Week 1, Wed, ICP

P2 (c) Proof. $rs = \sqrt{rs}\,\sqrt{rs}$

$rs = \sqrt{r}\sqrt{r}\sqrt{s}\sqrt{s} = \sqrt{rs}\sqrt{r}\sqrt{s}$ $\Big\}$ $\sqrt{rs} = \sqrt{r}\sqrt{s}$

(a) https://www.math.toronto.edu/mathnet/plain/falsdroots/secondleq2.html.

Week 1, Fri, ICP

P1.

Proof. We use proof by contradiction.

Suppose $\left.\begin{array}{l} a > \sqrt{n} \\ b > \sqrt{n} \end{array}\right\}$ $ab > n$, which contradicts $a \cdot b = n$.

Thus it must be that either a or $b \leq \sqrt{n}$.

P2. Proof. We use proof by contradiction.

Suppose $\sqrt{3} = \frac{n}{d} \Rightarrow 3d^2 = n^2 \Rightarrow n$ contains $3 \Rightarrow 3d^2 = 9k^2$

$\Rightarrow d^2 = 3k^2 \Rightarrow d$ contains 3 ~~$\Rightarrow \sqrt{3} = \frac{n}{3} = \frac{n}{d}$ contradict $\sqrt{3} = \frac{n}{d}$.~~

$\Rightarrow$ contradicts with n and d are primes

$\Rightarrow \sqrt{3}$ is irrational.

P4 Proof. We use proof by contradiction

Suppose $2\log_2 3 = \frac{n}{d} \Rightarrow \log_2 3 = \frac{n}{2d} \Rightarrow 2^{\frac{n}{2d}} = 3 \Rightarrow$

$2^n = 9^d \Rightarrow$ $\left\{\begin{array}{l} 2 \text{ is even} \Rightarrow 2^n \text{ is even} \Rightarrow \text{even} = \text{odd}. \\ 9 \text{ is odd} \Rightarrow 9^n \text{ is odd}. \end{array}\right.$

$\square$P1

contradiction $\Rightarrow 2\log_2 3$ is irrational.

lemma: ~~assume~~ assume any integer $n>1$,

(odd.)

$\swarrow$

$n = 2k+1$ (assume $k>1$, k is integer) $\Rightarrow n^2 = 4k^2+4k+1$

$= 4(k^2+1)+1$

$\Rightarrow$ n is even implies 凑

any product of n is even

(BTW n^2 is even $\rightarrow$ n is even)

Problem set 1

P1 Proof. We use proof by contradiction

$$\log_4 6 = \frac{n}{d} \Rightarrow 4^{\frac{n}{d}} = 6 \Rightarrow 4^n = 6^d \Rightarrow$$

$$2^{2n} = 2^d \cdot 3^d \Rightarrow 2^{2n-d} = 3^d \Rightarrow \begin{cases} 2n-d \le 0 \Rightarrow 2^{2n-d} \le 1 \\ 3^d > 1 \end{cases}$$

$2n-d > 0$

~~2 is even~~

} Contradiction

2^{2n-d} is even

3^d is odd } contradiction

} $\log_4 6$ is irrational.

Week 3, Thu, I cP

P3. $\exists x\ \exists y\ \exists z.\ E(x,y) \vee E(x,z) \wedge$

$x \neq y \wedge y \neq z \wedge x \neq z \wedge$

$\forall s, E(x,s) \rightarrow s = x \vee s = y \vee s = z$

ICP, Week 3, Wed.

實P2 (a) $\phi \in X$

(b) $y \in X$ AND $z \in X$

(c) $\forall z \cdot (z \in X \to z \in y)$

(d) $x = y$ or $x = z$

(e) $x = y$ and $x \notin z$

(f) $\forall X \exists P \forall u. u \subseteq X \leftrightarrow u \in P$

(g) $\forall z \exists u \forall x. (x \in u) \leftrightarrow (\exists y \text{ 套}. x \in y \text{ AND } y \in z)$

P3. TODO

Problem Set 2

Problem 1 ToDo

Problem 2.

$x \in \overline{A \cap B}$ iff NOT ($x \in A$ AND $x \in B$) iff $\overline{x \in A}$ OR $\overline{x \in B}$

iff $x \in \bar{A} \cup \bar{B}$

Problem 3

To do