Notations:

: a “neural point” (e.g. a neuron, or a column of neurons)

: the true retinal / visual field location associated with (e.g. the center of the receptive field of neuron )

: the presumed retinal / visual field location associated with , known up to a (prior) distribution (i.e. .

The visual field (i.e. the retinal space) is represented in the early stages of visual processing by topologically organized neural ensembles. We formalized such a representation as a dense set of neural points on a neural substrate (which can the retina, superior colliculus (SC), V1 cortex, etc.). The visual field or retinal position associated wtih a neural point (i.e. the center of its receptive field) is known to the visual system up to some uncertainty described as a (prior) probability distribution function . That is. (We refer to the random variable as the *presumed* visual field position of neural point .)

The visual system learns and tightens the priors and associated with a pair of neural points and after a saccade that intended to bring the retinal position associated with (“retinal locus”, ) to a target in the visual field currently at the position associated with (“saccade target”, ) (i.e. moving the target from the center of receptive field of to that of ). The visual system does so by measuring the discrepancy (vector difference) between post-saccade retinal location of the saccade target from the (true) receptive center .

(1)

where is a zero-mean Gaussian measurement number with a constant standard deviation , is the motor (saccade) command issued, is zero-mean Gaussian motor noise, with standard deviation proportional to saccade amplitude (), and the true visual-field position of the neural point .

Given and , has a Gaussian distribution with variance and a mean equals to signed difference of the discrepancy between the presumed retinal positions of the retinal locus and saccade target and their actual positions. therefore provides only relative position information about and .

Having observed , the posterior joint probability distribution of and is

(2)

where is the Gaussian pdf with mean and variance .

However, the visual system does not know the exact true visual-filed positions and . They are known only up to the current estimates of their probability distributions and . So to evaluate (2), the visual system has to marginalized the likelihood term:

(3)

To solve the integral, it is convenient to consider the pdf of , which is simply the cross correlation between and :

(4)

Eq. (3) can be rewritten as:

(5)

We further note that the integral is just a convolution between and Gaussian with mean and variance Let

(6)

and rewrite (5) as

(7)

Eq. 7 makes it clear that the (marginalized) likelihood function has the same value along the diagonals ().

is a normalization constant and can be replaced by a post-normalization step if the grid for and is sufficiently large. Alternatively, it can be evaluated explicitly as

After a saccade, we will update the priors by marginalizing the posterior probability. I.e.:

(8)

and similarly for .

<<I now think that these update rule is right.>>

**Simplifications**

If we assume the priors and are Gaussians, then and are both Gaussian. Let denote the Gaussian density function with mean and variance . Let and similarly for .From (4), it can be shown with a bit of arithmetic that

(4’)

and similarly,

(6’)

Eq. 7 can therefore be rewritten as:

(7’)

The numerator of Eq. 7’ can be readily evaluated with the Matlab function norm or exp.

The above equation can be applied to visual space with the following changes:

Where S is the control signal in visual coordinates and is the signal dependent noise constant (Van Beer et al 2007).

**Selecting the retinal locus for a saccade**

For each saccade, the visual system selects the neural point with the best expected acuity to receive the target. The expected acuity of is the expected acuity of the target seen by the visual system after the saccade targeting . Let be the actual acuity at retinal position . The expect acuity of targeting with a displacement from the position of in the absence of any motor noise is:

(9)

For now, we assume that (9) is implicitly known by each neural point . The expected acuity when is targeted with a saccade of amplitude is therefore a cross correlation between (9) and a Gaussian density describing the motor noise:

(10)

**The effect of the scotoma**

The saccade target can be in the scotoma and thus disappears. In this case, is undefined. The posterior density can be computed by integrating (marginalizing) of (7) over a uniform distribution of what are consistent with target being in the scotoma after the saccade.

**The forgetful function**

After each saccade, the prior probabilities associated with neural points and become sharper, using equation (7’). However, if a retinal locus is not used in performing the following saccades, there should be a mechanism by which the prior probability distributions return back to the initial distributions. In other words, we need a forgetful function which slows down the learning process.

If we define precision (κ) as 1/σ, the precision will decrease with time given the following equation:

(11)



Where τ is the time constant for precision decay. A big τ means there is no forgetting and a small τ means complete forgetting (on every trials, κ values are set to the initial values.)

Similarly, we can apply the forgetful function to the mean values of the prior probabilities by defining κ as .