ComS 311
Hw 4
Btholmes

Problem 1

$T(1) = 1$

(1) $T(n) = 3T(\frac{n}{2}) + n$

(a) $T(\frac{n}{2}) = 3T(\frac{n}{4}) + \frac{n}{2}$

$T(\frac{n}{4}) = 3T(\frac{n}{8}) + \frac{n}{4}$

$3(3(3T(\frac{n}{8}) + \frac{n}{4}) + \frac{n}{2}) + n$

$3(9T(\frac{n}{8}) + \frac{3n}{4} + \frac{n}{2}) + n$

$27T(\frac{n}{8}) + \frac{9n}{4} + \frac{3n}{2} + n$

$$= \sum_{k=0}^{\infty} 3^k T(\frac{n}{2^k}) + \frac{3^{(k-1)} n}{2^{(k-1)}} + \dots + \frac{3^{(k-k)} n}{2^{(k-k)}}$$

(b) $T(n) = T(\frac{n}{8}) + n$; $T(1) = 1$;

$T(\frac{n}{8}) = T(\frac{n}{8^2}) + \frac{n}{8} + n$

$T(\frac{n}{8^2}) = T(\frac{n}{8^3}) + \frac{n}{8^2} + \frac{n}{8} + n$

$$= \sum_{k=0}^{\infty} T(\frac{n}{8^k}) + \frac{n}{8^{(k-1)}} + \frac{n}{8^{(k-2)}} + \dots + \frac{n}{8^{(k-k)}}$$

② Recurrence Relation

$T(n) = 2T(\frac{n}{2}) + c$ $\qquad T(1) = 1$

$T(\frac{n}{2}) = 2T(\frac{n}{2^2}) + c$

$T(\frac{n}{2^2}) = 2T(\frac{n}{2^3}) + c$

$2(2(2T(\frac{n}{2^3}) + c) + c) + c$

$2^3 T(\frac{n}{2^3}) + c$

$\sum_{k=0}^{\infty} 2^k T(\frac{n}{2^k}) + c$

$= \sum_{k=0}^{\infty} 2^k \cdot 1 + c \Rightarrow \sum_{k=0}^{\infty} 2^{\log_2 n} \Rightarrow \sum_{k=0}^{\infty} n^{\log_2 2} \Rightarrow n$

$\boxed{O(n)}$

③ Recurrence Relation

$T(n) = 2T(\frac{n}{2}) + cn$

$= 2[2T(\frac{n}{2^2})] + cn(1+1)$

$= 2^2[2T(\frac{n}{2^3})] + cn(1+1+1)$

$= 2^{\log_2 n} T(\frac{n}{2^{\log_2(n)}}) + cn \cdot \log_2(n)$

$= nO(1) + cn\log_2(n) = \boxed{O(n\log_2(n))}$