

# Ray Tracing Using Shooting-Bouncing Technique to Model Mine Tunnels: Theory and Verification for a PEC Waveguide

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**Abstract**—We present a shooting-bouncing approach to ray-tracing as applied to signal propagation modeling in electrically large waveguides, such as underground mine tunnels at wireless communication frequencies. The method is verified for a dominant-mode rectangular metallic waveguide excited by a dipole antenna.

**Keywords**—communications, computational electromagnetics, high-frequency, microwave, ray tracing, signal propagation, waveguide.

## I. INTRODUCTION

This paper addresses application of computational electromagnetics (CEM) to signal propagation modeling in underground mines. One of our main approaches to the wireless propagation analysis of underground mines, which is an extremely challenging CEM problem, relies primarily on shooting-bouncing rays (SBR) ray-tracing (RT).

Using traditional full-wave EM solvers for microwave frequencies in an underground mine may prove impractical in many cases due to computation run time required, as well as memory requirements, depending on the particular technique employed. Ray-tracing provides a significant decrease in computational run time for these electrically large structures. Ray-tracing methods enable propagation modeling in very complicated scenarios such as railway stations, and they can provide useful prediction of signal loss characteristics [1,4].

## II. RAY TRACING THEORY

The shooting-bouncing rays approach in RT involves launching a set of test rays in all directions in which propagation from the source can be expected. These rays are then traced through the scene, and their intersections with objects in the scene recorded. This method is described in detail in [1]. The electric field at a desired location in the scene is then found by employing an ideal plane wave approximation for each ray. Then, using the reflection coefficients based on surface parameters for each reflection, the final electric field at the desired observation point can be approximated due to each ray path between the source and observation point [3]. This process may be repeated for several observation points to produce a 2D or 3D field profile at a desired location in the scene.

When a 2D field profile is desired, we discretize the plane of the desired field profile into a grid of uniform pixels or grid blocks. The complex-valued field vectors of all rays intersecting a given block are added to approximate the total field at that block due to the given source and scene geometry. This process naturally approximates interference. While this introduces phase and magnitude error, the error can be minimized by ensuring the grid blocks are small relatively to the wavelength, and that a large number of rays are used, such that each block has a sufficiently high sample density to accurately approximate the field.

The shooting-bouncing approach to ray-tracing is advantageous because it is conveniently parallelizable which allows for efficient and expeditious computations. This is essential because it enables analysis of problems that require very high ray counts to achieve sufficient sample density for field convergence. Another benefit to the acceleration (by parallelization) of ray-tracing is that larger structures can be evaluated for signal propagation characteristics more easily and more quickly. This technique may be further accelerated by reducing the total cost of ray to facet (environment objects) intersection tests. The rays that propagate in this model interact with environment objects that cause the rays to be reflected. These interactions with the environment can be optimized using space-partitioning trees that efficiently store and access obstacles located in the environment (similar to a binary search tree) [1,2].

## III. RESULTS AND DISCUSSION

Testing of the ray-tracing method we developed was conducted on a perfect electric conductor (PEC) rectangular waveguide. This scene was chosen because of the ability to compare with an analytic solution for verification. The waveguide dimensions are chosen to be  $0.5842 \text{ m} \times 0.2921 \text{ m}$ , and the waveguide was excited with a Hertzian dipole antenna of unit peak field magnitude and frequency 350 MHz. The observation plane was placed 50 cm from the source. Operation frequency was chosen to only propagate the dominant  $\text{TE}_{10}$  mode in this waveguide.

This waveguide embodies a very challenging case for ray-tracing, as it is PEC, so all reflections must be considered (this is a completely convex scene), and it is not electrically very large, as convenient in ray-tracing technique.

The analytical solution for the dominant mode in the rectangular PEC waveguide states that the electric field should be uniform in the direction parallel to the short axis of the waveguide, and vary with a half-cosine in the axis parallel with the long axis of the waveguide. Fig. 1 shows the result of the ray-tracing method on this scene.

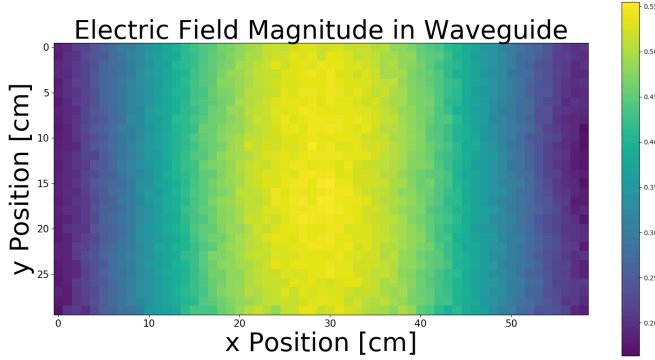


Fig. 1. Magnitude of the electric field for the waveguide excited with a Hertzian dipole at 350 MHz. The cutoff frequency for the waveguide is 256 MHz, which only allows propagation of the  $TE_{10}$  mode.

We observe in Fig. 1 the expected trends along both axes. The magnitude varies only slightly along  $y$  for any  $x$  coordinate in the waveguide, and the magnitude is peaked in  $x$  at the center of the waveguide, and is relatively symmetrical about the center of the waveguide.

The final electric field is found by summing a discrete number of uniform plane waves at the observation plane. The number of rays that intersect the observation plane determines the number of plane waves. The solution generated by a ray-tracing method should converge to the analytical solution as the number of rays increases. Fig. 2 shows the electric field magnitudes for a cross section of the waveguide for varying numbers of rays.

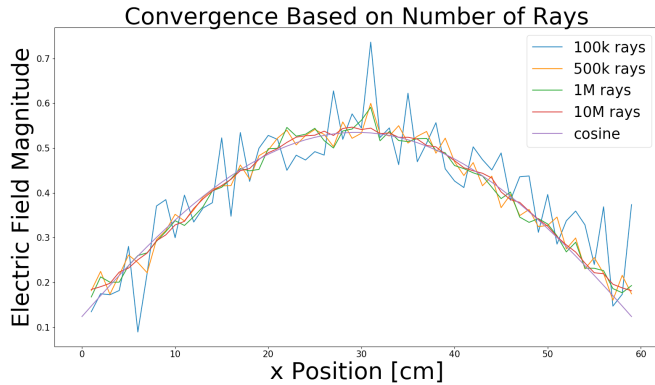


Fig. 2. Electric field magnitude in the cross section of the waveguide, with the waveguide parameters and excitation frequency remaining identical to Fig. 1. The number of rays was varied from 100 thousand rays to 10 million rays.

We observe from Fig. 2 the expected convergence of the ray tracing results with increasing the number of rays in the simulation. As the number of rays increases, the cross-section magnitude begins to smoothen to a cosine. The analytical solution states the electric field should be zero at the walls of the waveguide. The ray-tracing method results in a symmetrical offset of approximately 0.2 units on the edges. The offset is a result of the loss of accuracy from sending a finite number of rays resulting in a finite sampling density.

Each ray is terminated after a given number of reflections; if it did not reach the observation plane within the reflection limit, it will not contribute to a field at observation location. The solution should converge as the number of permissible reflections increases, as each additional ray that intersects the observation plane increases the sampling density. Fig. 3 shows the cross-sectional magnitude for varying number of reflections.

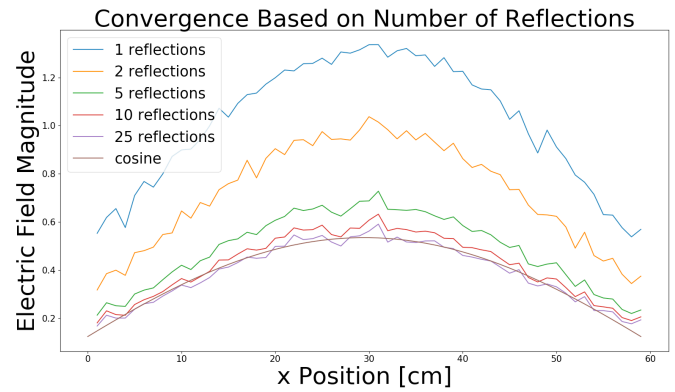


Fig. 3. Electric field magnitude based on the reflection order. The waveguide parameters remain identical to Fig. 1. The reflection order allowed varied from 1 to 25.

We observe from Fig. 3 a good convergence of the ray tracing results to the offset cosine as reflection order increases. The error is worst for low reflection order, and best for high reflection order, as expected.

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