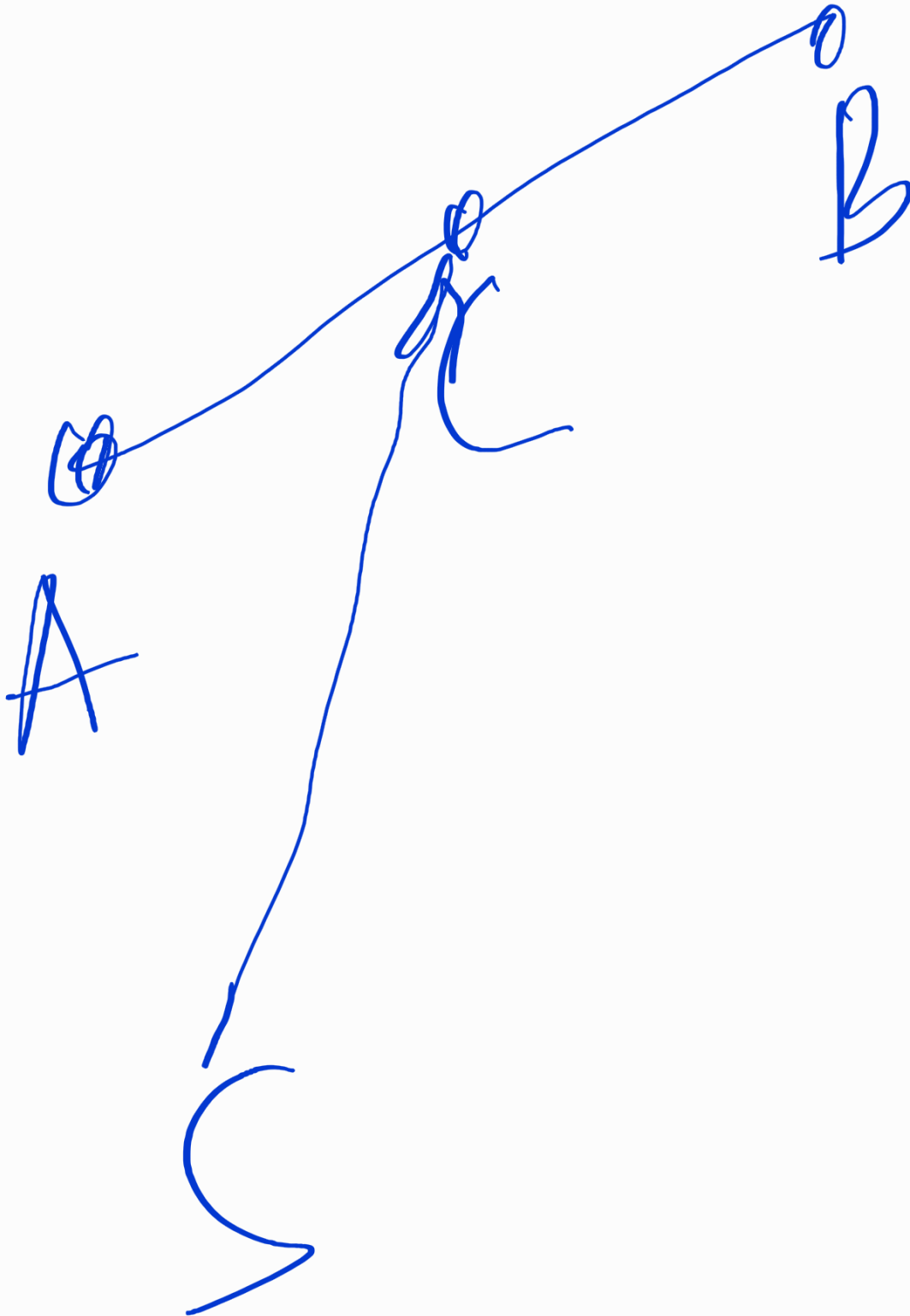


Problem 1: sniper problem!

The bridge $[A,B]$, a point C on the bridge;

car's speed - v (m/sec), the bullet's velocity - V (m/sec).

The sniper points the gun at the point C and pulls the trigger exactly at the moment the car enters the bridge at point A .



Our goal is to compute(as fast and simple as possible) the point C such that the car is hit(i.e. the car and bullet reach C at the same

time).

You input is 3 points A,B,S in 3D, in other words 9 real numbers(3 vectors of coordinates (x,y,z) in meters), the speed v and the velocity V . The output, i.e. the solution if exists, is point C or the "NO" if solution does not exist.

Hint/warning: the problem is not always solvable, geometry using coordinates(like the length of the vector etc.)...

Problem 2: basic calculator and Computing $\cos(n x)$.

Basic calculator:

[memory + algebraic operations(division, multiplication, addition, subtraction) + the square root].

You are given the value of $\cos(x)$. Your goal to compute EXACTLY using only basic calculator operations the following numbers (below N is an integer number).

1. $A(n) = \cos(N x)$. How the number of operations in your approach scales with N ? Is it $O(N)$ (the notation from the Algorithms Course)? $O(\log(N))$ (much faster than $O(N)$)?

2. $B(N) = \cos(x / 2^N)$:

$B(0) = \cos(x)$, $B(1) = \cos(x/2)$, $B(2) = \cos(x/4)$...

The same questions as above in 1.

(Here 2^N is the N -th power of 2: $2^0 = 1$, $2^1 = 2$, $2^2 = 4$ )

3. What about just $\cos(x/3)$, can we compute it EXACTLY using a finite number of basic calculator operations? It is a "creative

question" - explore it on the WEB,

PROBLEM 3: We need a trick...

$X(i+1) = (5X(i) + 6) \div (7X(i) + 13)$, $X(0) = 1$. How many divisions are needed to compute $X(1000000)$? And in general, try to compute on your laptops $X(2^{1000})$ using the (super-simple) recursion above... and share your observations.

Problem 4. Do you remember what is the determinant $\text{Det}(A)$ for $n \times n$ matrices (you certainly remember for $n = 2$, possibly for $n = 3$, but we will need for general n). If not - refresh your memory on the web (Wikipedia is fine!).

A creative question (i.e. explore the WEB for the answer): what is the connection between the determinant and the counting of spanning trees of connected graphs?

