FERM 6320 Spring 2019

Homework 1

1. Consider tossing a fair coin 20 times. Use simulations in Python to estimate:
   1. The probability of there being a sequence of exactly 5 heads in a row.
   2. The probability of there being a sequence of at least 5 heads or tails in a row?

Note: the exact solution to this is rather difficult to derive.

1. Suppose 2 teams A and B are playing a series of games and the first team to win 4 games wins the series. Suppose that team A has a 55% chance of winning each game and that the outcome of each game is independent.
   1. What is the probability that team A wins the series? Give an exact result and confirm it via simulation.
   2. What is the expected number of games played? Give an exact result and confirm it via simulation.
   3. What is the expected number of games played given that team A wins the series? Give an exact result and confirm it via simulation.
   4. Now suppose we only know that team A is more likely to win each game, but do not know the exact probability. If the most likely number of games played is 5, what does this imply about the probability that team A wins each game?
2. A bag contains 5 coins: a 2 two-headed coins, 2 two-tailed coin, and 1 standard fair coin. You reach into the bag, select one coin at random, flip the coin and observe a head. What is the probability that the selected coin is two-headed?
3. Three roommates are deciding who should take out the garbage. They want to ensure that everyone has an equal probability of being selected, but only have one fair coin. How can they use this coin to make their decision?