What I Know and When I Say It
How Trading Order and Informativeness Affect Market Prices

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Introduction

- Prediction markets are a belief aggregation mechanism designed to **elicit the personal beliefs** of traders about a future uncertain event and **aggregate those beliefs** into the market price
- These markets have been empirically observed to outperform polls as they have built-in **financial incentives** and **timely responses**
- We study the impact of traders' **informativeness** and the **sequence** in which they trade on **price convergence** properties and trader **compensation** under a new prediction market design

Market Design

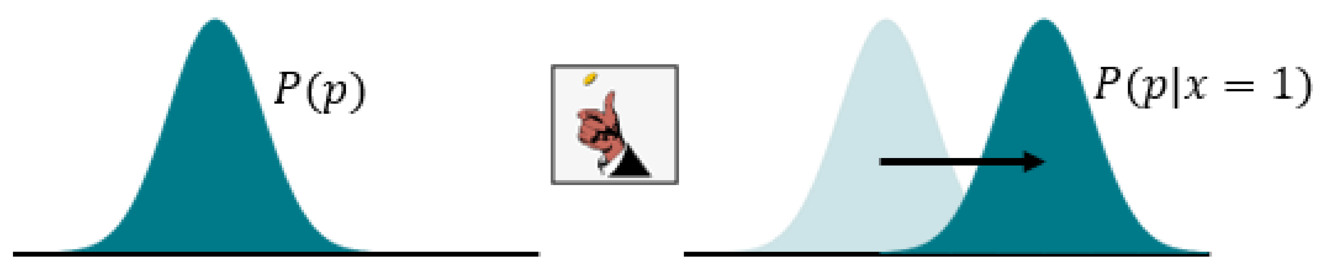
Suppose the random variable X under consideration in the market is binary and drawn from a **Bernoulli distribution** with success probability parameter p.

$$f(x;p) = p^x(1-p)^{1-x}, \quad \text{for } x \in 0, 1$$

Assuming traders are Bayesian, the market is set up to elicit the **conjugate Beta prior** on the Bernoulli success probability which has pdf

$$f(p;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1}$$

The market maker initializes the market with $\alpha^{(0)}$ and $\beta^{(0)}$, the initial prior beta distribution shape parameters. We elect to use a uniform prior distribution with $\alpha^{(0)} = \beta^{(0)} = 1$.



Trader Model

For each trader entering the market:

1) Trader's **private information** is modeled by sampling from the true distribution with N private sample observations

$$x_1, \ldots, x_N \overset{iid}{\sim} \text{Bern}(p)$$

2) Taking the market's current state as prior beliefs, trader uses private sample to **update Beta posterior** beliefs by:

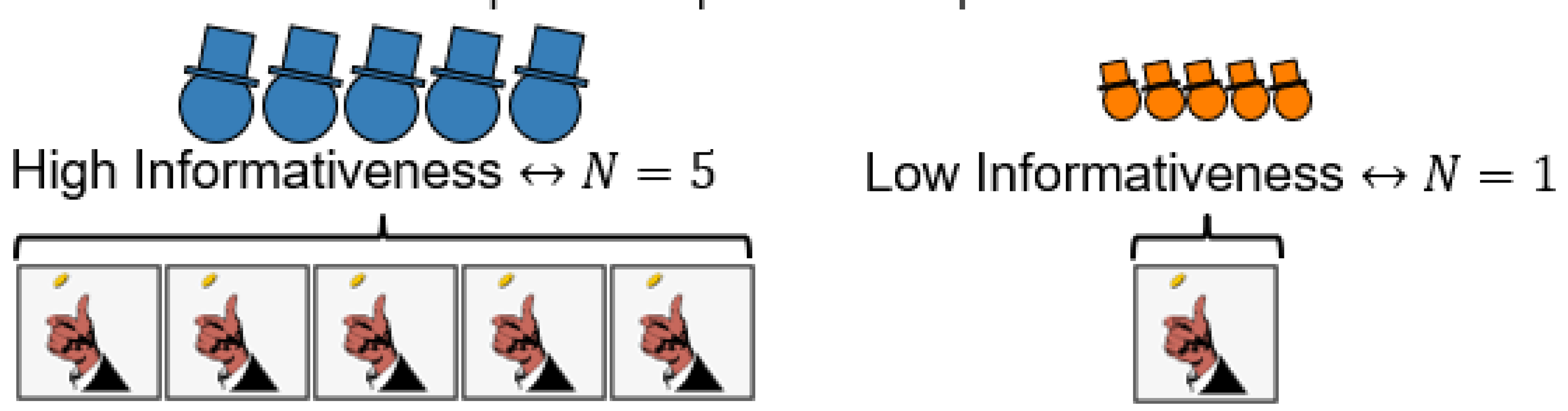
$$\hat{\alpha} := \alpha^{(t)} + \sum_{i=1}^{N} x_i$$
$$\hat{\beta} := \beta^{(t)} + \sum_{i=1}^{N} 1 - x_i$$

3) Trader purchases shares such that market parameters move to

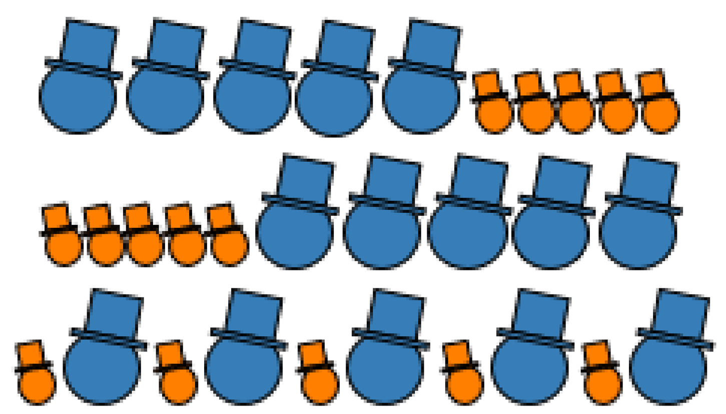
$$\alpha^{(t)} = \hat{\alpha}, \quad \beta^{(t)} = \hat{\beta}$$

Trader Informativeness and Sequencing

- The market has traders of **high and low informativeness**, where informativeness corresponds to private sample size N



- **Three sequences** of traders are tested



Experimental Results

Trader Characteristics	After 1 Round	After 10 Rounds	After 100 Rounds	After 1000 Rounds
10 High Info	2.43 ± 0.02	2.60 ± 0.00	2.61 ± 0.00	2.62 ± 0.00
10 Low Info	1.86 ± 0.07	2.52 ± 0.01	2.61 ± 0.00	2.62 ± 0.00

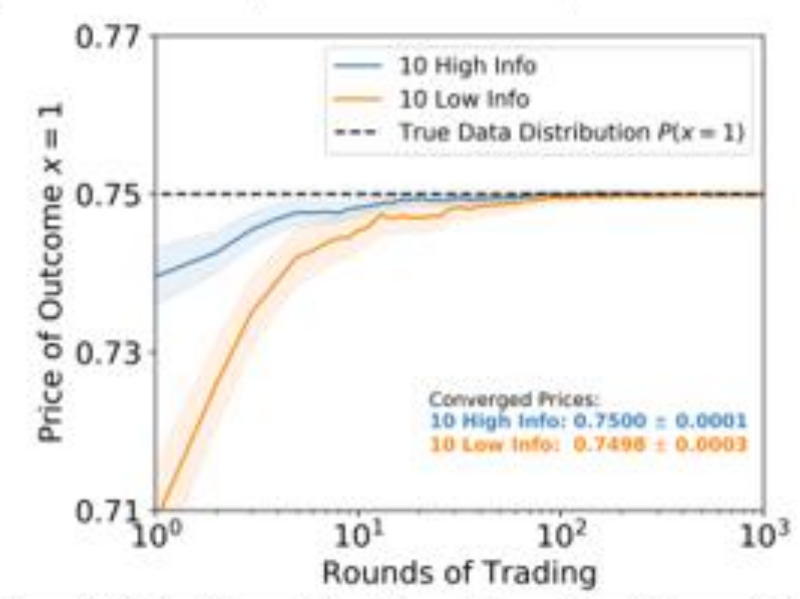


Figure 1: Market prices averaged over 1000 simulations with a 95% confidence interval show variation in convergence speed and early-round compensation ($\times 10^{-2}$) for traders with different levels of informativeness

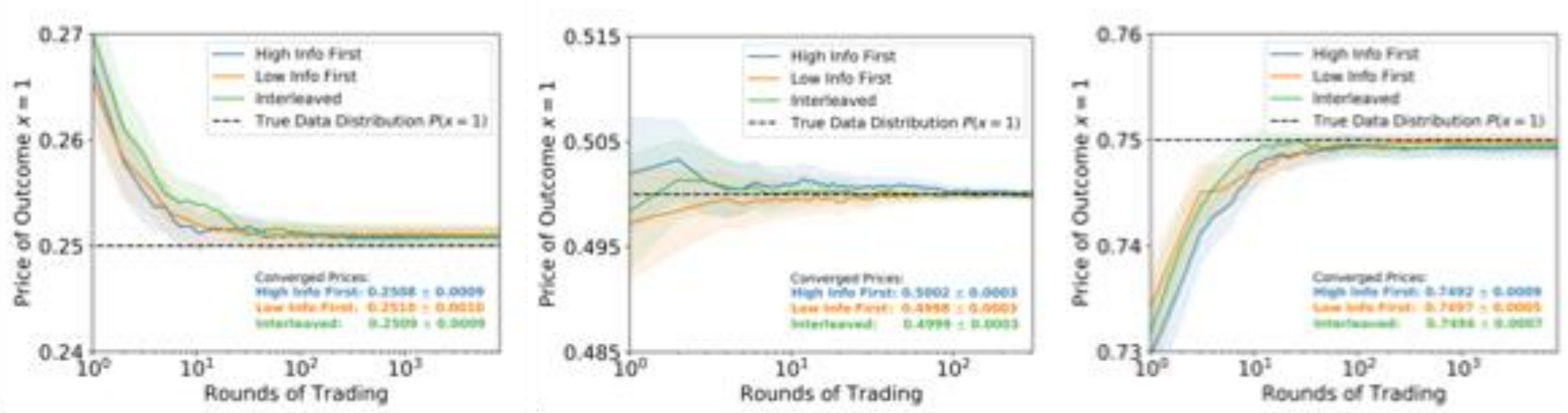


Figure 2: Market prices averaged over 1000 simulations with a 95% confidence interval show no significant variation with different sequences of traders

Experimental Results Cont.

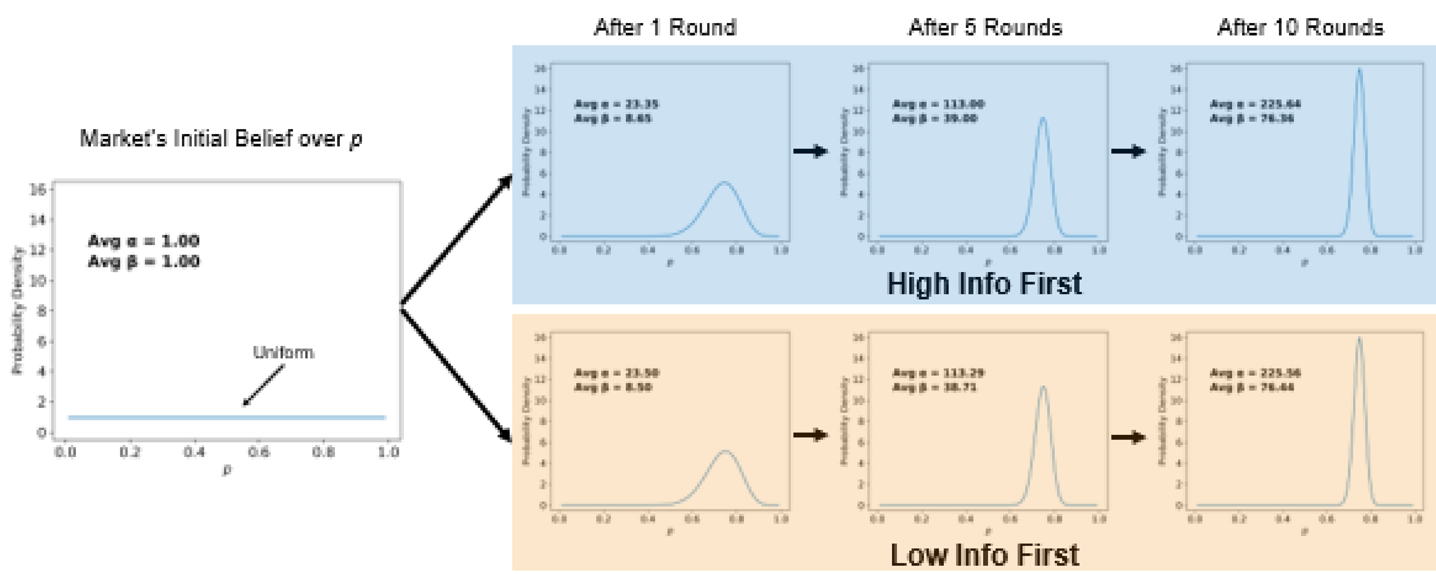


Figure 3: With shape parameters averaged over 1000 instances of the market, the evolution of the beta posterior distribution $P(p|x)$ over rounds of trading is practically unaffected by trader sequence

$P(X=1)$	Sequence	Avg Rounds	Avg High Info Compensation ($\times 10^{-2}$)	Avg Low Info Compensation ($\times 10^{-2}$)
0.25	High Info First	8225.9 ± 99.8	2.53 ± 0.02	0.08 ± 0.02
	Low Info First	8188.1 ± 103.6	0.99 ± 0.09	1.62 ± 0.09
	Interleaved	8271.2 ± 89.0	2.04 ± 0.19	0.57 ± 0.29
0.50	High Info First	341.3 ± 25.9	-0.08 ± 0.02	0.08 ± 0.02
	Low Info First	320.2 ± 25.8	1.15 ± 0.11	-1.15 ± 0.11
	Interleaved	313.9 ± 24.6	0.92 ± 0.07	-0.92 ± 0.07
0.75	High Info First	8186.2 ± 105.8	2.53 ± 0.02	0.08 ± 0.02
	Low Info First	8214.3 ± 99.7	1.00 ± 0.09	1.61 ± 0.09
	Interleaved	8177.5 ± 103.3	1.95 ± 0.19	0.66 ± 0.19

Table 1: Despite no significant impact on convergence time over 1000 market instances, sequence impacts the apportioning of compensation between high and low informativeness traders

Conclusions

1. Figure 1 confirms our intuition that **market convergence is faster with only highly informative traders** than with only low informativeness traders
2. Traders with the same overall informativeness induce the same **price convergence** characteristics **regardless of the sequence** in which they trade (Figs. 2-3)
3. The expected **compensation** of a trader depends not only on her **informativeness** but also strongly on the **sequence** as well as on prior parameters (Table 1)
4. Future work includes further experiments with other **market designs** and **trader models** as well as theoretical analysis to quantify the impact of the **sequencing-informativeness interplay**

Significant References

- Jacob Abernethy, Sindhu Kutty, Sébastien Lahaie, and Rahul Sami. "Information Aggregation in Exponential Family Markets." EC 2014
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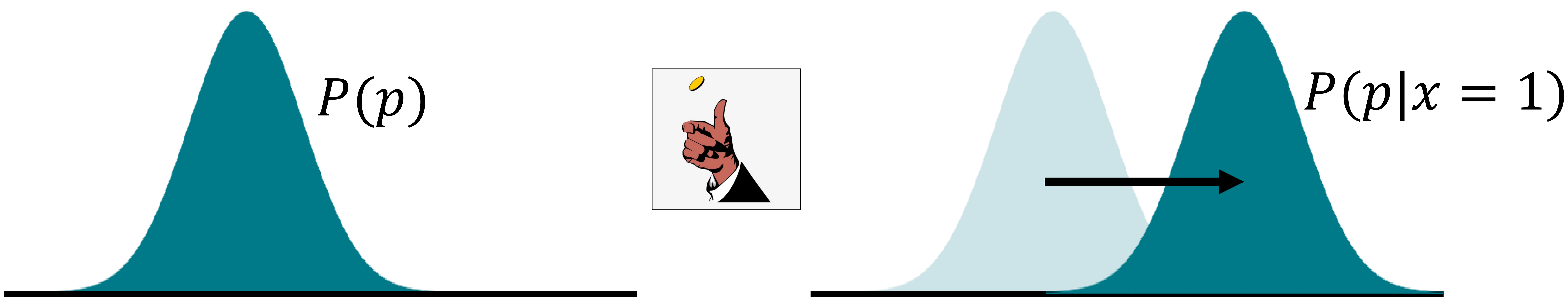

Prior Distribution

Posterior Distribution

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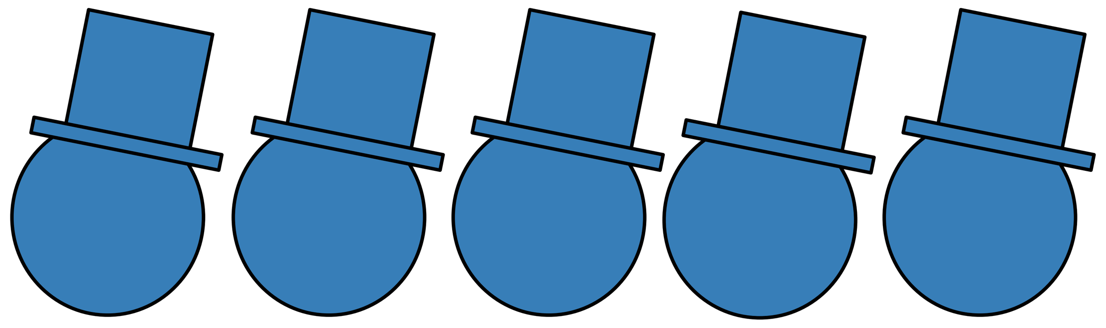
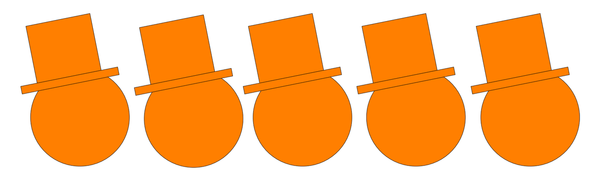
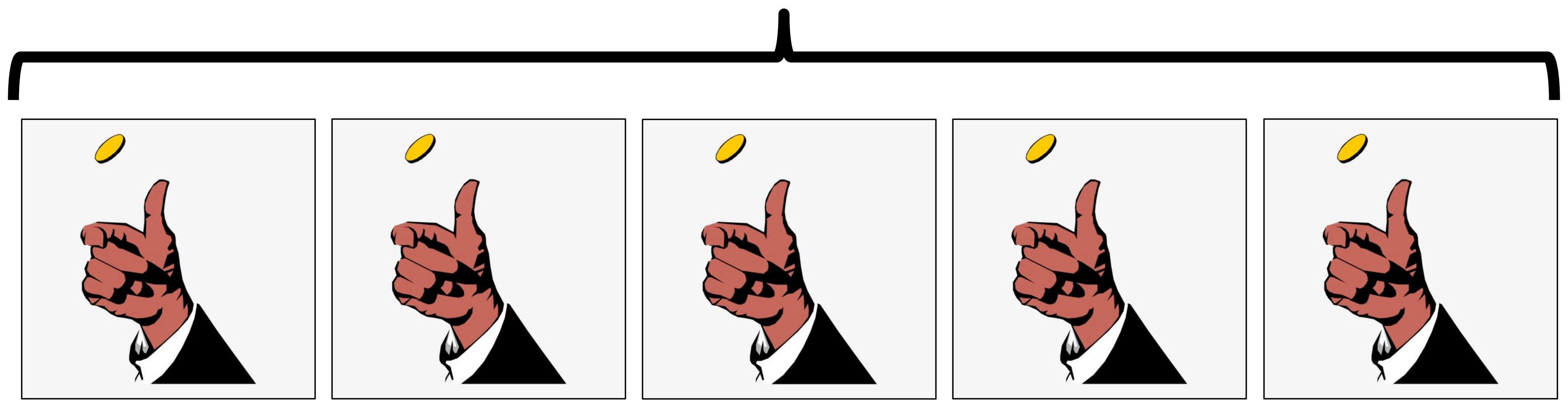




High Informativeness $\leftrightarrow N = 5$

Low Informativeness $\leftrightarrow N = 1$



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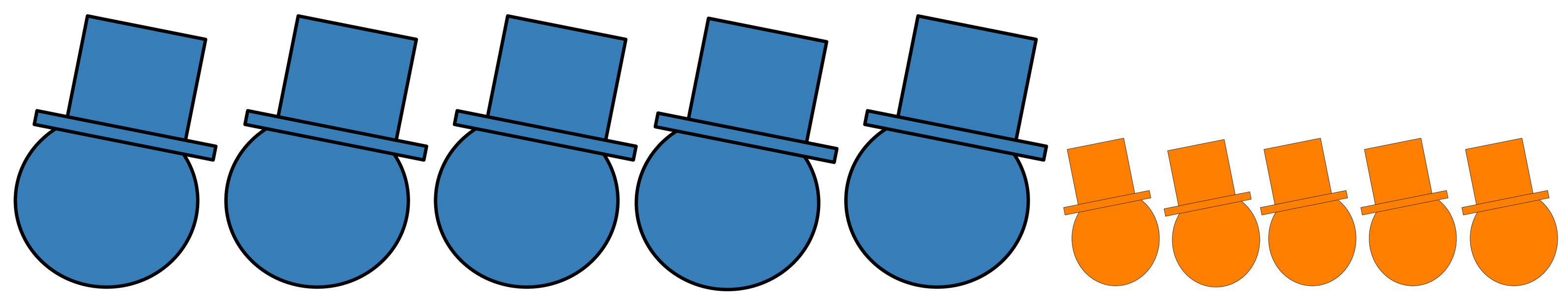
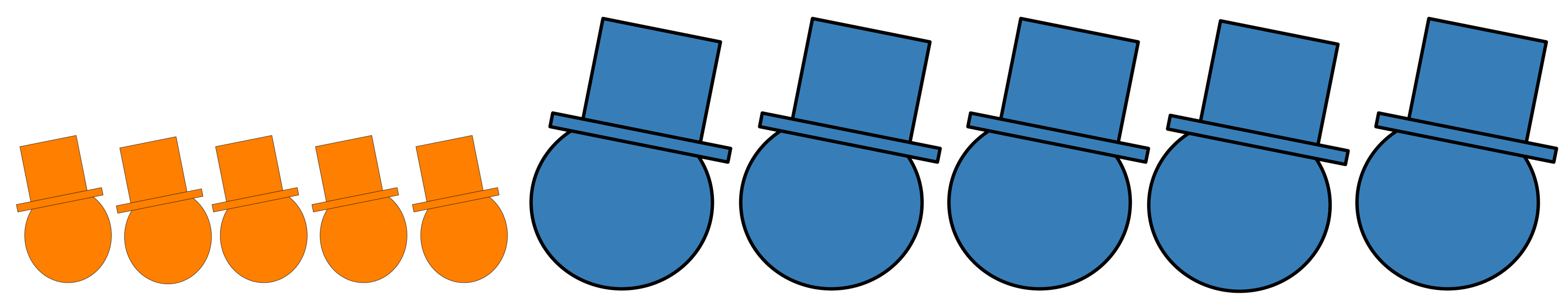
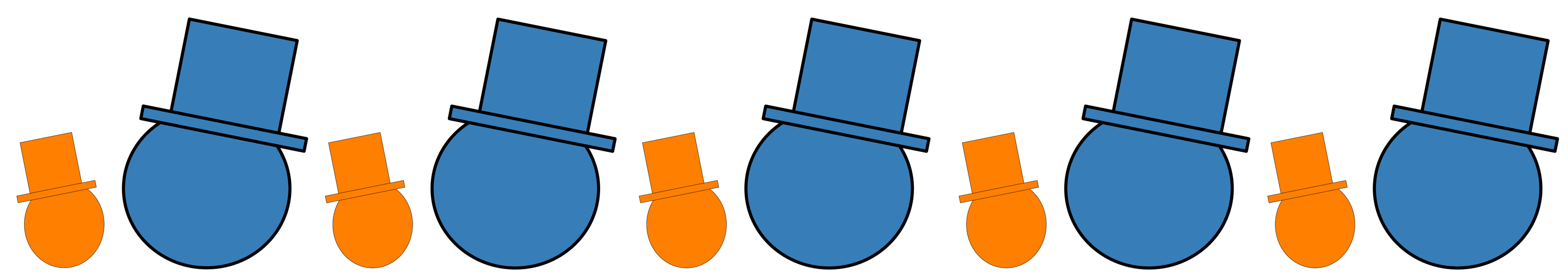
1) High Info First: 

2) Low Info First: 

3) Interleaved: 

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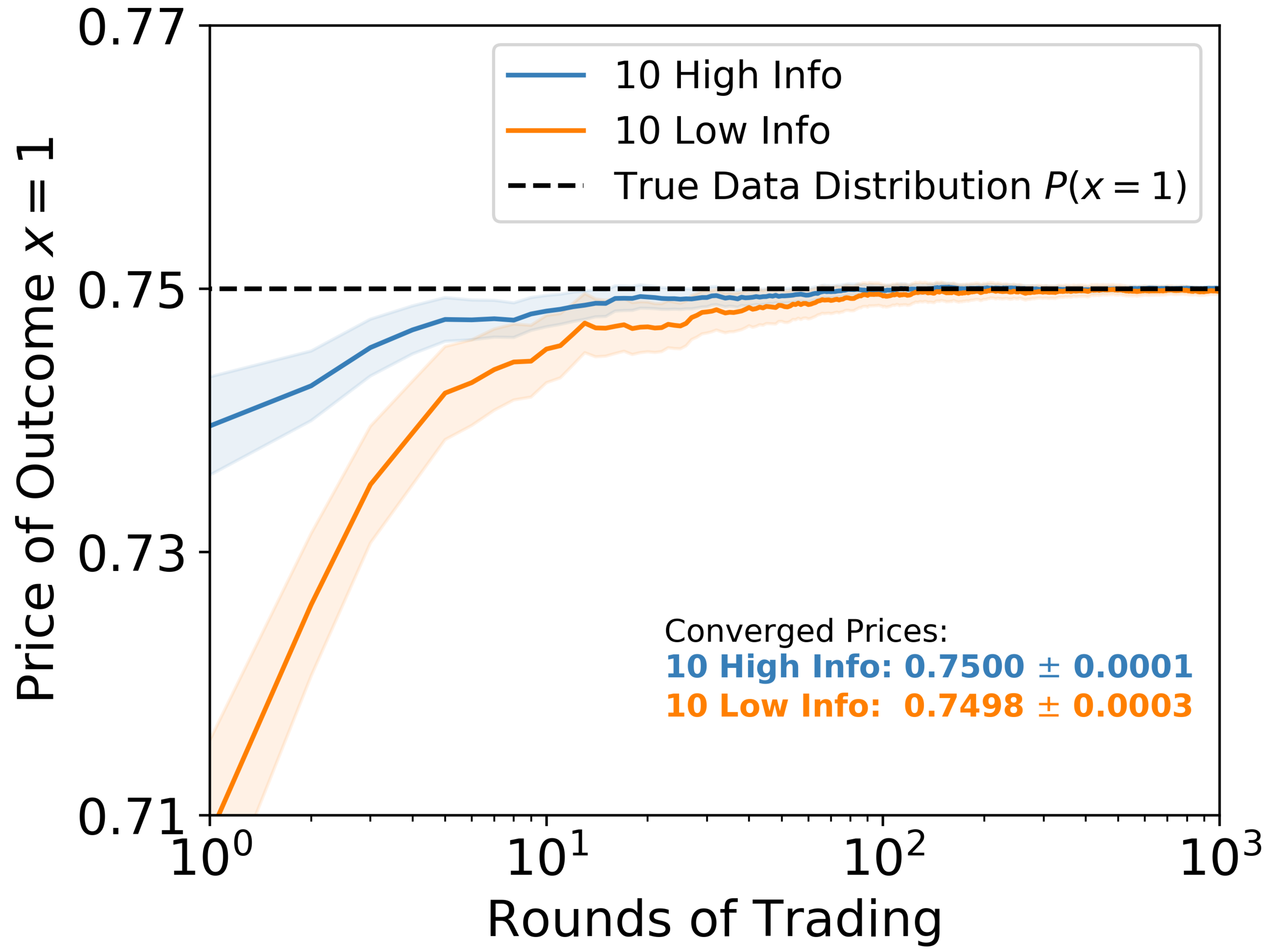

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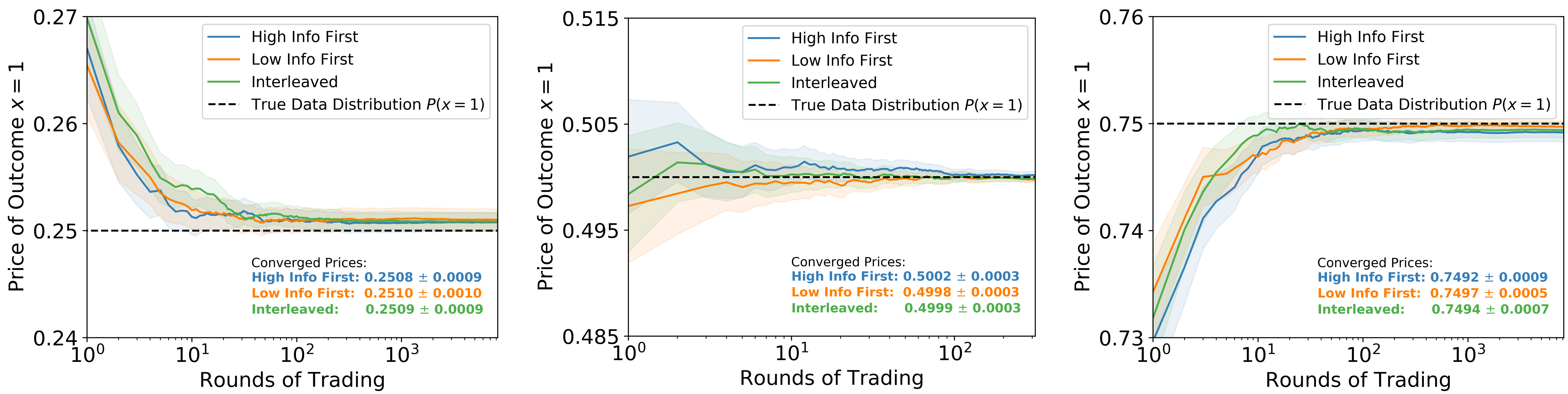

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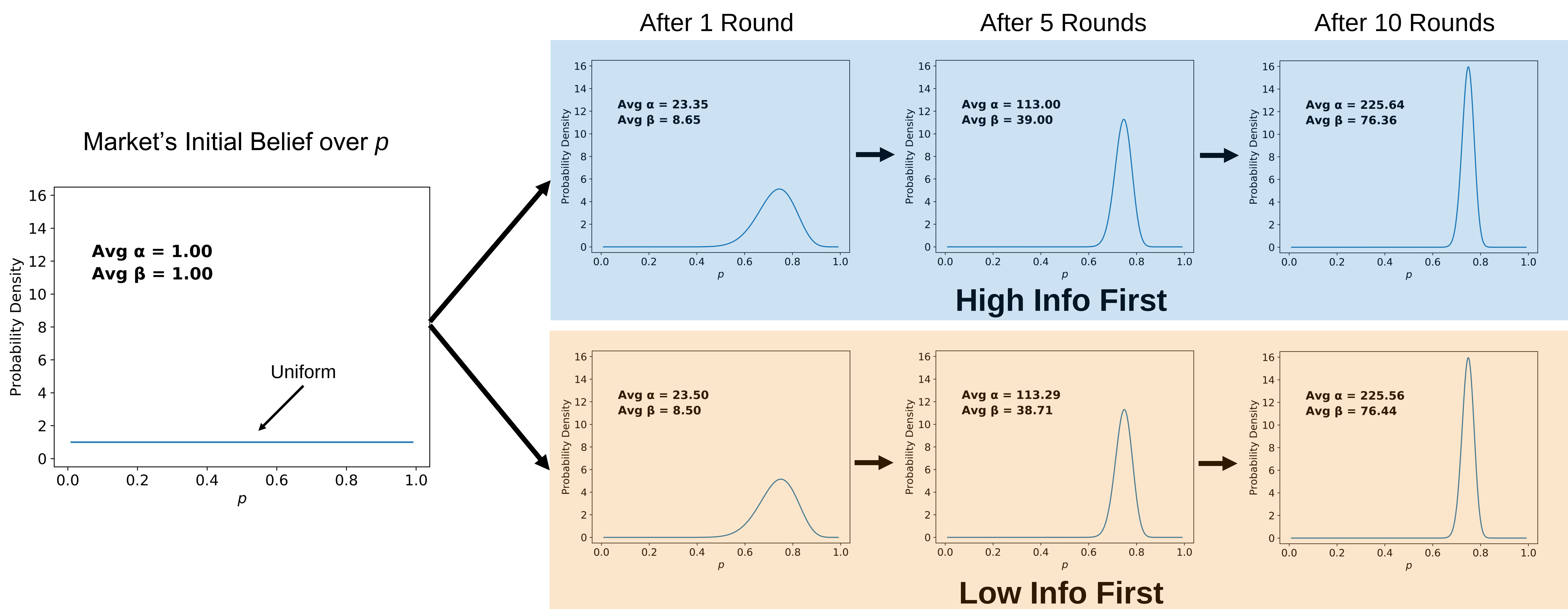

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