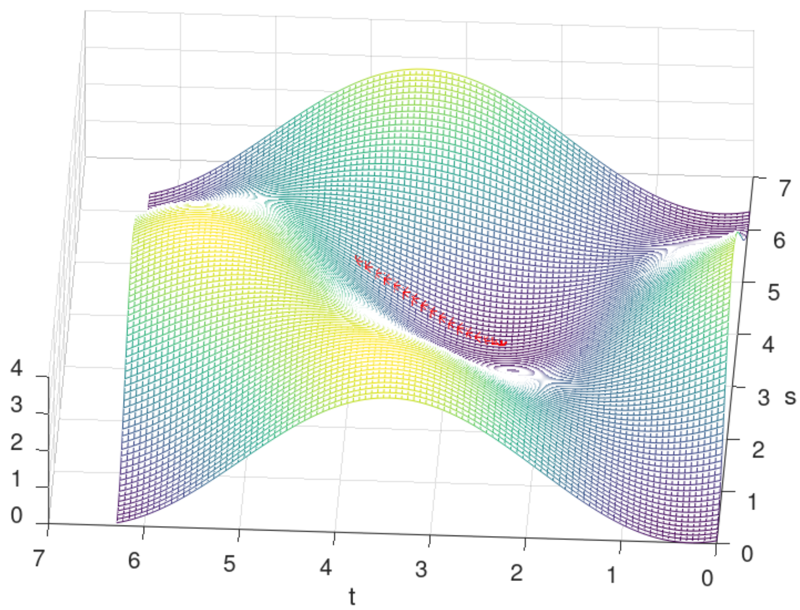


Figure 1: Resultant Trajectories
(Ho,dec,GHA)=(29.0, 13.0, 92.3) and (17.5, 13.0, 177.0) with initial values $(s, t) = (4, 4)$

I set the following three cases depending on the two measurements : Case 1 is the two circles of (Ho,dec,GHA)=(29.0, 13.0, 92.3) and (17.5, 13.0, 177.0), Case 2 is (Ho,dec,GHA)=(17.5, 13.0, 177.0) and (31.7, −2.3, 80.9), Case 3 is (Ho,dec,GHA)= (29.0, 13.0, 92.3) and (31.7, −2.3, 80.9). The table below shows the results of each case varying initial values. In each case, initial values seem not to affect results.

Considering the latitude and longitude of Arcata ($40.8665°N$, $124.0828°W$), the most accurate case is Case 1 that uses the measurement (Ho, dec, GHA)=(29.0, 13.0, 92.3) and (17.5, 13.0, 177.0). It indicates that the first two measurements (Ho,dec,GHA) are more accurate that the third measurements (Ho,dec,GHA)= (29.0, 13.0, 92.3).

Figure 2 shows that the three circles represented by each measurement (Ho,dec,GHA) do not intersect at a point, implying that the results of the three cases are not the same. So we can guess that there are measurement errors in Ho, dec, GHA and the calculation may be improved by reducing the measurement errors.

Case	Initial value		Result						
	s	t	x	y	z	s	t	latitude	longitude
1	4	4	-0.4259	-0.6307	0.6487	3.6170	2.4509	$40.3802°N$	$124.0149°W$
	4	3	-0.4259	-0.6307	0.6487	3.6170	2.4509	$40.3802°N$	$124.0149°W$
	1	3	-0.072983	-0.2734	-0.9591	-0.070901	6.5694	$73.3956°S$	$104.5646°W$
2	1	4	-0.4386	-0.5748	0.6908	2.5256	3.8051	$43.5337°N$	$127.2100°W$
	5	4	-0.4386	-0.5748	0.6908	2.5256	3.8051	$43.5337°N$	$127.2100°W$
	5	7	-0.091528	-0.5802	-0.8093	6.9298	6.0674	$54.1239°S$	$98.5752°W$
3	4	3	-0.5764	-0.6024	0.5521	3.8242	4.0378	$33.4123°N$	$133.4411°W$
	4	6	-0.5762	-0.6024	0.5523	3.8239	4.0374	$33.4213°N$	$133.4335°W$
	1	3	0.7848	-0.4253	-0.4509	1.1580	7.2654	$26.5718°S$	$28.2715°W$

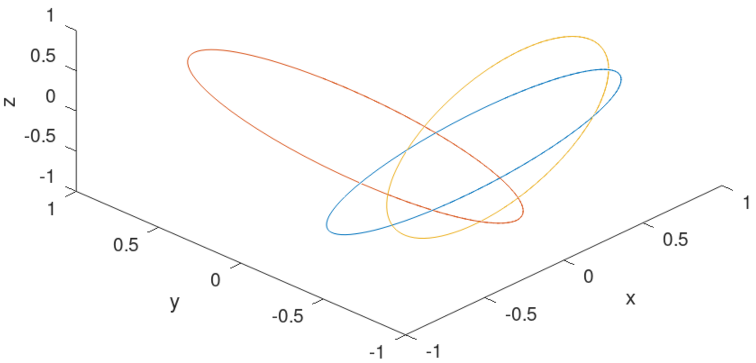


Figure 2: Three circles represented by measurements
(Ho,dec,GHA)=(29.0, 13.0, 92.3) and (17.5, 13.0, 177.0) and (31.7, -2.3, 80.9)

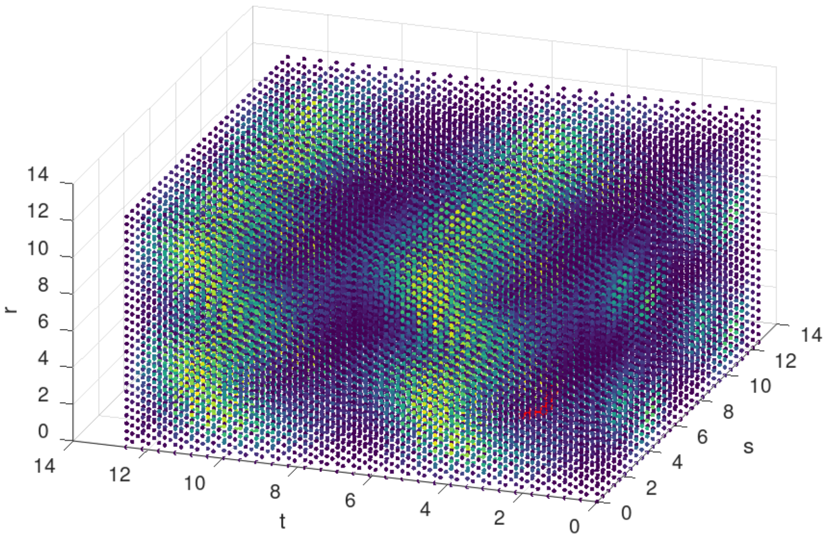
Method 2: Gradient descent using Cross product between three circles

This time, I minimized the cross product of the points in the three circles using gradient descent: I generated two vectors $c_2 - c_2$ and $c_3 - c_1$ to get the cross product. The measurements to calculate the three circles are the following: (Ho,dec,GHA)=(29.0, 13.0, 92.3) and (17.5, 13.0, 177.0) and (31.7, -2.3, 80.9).

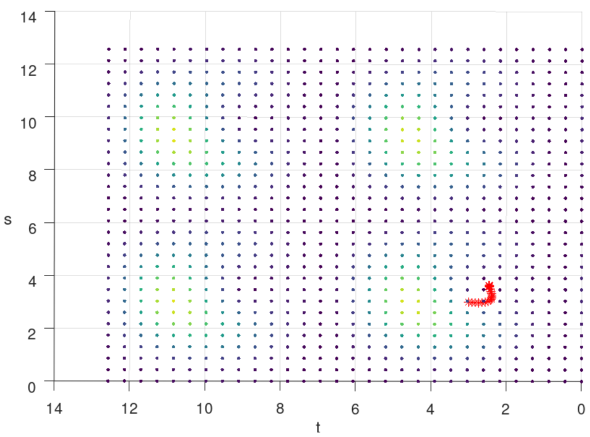
I found that the best result is $45.4640°N$, $119.0639°W$, the closest to the latitude and longitude of Arcata. Figure 3 shows that the trajectory goes to a minimum: the darker the points are, the smaller value they have. However, the results varies depending on initial values as shown in the following table. Thus I think Method 2 is sensitive to algorithmic errors.

We already showed in Figure 2 that the three circles do not intersect at a point. Hence the trajectories of the three circles clearly do not intersect at a point, either. So some measurement error is inevitable.

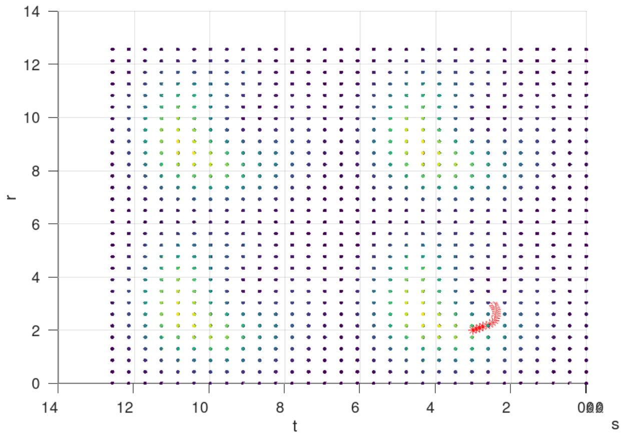
Initial value			Result							
s	t	r	x	y	z	s	t	r	latitude	longitude
3	2	2	-0.3382	-0.5954	0.7110	3.6170	2.4509	3.4319	$46.1619°N$	$119.3551°W$
3	2	1	$-6.0788e-03$	-0.5585	0.3106	3.6170	2.4509	1.0986	$2.5457°N$	$174.4502°W$
3	2	4	-0.3365	-0.6043	0.7059	3.2621	2.5250	3.8060	$45.4640°N$	$119.0639°W$
4	2	2	-0.010052	-0.5484	0.5728	3.6170	2.4509	2.0608	$46.2902°N$	$91.0300°W$
2	2	3	-0.3352	-0.5949	0.7119	3.6168	2.4509	3.4210	$46.2318°N$	$119.2357°W$



(a) Cross product values and trajectories



(b) s vs t



(c) t vs r

Figure 3: Resultant Trajectories of three circles
initial values $(s, t, r) = (3, 2, 2)$

Discussion

Case 1 of Method 1 performed the best with the least error, because the first two circles have smaller measurement errors in Ho, dec and GHA compared to the third one. Due to the measurement error of the third circle, Method 2 that uses the three circles simultaneously does not yield good results. If the measurements are improved, then the two gradient descent methods may generate more accurate results.

The two methods also showed that the selection of initial values is important in gradient descent. So we need to examine how a gradient descent code behaves before actually running it.