Final Project

Soeon Park

December 18, 2021

1. Derivation of the Crank-Nicolson algorithm

$$\frac{u_i^{n+1}-u_i^n}{\Delta t} = \frac{1}{2}\left(\frac{\frac{u_{i+1}^{n+1}-u_i^{n+1}}{\Delta x} - \frac{u_i^{n+1}-u_{i-1}^{n+1}}{\Delta x}}{\Delta x} + \frac{\frac{u_{i+1}^{n}-u_i^{n}}{\Delta x} - \frac{u_i^{n}-u_{i-1}^{n}}{\Delta x}}{\Delta x}\right)$$

$$\frac{u_i^{n+1}-u_i^n}{\Delta t} = \frac{1}{2}\left(\frac{u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}+u_{i+1}^{n}-2u_i^{n}+u_{i-1}^{n}}{\Delta x^2}\right)$$

$$\frac{2\Delta x^2}{\Delta t}(u_i^{n+1}-u_i^n) = u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}+u_{i+1}^{n}-2u_i^{n}+u_{i-1}^{n}$$

$$r(u_i^{n+1}-u_i^n) = u_{i+1}^{n+1}-2u_i^{n+1}+u_{i-1}^{n+1}+u_{i+1}^{n}-2u_i^{n}+u_{i-1}^{n} \qquad \text{Let } r = \frac{2\Delta x^2}{\Delta t}$$

$$-u_{i-1}^{n+1}+(r+2)u_i^{n+1}-u_{i+1}^{n+1} = u_{i-1}^{n}+(r-2)u_i^{n}+u_{i+1}^{n}$$

2. A plot of u(x,t) for dt=0.1

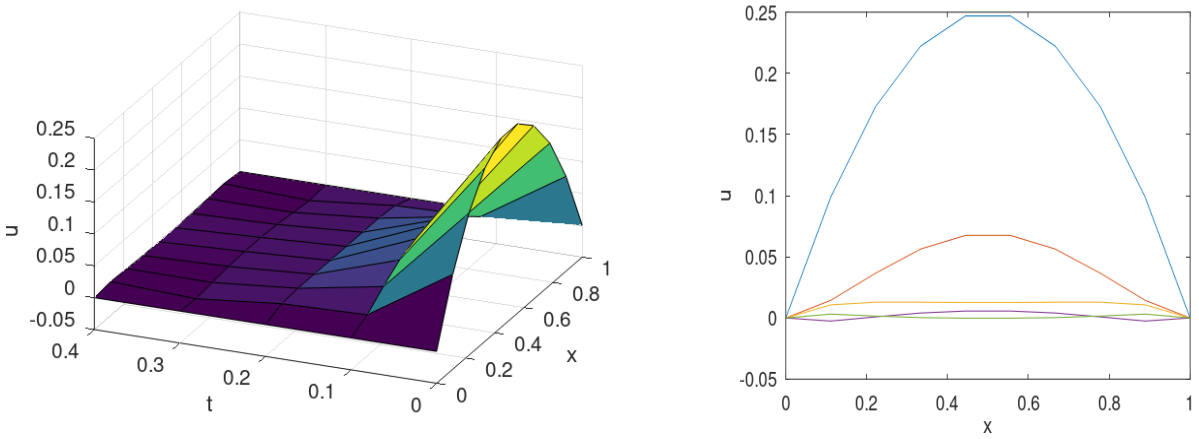

Figure 1: $\Delta x = 0.1$, $\Delta t = 0.1$, $\alpha = 10$